Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Model Predictive Traffic Control Based on a New Multi-Class METANET Model ⋆

Shuai Liu, Bart De Schutter, Hans Hellendoorn

Delft Center for Systems and Control, Delft University of Technology
Mekelweg 2, 2628 CD Delft, The Netherlands

Email: {s.liu-1,b.deschutter,j.hellendoorn}@tudelft.nl

Abstract: Mutli-class traffic flow models account for the heterogeneous characteristics of traffic networks. This leads to higher accuracy when applying them for on-line model-based control. We propose a new multi-class METANET model. The proposed model is an extension of the single-class macroscopic traffic flow model METANET. In the new model, each vehicle class is subject to its own single-class fundamental diagram, and is limited within an assigned space. In this paper, model predictive control is used for on-line traffic control based on the newly proposed model. A case study is implemented for illustrating the efficiency of the new multi-class model. More specifically, the simulation results show that the new multi-class METANET model leads to a better performance than single-class METANET model.

Keywords: traffic control, multi-class traffic model, METANET, on-line model-based control, model predictive control.

1. INTRODUCTION

Due to the increasing traffic demand, traffic management has become more and more important. On-line model-based traffic control is one traffic management approach that can often lead to satisfying performance, since it takes into account the evolution of traffic flows. The models used in on-line model-based traffic control affect the control performance significantly. In general, the more accurate the models are, the better the control performance will be.

Macroscopic traffic flow models are popular in on-line model-based control, since the accuracy and the computation speed can be balanced well. However, many macroscopic traffic flow models do not consider the heterogeneous nature of traffic networks. In fact, traffic networks often include multiple classes of vehicles, such as cars, vans, and trucks etc. Although microscopic models usually include multi-class behavior, they are not suitable for on-line model-based control, because they can lead to low computation speeds. Hence, the development of multi-class macroscopic traffic flow models is necessary.

Some macroscopic multi-class traffic flow models have already been developed. Wong and Wong (2003) proposed a multi-class traffic flow model that is an extension of the Lighthill-Whitham-Richards (LWR) model (Lighthill and Whitham, 1955; Richards, 1956). In this multi-class model, the traffic flow for each vehicle class is computed through its own fundamental diagram by using the total density. Thus, the essential characteristics of each vehicle class remain unchanged. However, the total flow is considered when estimating traffic state variables. Logghe (2003) also extended the LWR model to a multi-class version, with each class described by an individual fundamental diagram. In this extended model, three traffic regimes are considered: free flow, semi-congestion, and congestion. Each vehicle class is limited within an assigned space, and the space fractions are determined according to the three traffic regimes. The FASTLANE model proposed by van Lint et al. (2008) is a first-order multi-class macroscopic network traffic flow model. In this model, dynamic passenger car equivalents are used. This means that differences in the space occupied by a vehicle under different traffic conditions (e.g. free flow or congested flow) are considered. Caligaris et al. (2008) developed a traffic flow model considering two classes of vehicles on the basis of the macroscopic model in Papageorgiou (1983). The interference between these two vehicle classes is represented by the steady-state relation between speed and density. Deo et al. (2009) extended the METANET model of Messmer and Papageorgiou (1990) into a multi-class version, using passenger car equivalents. However, this multi-class METANET model uses a convex combination of the desired speeds of all vehicle classes for computing the desired speed of each vehicle class. This reduces the heterogeneity of this model. Note that the METANET model is a second-order model which in general is more accurate than first-order models since second-order models capture phenomena that cannot be captured by first-order models (see Papageorgiou (1998), but also the points raised by Daganzo (1995) and Helbing and Johansson (2009)). More specifically, the METANET model can reproduce capacity drop near on-ramps and in shock waves, which is important for model-based traffic control (Papageorgiou, 1998). In this paper, we develop a new multi-class METANET model via an approach that is inspired by the approach used by Logghe (2003) for deriving a multi-class version of the LWR model. In this new model, desired speeds are computed in a different way from Deo et al.

⋆ Research supported by the China Scholarship Council, the European COST Action TU1102, and the European Union Seventh Framework Program [FP7/2007-2013] under grant agreement no. 257462 HYCON2 Network of Excellence.

(2009), aiming to obtain better heterogeneity. In order to investigate the advantages of this new model, it is applied in on-line model-based traffic control. Model Predictive Control (MPC) is selected as control approach, since it can deal with nonlinear system, multi-criteria optimization, and constraints. Besides, MPC is closed-loop control approach, and as such the prediction errors that are resulted from model mismatch can be corrected. For comparison, MPC is implemented based on two kinds of prediction models: the single-class METANET model and the new multi-class METANET model.

This paper is organized as follows. In Section 2, we summarize the multi-class LWR model developed by Logghe (2003). Next, we recapitulate the single-class METANET model in Section 3. Based on the method in Section 2 and the single-class METANET model, we propose the new multi-class METANET model in Section 4. Then, on-line traffic control based on this new model is developed in Section 5. Next, a case study is implemented in Section 6.

