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Periodic Event-Triggered Distributed Receding Horizon Control of Dynamically Decoupled Linear Systems

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Abstract: This paper is concerned with the distributed receding horizon control (DRHC) problem for large-scale linear systems with disturbances. A periodic event-triggered DRHC scheme is proposed to reduce communication and computation load. A detailed dual-mode periodic event-triggered DHRC algorithm is designed, and sufficient conditions for ensuring feasibility and stability are established, respectively. We show that the feasibility depends on the testing period, and that the stability is related with the testing period, and the cooperation matrices. The overall system is stable and the system state converges to a set under the designed algorithm.

Keywords: Periodic event-triggering, distributed receding horizon control (DRHC), linear systems, large-scale systems, disturbance

1. INTRODUCTION

The control of large-scale and complex systems is becoming a new frontier in the areas of systems and control, due to the trend of developing many large-scale systems, such as multi-agent systems, complex process control systems, smart grid and power systems, and cyber-physical systems. The receding horizon control (RHC), also known as model predictive control, has been widely used in the industrials Qin and Badgwell (2003, 2000), and it is one of the most promising approaches to large-scale systems. There are three RHC schemes for the control of large-scale systems, namely, centralized RHC, distributed RHC (DRHC) and decentralized RHC. The implementation of centralized RHC normally requires solving a high-dimensional optimization problem in real time, which is computationally expensive or practically infeasible. On the other hand, the decentralized RHC generally decouples large-scale systems into many subsystems by ignoring couplings, which may bring poor control performance or undesired results. The DRHC is able to convert the large-scale optimization problem into several small-size optimization problems while considering couplings among subsystems by using communication links, and thus it is computationally efficient and can achieve prescribed control performance.

In recent years, the study of the DRHC problem for large-scale systems has received much attention, and many results have been reported for large-scale linear systems, nonlinear systems and their applications. For example, the investigation of the DRHC problem for large-scale linear systems has been conducted in Camponogara et al. (2002); Jia and Krogh (2002); Motee and Sayyar-Rodsari (2003); Izadi et al. (2009); Maestre et al. (2011); Venkat et al. (2008); Richards and How (2007); Franco et al. (2007); Borrelli and Keviczky (2008); Stewart et al. (2010); Li and Shi (2013b). In these results, Jia and Krogh (2002); Camponogara et al. (2002); Stewart et al. (2010); Maestre et al. (2011); Borrelli and Keviczky (2008) are focused on large-scale systems with coupled dynamics among subsystems; while Franco et al. (2007); Izadi et al. (2009); Richards and How (2007); Li and Shi (2013b) study the DRHC problems of large-scale dynamically decoupled linear systems, with couplings presenting in the constraints or objective functions. Izadi et al. (2009) reports the application result for the cooperative control of multi-vehicle systems, and Venkat et al. (2008) investigates the DRHC problem of a complex power system.

The DRHC of large-scale nonlinear systems and its applications have been studied in Raimondo et al. (2007); Keviczky et al. (2008); Dunbar and Murray (2006); Keviczky et al. (2006); Dunbar (2007); Stewart et al. (2011); Dunbar and Caveney (2012); Franco et al. (2008); Li and

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Shi (2013a); Liu et al. (2009, 2010, 2012). In particular, Raimondo et al. (2007); Keviczky et al. (2006) approach the DRHC problems of discrete-time nonlinear systems, where Raimondo et al. (2007) establishes the input-to-state stability and Keviczky et al. (2006) presents the asymptotic stability. Dunbar and Murray (2006); Dunbar (2007); Dunbar and Caveney (2012) investigate the DRHC problem of continuous-time decoupled nonlinear systems, coupled nonlinear systems and inter-connected robot systems, respectively. In Franco et al. (2008), the DRHC problem of decoupled nonlinear systems with constant communication delays is studied, and the results for decoupled nonlinear systems with disturbances and time-varying communication delays is reported in Li and Shi (2013a). Liu et al. (2009, 2010, 2012) investigate the DRHC problem of coupled nonlinear systems based on the Lyapunov-based model predictive control concept.

Recently, in Åström and Bernhardsson (2002), the event-triggered control strategy is proven to be computationally more efficient than the traditional periodic sampling and update scheme. Thus, many interesting results have reported in the literature, such as Tabuada (2007); Heemels et al. (2008); Donkers and Heemels (2012); Wang and Lemmon (2011). Note that the existing results of DRHC rely on the communication links to exchange information among subsystems. By using the communication link, the information transmission is synchronized periodically. With that period, the optimization problem is solved, and the control signal is updated accordingly. Thus, the traditional periodic scheme in DRHC may be inefficient. To solve such an issue, the event-triggered strategy has been proposed in MPC Eqtami et al. (2011b); Varutti et al. (2009) and in decentralized MPC Eqtami et al. (2011a). However, in these results, the triggering conditions need to be examined continuously, which may be not practical or infeasible. Most recently, a periodic event-triggered strategy is proposed in Heemels et al. (2013), where the triggering condition only requires being testing periodically. Therefore, the periodic event-triggered strategy is more practical while computationally more efficient. Motivated by this fact, we propose to study the DRHC problem of linear systems based on the periodic event-triggered strategy.

In this paper, we will investigate the the event-triggered DRHC problem of large-scale decoupled linear systems with disturbances. The main contributions of this paper are two-fold:

- A periodic event-triggered DRHC scheme is proposed and the detailed dual-model periodic event-triggered DRHC algorithm has been designed.
- The feasibility of the designed algorithm and the stability of the closed-loop system are rigorously analyzed. The conditions for ensuring feasibility and stability are established, respectively. It is shown that the closed-loop system is stable and the system state converges to a set.

