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A Stable Nonlinear in Parameter Neural Network Controller for a Class of Saturated Nonlinear Systems

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Abstract: This paper deals with the tracking problem for a class of uncertain nonlinear systems subjected to actuator saturation constraint. Despite most of proposed control schemes for saturated systems which employed linear in parameter neural networks (LPNNs), in the present work nonlinear in parameter neural network (NLPNN) is introduced to support global approximation property. To compensate the effect of the input saturation constraint an auxiliary system is introduced and the error dynamics are modified based on the auxiliary states. Then, learning rules are achieved based on the back propagation (BP) algorithm and by adding two robustifying terms to the standard BP learning rules the stability of the overall system is ensured via Lyapunov direct method. Finally simulations performed on a ''generalized pendulum'' nonlinear system to illustrate the effectiveness of the proposed tracking control scheme.

Keywords: Adaptive tracking control, Back propagation algorithm, Input saturation, Neural networks, Nonlinear systems.

1. INTRODUCTION

Feedback linearization technique is known to be effective in designing controller for nonlinear systems. In standard form, it is based on the cancellation of known nonlinearities (Slotine, 1991). In order to handle uncertainty, robust feedback linearization schemes have been developed (Kanellakopoulos et al., 1991). However a drawback with the given schemes is that uncertainties are assumed to be bounded with known or partially known bounds. Uncertainty with known or partially known bounds is a restrictive constraint which may not be satisfied for all practical systems, therefore relaxing this condition makes control of the system challenging.

Neural network (NN) capability in approximating uncertain nonlinear functions makes it a valuable tool in control of systems with high degree of nonlinearities. Different control schemes have been introduced by incorporating feedback linearization approach and NN universal approximation capabilities (Goa and Selmic, 2004; Yuan et al., 2011). Saturation is one of the most common actuator nonlinearities which has a theoretical and practical significant aspects. Adaptive controllers which are designed without considering saturation constraint may cause loosing good performance, damaging actuators and instability of the system. Hence, guaranteeing that control signal remains in the desired range while simultaneously achieving good tracking performance and ensuring stability of the close loop system is a desired goal to be achieved. One approach to compensate saturation is adjustment of command or feedback signal. In Goa and Selmic (2004), Li et al., (2011b), Wen and Ren (2012) authors used LPNN and introduced an approach to control uncertain saturated systems. In Kiirason and Annaswamy (1994) an auxiliary system and modified tracking error are used in order to compensate saturation constraint in linear time invariant (LTI) systems. In Li et al., (2011a) authors used auxiliary system and backstepping method to design a LPNN based adaptive controller for nonlinear saturated systems.

In Chen et al., (2011), Kim and Ha (2000), Goa and Selmic, (2004) authors assumed that nonlinear functions are completely or partially known and also bound of nonlinear functions is available. In the present approach we have relaxed these restrictive assumptions. Hence, the proposed approach is applicable to a large class of nonlinear systems. In order to use global approximation properties of NN, unlike most of the previous approaches in the literatures on the saturated systems, we will use NLPNN in the designing procedure which makes stability analysis more complicated. By employing NLPNN our proposed approach is applicable to the systems with high degree of uncertainties. It has been proven that BP algorithm is a valuable learning rule which is applicable to the different engineering problems. Therefore, in this paper learning rules are derived based on the standard BP algorithm and by adding two robustifying terms to the standard BP algorithm, the stability analysis is presented via Lyapunov direct method.

Some of the prominent characteristics of the proposed approach lies in (a) NLPNN is employed in designing procedure (b) uniformly ultimately boundedness (UUB) of all signals is guaranteed using Lyapunov direct method (c) due to the description of the system and employing NLPNN, proposed scheme is applicable to a large class of nonlinear systems (d) tracking problem of nonlinear systems has been solved using BP learning algorithm (e) there is no need to have a lot of priori knowledge about uncertain nonlinear functions.

The rest of the paper is organized as follows: problem description and some preliminaries are introduced in Section 2, adaptive NLPNN-based controller is presented in Section 3, and stability analysis is presented in Section 4. To illustrate effectiveness of the proposed scheme, simulations performed on the generalize pendulum in Section 5. Section 6 contains conclusion.