2. MULTI-CLASS LWR MODEL

2.1 Basic concepts

The multi-class LWR model of Logghe (2003) is an extension of the well-known LWR model. The model describes multi-class traffic flow, where each vehicle class is described on the basis of a triangular fundamental diagram:

$$Q_c(\rho_c) = \begin{cases} \rho_c v_{\text{free},c} & \text{if} \quad \rho_c \leqslant \rho_{\text{crit},c} \\ \dfrac{\rho_{\text{crit},c} v_{\text{free},c}}{\rho_{\text{crit},c} - \rho_{\text{max},c}} (\rho_c - \rho_{\text{max},c}) & \text{if} \quad \rho_c > \rho_{\text{crit},c} \end{cases} \tag{1}$$

where Q_c is the flow of vehicles of class c, ρ_c is the density of vehicles of class c, $v_{\text{free},c}$ is the free-flow speed of vehicles of class c, $\rho_{\text{crit},c}$ is the critical density of vehicles of class c, and $\rho_{\text{max},c}$ is the maximum density of vehicles of class c. It is assumed that a vehicle class constrains itself to an assigned fraction of the whole road, being subject to its fundamental diagram:

$$q_c = \alpha_c Q_c \left(\frac{\rho_c}{\alpha_c} \right) \tag{2}$$

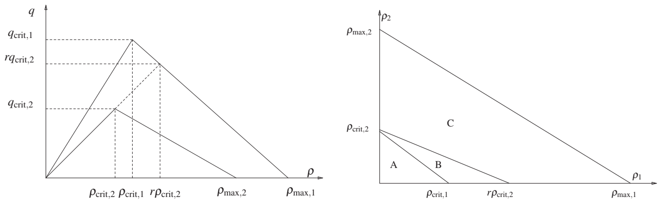
The road fractions for different classes of vehicles are always positive ($\alpha_c \geqslant 0$), and the sum of all fractions cannot exceed 1:

$$\sum \alpha_c \leqslant 1 \tag{3}$$

Several ways have already been developed to determine α_c. Wong and Wong (2003); Logghe (2003) equated the road fraction α_c to the relative class density. Benzoni-Gavage and Colombo (2003) obtained road fractions by assuming that the distance gaps for all vehicles in a heterogeneous traffic flow are identical. Chanut and Buisson (2003) computed road fractions by setting the distance gap to be proportional to the vehicle length. Besides, Chanut and Buisson (2003) also obtained space fractions through equating the speeds of different classes of vehicles. This method was also used by Logghe (2003) for vehicles in congestion mode. Similarly, we will also use it for obtaining space fractions for vehicles in congestion mode.

2.2 Combination of two fundamental diagrams

Assumed that there are two classes of vehicles. The combination of the fundamental diagrams of these two vehicle classes is obtained based on the following assumptions (Logghe, 2003):



(a) Fundamental diagrams for two vehicle classes

(b) Three traffic regimes

- User optimum: no vehicle can increase its speed without reducing the speed of slower vehicles. The drivers are only influenced by traffic circumstance ahead of them.
- Optimal road use: it is assumed that a vehicle class never occupies more space than is necessary.

A scale factor r for class 2 (the slower class) linking both fundamental diagrams is defined as follows (Logghe, 2003). The capacity point for class 2 lies on the congestion branch of class 1 (the faster class) when class 2 is scaled by this factor r (as shown in Fig. 1(a)):

$$r = \frac{w_1 \rho_{\text{max},1}}{\rho_{\text{crit},2}(w_1 - v_{\text{free},2})} \tag{4}$$

where

$$w_1 = \frac{\rho_{\text{crit},1} v_{\text{free},1}}{\rho_{\text{crit},1} - \rho_{\text{max},1}} \tag{5}$$

According to different densities, three traffic regimes are distinguished (Logghe, 2003): free flow, semi-congestion, and congestion (as shown in Fig. 1(b)). These regimes are defined in the ensuing.

Free flow (A) In free-flow regime, both classes of vehicles are in free-flow. The constraint that separates the free-flow regime with the semi-congestion regime (see below) is

$$\frac{\rho_1}{\rho_{\text{crit},1}} + \frac{\rho_2}{\rho_{\text{crit},2}} \leqslant 1 \tag{6}$$

In the free-flow regime, the following relation holds:

$$\frac{\rho_c}{\alpha_c} \leqslant \rho_{\text{crit},c} \qquad \text{for} \quad c = 1,2 \tag{7}$$

The constraint (6) is derived from (3) and (7). In this regime, the flow of each class of vehicles is computed through

$$q_c = \rho_c v_{\text{free},c} \qquad \text{for} \quad c = 1,2 \tag{8}$$

Semi-congestion (B) Vehicles of class 1 are in the congestion regime, but vehicles of class 2 are in the free-flow regime. However, the speed of class 1 is still greater than the free-flow speed of class 2. The constraint that distinguishes the semi-congestion regime with the congestion regime is defined by:

$$\frac{\rho_1}{r\rho_{\text{crit},2}} + \frac{\rho_2}{\rho_{\text{crit},2}} \leqslant 1 \tag{9}$$