The remainder of the paper is organized as follows. Section 2 formulates the distributed RHC problem and presents a preliminary result. In Section 3, the event-triggered strategy is designed and the periodic event-triggered DRHC algorithm is presented. In Section 4, the feasibility and stability issues are analyzed, and the sufficient conditions for guaranteeing feasibility and stability are established, respectively. Finally, the conclusion remarks are given in Section 5.

The following notations are adopted in this paper. The real space is denoted by the symbol $\mathbb{R}$; the set of all integers is denoted by $\mathbb{Z}$ and $\mathbb{Z}_{\geqslant 0} \triangleq \{n \in \mathbb{Z} : n \geqslant 0\}$. Given a matrix P, its transpose and inverse (if invertible) are denoted as P^{T} and P^{-1}, respectively. $P > 0$ means that the matrix P is positive definite. The symbols $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ stands for the maximum and the minimum eigenvalues of P, respectively. The symbol $\bar{\sigma}(P)$ represents the maximum singular eigenvalue of P. Given two matrices $P > 0$ and $Q > 0$, $\lambda_{P,Q} \triangleq \lambda_{\max}(P)/\lambda_{\min}(Q)$. Given a column vector v and a matrix $Q > 0$ with appropriate dimension, $\|v\|$ stands for the Euclidean norm and $\|v\|_Q \triangleq \sqrt{v^{\mathrm{T}}Qv}$ represents the Q-weighted norm. For column vectors $v_1, \cdots, v_n$, $\mathrm{clo}(v_1, \cdots, v_n) = [v_1^{\mathrm{T}}, \cdots, v_n^{\mathrm{T}}]^{\mathrm{T}}$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the distributed receding horizon control problem for a group of linear agents. For each agent i, the system dynamics is modeled as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \omega_i(t), i = 1, \cdots, M, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is system state, $u_i(t) \in \mathbb{R}^m$ is control input, $\omega_i(t) \in \mathbb{R}^n$ is disturbance and M is the number of the agents. The control input is required to satisfy constraint

$$u_i(t) \in \mathcal{U}_i, \tag{2}$$

where $\mathcal{U}_i \subseteq \mathbb{R}^m$ is a convex and compact set containing the origin. The disturbance has an energy bound $\|\omega_i(t)\| \leqslant \rho_i$.

There is a communication network among the agent system, in which each agent can communicate with some of the agents. For each agent i, its neighbors are defined as the agents from which it can receive information. Denote the neighbors' index set of agent i by $\mathcal{N}_i$, where $\mathcal{N}_i \neq \emptyset$. The collection of agent i's neighbors' states is denoted by $x_{-i}(t)$.

Based on (1), the overall system can be represented as

$$\dot{x}(t) = Ax(t) + Bu(t) + \omega(t), \tag{3}$$

with $u(t) \in \mathcal{U}$, where $x = \mathrm{clo}(x_1, \cdots, x_M)$, $u = \mathrm{clo}(u_1, \cdots, u_m)$, $\omega = \mathrm{clo}(\omega_1, \cdots, \omega_M)$, $A = \mathrm{diag}(A_1, \cdots, A_M)$, $B = \mathrm{diag}(B_1, \cdots, B_M)$, and $\mathcal{U} = \mathcal{U}_1 \times \cdots \times \mathcal{U}_M$.

The nominal system of (1) is defined as follows

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t). \tag{4}$$

Assumption 1. For the system in (1), there exists a robustly control invariant set Blanchini (1999) $\mathcal{X}_i$, i.e., for all $u_i(t) \in \mathcal{U}_i$, if $x_i(t_0) \in \mathcal{X}_i$, then $x_i(t) \in \mathcal{X}_i$, for all bounded disturbances with $\|\omega_i(t)\| \leqslant \rho_i$.

The conditions for ensuring such an $\mathcal{X}_i$ can be referred to Blanchini (1999). Define $\bar{\delta}_i = \sup_{x_i(t) \in \mathcal{X}_i} \|x_i(t)\|$.

Assumption 2. For each agent i, the pair (A_i, B_i) is controllable.

Using Assumption 2, it is well known that there exists a state feedback control law $u_i(t) = K_i x_i(t)$ such that $\bar{A}_i = A_i + B_i K_i$ is stable.

Theorem 3. For the system in (4), given a stabilizing control law $u_i(t) = K_i x_i(t)$ and two symmetric matrices $Q_i > 0$, $R_i > 0$, there exists a parameter $\varepsilon_i > 0$, such that $x_i(t_0) \in \Omega_i(\varepsilon_i)$ implies $x_i(t) \in \Omega_i(\varepsilon_i)$, and the constraint in (2) is satisfied, for all $t \geqslant t_0$. Here, $\Omega_i(\varepsilon_i) \triangleq \{x_i(t) \in \mathbb{R}^n : \|x_i(t)\|_{P_i} \leqslant \varepsilon_i\}$, and P_i is the solution to the Lyapunov equation $\bar{A}_i^{\mathrm{T}} P_i + P_i \bar{A}_i + Q_i + K_i^{\mathrm{T}} R_i K_i = 0$.

Proof. The proof can be derived by following the similar lines in Dunbar (2007); Chen and Allgöwer (1998); Michalska and Mayne (1993); Li and Shi (2014b,a), and thus is omitted here.

3. PERIODIC EVENT-TRIGGERED DRHC ALGORITHM

In this section, the optimization problem associated with each agent i is firstly formulated. Then the periodic event-triggered strategy is proposed and the detailed periodic event-triggered DRHC is designed.