2. PROBLEM FORMULATION AND SOME PRELIMINARIES

2.1 System description

Consider a class of uncertain saturated nonlinear systems in the following form:

$$\dot{x} = f_o(x) + g(x)\,\mathrm{sat}(u) + d \tag{1}$$

where $x \in R^{n\times 1}$ is the state vector, $f_o(x): R^{n\times 1} \to R^{n\times 1}$ and $g(x): R^{n\times 1} \to R^{n\times 1}$ are unknown nonlinear functions, $d \in R^{n\times 1}$ is the unknown but bounded disturbance vector that $\|d\| \le \bar{d}$ (Wen et al., 2011; Yuan et al., 2011; Li et al., 2011b) and $\mathrm{sat}(u)$ denotes the plant input subjected to saturation nonlinearities. Throughout this paper, the following assumptions are made:

Assumption 1. The plant input $sat(u)$ satisfies saturation nonlinearity expressed by

$$\mathrm{sat}(u(t)) = \begin{cases} u_{\max} & u(t) > u_{\max} \\ mu(t) & u_{\min} \le u(t) \le u_{\max} \\ u_{\min} & u(t) < u_{\min} \end{cases} \tag{2}$$

where $u_{\max}$ and $u_{\min}$ are known bounds of saturation.

Assumption 2 (Chen et al., 2011)**.** $g(x)$ is an unknown nonlinear function, and there exists unknown constant $\bar{g}$ such that $\|g(x)\| \le \bar{g}$.

Assumption 3 (Li et al., 2011a,b; Wen 2011). The desired trajectory vector x_d is assumed to be bounded.

Remark 1. Assumption 1 is common in the literatures. In Chen et al. (2011), Kim and Ha (2000), $g(x)$ is assumed partially known and its bound is also supposed to be known. In Li et al. (2011b), it is assumed that $g(x)$ has an unknown constant upper and lower bounds or it is assumed that $g(x)$ is completely known (Goa and Selmic, 2004).

The control objective is to design a NN-based controller for system (1) such that the control signal respects saturation constraints and all the states track the desired trajectories in the presence of uncertainties and unknown bounded disturbances.

2.2 Neural network

The motivation of using NN in this paper is to take advantages of NN global approximation capabilities to handle nonlinearities and identify a feedback linearization controller. Based on the universal approximation theorem a wide range of nonlinear functions can be estimated by a NN with sufficient neurons, at least one hidden layer and a linear combination of sigmoidal functions, as follows:

$$f(x_{NN}) = W^T \sigma(V x_{NN}) + \varepsilon(x_{NN}) \tag{3}$$

where W and V are the output and hidden layers ideal weights, respectively, x_{NN} is the input vector and $\varepsilon(x_{NN})$ is the NN approximation error. The NN approximation error is bounded on a compact set S by $|\varepsilon(x_{NN})| \le \bar{\varepsilon}$. $\sigma(.)$ is the activation function of hidden layer that is usually considered sigmoidal function (Cybenko, 1989).

$$\sigma_i(V x_{NN}) = -1 + \frac{2}{1 + e^{-2V_i x_{NN}}}$$

where V_i is the *i*th row of V and $\sigma_i(.)$ is the *i*th element of $\sigma(.)$. Hence the nonlinear function $f(x_{NN})$ can be approximated by $\hat{f}(x_{NN}) = \hat{W}^T \sigma(\hat{V} x_{NN})$. Note that it is assumed that ideal weights are not known a priori, NN approximation error is bounded and first and second layer weights are tunable.

3. ADAPTIVE NLPNN-BASED CONTROLLER

By adding Ax to and subtracting it from (1), we obtain:

$$\dot{x} = Ax + f(x) + g(x)\,\mathrm{sat}(u) + d \tag{4}$$

where $f(x) = f_o(x) - Ax$ while A is a Hurwitz matrix. If the nonlinear functions $f(x)$ and $g(x)$ were known and there were no disturbances and saturation constraint, then the feedback linearization based controller, defined by $u = g^T(x)(g(x)g^T(x))^{-1}(-f(x) + \dot{x}_d - Ax_d)$, was able to linearize (4) and make the states to track desired trajectories.