The minimum fraction that the vehicles of class 2 need is

$$\alpha_2 = \frac{\rho_2}{\rho_{\text{crit},2}} \tag{10}$$

According to the assumption of optimal road use, this fraction is assigned to class 2, and

$$\alpha_1 = 1 - \frac{\rho_2}{\rho_{\text{crit},2}} \tag{11}$$

Considering $v_1 \geqslant v_{\text{free},2}$, expressions (1), (2), (10), and (11) are substituted for this inequality. Thus the constraint (9) is obtained. Now we have

$$q_1 = \alpha_1 \frac{\rho_{\text{crit},1} v_{\text{free},1}}{\rho_{\text{crit},1} - \rho_{\text{max},1}} \left(\frac{\rho_1}{\alpha_1} - \rho_{\text{max},1} \right) \tag{12}$$

$$q_2 = \rho_2 v_{\text{free},2} \tag{13}$$

Congestion (C) The traffic-flow speed is less than the free-flow speed of class 2, and the speeds of both classes are equal. Otherwise, the slower class could increase its speed by taking road space from the faster class. The constraint of the congestion regime is the maximum density restriction:

$$\frac{\rho_1}{\rho_{\text{max},1}} + \frac{\rho_2}{\rho_{\text{max},2}} \leqslant 1 \tag{14}$$

The speed during congestion is

$$v = v_c = \alpha_c Q_c \left(\frac{\rho_c}{\alpha_c} \right) \qquad \text{for} \quad c = 1,2 \tag{15}$$

The fractions can be extracted by equating the class speeds:

$$\frac{\alpha_1}{\rho_1} \frac{\rho_{\text{crit},1} v_{\text{free},1}}{\rho_{\text{crit},1} - \rho_{\text{max},1}} \left(\frac{\rho_1}{\alpha_1} - \rho_{\text{max},1} \right) \tag{16}$$
$$= \frac{1-\alpha_1}{\rho_2} \frac{\rho_{\text{crit},2} v_{\text{free},2}}{\rho_{\text{crit},2} - \rho_{\text{max},2}} \left(\frac{\rho_2}{1-\alpha_1} - \rho_{\text{max},2} \right)$$

The space fractions are

$$\alpha_1 = \frac{A}{B} \tag{17}$$

$$\alpha_2 = 1 - \alpha_1 \tag{18}$$

with

$$\begin{aligned} A = {} & ((\rho_{\text{crit},2} - \rho_{\text{max},2})\rho_{\text{crit},1} v_{\text{free},1} \\ & - (\rho_{\text{crit},1} - \rho_{\text{max},1})\rho_{\text{crit},2} v_{\text{free},2})\rho_1 \rho_2 \\ & + (\rho_{\text{crit},1} - \rho_{\text{max},1})\rho_{\text{crit},2} \rho_{\text{max},2} v_{\text{free},2} \rho_1 \end{aligned} \tag{19}$$

$$\begin{aligned} B = {} & (\rho_{\text{crit},1} - \rho_{\text{max},1})\rho_{\text{crit},2} \rho_{\text{max},2} v_{\text{free},2} \rho_1 \\ & + (\rho_{\text{crit},2} - \rho_{\text{max},2})\rho_{\text{crit},1} \rho_{\text{max},1} v_{\text{free},1} \rho_2 \end{aligned} \tag{20}$$

Now we have

$$q_c = \rho_c v \qquad \text{for} \quad c = 1,2 \tag{21}$$

The traffic states of both classes of vehicles can be described through these three traffic regimes. In addition, the total traffic state is the vector addition of these two classes of vehicles. The total fundamental diagram relation can be formulated as follows

$$Q_{\text{tot}}(\rho_{\text{tot}}) = \alpha_1 Q_1 \left(\frac{\rho_1}{\alpha_1} \right) + \alpha_2 Q_2 \left(\frac{\rho_2}{\alpha_2} \right) \tag{22}$$

3. SINGLE-CLASS METANET

The METANET model (Messmer and Papageorgiou, 1990) is a macroscopic traffic flow model. In this model, freeway stretches are represented by links (indexed by m), and each link can be divided into several segments (indexed by i). Besides, nodes are used to represent on-ramps, off-ramps, and changes in geometry. Traffic states are described with segment average values: density ($\rho_{m,i}$), space mean speed ($v_{m,i}$), and outflow($q_{m,i}$). These traffic states and their evolution are described through the following equations [1] :

$$q_{m,i}(k) = \rho_{m,i}(k) v_{m,i}(k) \lambda_m \tag{23}$$

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m} (q_{m,i-1}(k) - q_{m,i}(k)) \tag{24}$$

$$\begin{aligned} v_{m,i}(k+1) = {} & v_{m,i}(k) + \frac{T}{\tau_m} (V(\rho_{m,i}(k)) - v_{m,i}(k)) \\ & + \frac{T}{L_m} v_{m,i}(k)(v_{m,i-1}(k) - v_{m,i}(k)) \\ & - \frac{T \eta_m}{L_m \tau_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa_m} \end{aligned} \tag{25}$$

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \exp\left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right] \tag{26}$$

where k is the time step counter, T is the simulation time step, λ_m is the number of lanes of link m, τ_m, η_m, κ_m, and a_m are model parameters, $\rho_{\text{crit},m}$ is the critical density, $v_{\text{free},m}$ is the average free-flow speed, and $V(\rho)$ is the desired speed depending on density ρ.