3.1 Distributed Optimization

For each agent i, at each time instant $t_{k_p^i}$, define an optimization problem $\mathcal{P}_i$:

$$\hat{u}_i^*(s; t_{k_p^i}) = \arg \min_{\hat{u}_i(s;t_{k_p^i})} J_i(\hat{x}_i(s; t_{k_p^i}), \hat{u}_i(s; t_{k_p^i})), \text{subject to:}$$

$$\begin{aligned}
&\dot{\hat{x}}_i(s; t_{k_p^i}) = A_i \hat{x}_i(s; t_{k_p^i}) + B_i \hat{u}_i(s; t_{k_p^i}),\\
&\dot{\hat{x}}_j^a(s; t_{k_p^i}) = A_j \hat{x}_j^a(s; t_{k_p^i}) + B_j \hat{u}_j^a(s; t_{k_p^i}),\\
&\hat{u}_i(s; t_{k_p^i}) \in \mathcal{U}_i, s \in [t_{k_p^i}, t_{k_p^i} + T],\\
&\|\hat{x}_i(t_{k_p^i} + T; t_{k_p^i})\|_{P_i} \leqslant \alpha_i \varepsilon_i. \qquad (5)
\end{aligned}$$

Here, the cost function is designed as follows

$$\begin{aligned}
&J_i(\hat{x}_i(s; t_{k_p^i}), \hat{u}_i(s; t_{k_p^i}))\\
&\triangleq \int_{t_{k_p^i}}^{t_{k_p^i}+T} \|\hat{x}_i(s; t_{k_p^i})\|_{Q_i}^2 + \|\hat{u}_i(s; t_{k_p^i})\|_{R_i}^2\\
&\quad + \sum_{j \in \mathcal{N}_i} \|\hat{x}_i(s; t_{k_p^i}) - \tilde{x}_j^a(s; t_{k_p^i})\|_{Q_{ij}}^2 ds\\
&\quad + \|\hat{x}_i(t_{k_p^i} + T; t_{k_p^i})\|_{P_i}^2, \qquad (6)
\end{aligned}$$

where $Q_i > 0$, $R_i > 0$, and Q_{ij} are symmetric matrices, and P_i and ε_i are designed according Theorem 3. $\alpha_i \in (0, 1)$ is the shrinkage rate Li and Shi (2013a). $T = n_0 \tau$ is the prediction horizon, with $n_0 > 1$, being a given integer and $\tau > 0$ being the testing period. In (6), $\hat{x}_i(s; t_{k_p^i})$ is called predicted state trajectory and it is generated by

$$\dot{\hat{x}}_i(s; t_{k_p^i}) = A_i \hat{x}_i(s; t_{k_p^i}) + B_i \hat{u}_i(s; t_{k_p^i}), s \in [t_{k_p^i}, t_{k_p^i} + T].$$

$\tilde{x}_j^a(s; t_{k_p^i})$ is called assumed state trajectory which is generated by agent j and transmitted to agent i. In particular, $\tilde{x}_j^a(s; t_{k_p^i})$ fulfills the following equation

$$\dot{\tilde{x}}_j^a(s; t_{k_p^i}) = A_j \tilde{x}_j^a(s; t_{k_p^i}) + B_j \tilde{u}_j^a(s; t_{k_p^i}), s \in [t_{k_p^i}, t_{k_p^i} + T],$$

where $\tilde{u}_j^a(s; t_{k_p^i})$ is produced as follows:

$$\tilde{u}_j^a(s; t_{k_p^i}) = \begin{cases} \hat{u}_j^*(s; t_{k_q^j}), & \text{if} \quad s \in [t_{k_p^i}, t_{k_q^j} + T] \\ K_j \tilde{x}_j^a(s; t_{k_q^j} + T), & \text{if} \quad s \in [t_{k_q^j} + T, t_{k_p^i} + T]. \end{cases}$$

Here, $t_{k_q^j} = \sup\{k_n^j \in \mathbb{Z}_{\geq 0} : t_{k_p^i} \geqslant t_{k_n^j}\}$.

3.2 Periodic Event-triggered Strategy

Unlike the classical DRHC using the fixed period to update the control input in Dunbar (2007); Franco et al. (2008), for each agent i, we design an event-triggered strategy to determine the time interval for updating control input and transmitting information. In particular, given some $t_{k_p^i}$, $p = 0, 1, \cdots$, the next time instant $t_{k_{p+1}^i}$ for update is determined as follows: a) Test the triggering condition

$$\|x_i(t_{k_p^i} + n\tau) - \hat{x}_i^*(t_{k_p^i} + n\tau; t_{k_p^i})\|_{P_i} \geqslant \sigma_i, \qquad (7)$$

at $t = t_{k_p^i} + n\tau$, $n = 1, \cdots, n_0$, where $\sigma_i > 0$ is the triggering level. If the triggering condition in (7) holds for some n, take n_t as the minimum value of such n; otherwise, take $n_t = n_0$. b) Determine $t_{k_{p+1}^i} = t_{k_p^i} + n_t \tau$.

Due to the existence of disturbances, it would be computationally costly to execute the optimal control input when the system state trajectory enters a small region around the origin. Like the robust RHC in Michalska and Mayne (1993) and DRHC in Dunbar (2007); Li and Shi (2013a), we also take the so-called dual-mode strategy here. That is, for each agent i, if the optimal state trajectory $x_i^*(s; t_{k_p^i})$, $s \in [t_{k_p^i}, t_{k_p^i} + T]$, enters the set $\Omega_i(r_i \alpha_i \varepsilon_i)$ at some time instant $s = t_o + t_{k_p^i}$, where $r_i \in (0, 1)$, then the control input is switched to the state feedback control law $u_i(t) = K_i x_i(t)$, for all $t > t_o + t_{k_p^i}$.