3.1 Nonlinear in Parameter Neural Network (NLPNN)

Since the nonlinear functions $f(x)$ and $g(x)$ are unknown, a two-layer NN given in (3) is employed for nonlinear control signal:

$$u = -W\sigma(V x_{NN}) - \varepsilon(x_{NN}) \tag{5}$$

The implemented NN is an approximation of the ideal NN and can be expressed as follows

$$\hat{u} = -\hat{W}\sigma(\hat{V}x_{NN}) \tag{6}$$

In order to take full advantages of global approximation theory NLPNN is considered, where $\tilde{W} = W - \hat{W}$, $\tilde{V} = V - \hat{V}$ denote weights approximation errors.

3.2 Saturation compensation

To remove the effect of the input saturation constraint the following auxiliary system is proposed

$$\dot{\varsigma} = K\varsigma + h\Delta u,\ \varsigma(0) = 0 \tag{7}$$

where $\Delta u = u - \mathrm{sat}(u)$ is the control signal that cannot be implemented by the actuator, $\varsigma \in R^{n\times 1}$ is the auxiliary state vector, K is Hurwitz matrix and h is considered as a constant vector. It is clear that $\varsigma(t)$ is convolution of an exponential term with $\Delta u(t)$ and converges to zero if and only if $\Delta u(t)$ converges to zero. In other words, ς denotes a filtered version of Δu and remains zero as long as $\Delta u = 0$, and once $\Delta u \neq 0$ auxiliary states becomes nonzero. Hence, by modifying tracking error as $\tilde{e} = q(x - x_d) - \varsigma$ (where q is a constant which let us to give different importance to the tracking error), we can remove saturation constraint and rewrite system (4) as follows:

$$\dot{x} = Ax + f(x) + g(x)u + d \tag{8}$$

Note that since we do not have any knowledge about $\Delta\dot{u}$, we did not modify error based on Δu.

4. STABILITY ANALYSIS

In this section BP algorithm learning rule, which seems the most popular learning rule for control problems, is used in designing procedure of NLPNN-based controller. As we know the most important issue in the control problems is to ensure the stability of the system. Most of the previous approaches using BP algorithm for control suffers from lack of mathematical stability analysis (e.g. Talebi et al., 2000). Therefore, we will present a novel learning rule based on BP algorithm for tracking problem then the stability of the overall system is proved in the presence of saturation and external disturbances based on Lyapunov direct method.

Theorem 1. Consider the uncertain nonlinear system (1) with input constraint (2) and auxiliary system (7). Given that full state measurement is available. Controller given in (6) makes the states to track the desired trajectories and keeps all the signals of the closed loop system UUB (Uniformly Ultimately Bounded) under the NLPNN weights learning rules (9), (10) and Assumptions 1-3.

$$\dot{\hat{W}} = -\eta_1 \frac{\partial J}{\partial \hat{W}} - \rho_1 \|e\| \hat{W} - \rho_2 \|\varsigma\| \hat{W} \tag{9}$$

$$\dot{\hat{V}} = -\eta_2 \frac{\partial J}{\partial \hat{V}} - \rho_3 \|e\| \hat{V} - \rho_4 \|\varsigma\| \hat{V} \tag{10}$$

where η_i, $i = 1,2$ are positive learning rate parameters, ρ_i, $1 \le i \le 4$ are small positive design parameters and $J = \frac{1}{2}\tilde{e}^T\tilde{e}$ is the cost function which should be minimized.

Moreover with appropriate choice of parameters, tracking error and NN weights error can be made arbitrarily small while control signal will respect saturation constraint.

Proof:

Without loss of generality, let us assume $q, m = 1$ and derive $\frac{\partial J}{\partial \hat{W}}$, $\frac{\partial J}{\partial \hat{V}}$ separately by employing chain rule as follows

$$\frac{\partial J}{\partial \hat{W}} = \frac{\partial J}{\partial \tilde{e}}\frac{\partial \tilde{e}}{\partial \hat{u}}\frac{\partial \hat{u}}{\partial \hat{W}} = \tilde{e}^T \frac{\partial \tilde{e}}{\partial \hat{u}}\frac{\partial \hat{u}}{\partial \hat{W}} \tag{11}$$

$$\frac{\partial J}{\partial \hat{V}} = \frac{\partial J}{\partial \tilde{e}}\frac{\partial \tilde{e}}{\partial \hat{u}}\frac{\partial \hat{u}}{\partial(\hat{V}\tilde{e})}\frac{\partial(\hat{V}\tilde{e})}{\partial \hat{V}} = \tilde{e}^T \frac{\partial \tilde{e}}{\partial \hat{u}}\frac{\partial \hat{u}}{\partial(\hat{V}\tilde{e})}\tilde{e} \tag{12}$$