Referring to Hegyi et al. (2005), the desired speed with a dynamic speed limit is computed as follows:

$$\begin{aligned} V(\rho_{m,i}(k)) = \min \Bigg(& v_{\text{free},m} \exp\left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right], \\ & (1+\chi_m) v_{\text{control},m,i}(k) \Bigg) \end{aligned} \tag{27}$$

where $v_{\text{control},m,i}$ is the dynamic speed limit in segment i of link m, and $1+\chi_m$ is the non-compliance factor.

Origins are modeled with a simple queue model:

$$w_o(k+1) = w_o(k) + T(d_o(k) - q_o(k)) \tag{28}$$

where w_o is the queue length at the origin o, d_o is the origin demand, and q_o is the origin outflow.

The outflow of origin o is estimated through the following equation:

$$q_o(k) = \min \left[d_o(k) + \frac{w_o(k)}{T}, C_o r_o(k), C_o \left(\frac{\rho_{\text{max},m} - \rho_{m,1}(k)}{\rho_{\text{max},m} - \rho_{\text{crit},m}} \right) \right] \tag{29}$$

in which C_o is the capacity of origin o, r_o is the ramp metering rate, $\rho_{\text{max},m}$ is the maximum density of the link m that the origin is connected to, and $\rho_{m,1}$ is the density of the segment that the origin is connected to.

4. NEW MULTI-CLASS METANET MODEL

On the basis of the method that is used for multi-class LWR model (see Section 2), we develop a new multi-class METANET model here. We assume that each vehicle class is subject to its own fundamental diagram, and constrains itself on an assigned space (expressed as a fraction $\alpha_{m,i,c}$ of the whole road). The traffic state variables for vehicle class c are $q_{m,i,c}$, $\rho_{m,i,c}$, $w_{o,c}$, $v_{m,i,c}$, and $q_{o,c}$. The state variables $q_{m,i,c}$, $\rho_{m,i,c}$, and $w_{o,c}$ are updated through the single-class equations. However, the speed $v_{m,i,c}$ and the origin flow $q_{o,c}$ are computed with class-dependent parameters $\tau_{m,c}$, $\eta_{m,c}$, $\kappa_{m,c}$, $\rho_{\text{crit},m,c}$, $v_{\text{free},m,c}$, $a_{m,c}$, and $\chi_{m,c}$. The speed equation is as follows:

[1] Only the basic equations are given in this paper, for the full model and for extensions, the reader is referred to Messmer and Papageorgiou (1990); Hegyi et al. (2005).

$$v_{m,i,c}(k+1) = v_{m,i,c}(k) + \frac{T}{\tau_{m,c}}\left(V_{m,c}\left(\frac{\rho_{m,i,c}(k)}{\alpha_{m,i,c}(k)}\right) - v_{m,i,c}(k)\right) + \frac{T}{L_m}v_{m,i,c}(k)(v_{m,i-1,c}(k) - v_{m,i,c}(k)) - \frac{T\eta_{m,c}}{L_m\tau_{m,c}}\frac{\rho_{m,i+1,c}(k)-\rho_{m,i,c}(k)}{\rho_{m,i,c}(k)+\rho_{\mathrm{crit},m,c}\kappa_{m,c}} \quad (30)$$

with

$$V_{m,c}\left(\frac{\rho_{m,i,c}(k)}{\alpha_{m,i,c}(k)}\right) = v_{\mathrm{free},m,c}\exp\left(\frac{-1}{a_{m,c}}\left(\frac{\rho_{m,i,c}(k)/\alpha_{m,i,c}(k)}{\rho_{\mathrm{crit},m,c}}\right)^{a_{m,c}}\right) \quad (31)$$

If there is a speed limit, the desired speed becomes

$$V_{m,c}\left(\frac{\rho_{m,i,c}(k)}{\alpha_{m,i,c}(k)}\right) = \min\left(v_{\mathrm{free},m,c}\exp\left(\frac{-1}{a_{m,c}}\left(\frac{\rho_{m,i,c}(k)/\alpha_{m,i,c}(k)}{\rho_{\mathrm{crit},m,c}}\right)^{a_{m,c}}\right),\ (1+\chi_{m,c})v_{\mathrm{control},m,i}(k)\right) \quad (32)$$

