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UAVs in Formation and Dynamic Encirclement via Model Predictive Control

A. T. Hafez * M. Iskandarani ** S. N. Givigi ** S. Yousefi *
A. Beaulieu **

* Department of Electrical and Computer Engineering, Queens University, Kingston, ON. Canada (e-mail: 11edhk@queensu.ca, s.yousefi@queensu.ca).
** Royal Military College of Canada, Kingston, ON. Canada (e-mail: mohamad.iskandarani@rmc.ca, sidney.givigi@rmc.ca, alain.beaulieu@rmc.ca)

Abstract: Switching between the formation flight tactic and the dynamic encirclement tactic for a team of Unmanned Aerial Vehicles (UAVs) is done using a decentralized approach. A team formed from N UAVs, accomplishes a line-of-breast formation then dynamic encirclement around a desired target. A high-level Linear Model Predictive Control (LMPC) policy is used to control the UAV team during the execution of the required formation tactic, while a combination of decentralized LMPC and Feedback Linearization (FL) is implemented on the UAV team to accomplish dynamic encirclement. During the simulations, Reynold's rules of flocking are respected. The linear plant, representing each UAV, is found through System Identification. The main contribution of this paper lies in the use of LMPC to implement multiple UAV tactics while ensuring stability and robustness of the system during tactic switching.

Keywords: Model Predictive Control, Unmanned Aerial Vehicles, Cooperative robotics

1. INTRODUCTION

UAV Tactics are defined as the general strategies used by individuals in a UAV team to achieve a desired outcome (Marasco, 2012). Research and experiments have dealt with the different tactics that can be carried out by a team of UAVs, and a wide variety of approaches to effectively implement these tactics have been proposed. From different point of views, tactics of UAVs can be divided into four main categories: swarming, task assignment , formation reconfiguration and dynamic encirclement (Marasco, 2012).
On one hand, formation reconfiguration is defined as the dynamic ability of a UAV team to change its formation according to the surrounding circumstances and due to the response to different external factors such as type of mission, UAV populations and environments. The UAV formation must guarantee all vehicles safety and be compatible with the UAV dynamics and may be governed by time constraints to pass between obstacles. Formation Reconfigurations have different behaviours according to the factors affecting the UAV team, among them: changing the position of UAVs, combining small groups to form a large one or breaking a large group into smaller groups. These behaviours are activated to avoid the failure of a mission due to external factors (Ryan et al., 2004).
Much of the research discussing formation reconfiguration focuses on the control of the formation itself and many solutions have been presented (Bhattacharya and Basar, 2010; Léchevin et al., 2009). An autonomous configuration control algorithm is used to control the formation of the fleet to overcome the threats of an early warning radar and its fire system to execute the required mission. Formation reconfiguration is applied by the UAV team to reduce the effect of the aerial jammer on the communication channel, minimizing the use of munitions and the loss of UAVs facing hostile environments (Hattenberger et al., 2007).
The field of formation control holds multiple structure approaches. For instance, the leader-follower strategy has one of the UAVs as leader while the rest are considered followers (Do and Pan, 2007), however, the failure of the leader means the failure of the whole mission. Another formation control approach is a virtual structure, where the UAVs follow a certain moving point acting like a rigid body, which makes it difficult to avoid obstacles and to do collision avoidance and disturbance rejection (Ren and Beard, 2003), while in the behaviour approach, the goal of the mission and its corresponding constraints are laid out for the team who decides the manner with which to accomplish the task in a decentralized way (Lawton et al., 2003). The latter strategy is suitable for uncertain environments but lacks theoretical guaranties of stability. In our paper, we will adopt a control strategy within the leader-follower umbrella using LMPC as a higher level control.
Dynamic encirclement is a military term defined to be the situation in which a target is isolated and surrounded by enemy forces. In the field of multiple UAVs, encirclement is considered to be a strategy in which UAVs assume positions around a target to restrict its movements. In real life, this tactic is used in defending a secure airspace against an invading aircraft, maintaining surveillance over a ground target and in protecting the borders against invading targets (Sharma et al., 2010).