Remark 4. From the testing criterion in (7), it can been seen that, for each agent i, the triggering condition is only examined with a fixed interval τ, and the optimization problem $\mathcal{P}_i$ is only required to be solved with time-varying intervals $n(t_{k_p^i})\tau$ when the triggering condition is satisfied, where $1 \leqslant n(t_{k_p^i}) \leqslant n_0$ is an integer depending on time instant $t_{k_p^i}$. The information sent from agent i to its neighbors also follows such a time-varying interval. This, in fact, reduces the computation and communication load.

3.3 Periodic Event-triggered DRHC

Combining the periodic event-triggered strategy and the dual-mode strategy, the periodic event-triggered DRHC algorithm is summarized as follows:

Algorithm 1. For each agent i, $i = 1, \cdots, M$,

- S1) Initialize the optimization problem at $t = t_{k_0^i}$.
- S2) Solve Problem $\mathcal{P}_i$ at $t_{k_p^i}$ to generate $\hat{u}_i^*(s; t_{k_p^i})$, $p \geqslant 0$.
- S3) If $\|\hat{x}_i^*(s; t_{k_p^i})\|_{P_i} \leqslant r_i \alpha_i \varepsilon_i$ for some $s_0 \in [t_{k_p^i}, t_{k_p^i} + T]$, then apply the control input $u_i^*(s; t_{k_p^i})$ for $s \in [t_{k_p^i}, s_0]$ and go to S6).
- S4) Apply $\hat{u}_i^*(s; t_{k_p^i})$ and at time $s = t_{k_p^i} + n\tau$, $n = 1, \cdots, n_0$, test (7) to determine $t_{k_{p+1}^i}$.
- S5) Set $t_{k_p^i} = t_{k_{p+1}^i}$, and go to S2).
- S6) Apply $u_i(t) = K_i x_i(t)$.

4. ANALYSIS

In order to make the designed algorithm practically useful, we need to investigate the feasibility and stability issues. This section provides conditions on how to design the parameters to ensure feasibility and stability.

4.1 Feasibility Analysis

Before proceeding to conduct the feasibility analysis, a feasible control trajectory candidate should be constructed. Given the optimal control trajectory $u_i^*(s;t_{k_p^i})$, at time instant $t_{k_p^i}$, a feasible control trajectory candidate $\tilde{u}_i(s;t_{k_{p+1}^i})$ at time instant $t_{k_{p+1}^i}$ is constructed as follows Li and Shi (2013a), Michalska and Mayne (1993):

$$\tilde{u}_i(s;t_{k_{p+1}^i}) = \begin{cases} \hat{u}_i^*(s;t_{k_p^i}), & s \in [t_{k_{p+1}^i}, t_{k_p^i}+T], \\ K_i\tilde{x}_i(s;t_{k_{p+1}^i}), & s \in (t_{k_p^i}+T, t_{k_{p+1}^i}+T]. \end{cases}$$

Theorem 5. For each agent i with dynamics in (1), given $u_i^*(s;t_{k_p^i})$, at $t_{k_p^i}$, if the testing period τ is designed, such that

$$\rho_i n\tau\bar{\sigma}(\sqrt{P_i}e^{A_i(n-n_0)\tau}) \leqslant (1-\alpha_i)\varepsilon_i, \tag{8}$$

$$\bar{\sigma}(\sqrt{P_i}e^{\bar{A}_i n\tau}(\sqrt{P_i})^{-1}) \leqslant \alpha_i, \tag{9}$$

for $n = 1, \cdots, n_0$, then $\tilde{u}_i(s;t_{k_{p+1}^i})$, $s \in [t_{k_{p+1}^i}, t_{k_{p+1}^i}+T]$, is a feasible solution to Problem $\mathcal{P}_i$ at $t_{k_{p+1}^i}$.

Proof. We know that $\tilde{u}_i(s;t_{k_{p+1}^i})$, $s \in [t_{k_{p+1}^i}, t_{k_p^i}+T]$, makes the constraint in (2) be fulfilled. We next need to show that $\tilde{u}_i(s;t_{k_{p+1}^i})$, $s \in [t_{k_p^i+T}, t_{k_{p+1}^i}+T]$ renders the constraints in (2) being satisfied.

According to (4), we have

$$\begin{aligned} \hat{x}_i^*(s;t_{k_p^i}) =& e^{A_i(s-t_{k_{p+1}^i})}\hat{x}_i^*(t_{k_{p+1}^i};t_{k_p^i}) \\ &+ \int_{t_{k_{p+1}^i}}^{s} e^{A_i(s-t)}B_i\hat{u}_i^*(t;t_{k_p^i})dt, s \geqslant t_{k_{p+1}^i}, \\ \tilde{x}_i^*(s;t_{k_{p+1}^i}) =& e^{A_i(s-t_{k_{p+1}^i})}x_i(t_{k_{p+1}^i}) \\ &+ \int_{t_{k_{p+1}^i}}^{s} e^{A_i(s-t)}B_i\hat{u}_i^*(t;t_{k_p^i})dt, s \geqslant t_{k_{p+1}^i}. \end{aligned}$$