Besides, the outflow of vehicles of class c at origin o is

$$q_{o,c}(k) = \min\left[d_{o,c}(k) + \frac{w_{o,c}(k)}{T}, \alpha_{m,1,c}(k)C_{o,c}r_o(k), \alpha_{m,1,c}(k)C_{o,c}\left(\frac{\rho_{\max,m,c}-\rho_{m,1,c}(k)/\alpha_{m,1,c}(k)}{\rho_{\max,m,c}-\rho_{\mathrm{crit},m,c}}\right)\right] \quad (33)$$

in which $C_{o,c}$ is the theoretical maximum capacity of origin o if there would be only vehicles of class c, $d_{o,c}$ is the demand of vehicles of class c at origin o, $\alpha_{m,1,c}$ is the space fraction of vehicle class c in the segment that is connected to the origin, $\rho_{m,1,c}$ is the density of the segment that the origin is connected to, and $\rho_{\max,m,c}$ is the maximum density of the link m that the origin is connected to.

Three traffic regimes are considered: free flow, congestion, and semi-congestion. In the description of the three traffic regimes, the speed means the above-mentioned desired speed $V_{m,c}$.

Free-flow All classes of vehicles are in free-flow regime. Here the free flow means that the density of each class of vehicles in the assigned space is less than or equal to the critical density of that class. The constraint distinguishing free-flow regime with semi-congestion regime is

$$\sum_{c=1}^{n_c}\frac{\rho_{m,i,c}(k)}{\rho_{\mathrm{crit},m,c}} \leqslant 1 \quad (34)$$

where n_c is the number of vehicle classes. Expression (34) is derived from (3) and the following sufficient and necessary condition of free-flow regime:

$$\frac{\rho_{m,i,c}(k)}{\alpha_{m,i,c}(k)} \leqslant \rho_{\mathrm{crit},m,c} \quad (35)$$

In order to keep both classes of vehicles in free-flow regime, the relation (35) has to hold. According to (34) and (35), the space fraction of vehicle class c is taken as

$$\alpha_{m,i,c}(k) = \frac{\rho_{m,i,c}(k)/\rho_{\mathrm{crit},m,c}}{\sum_{j=1}^{n_c}\rho_{m,i,j}(k)/\rho_{\mathrm{crit},m,j}} \quad (36)$$

Semi-congestion Not all classes of vehicles are in free flow, and not all classes of vehicles are in congested mode. The constraint that distinguishes semi-congestion with congestion is

$$\sum_{c=1}^{n_c}\frac{\rho_{m,i,c}(k)}{\rho^*_{\mathrm{crit},m,c}} \leqslant 1 \quad (37)$$

where $\rho^*_{\mathrm{crit},m,c}$ is a parameter for vehicle class c defined as

$$\rho^*_{\mathrm{crit},m,c} = \rho_{\mathrm{crit},m,c}\left[-a_{m,c}\ln\left(\frac{v_{\mathrm{free},m,c^*_m}}{v_{\mathrm{free},m,c}}\exp\left(\frac{-1}{a_{m,c^*_m}}\right)\right)\right]^{\frac{1}{a_{m,c}}} \quad (38)$$

in which $c^*_m = \arg\min_{c=1,\ldots,n_c}\{v_{\mathrm{free},m,c}\exp(-1/a_{m,c})\}$ is the vehicle class with the slowest speed in free flow. The constraint (37) is constructed as follows. Class c^*_m is at the verge of getting in congested mode, and all the other classes are congested. In addition, the speed of the vehicles of class c^*_m in free flow is less than or equal to the speed of other congested classes of vehicles:

$$V_{c^*_m}\left(\frac{\rho_{m,i,c^*_m}(k)}{\alpha_{m,i,c^*_m}(k)}\right) \leqslant V_c\left(\frac{\rho_{m,i,c}(k)}{\alpha_{m,i,c}(k)}\right) \quad \text{for } c = 1,\ldots,n_c \text{ with } c \neq c^*_m \quad (39)$$

According to the assumption of optimal road use, the space fraction of vehicle class c^*_m is

$$\alpha_{m,i,c^*_m}(k) = \frac{\rho_{m,i,c^*_m}(k)}{\rho_{\mathrm{crit},m,c^*_m}} \quad (40)$$

Considering (3), (39), and (40), we obtain the constraint (37). Suppose that $S_{m,i,\mathrm{cong}}(k)$ is the set of all vehicle classes that are in congested mode in segment i of link m at time step k, and $S_{m,i,\mathrm{free}}(k)$ is the set of all vehicle classes that are in free flow in segment i of link m at time step k. Then the space fractions for those vehicle classes in free flow are

$$\alpha_{m,i,c} = \frac{\rho_{m,i,c}(k)}{\rho_{\mathrm{crit},m,c}} \quad \text{for} \quad c \in S_{m,i,\mathrm{free}}(k) \quad (41)$$