Different methods of control were used to solve the problem of dynamic encirclement for multiple UAVs. A fleet of UAVs controls the movement of a target by restricting its trajectory (Sharma et al., 2010), while in (Kim et al., 2010), a distributed cooperative control method is proposed to capture a target based on a cyclic pursuit strategy. Dynamic encirclement of a stationary target with multiple UAVs using a decentralized NMPC was successfully presented, also, the effect of communications between the vehicles was discussed, and a stabilizing control policy was derived and proved by simulation results (Marasco et al., 2012, 2013). Furthermore, a combination between Taylor Series Linearization and decentralized LMPC is used on a team of UAVs for dynamic encirclemenet of a stationary target. This control policy allowed real-time implementation on the Qball-X4 quadrotor aircraft (Iskandarani et al., 2013).

UAV dynamics are nonlinear, multivariable and are exposed to parameter uncertainties, external noise and external disturbances. The controller needs to overcome the nonlinearities of the system to reach stability, robustness, and the desired dynamics properties (Benallegue et al., 2006). An FL technique is a common approach used in the control of different nonlinear systems; furthermore, different combinations of FL and various types of controllers are used to overcome the nonlinear dynamics of UAVs during their flights (Lee et al., 2009; Fang et al., 2008; Voos, 2009). Also, MPC is a common controller that can handle multivariable cases which helps in solving complex control problems (Camacho and Bordons, 2007). Different types of decentralized MPC are used in the field of control, such as UAVs performing various missions and different tactics (Richards and How, 2004; Kim et al., 2003; Shin and Thak, 2011).

Our main contribution in this paper is solving the problem of UAV tactic switching while ensuring the stability and robustness of the system. An LMPC is used to control the UAV team during formation flight, while a combination of decentralized LMPC and FL is used to solve the problem of dynamic encirclement in simulation. We occupy ourselves with a decentralized high-level controller, where each team member generates the required path necessary to respect the line-of-breast formation and encirclement conditions. All formation and encirclement tasks are accomplished autonomously.

The paper is organized as follows. In Section II, the problems of line breast formation and dynamic encirclement are formulated, the specific control objectives are defined and the dynamic feedback linearization is introduced. In Section III, we develop our control policy and show all constraints. Section IV presents the results of our simulations while Section V concludes the work and presents some future objectives.

2. PROBLEM FORMULATION

A team of N UAVs form a linear formation and advance in a leader-follower manner then switch to dynamic encirclement tactic to encircle a desired target. The goal is to ensure the stability of the UAV system during switching from the line of breast formation to dynamic encirclement. In order to simplify our problem, we consider that the UAVs act in a two dimensional space such that the height and yaw controllers have no influence on the lateral movement of the vehicles, although the problem could be extended to the three dimensional case with increased computational demands according to (Sharma et al., 2010).

By collecting flight data from a Qball-X4 quadrotor describing the Cartesian movement of a UAV, system identification, based on a least-squares algorithm, was used to construct a second order system representing x and y for each UAV in the team (Iskandarani et al., 2013).

2.1 Formation Flight

The UAVs form a linear formation and advance in a leader-follower manner while maintaining the desired separation distance and matching speed with its neighbors. By subtracting the position of each UAV in the fleet with its neighbor, we get the following error dynamics in state-space form:

$$\begin{bmatrix} \ddot{E}x \\ \ddot{E}y \\ \dot{E}x \\ \dot{E}y \end{bmatrix} = \begin{bmatrix} -2.62 & 0 & -1.72 & 0 \\ 0 & -1.38 & 0 & -0.49 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{E}x \\ \dot{E}y \\ Ex \\ Ey \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{d1} \\ X_{d2} \\ Y_{d1} \\ Y_{d2} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} Ex \\ Ey \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1.7452 & 0 \\ 0 & 0 & 0 & 0.497 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{E}x \\ \dot{E}y \\ Ex \\ Ey \end{bmatrix} \quad (2)$$

where E_x and E_y are the errors in x and y respectively between each two successive UAVs in the formation, while $\dot{E}_x$ and $\dot{E}_y$ are their respective time derivatives. Equation (2) is the output equation that give us the distance in x and y between the UAVs. LMPC is used to maintain these errors at a desired value. The choice of leader UAV helps us form the desired line abreast formation during the formation tactic phase.