In order to compute $\frac{\partial \tilde{e}}{\partial \hat{u}} = \frac{\partial(x - x_d)}{\partial \hat{u}} - \frac{\partial \varsigma}{\partial \hat{u}} = \frac{\partial x}{\partial \hat{u}} - \frac{\partial \varsigma}{\partial \hat{u}}$, let us consider static approximation ($\dot{x} = 0$, $\dot{\varsigma} = 0$) (Ye, 2008; Abdollahi et al., 2006). Using this approximation and considering (7), (8) yields $\frac{\partial x}{\partial \hat{u}} \approx -A^{-1}g(x) \approx -A^{-1}1_{n.1}$ and $\frac{\partial \varsigma}{\partial \hat{u}} == -K^{-1}h\Delta\overline{u}$.

where $\Delta\overline{u}$ is defined as follows

$$\Delta\overline{u} = \begin{cases} 1 & u > u_{\max}\ or\ u < u_{\min} \\ 0 & u_{\min} \le u \le u_{\max} \end{cases}$$

Therefore, by using described static approximation, considering (11), (12) and performing some manipulations we have

$$\frac{\partial J}{\partial \hat{W}} = \tilde{e}^T(-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u})(-\sigma(\hat{V}\tilde{e}))^T \tag{13}$$

$$\frac{\partial J}{\partial \hat{V}} = \left[\tilde{e}^T(-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u})(-\hat{W}(I - \Lambda))\right]^T \tilde{e}^T \tag{14}$$

where Λ denotes $diag(\sigma_i(\hat{V}\tilde{e})^2)$.

By substituting (13), (14) in to the (9), (10) learning rules will be as follows:

$$\begin{aligned}\dot{\hat{W}} = &-\eta_1\tilde{e}^T(-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u})(-\sigma(\hat{V}\tilde{e}))^T \\ &- \rho_1\|e\|\hat{W} - \rho_2\|\varsigma\|\hat{W}\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{\hat{V}} = &-\eta_2\left[\tilde{e}^T(-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u})(-\hat{W}(I - \Lambda))\right]^T \tilde{e}^T \\ &- \rho_3\|e\|\hat{V} - \rho_4\|\varsigma\|\hat{V}\end{aligned} \tag{16}$$

Now let e is defined as the tracking error vector, taking derivative of e and substituting (8) in it gives

$$\dot{e} = \dot{x} - \dot{x}_d = Ax + f(x) + g(x)\hat{u} + d - \dot{x}_d \quad (17)$$

Further, since u puts the system in the feedback linearization form and makes the states to track desired trajectories, we have $f(x) = -g(x)u + \dot{x}_d - Ax_d$. Considering this fact and substituting (5) and (6) in to (17) leads to

$$\dot{e} = Ae + g(x)(\tilde{W}\sigma(\hat{V}\tilde{e}) + a(t)) + d \quad (18)$$

where $\tilde{W} = W - \hat{W}$ is the NN weights error and $a(t) = W(\sigma(V\tilde{e}) - \sigma(\hat{V}\tilde{e})) + \varepsilon$.