Therefore, it can be obtained that

$$\begin{aligned} &\tilde{x}_i^*(t_{k_p^i}+T;t_{k_{p+1}^i}) - \hat{x}_i^*(t_{k_p^i}+T;t_{k_p^i}) \\ =& e^{A_i(t_{k_p^i}+T-t_{k_{p+1}^i})}[x_i(t_{k_{p+1}^i}) - \hat{x}_i^*(t_{k_{p+1}^i};t_{k_p^i})] \\ =& e^{A_i(t_{k_p^i}+T-t_{k_{p+1}^i})}[\int_{t_{k_p^i}}^{t_{k_{p+1}^i}} \omega_i(t;t_{k_p^i})dt], \end{aligned}$$

where the solution to the differential equation in (1) is used. As a result, we can get

$$\begin{aligned} &\|\tilde{x}_i^*(t_{k_p^i}+T;t_{k_{p+1}^i})\|_{P_i} \\ \leqslant& \|\hat{x}_i^*(t_{k_p^i}+T;t_{k_p^i})\|_{P_i} \\ &+ \|e^{A_i(t_{k_p^i}+T-t_{k_{p+1}^i})}[\int_{t_{k_p^i}}^{t_{k_{p+1}^i}} \omega_i(t;t_{k_p^i})dt]\|_{P_i} \\ \leqslant& \alpha_i\varepsilon_i + \rho_i n\tau\sqrt{\lambda_{\max}((e^{A_i(n-n_0)\tau})^{\mathrm{T}}P_i e^{A_i(n-n_0)\tau})}, \end{aligned}$$

where $t(k_{p+1}^i) - t_{k_p^i} = n\tau$, $n = 1, \cdots, n_0$ is used. By using the condition in (8), it follows $\|\tilde{x}_i^*(t_{k_p^i}+T;t_{k_{p+1}^i})\|_{P_i} \leqslant \varepsilon_i$. As a consequence, the result in Theorem 3 can be used, and it implies $\tilde{u}_i(s;t_{k_{p+1}^i}) \in \mathcal{U}_i$, $s \in [t_{k_p^i+T}, t_{k_{p+1}^i}+T]$. Furthermore, we have

$$\begin{aligned} &\|\tilde{x}_i(t_{k_{p+1}^i}+T;t_{k_{p+1}^i})\|_{P_i} \\ =& \|e^{\bar{A}_i(t_{k_{p+1}^i}-t_{k_p^i})}\tilde{x}_i(t_{k_p^i}+T;t_{k_{p+1}^i})\|_{P_i} \\ \leqslant& \sqrt{\lambda_{\max}(((\sqrt{P_i})^{-1})^{\mathrm{T}}(e^{\bar{A}_i n\tau})^{\mathrm{T}}P_i e^{\bar{A}_i n\tau}(\sqrt{P_i})^{-1})}\varepsilon_i \\ \leqslant& \alpha_i\varepsilon_i, \forall n = 1, \cdots, n_0, \end{aligned}$$

where the condition in (9) is applied. Thus, $\tilde{x}_i(t_{k_{p+1}^i}+T;t_{k_{p+1}^i}) \in \Omega(\alpha_i\varepsilon_i)$ and the terminal constraint is satisfied. This completes the proof.

Remark 6. It is worth noting that Theorem 5 provides sufficient conditions on how to design τ to guarantee feasibility. From (8) and (9), it can been seen that the design of τ depends on the choice of the parameter α_i and the disturbance bound ρ_i after the parameter ε_i, P_i and K_i are given.

4.2 Stability Analysis

Using the designed Algorithm 1, we can show that the closed-loop system is stable and converges to a robustly invariant set. To facilitate the presentation, we define two terms as follows:

$$\begin{aligned} C_1(\tau,n) \triangleq& (1-\alpha_i)^2\varepsilon_i^2 + (n_0-n)\tau\lambda_{Q_i,P_i}\delta_i^2 \\ &- \beta_1 n\tau\lambda_{P_i,Q_i}^{-1}(1-r_i)^2\alpha_i^2\varepsilon_i^2, \\ C_2(Q_{ij},n) \triangleq& \lambda_{Q_{ij},P_i}[(n_0-n)\tau\delta_i^2 + n\tau\alpha_j^2\varepsilon_j^2] \\ &+ \lambda_{Q_{ij},P_j}[(n_0-1)\tau\delta_j^2 + n_0\tau\alpha_j^2\varepsilon_j^2] \\ &- \beta_2 n\tau\lambda_{P_i,Q_i}^{-1}(1-r_i)^2\alpha_i^2\varepsilon_i^2, \end{aligned}$$

where $\beta_1 \in (0,1)$, $\beta_2 \in (0,1)$ and $\beta_1 + \beta_2 < 1$, and $\delta_i = \lambda_{\max}(\sqrt{P_i})\bar{\delta}_i$. The stability result is reported in the following theorem.

Theorem 7. For the overall system in (3), if (A): The testing interval τ is designed, such that (8) and (9) holds; (B): $C_1(\tau,n) \leqslant 0$ for $n = 1, \cdots, n_0$, and the cooperation matrices Q_{ij}, $j \in \mathcal{N}_i$, are designed such that $C_2(Q_{ij},n) \leqslant 0$, for $n = 1, \cdots, n_0$, then closed-loop system is stable and the system state converges to a robustly invariant set.