2.2 Dynamic encirclement

The linear system found in (Iskandarani et al., 2013) has a state vector $[x\ y\ \dot{x}\ \dot{y}]^T$ and inputs $[X_d\ Y_d]^T$, where X_d and Y_d are the desired positions of the Qball-X4, x and y are the current positions of the Qball-X4 during flight and $\dot{x}$ and $\dot{y}$ are the corresponding speeds. Since the control of each UAV during dynamic encirclement depends on using the radius of encirclement and the angular velocity, a state transformation from Cartesian coordinates to Polar coordinates is applied. The resultant system is represented in the following form:

$$\dot{\bar{X}} = f(r, \phi, \dot{r}, \dot{\phi}, x_o, y_o, X_d, Y_d) \quad (3)$$

where $f(\cdot)$ is a nonlinear function combining the inputs and states of the system $\bar{X}$, x_o is the translation in x-direction, and y_o is the translation in y-direction. This translation represents the coordinates of the moving target. Moreover, r represents the radius of encirclement, ϕ the angle between the target and the encircling UAV, and $\dot{\phi}$ (or ω), is the angular velocity of the UAV. To complete the transformation we first replace the states x, y, $\dot{x}$ and $\dot{y}$ with their Polar equivalents:

$$x = r\cos\phi + x_o \quad (4a)$$

$$y = r\sin\phi + y_o \quad (4b)$$

$$\dot{x} = \dot{r}\cos\phi - r\dot{\phi}\sin\phi \quad (4c)$$

$$\dot{y} = \dot{r}\sin\phi + r\dot{\phi}\cos\phi \quad (4d)$$

Then, the resultant set of equations characterized by $[\dot{x}\ \dot{y}\ \ddot{x}\ \ddot{y}]^T$ are multiplied using a transformation matrix $\mathbf{T}$ as follows (Iskandarani et al., 2013).

$$\begin{bmatrix} \dot{r} \\ \dot{\phi} \\ \ddot{r} \\ \ddot{\phi} \end{bmatrix} = \mathbf{T} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} \quad (5)$$

We now have a nonlinear model with outputs states, r and ω, for each UAV in the team to be applied during the encirclement tactic phase. Marasco (2012) presents an accurate description of UAV nonlinear dynamics while encircling a target, where each UAV in the two-dimensional Cartesian space is represented by the state vector:

$$\bar{q}_i(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \quad (6)$$

In order to form a dynamic encirclement around a target, a new state vector in a polar coordinate reference frame centered at the origin is introduced. The new vector state for each UAV is as follows:

$$\bar{q}_i(t) = \begin{bmatrix} r_i(t) \\ \phi_i(t) \end{bmatrix} = \begin{bmatrix} \|\tilde{q}_i(t)\| \\ \angle\tilde{q}_i(t) \end{bmatrix} \quad (7)$$

where i represents the number of UAV in the team, $r_i(t)$ represents the distance between the UAV and the target (equal to the required radius of dynamic encirclement at time t) and $\phi_i(t)$ represents the angle between the UAV and the target at time t. The main objective of our designed controller is to achieve the required constraints mentioned in the following equations (Kawakami and Namerikawa, 2009):

$$C1)\ \lim_{t\to\infty} |r_i(t) - R_D| = 0 \quad \forall i \leq N \quad (8)$$

$$C2)\ \lim_{t\to\infty} |\dot{\phi}_i(t) - \dot{\phi}_D| = 0 \quad \forall i \leq N \quad (9)$$

$$C3)\ \lim_{t\to\infty} |\phi_{i+1}(t) - \phi_i(t)| = \frac{2\pi}{N} \quad \forall i \leq N \quad (10)$$

The above equations represent the model objectives that must be achieved by the controller: Condition *C1* states that each UAV in the team maintains a desired distance R_D from the target, while Condition *C2* states that each UAV in the team maintains a desired angular velocity $\dot{\phi}_D$ around the target. Finally, Condition *C3* states that each member in the team spreads itself evenly in a circular formation around the target. The LMPC controller respects these constraints when accomplishing dynamic encirclement.

Finally, the goal of our simulation is to guarantee the stability and robustness of the system after switching from the formation tactic to the encirclement tactic.