Let $l_{m,i}(k)$ be in the set $S_{m,i,\mathrm{cong}}(k)$. The space fractions for the congested vehicle classes are obtained through solving the following equation set:

$$\begin{cases} V_{m,c}\left(\frac{\rho_{m,i,c}(k)}{\alpha_{m,i,c}(k)}\right) = V_{m,l_{m,i}(k)}\left(\frac{\rho_{m,i,l_{m,i}(k)}}{\alpha_{m,i,l_{m,i}(k)}}\right) \\ \qquad \text{for} \quad c \in S_{m,i,\mathrm{cong}}(k)/\{l_{m,i}(k)\} \\ \sum_{c\in S_{m,i,\mathrm{cong}}(k)}\alpha_{m,i,c}(k) = 1 - \sum_{j\in S_{m,i,\mathrm{free}}(k)}\alpha_{m,i,j}(k) \end{cases} \quad (42)$$

Congestion All classes of vehicles are congested, and the speeds of all classes of vehicles are equal. The constraint of the congestion regime is the maximum density restriction:

$$\sum_{c=1}^{n_c}\frac{\rho_{m,i,c}}{\rho_{\max,m,c}} \leqslant 1 \quad (43)$$

The fractions can be extracted by equating the speeds of all classes of vehicles:

$$\begin{cases} V_1(\rho_{m,i,1}/\alpha_{m,i,1}) = V_2(\rho_{m,i,2}/\alpha_{m,i,2}) \\ \quad\vdots \\ V_{n_c-1}(\rho_{m,i,n_c-1}/\alpha_{m,i,n_c-1}) = V_{n_c}(\rho_{m,i,n_c}/\alpha_{m,i,n_c}) \\ \sum_{c=1}^{n_c}\alpha_{m,i,c} = 1 \end{cases} \quad (44)$$

The total fundamental diagram relation is obtained through the following equation:

$$Q_{m,i} = \sum_{c=1}^{n_c} \rho_{m,i,c} V_{m,c}\left(\frac{\rho_{m,i,c}}{\alpha_{m,i,c}}\right) \tag{45}$$

where $Q_{m,i}$ is the total flow. This total flow $Q_{m,i}$ depends on the densities of all classes of vehicles. More specifically, in this new multi-class METANET model, the total flow $Q_{m,i}$ is uniquely determined, when the densities of all classes of vehicles are given.

5. MODEL PREDICTIVE CONTROL BASED ON NEW MULTI-CLASS METANET MODEL

Model Predictive Control (MPC) (Camacho and Bordons, 1995) is an on-line control approach based on dynamic model prediction and receding horizon. The objective function of MPC captures the predicted performance of the controlled systems over a given prediction horizon. The controller finds the optimal control input sequence, and only the first element of the input sequence is applied to the controlled system. In addition, MPC can be used for nonlinear system, multi-criteria optimization, and constraints handling. Hence, MPC is selected for on-line model-based traffic control here.

The newly proposed multi-class METANET model is used as prediction model. Here we denote the prediction horizon by N_p, and denote the control horizon by N_c. We also distinguish the simulation time step length T and the controller time step length T_c, and $M = T/T_c$ is assumed to be a positive integer.

The Total Time Spent (TTS) is considered as the main term to be optimized. Here it is estimated as follows:

$$\mathrm{TTS}(k_c) = T \sum_{j=k_c M}^{(k_c+N_p)M-1} \sum_{c=1}^{n_c}\left(\sum_{(m,i)\in I_{\mathrm{all}}} L_m \lambda_m \rho_{m,i,c}(j) + \sum_{o\in O_{\mathrm{all}}} w_{o,c}(j)\right) \tag{46}$$

in which k_c is the controller time step counter, I_{all} is the set of all link-segment pairs (m,i) in the network, O_{all} is the set of all origins, and $w_{o,c}$ is the queue length of vehicles of class c at the origin o.

The objective function at controller time step k_c is defined as

$$J(k_c) = \xi_{\mathrm{TTS}} \frac{\mathrm{TTS}(k_c)}{\mathrm{TTS}_{\mathrm{nom}}} + \xi_{\mathrm{ramp}} \sum_{l=k_c}^{k_c+N_c-1} \sum_{o\in O_{\mathrm{ramp}}} (r_{\mathrm{ctrl},o}(l) - r_{\mathrm{ctrl},o}(l-1))^2 + \xi_{\mathrm{speed}} \sum_{l=k_c}^{k_c+N_c-1} \sum_{(m,i)\in I_{\mathrm{speed}}} \left(\frac{v_{\mathrm{ctrl},m,i}(l) - v_{\mathrm{ctrl},m,i}(l-1)}{v_{\mathrm{free},m,\mathrm{max}}}\right)^2 \tag{47}$$

in which ξ_{TTS}, ξ_{ramp}, and ξ_{speed} are nonnegative weights, $\mathrm{TTS}_{\mathrm{nom}}$ is the 'nominal' TTS for some nominal control profile (here we take the TTS in no control case), O_{ramp} is the set of all metered origins, I_{speed} is the set of all link-segment pairs (m,i) where speed limit is present, $v_{\mathrm{free},m,\mathrm{max}} = \max_c v_{\mathrm{free},m,c}$, $r_{\mathrm{ctrl},o}$ is the ramp metering rate of origin o at controller time step, and $v_{\mathrm{ctrl},m,i}$ is the speed limit in segment i of link m at controller time step. For a given simulation time step k, we have