Proof. The proof consists of two parts. Firstly, we show that the system trajectory of each agent i will enter a set in finite time. Define

$$\begin{aligned} \Delta_i \triangleq& J_i(\tilde{x}_i(s;t_{k_{p+1}^i}), \tilde{u}_i(s;t_{k_{p+1}^i}), \tilde{x}_{-i}^a(s;t_{k_{p+1}^i})) \\ &- J_i(\hat{x}_i^*(s;t_{k_p^i}), \hat{u}_i^*(s;t_{k_p^i}), \tilde{x}_{-i}^a(s;t_{k_p^i})). \end{aligned}$$

By substituting all the terms in Δ_i, we have

$$\begin{aligned} \Delta_i \leqslant& \int_{t_{k_p^i}+T}^{t_{k_{p+1}^i}+T} \|\tilde{x}_i(s;t_{k_{p+1}^i})\|_{Q_i}^2 + \|\tilde{u}_i(s;t_{k_{p+1}^i})\|_{R_i}^2 ds \\ &+ \|\tilde{x}_i(t_{k_{p+1}^i}+T;t_{k_{p+1}^i})\|_{P_i}^2 - \|\hat{x}_i^*(t_{k_p^i}+T;t_{k_p^i})\|_{P_i}^2 \\ &+ \int_{t_{k_{p+1}^i}}^{t_{k_p^i}+T} \|\tilde{x}_i(s;t_{k_{p+1}^i})\|_{Q_i}^2 - \|\hat{x}_i^*(s;t_{k_p^i})\|_{Q_i}^2 ds \\ &+ \int_{t_{k_{p+1}^i}}^{t_{k_{p+1}^i}+T} \sum_{j\in\mathcal{N}_i} \|\tilde{x}_i(s;t_{k_{p+1}^i}) - \hat{x}_j^a(s;t_{k_{p+1}^i})\|_{Q_{ij}}^2 ds \\ &- \int_{t_{k_p^i}}^{t_{k_{p+1}}^i} \|\hat{x}_i^*(s;t_{k_p^i})\|_{Q_i}^2 ds. \end{aligned} \tag{10}$$

Next, we consider the terms in the right hand side of (10) one by one. According to Theorem 5, $\tilde{x}_i(t_{k_p^i}+T; t_{k_{p+1}^i}) \in \Omega_i(\varepsilon_i)$ and $\tilde{u}_i(s; t_{k_{p+1}^i}) = K_i\tilde{x}_i(s; t_{k_{p+1}^i}), s \in [t_{k_p^i}+T, t_{k_{p+1}^i}]$. By using Theorem 3, it can be obtained

$$\begin{aligned}&\int_{t_{k_p^i}+T}^{t_{k_{p+1}^i}+T} \|\tilde{x}_i(s; t_{k_{p+1}^i})\|_{Q_i}^2 + \|\tilde{u}_i(s; t_{k_{p+1}^i})\|_{R_i}^2 ds \\ &\quad + \|\tilde{x}_i(t_{k_{p+1}^i}+T; t_{k_{p+1}^i})\|_{P_i}^2 - \|\hat{x}_i^*(t_{k_p^i}+T; t_{k_p^i})\|_{P_i}^2 \\ &=\|\tilde{x}_i(t_{k_p^i}+T; t_{k_{p+1}^i})\|_{P_i}^2 - \|\hat{x}_i^*(t_{k_p^i}+T; t_{k_p^i})\|_{P_i}^2 \\ &\leqslant (1-\alpha_i^2)\varepsilon_i^2. \end{aligned} \tag{11}$$

In terms of the fact that $\|\hat{x}_i^*(s; t_{k_p^i})\|_{P_i} \geqslant (1-r_i)\alpha_i\varepsilon_i$, $s \in [t_{k_{p+1}^i}, t_{k_{p+1}^i}+T]$, and $\|\tilde{x}_i(s; t_{k_{p+1}^i})\|_{P_i} \leqslant \delta_i$, we get

$$\begin{aligned}&\int_{t_{k_{p+1}^i}}^{t_{k_p^i}+T} \|\tilde{x}_i(s; t_{k_{p+1}^i})\|_{Q_i}^2 - \|\hat{x}_i^*(s; t_{k_p^i})\|_{Q_i}^2 ds \\ &\leqslant (n_0-n)\tau(\lambda_{Q_i,P_i}\delta_i^2 - \lambda_{P_i,Q_i}^{-1}(1-r_i)^2\alpha_i^2\varepsilon_i^2). \end{aligned} \tag{12}$$

By using the same reasoning, we can obtain

$$\begin{aligned}&\int_{t_{k_{p+1}^i}}^{t_{k_{p+1}^i}+T} \sum_{j\in\mathcal{N}_i} \|\tilde{x}_i(s; t_{k_{p+1}^i}) - \hat{x}_j^a(s; t_{k_{p+1}^i})\|_{Q_{ij}}^2 ds \\ &\leqslant 2\sum_{j\in\mathcal{N}_i} \int_{t_{k_{p+1}^i}}^{t_{k_{p+1}^i}+T} \|\tilde{x}_i(s; t_{k_{p+1}^i})\|_{Q_{ij}}^2 + \|\hat{x}_j^a(s; t_{k_{p+1}^i})\|_{Q_{ij}}^2 ds \\ &\leqslant 2|\mathcal{N}_i|[\lambda_{Q_{ij},P_i}((n_0-n)\tau\delta_i^2 + n\tau\alpha_i^2\varepsilon_i^2) \\ &\quad + (\lambda_{Q_{ij},P_j}(n_0-n)\tau\delta_j^2 + \lambda_{Q_{ij},P_j}n\tau\alpha_i^2\varepsilon_i^2)]. \end{aligned} \tag{13}$$

Finally, it can be obtained

$$\int_{t_{k_p^i}}^{t^i_{k_{p+1}}} \|\hat{x}_i^*(s; t_{k_p^i})\|_{Q_i}^2 ds \geqslant n\tau\lambda_{P_i,Q_i}^{-1}(1-r_i)^2\alpha_i^2\varepsilon_i^2. \tag{14}$$