3. FEEDBACK LINEARIZATION

In section 2, we identify the model for each UAV and setup the transformation to the new set of states that allows us to output the required radius of encirclement and angular velocity for each UAV in the team. The objective of FL is to linearize (3) and represent it in the standard state-space form allowing us to use the decentralized LMPC during the encirclement phase. A combination of two new inputs u_1 and u_2 is used in replacing the original inputs X_d and Y_d. This substitution cancels the nonlinearities in (3) introducing a linear system with new inputs u_1 and u_2. The resultant linear system can be represented as follows:

$$\dot{\bar{X}} = g(r, \theta, \dot{r}, \dot{\theta}, u_1, u_2) \quad (11)$$

where $g(\cdot)$ is a linear function found through FL substitution. The final linear process model may be represented in state-space form:

$$\dot{\bar{X}} = A\bar{X} + BU; \bar{Y} = C\bar{X} \quad (12)$$

where U is $[u_1\ u_2]^T$, $\bar{X}$ is $\begin{bmatrix} r\ \theta\ \dot{r}\ \dot{\theta} \end{bmatrix}^T$, $\bar{Y}$ is the ouput vector holding r and $\omega = \dot{\theta}$ and matrices A, B, C ensure controllability and observability of the state-space.

4. CONTROL DESIGN

Our control design is divided into two parts; the first part deals with the UAV team while accomplishing the required line-of-breast formation, while the other part deals with each UAV in a decentralized manner according to its decision of encirclement. A smoothing time t_{sm}, used in the piece-wise control signal mixing, is introduced to guarantee the stability of the system at the time of switching from the formation controller to the encirclement controller.

4.1 Formation flight

An LMPC is used to maintain the desired distance between each UAV and its neighbor during the flight. The first UAV is considered the leader of the fleet while the other UAVs maintain the desired distance in x and y coordinates.

For each UAV in the team, the cost function of the LMPC controller is minimized according to the weights of the outputs, and is given as follows:

$$J(\bar{Z}, \Delta u) = \sum_{i=0}^{M-1} \Gamma^T Q \Gamma + \Delta u(k+i|k)^T R_{\Delta u}(k+i|k) \quad (13)$$

The components of the cost function are:

$$\Gamma = \bar{Z}(k+i+1|k) - d(k+i+1|k) \quad (14)$$

where M is the prediction horizon and $\bar{Z}(k+i+1|k)$ is the state vector containing the error dynamics as highlighted in (1) and (2). These states are predicted for time $k+i+1$ at time k. Furthermore, $r(k+i+1|k)$ is the reference sampled for time $(k+i+1)$ at time k; $\Delta u(k+i|k)$ is the manipulated variables rate calculated for time $k+i$ at time k; Q and $R_{\Delta u}$ are positive semi-definite matrices that hold the weights for the output variables and the manipulated variables rate respectively. The references d represent the desired distances between the UAVs in x and y and their desired rate of change.

4.2 Dynamic encirclement

The FL method is used to linearize the encirclement model of each UAV in the team, which allows the use of a suitable LMPC in real-time. The optimization problem is convex in nature which allows for faster computational performance compared to its nonlinear counterparts. Moreover, the UAVs accomplish the task of encircling a stationary and moving target autonomously with the user only initiating the system.

In order to accomplish dynamic encirclement using a team of N UAVs, conditions *(C1,C2,C3)* in section 2.2 must be met. Each UAV in the team is cognisant of the leading and lagging UAV in the formation.

We will derive the controller for N vehicles as follows, we first define the desired angular separation

$$\Delta\phi_D = 2\pi/N \tag{15}$$

$$\Delta\phi_{i,j}(t) = \phi_j(t) - \phi_i(t) \in [0, 2\pi] \tag{16}$$

where θ_i represents the angle of the i^{th} UAV, θ_j is the angle of the j^{th} UAV, $\Delta\theta_D$ is the desired angular separation between two UAVs, and N is the number of UAVs in the formation.