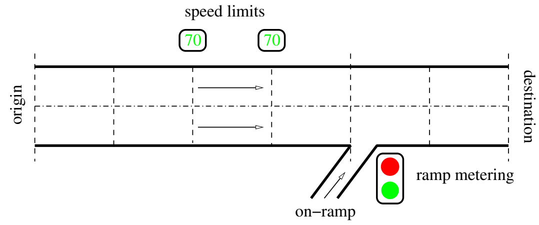


Fig. 1. The benchmark network

$$v_{\mathrm{control},m,i}(k) = v_{\mathrm{ctrl},m,i}\left(\left\lfloor \frac{k}{M} \right\rfloor\right) \tag{48}$$

$$r_o(k) = r_{\mathrm{ctrl},o}\left(\left\lfloor \frac{k}{M} \right\rfloor\right) \tag{49}$$

6. CASE STUDY

6.1 Benchmark Network

We select a benchmark network (Hegyi et al., 2005) for the case study. The network consists of two double-lane links, one single-lane on-ramp, one origin, and one destination with unrestricted outflow. The first link is divided into four homogeneous segments, and the second link is divided into two homogeneous segments. Fig. 1 presents a sketch of this network.

It is assumed that there are two classes of vehicles in this network, and the parameters are selected according to Hegyi et al. (2005); Logghe (2003): $v_{\mathrm{free},m,1} = 106.34$ km/h, $a_{m,1} = 1.6761$, $\chi_1 = 0.12$, $\rho_{\mathrm{crit},m,1} = 34.7349$ veh/km/lane, $\rho_{\mathrm{max},1} = 175$ veh/km/lane, $C_{\mathrm{mainstream},1} = 2034$ veh/h/lane; $v_{\mathrm{free},2} = 82.80$ km/h, $a_{m,2} = 2.1774$, $\chi_2 = 0.0533$, $\rho_{\mathrm{crit},m,2} = 18.9261$ veh/km/lane, $\rho_{\mathrm{max},2} = 75$ veh/km/lane, $C_{\mathrm{mainstream},2} = 990$ veh/h/lane. The capacity of the on-ramp is $C_{\mathrm{onramp}} = C_{\mathrm{mainstream}} - 100$ veh. Other parameters are: $L = 1$ km. $\tau = 18$ s, $\kappa = 40$ veh/h/km, $\eta = 60$ km^2/h, $\delta = 0.0122$.

The single-class parameters are generated as follows:

$\mathrm{Parameters_{nom}} = \theta^1_{\mathrm{nom}}\mathrm{Parameters_1} + (1-\theta^1_{\mathrm{nom}})\mathrm{Parameters_2}$.

with $\theta^1_{\mathrm{nom}} = 0.7$.

The control parameters are $\xi_{\mathrm{TTS}} = 1$, $\xi_{\mathrm{ramp}} = \xi_{\mathrm{speed}} = 0.01$, $T = 10$ s, $T_c = 60$ s, $N_p = 7$, $N_c = 5$.

6.2 Scenarios

The demand scenario shown in Fig. 2 is used for the case study (see also Hegyi et al. (2005)). Here we aim to examine the performance of the newly proposed multi-class METANET model with the same demand scenario and for several proportions of vehicles of different classes. Hence, the total demand scenario is represented in passenger car equivalents. Here passenger car equivalents mean that the number of vehicles of class 2 is transformed to the number of vehicles of class 1, using the vehicle length ratio $L_1^{\mathrm{veh}} = 3/7 L_2^{\mathrm{veh}}$. The proportions of vehicles of class 1 are (computed in pce): $\theta_1 \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. Besides, we assume that the queue length at O_2 cannot exceed 100 pce.

Multi-class METANET is used as simulation model. For comparison, the following control scenarios are implemented:

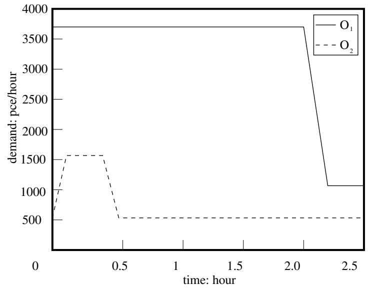


Fig. 2. The total demand scenario for two vehicle classes

- No control: here no control is applied for the given benchmark network.
- Single-class MPC: here the single-class METANET is used as the prediction model.
- Multi-class MPC: here the multi-class METANET is used as the prediction model.