Lemma 1. For a Hurwitz matrix A and a symmetric positive definite matrix Q there exists a positive definite matrix P such that satisfies Lyapunov equation:

$$A^T P + PA = -Q$$

In order to guarantee stability of the overall system the following Lyapunov candidate is proposed

$$L = \frac{1}{2}e^T P_1 e + \frac{1}{2}\varsigma^T P_2 \varsigma + \frac{1}{2}tr(\tilde{W}^T \rho_1^{-1}\tilde{W}) \quad (19)$$

where P_1 and P_2 are positive definite matrices such that triples of A, P_1, Q_1 and K, P_2, Q_2 satisfy Lemma 1. Throughout the proof the following equalities, inequalities and facts are used:

$$\sup(\sigma(.)) = \sigma_m,\ \sup(W) = W_M,,\ \hat{W} = W - \tilde{W},$$
$$\|\tilde{e}\| = \|e - \varsigma\| \le \|e\| + \|\varsigma\|,\ -e^T Q e \le -\|e\|^2 \lambda_{\min}(Q),$$
$$tr(\tilde{W}^T(W - \tilde{W})) \le \|\tilde{W}\|(W_M - \|\tilde{W}\|).$$

Fact 1. Since ideal weights, $\sigma(.)$ and NN approximation error ε are bounded, there exists an unknown constant $\overline{a}$ such that $\|a(t)\| \le \overline{a}$.

Fact 2. Since $\sigma_i(\hat{V}\tilde{e})$ is bounded, regardless of boundedness of $\hat{V}$, $\Lambda = diag(\sigma_i(\hat{V}\tilde{e})^2)$ is always bounded.

Fact 3. Since W are ideal fixed weights hence $\dot{\tilde{W}} = -\dot{\hat{W}}$.

Differentiating (19) along the system trajectory, yields

$$\dot{L} = \frac{1}{2}\dot{e}^T P_1 e + \frac{1}{2}e^T P_1 \dot{e} + \frac{1}{2}\dot{\varsigma}^T P_2 \varsigma + \frac{1}{2}\varsigma^T P_2 \dot{\varsigma} + tr(\tilde{W}^T \rho_1^{-1}\dot{\tilde{W}})$$

By using (6), (7), (15), (18) and employing Lemma 1 and Fact 3 we have

$$\begin{aligned}\dot{L} = &-\frac{1}{2}e^T Q_1 e + e^T P_1\left[g(x)(\tilde{W}\sigma(\hat{V}\tilde{e}) + a(t)) + d\right] \\ &-\frac{1}{2}\varsigma^T Q_2 \varsigma + \varsigma^T P_2 h(-\hat{W}\sigma(\hat{V}\tilde{e}) - \text{sat}(u)) + \\ &tr(\tilde{W}^T \rho_1^{-1}(-\eta_1 \tilde{e}^T(-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u})(\sigma(\hat{V}\tilde{e}))^T \\ &+ \rho_1\|e\|\hat{W} + \rho_2\|\varsigma\|\hat{W}))\end{aligned}$$

By selecting designing parameters ρ_i, $i = 1,2$ such that $\rho_1 = \rho_2$ and using Fact 1, Assumption 2 and the described inequalities we rewrite the above equation as follows

$$\begin{aligned}\dot{L} \le &\|e\|.\left[-\frac{1}{2}\lambda_{\min}(Q_1)\|e\| + \|\tilde{W}\|(C - \|\tilde{W}\|) + B\right] \\ &+\|\varsigma\|.\left[-\frac{1}{2}\lambda_{\min}(Q_2)\|\varsigma\| + \|\tilde{W}\|(E - \|\tilde{W}\|) + F\right]\end{aligned} \quad (20)$$

where

$$B = \|P_1\|(\overline{g}.\overline{a} + \overline{d}),$$
$$C = \|P_1\|\overline{g}\sigma_m + \eta_1\rho_1^{-1}\sigma_m\|-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u}\| + W_M,$$
$$E = \|P_2 h\|\sigma_m + +\eta_1\rho_1^{-1}\sigma_m\|-A^{-1}1_{n.1} + K^{-1}h\Delta\overline{u}\| + W_M,$$
$$F = \|P_2 h\|(W_M\sigma_m + \overline{u}),\ \ \overline{u} = \max\{|u_{\min}|, |u_{\max}|\}.$$

By completing squares in (20) we have

$$\begin{aligned}\dot{L} \le &\|e\|.\left[-\frac{1}{2}\lambda_{\min}(Q_1)\|e\| - (\|\tilde{W}\| - \frac{C}{2})^2 + \frac{C^2}{4} + B\right] \\ &+\|\varsigma\|.\left[-\frac{1}{2}\lambda_{\min}(Q_2)\|\varsigma\| - (\|\tilde{W}\| - \frac{E}{2})^2 + \frac{E^2}{4} + F\right]\end{aligned} \quad (21)$$