Thus, the error between two of the UAVs in a team is given as follows:

$$e_i(t) = \Delta\phi_{i,j}(t) - \Delta\phi_D \tag{17}$$

The suitable Lyapunov candidate function is defined as

$$\begin{aligned} V(t) &= 1/2(e_1^2(t) + e_2^2(t) + e_3^2(t) + ...e_N^2(t)) \\ \dot{V}(t) &= e_1(t)\dot{e_1}(t) + e_2(t)\dot{e_2}(t) + ...e_N(t)\dot{e_N}(t) \end{aligned} \tag{18}$$

By choosing the suitable condition

$$\dot{\phi}_j(t) - \dot{\phi}_i(t) = -\gamma e_i(t) \tag{19}$$

and by choosing γ as a positive constant, Lyapunov stability is achieved:

$$\dot{V}(t) = -\gamma[e_1(t)^2 + e_2(t)^2 + ... + e_N(t)^2] \tag{20}$$

Moreover, the chosen Lyapunov candidate function allows the errors to decrease as time goes by, despite the fact that the UAVs are allowed to vary their speeds initially to achieve encirclement. We now add the desired angular speed to (19) using the following equation

$$\dot{\phi}_D = \frac{\dot{\phi}_1(t) + \dot{\phi}_2(t) + \dot{\phi}_3(t)}{3} \tag{21}$$

We can use (19) and (21) to calculate the desired angular speed for each member in the formation.

In general, the desired angular speed for each member in the team is calculated by observing the current angular positions of the leading and lagging UAV in the formation respectively according to the following equation:

$$\dot{\phi}_{Di}(t) = \frac{3 * \dot{\phi}_D + \gamma(\phi_{lead}(t) - \phi_{lag}(t))}{3}, \forall i \in [1, N] \tag{22}$$

where ϕ_{lead} represents the angular difference between the UAV being considered and the one in front of it, while ϕ_{lag} represents the angular difference with the one behind it. Finally, the state $\Delta\phi_{i,j}$, angular separation between two UAVs is added to the linear system presented in section 2 so that it may be included in the cost function. Moreover, the desired angular velocity $\dot{\phi}_{Di}$ is fed back as a reference to the process model. The prediction horizon and control horizon used are eight and two respectively. This leads to some modifications to our cost function so that it reflects the conditions highlighted in section 2.2:

$$J_i = J_i^{C1} + J_i^{C2} + J_i^{C3} \tag{23}$$

where *C1*, *C2*, and *C3* are the encirclement objectives that must be achieved by the controller and are given by:

$$J_i^{C1} = \sum_{j=1}^{M}(\alpha(r_i(t+j\delta) - R_D)^2 + \beta(\dot{\phi}_i(t+j\delta) - \dot{\phi}_{Di})^2) \tag{24a}$$

$$J_i^{C2} = \sum_{j=1}^{M}(\rho(((\phi_i(t+j\delta) - \phi_{lag}(t+j\delta)) - \frac{2\pi}{N})^2) \tag{24b}$$

$$J_i^{C3} = \sum_{j=1}^{M}(\zeta((\phi_{lead}(t+j\delta) - \phi_i(t+j\delta)) - \frac{2\pi}{N})^2) \tag{24c}$$

where $r_i(t+j\delta)$ and $\dot{\phi}_i(t+j\delta)$ are the predicted radius and angular speed. Also α, β , ρ and ζ are positive constants where α is the distance gain, β is the angular speed gain, ρ is the angular separation gain for lagging UAV while ζ is the angular separation gain for leading UAV, $\phi_{lag}(t + j\delta)$ and $\phi_{lead}(t + j\delta)$ are the angles of the lagging and leading UAVs in the formation respectively. Since only the current position of the leading and lagging UAVs are known, $\phi_{lag}(t)$ and $\phi_{lead}(t)$, the remaining future angles must be estimated by solving the following differential equations:

$$\dot{\phi}_{lead}(t) = \dot{\phi}_D \tag{25}$$

$$\dot{\phi}_{lag}(t) = \dot{\phi}_D \tag{26}$$

5. SIMULATION RESULTS

The control strategy discussed in sections 2 and 4 is successfully implemented in simulation on a multi-UAV team consisting of three vehicles. The objective of these simulations is to show that the LMPC policy designed is fit for tactic switching.