6.3 Results

The simulation results are shown in Table 1. The J values listed in Table 1 are calculated for the entire simulation period of 2.5 h.

Table 1 Simulation results

Scenario	$J(0.1)$	$J(0.3)$	$J(0.5)$	$J(0.7)$	$J(0.9)$
No control	23.4048	26.8020	32.1822	36.9166	41.8598
Single-class MPC	23.4148	26.4550	25.7570	31.6012	39.0200
Multi-class MPC	20.7174	21.6241	24.2127	32.4786	38.6585

The results show that multi-class MPC leads to a better performance (smaller J) than single-class MPC. For $\theta_1 = 0.1$, single-class MPC leads to a worse result in comparison with no control case. This is probably due to that single-class MPC generates optimal inputs based on single-class METANET, which is different from the multi-class network. However, we noticed that the single-class MPC obtains a smaller J than the multi-class MPC for $\theta_1 = 0.7$. On reason may be that the single-class parameters have been selected using $\theta_1 = 0.7$, so the multi-class and single-class models are close to each other in this case. Another reason is probably that in the receding horizon approach in MPC sometimes earlier more optimal decisions may result in negative effects later on. In fact, our numerical experiments confirm that when the prediction horizon is increased (e.g. $N_p = 30$), multi-class MPC actually leads to a smaller J than single-class MPC for $\theta_1 = 0.7$.

7. CONCLUSIONS AND FUTURE TOPICS

In this paper, we have developed a new multi-class METANET model, using an approach that is similar to the one that Logghe (2003) used for developing multi-class LWR model. In this newly proposed model, each vehicle class constrains itself on an assigned space, being subject to its own fundamental diagram. We developed equations for computing the space fractions for different classes of vehicles based on three traffic regimes: free flow, semi-congestion, and congestion. Next, we used the newly proposed multi-class METANET model as prediction model in an on-line traffic control approach that is based on model predictive control. A case study was implemented to illustrate the efficiency of the new multi-class METANET model. For the given set-up and demand scenarios, the simulation results show that on-line traffic control based on the new multi-class METANET model leads to a better performance.

In the future, a comparison with other multi-class traffic flow models will be considered. We will also search for suitable emission models, and aim to realize a balance between total time spent and total emissions.

REFERENCES

Benzoni-Gavage, S. and Colombo, R.M. (2003). An n-populations model for traffic flow. *European Journal of Applied Mathematics*, 14(05), 587–612.

Caligaris, C., Sacone, S., and Siri, S. (2008). Multiclass freeway trafc: Model predictive control and microscopic simulation. In *Proceedings of 16th Mediterranean Conference on Control and Automation*, 1862–1867. Ajaccio, France.

Camacho, E.F. and Bordons, C. (1995). *Model Predictive Control in the Process Industry*. Springer-Verlag, Berlin, Germany.

Chanut, S. and Buisson, C. (2003). Macroscopic model and its numerical solution for two-flow mixed traffic with different speeds and lengths. *Transportation Research Board*, 1852(1), 209–219.

Daganzo, C.F. (1995). Requiem for second-order fluid approximations of traffic flow. *Transportation Research Part B*, 29B(4), 277–286.

Deo, P., De Schutter, B., and Hegyi, A. (2009). Model predictive control for multi-class traffic flows. In *Proceedings of the 12th IFAC Symposium on Transportation Systems*, 25–30. Redondo Beach, California, USA.

Hegyi, A., De Schutter, B., and Hellendoorn, H. (2005). Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C*, 13(3), 185–209.

Helbing, D. and Johansson, A. (2009). On the controversy around daganzos requiem for and aw-rascles resurrection of second-order traffic flow models. *The European Physical Journal B*, 69(4), 549–562.

Lighthill, M.J. and Whitham, G.B. (1955). On kinematic waves. II. a theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 229, 317–345.

Logghe, S. (2003). *Dynamic Modeling of Heterogeneous Vehicular Traffic*. Ph.D. thesis, University of Leuven, Heverlee, Belgium.

Messmer, A. and Papageorgiou, M. (1990). METANET: A macroscopic simulation program for motorway networks. *Traffic Engineering and Control*, 31(9), 466–470.

Papageorgiou, M. (1983). Applications of automatic control concepts to traffic flow modeling and control. Lecture Notes in Control and Information Sciencies. Springer-Verlag, Berlin, Germany.

Papageorgiou, M. (1998). Some remarks on macroscopic traffic flow modelling. *Transportation Research Part A*, 32(5), 323–329.

Richards, P.I. (1956). Shock waves on the highway. *Operations Research*, 4, 42–51.

van Lint, J.W.C., Hoogendoorn, S.P., and Schreuder, M. (2008). Fastlane: New multiclass first-order traffic flow model. *Transportation Research Record*, (1), 177–187.

Wong, G.C.K. and Wong, S.C. (2003). A multi-class traffic flow model-an extension of LWR model with heterogeneous drivers. *Transportation Research Part A*, 36, 827–841.