By plugging (11) - (14) into (10), and using the condition of $C_1(\tau, n) \leqslant 0$ and $C_2(Q_{ij}, n) \leqslant 0$, we can obtain

$$\Delta J_i \leqslant -(1-\beta_1-\beta_2)n\tau\lambda_{P_i,Q_i}^{-1}(1-r_i)^2\alpha_i^2\varepsilon_i^2.$$

Due to the optimality, we have

$$\begin{aligned}&J_i(\hat{x}_i^*(s; t_{k_{p+1}^i}), \hat{u}_i^*(s; t_{k_{p+1}^i}), \tilde{x}_i^a(s; t_{k_{p+1}^i})) \\ &\quad - J_i(\hat{x}_i^*(s; t_{k_p^i}), \hat{u}_i^*(s; t_{k_p^i}), \tilde{x}_{-i}^a(s; t_{k_p^i})) \\ &\leqslant \Delta J_i \leqslant -(1-\beta_1-\beta_2)n\tau\lambda_{P_i,Q_i}^{-1}(1-r_i)^2\alpha_i^2\varepsilon_i^2. \end{aligned}$$

By using the same argument in Michalska and Mayne (1993), it can be shown that for some $s > 0$, $\hat{x}_i^*(s; t_{k_p^i})$ enters $\Omega_i(r_i\alpha_i\varepsilon_i)$ in finite time, i.e., $x_i(s)$ enters $\Omega_i((1-r_i\alpha_i)\varepsilon_i)$.

Secondly, we show that the closed-loop system under the control law $u_i(t) = K_ix_i(t)$ is stable and the system state converges to a set after $x_i(t) \in \Omega_i((1-r_i\alpha_i)\varepsilon_i)$. Taking $V_i(x_i(t)) = \|x_i(t)\|_{P_i}$ as a Lyapunov function candidate, we have

$$\begin{aligned}\dot{V}_i(x_i(t)) =& x_i^{\mathrm{T}}(t)(\bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i)x_i(t) + 2x_i^{\mathrm{T}}\bar{A}_i^{\mathrm{T}}\omega_i(t) \\ =& -x_i^{\mathrm{T}}(t)Q_i^*x_i(t) + 2x_i^{\mathrm{T}}\bar{A}_i^{\mathrm{T}}\omega_i(t) \\ \leqslant& -\|x_i(t)\|_{Q_i^*}^2 + 2\rho_i\lambda_{\max}(\sqrt{Q_i^*})\|\bar{A}_i\|\|x_i(t)\|_{Q_i^*}, \end{aligned}$$

where $Q_i^* = Q_i + K_i^{\mathrm{T}}R_iK_i$. Thus, the closed-loop system is stable as desired and the system state of agent i will converge to the set $\{x_i(t) \in \mathbb{R}^n : \|x_i(t)\|_{Q_i^*} \leqslant 2\rho_i\lambda_{\max}(\sqrt{Q_i^*})\|\bar{A}_i\|\}$.

Remark 8. Theorem 7 reals that the stability of the closed-loop system is related with the testing period τ, the parameter δ_i, α_i, and the cooperation matrices Q_{ij} when the other parameters are fixed.

5. CONCLUSION

In this paper, we have investigated the event-triggered DRHC problem of decoupled linear systems. The periodic event-triggered DRHC algorithm has been proposed to reduce the communication and computation load. The conditions for guaranteeing the feasibility and stability have been established. The future work will be the consideration of the event-triggered DRHC problem of large-scale nonlinear systems.

REFERENCES

Åström, K.J. and Bernhardsson, B.M. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of the 41st IEEE Conference on Decision and Control*, volume 2, 2011–2016.

Blanchini, F. (1999). Set invariance in control. *Automatica*, 35(11), 1747–1767.

Borrelli, F. and Keviczky, T. (2008). Distributed LQR design for identical dynamically decoupled systems. *IEEE Transactions on Automatic Control*, 53(8), 1901–1912.

Camponogara, E., Jia, D., Krogh, B.H., and Talukdar, S. (2002). Distributed model predictive control. *IEEE Control Systems Magazine*, 22(1), 44–52.

Chen, H. and Allgöwer, F. (1998). A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. *Automatica*, 34(10), 1205–1217.

Donkers, M.C.F. and Heemels, W.P.M.H. (2012). Output-based event-triggered control with guaranteed $\mathcal{L}_2$-gain and improved and decentralized event-triggering. *IEEE Transactions on Automatic Control*, 57(6), 1362–1376.

Dunbar, W.B. (2007). Distributed receding horizon control of dynamically coupled nonlinear systems. *IEEE Transactions on Automatic Control*, 52(7), 1249–1263.

Dunbar, W.B. and Caveney, D.S. (2012). Distributed receding horizon control of vehicle platoons: Stability and string stability. *IEEE Transactions on Automatic Control*, 57(3), 620–633.

Dunbar, W.B. and Murray, R.M. (2006). Distributed receding horizon control for multi-vehicle formation stabilization. *Automatica*, 42(4), 549–558.

Eqtami, A., Dimarogonas, D.V., and Kyriakopoulos, K.J. (2011a). Event-triggered strategies for decentralized model predictive controllers. In *Proceedings of the 18th IFAC World Congress*. Milano, Italy.

Eqtami, A., Dimarogonas, D.V., and Kyriakopoulos, K.J. (2011b). Novel event-triggered strategies for model predictive controllers. In *Proceedings of 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, 3392–3397.