During each flight test, the team flies in formation until its sensors indicate the presence of a target. The UAV which met the target first, checks its communication radius and sends a package of information including the location of the target, the importance of the target η and the degree of threat of the target ϵ to the other members of the team in its zone of communication. According to this information, each UAV takes its own decision to either switch to the encirclement tactic or continue in the formation tactic. The decision to encircle will happen if $\eta > 0.6$, $\epsilon < 0.5$ and the distance between the UAV and the target is d <12 m.

This section will highlight two pertinent cases. The first case considers one UAV taking a decentralized decision to encircle a stationary target while the other two members continue in the formation phase, while in the second case all three vehicles deciding independently to encircle a moving target at a rate of 0.1 m/s initially located at (35,-5). The objective of the second case is to prove the robustness and scalability of our designed controller. In these results, UAV 1 is represented in blue, UAV 2 in red and UAV 3 in green.

5.1 Case #1: One UAV encircling a stationary target

A set of three UAVs with initial positions $(0, 4)$, $(0, 0)$ and $(0, -4)$ successfully form a line-of-breast formation with a desired distance of 5m between each other. The target is located at (35,-2) but the UAVs are not aware of it in the beginning of the flight test. UAV1 senses the target first and informs the other UAVs in its communication radius of the presence of the target. UAV1 takes the decision to switch to the encirclement tactic while the other two members in the team take the decision to continue in the formation tactic. This simulation runs for 240 seconds and its overall result may be seen in Fig. 1. The main parameters used in simulation are desired separation distance of 5m, desired radius of encirclement 10 m and angular velocity of 0.15 rad/s. Fig. 2 represents the speed of each UAV in the team during the formation flight phase where we can see that each UAV matches the other flockmate's speed of 0.2 m/s. Moreover, Fig. 3 shows the desired radius and angular velocity for UAV1 during the encirclement phase. During both tactics, the UAVs converge to the proper requirements highlighted above.

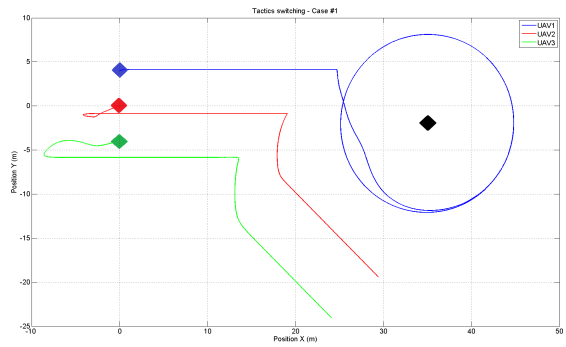

Fig. 1. UAV team members switching from formation flight to dynamic encirclement tactic while maintaining the required separating distance during the formation phase and the desired radius and angular velocity during the encirclement phase. UAV 1, 2 and 3 are represented by the blue, red and green diamonds respectively while the target is the black diamond.

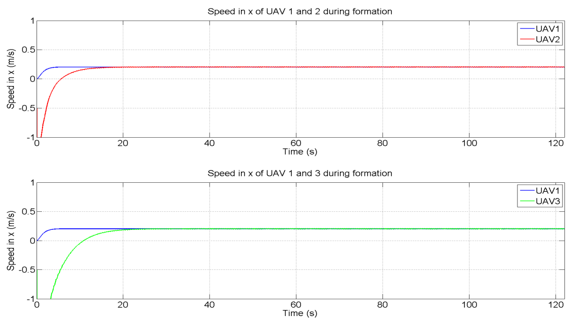

Fig. 2. The speed of the UAV team in the x-direction during formation phase.