Franco, E., Magni, L., Parisini, T., Polycarpou, M.M., and Raimondo, D.M. (2008). Cooperative constrained control of distributed agents with nonlinear dynamics and delayed information exchange: A stabilizing receding-horizon approach. *IEEE Transactions on Automatic Control*, 53(1), 324–338.

Franco, E., Parisini, T., and Polycarpou, M.M. (2007). Design and stability analysis of cooperative receding-horizon control of linear discrete-time agents. *International Journal of Robust and Nonlinear Control*, 17(10-11), 982–1001.

Heemels, W.P.M.H., Donkers, M.C.F., and Teel, A.R. (2013). Periodic event-triggered control for linear systems. *IEEE Transactions on Automatic Control*, 58(4), 847–861.

Heemels, W.P.M.H., Sandee, J.H., and Van Den Bosch, P.P.J. (2008). Analysis of event-driven controllers for linear systems. *International Journal of Control*, 81(4), 571–590.

Izadi, H.A., Gordon, B.W., and Zhang, Y. (2009). Decentralized receding horizon control for cooperative multiple vehicles subject to communication delay. *Journal of Guidance, Control, and Dynamics*, 32(6), 1959–1965.

Jia, D. and Krogh, B. (2002). Min-max feedback model predictive control for distributed control with communication. In *Proceedings of the 2002 American Control Conference*, volume 6, 4507–4512.

Keviczky, T., Borrelli, F., Fregene, K., Godbole, D., and Balas, G.J. (2008). Decentralized receding horizon control and coordination of autonomous vehicle formations. *IEEE Transactions on Control Systems Technology*, 16(1), 19–33.

Keviczky, T., Borrelli, F., and Balas, G.J. (2006). Decentralized receding horizon control for large scale dynamically decoupled systems. *Automatica*, 42(12), 2105–2115.

Li, H. and Shi, Y. (2013a). Distributed model predictive control of constrained nonlinear systems with communication delays. *Systems & Control Letters*, 62(10).

Li, H. and Shi, Y. (2013b). Distributed receding horizon control of constrained linear systems with communication delays. In *The Proceedings of The 2013 America Control Conference (ACC)*. Washinton DC, USA.

Li, H. and Shi, Y. (2014a). Distributed receding horizon control of large-scale nonlinear systems: handling communication delays and disturbances. *Automatica*, http://dx.doi.org/10.1016/j.automatica.2014.02.031.

Li, H. and Shi, Y. (2014b). Robust distributed model predictive control of constrained continuous-time nonlinear systems: A robustness sonstraint approach. *IEEE Transactions on Automatic Control*, DOI:10.1109/TAC.2013.2294618.

Liu, J., Chen, X., de la Peña, D.M., and Christofides, P.D. (2010). Sequential and iterative architectures for distributed model predictive control of nonlinear process systems. *AIChE Journal*, 56(8), 2137–2149.

Liu, J., Chen, X., Mu, de la Peña, D.M., and Christofides, P.D. (2012). Iterative distributed model predictive control of nonlinear systems: Handling asynchronous, delayed measurements. *IEEE Transactions on Automatic Control*, 57(2), 528–534.

Liu, J., de la Peña, D.M., and Christofides, P.D. (2009). Distributed model predictive control of nonlinear process systems. *AIChE Journal*, 55(5), 1171–1184.

Maestre, J.M., de la Peña, D.M., and Camacho, E.F. (2011). Distributed model predictive control based on a cooperative game. *Optimal Control Applications and Methods*, 32(2), 153–176.

Michalska, H. and Mayne, D.Q. (1993). Robust receding horizon control of constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 38(11), 1623–1633.

Motee, N. and Sayyar-Rodsari, B. (2003). Optimal partitioning in distributed model predictive control. In *Proceedings of the 2003 American Control Conference*, volume 6, 5300–5305.

Qin, S.J. and Badgwell, T.A. (2000). An overview of nonlinear model predictive control applications: nonlinear model predictive control. volume 26 of *Progress in Systems and Control Theory*, 369–392.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733–764.

Raimondo, D.M., Magni, L., and Scattolini, R. (2007). Decentralized MPC of nonlinear systems: An input-to-state stability approach. *International Journal of Robust and Nonlinear Control*, 17(17), 1651–1667.

Richards, A. and How, J.P. (2007). Robust distributed model predictive control. *International Journal of Control*, 80(9), 1517–1531.

Stewart, B.T., Venkat, A.N., Rawlings, J.B., Wright, S.J., and Pannocchia, G. (2010). Cooperative distributed model predictive control. *Systems & Control Letters*, 59(8), 460–469.

Stewart, B.T., Wright, S.J., and Rawlings, J.B. (2011). Cooperative distributed model predictive control for nonlinear systems. *Journal of Process Control*, 21(5), 698–704.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52(9), 1680–1685.

Varutti, P., Kern, B., Faulwasser, T., and Findeisen, R. (2009). Event-based model predictive control for networked control systems. In *Proceedings of the 48th IEEE Conference on Decision and Control and jointly with the 28th Chinese Control Conference (CDC/CCC)*, 567–572.

Venkat, A.N., Hiskens, I.A., Rawlings, J.B., and Wright, S.J. (2008). Distributed MPC strategies with application to power system automatic generation control. *IEEE Transactions on Control Systems Technology*, 16(6), 1192–1206.

Wang, X. and Lemmon, M.D. (2011). Event-triggering in distributed networked control systems. *IEEE Transactions on Automatic Control*, 56(3), 586–601.