5.2 *Case #2: Three UAVs encircle a moving target*

A team of three UAVs flying in a line-of-breast formation with a distance of 5m between each other switch to a dynamic encirclement tactic around a moving target with

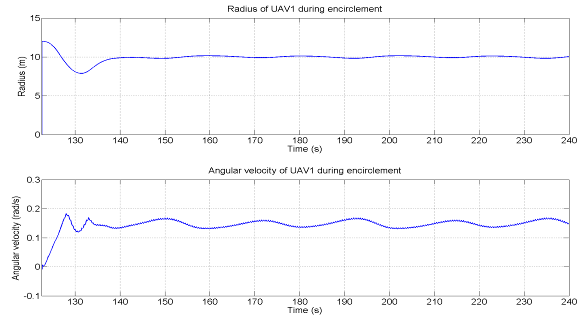

Fig. 3. The radius of encirclement and angular velocity for UAV1 encircling a stationary target at (35,-2).

a desired radius of encirclement of 10 m, angular velocity of 0.05 rad/s and an angular separation of $\frac{2\pi}{3}$ rad. The initial positions of the UAVs are $(0, 4)$, $(0, 0)$ and $(0, -4)$ and the target is located at (35,-5). The objective here is to show that the UAVs may accomplish dynamic encirclement by controlling their own radii with respect to the target, their angular velocities and the angle of separation. This simulation was run for 600 seconds, Fig. 4 shows the overall performance of the team. The desired radii and angular velocities are respected as seen in Fig. 5, while Fig. 6 shows that the UAVs converge to the desired angle of separation while orbiting the moving target.

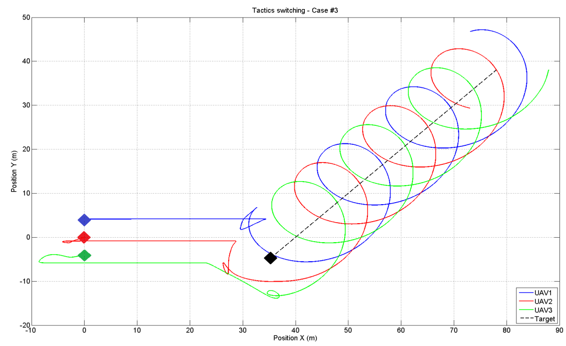

Fig. 4. Three UAVs, switch from the formation tactic to encirclement tactic around a moving target. UAV 1, 2 and 3 are represented by the blue, red and green diamonds respectively while the target is the black diamond.

6. CONCLUSION

In this paper, we proposed a control strategy for tactic switching, going from line abreast formation to dynamic encirclement. Our results show that applying the MPC strategy solves the problem of tactic switching for a team of UAVs in simulation. Our simulations results show that our control policy succeeded to control the UAV system by converging to the desired requirements for formation and encirclement. This control policy is characterized by robustness, scalability, and stability.
In the future, we will apply the designed controllers derived in this paper combined with a learning algorithm to solve the tactic switching problem for multiple UAVs aiming

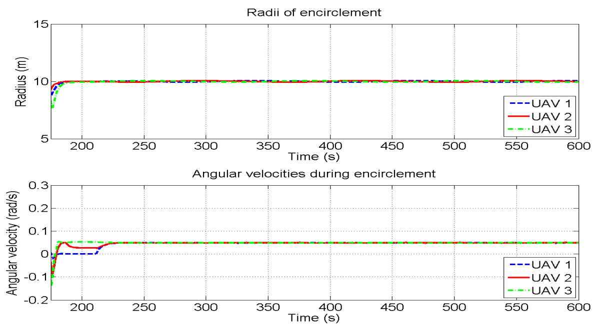

Fig. 5. The radii of encirclement and angular velocities for three UAVs encircling a target moving in a straight line, starting at (35,-5).

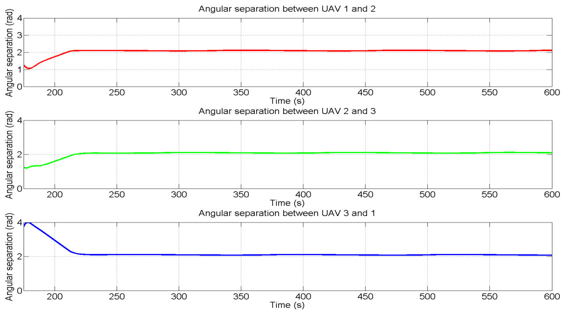

Fig. 6. Angular separation for three UAVs encircling a target moving in a straight line. All vehicles converge to an angle of separation of $\frac{2\pi}{3}$ rad.

to reduce the imperfections between the linearized system and the perfect model. We see as a necessary step to better the stability, robustness, convergence and safety of these autonomous applications.

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