Since $-\frac{1}{2}\lambda_{\min}(Q_i)\|.\|$ is always negative, we neglect this term and conclude that $\dot{L}$ is guaranteed to be negative as long as

$$\|\tilde{W}\| > \max\left\{\frac{C}{2} + \sqrt{\frac{C^2}{4} + B}, \sqrt{\frac{E^2}{4} + F} + \frac{E}{2}\right\} = c \quad (22)$$

So far we conclude that if norm of NN weights error increase then $\dot{L}$ is guaranteed to be negative. In order to show that if norm of e or ς is also increased then $\dot{L}$ will become negative, let us rewrite (21) as follows

$$\begin{aligned}\dot{L} \le &-\frac{1}{2}\lambda_{\min}(Q_1)\|e\|^2 - \frac{1}{2}\lambda_{\min}(Q_2)\|\varsigma\|^2 \\ &+(\frac{C^2}{4} + B)\|e\| + (\frac{E^2}{4} + F)\|\varsigma\|\end{aligned}$$

In other words the square terms in (21) are eliminated. By completing squares, $\dot{L}$ is guaranteed to be negative when

$$\|e\| > \frac{\frac{C^2}{4} + B}{\lambda_{\min}(Q_1)} + \sqrt{(\frac{\frac{C^2}{4} + B}{\lambda_{\min}(Q_1)})^2 + \frac{\lambda_{\min}(Q_2)}{\lambda_{\min}(Q_1)}(\frac{\frac{E^2}{4} + F}{\lambda_{\min}(Q_2)})^2} = a \quad (23)$$

or

$$\|\varsigma\| > \frac{\frac{E^2}{4} + F}{\lambda_{\min}(Q_2)} + \sqrt{\frac{\lambda_{\min}(Q_1)}{\lambda_{\min}(Q_2)}(\frac{\frac{C^2}{4} + B}{\lambda_{\min}(Q_1)})^2 + (\frac{\frac{E^2}{4} + F}{\lambda_{\min}(Q_2)})^2} = b \quad (24)$$

Therefore if $\|e\|$ or $\|\varsigma\|$ or $\|\tilde{W}\|$ become greater than a, b, c respectively, then $\dot{L}$ is guaranteed to become negative. In other words, they are uniformly ultimately bounded (UUB). It should be mentioned that by properly choosing parameters we are able to make the bounds arbitrary small.

To complete the proof we should guarantee that $\tilde{V}$ is also bounded. Since V is ideal hidden layer weight, hence it is fixed and we have $\dot{\tilde{V}} = -\dot{\hat{V}}$. By employing (16) we have

$$\dot{\tilde{V}} = -\dot{\hat{V}} = h_o(\tilde{e}, A, K, h, \sigma(.), \hat{W}) + (\rho_3\|e\| + \rho_4\|\varsigma\|)\hat{V} \quad (25)$$

where $h_o(.) = \eta_2\left[\tilde{e}^T(-A^{-1}1_{n.1} + K^{-1}h\Delta\bar{u})(-\hat{W}(I-\Lambda))\right]^T \tilde{e}^T$.

We have proved boundedness of $\tilde{W}$ and since W are ideal bounded weights, hence $\hat{W}$ is ensured to be bounded. According to (23), (24) e and ς are also bounded therefore $\tilde{e}$ is guaranteed to be bounded and since A, K are Hurwitz matrices, we conclude that $h_o(.)$ is always bounded . Now we rewrite (25) in the following form

$$\dot{\tilde{V}} = -(\rho_3\|e\| + \rho_4\|\varsigma\|)\tilde{V} + h_o(. \ + (\rho_3\|e\| + \rho_4)\|\varsigma\|)V \quad (26)$$

Based on the boundedness of $h_o(.) + (\rho_3\|e\| + \rho_4\|\varsigma\|)V$ and positive definiteness of $(\rho_3\|e\| + \rho_4\|\varsigma\|)$, we can consider (26) as a linear system with bounded input, that guarantees boundedness of $\tilde{V}$. This completes the proof. ■

5. SIMULATION RESULTS

To verify the effectiveness of the proposed tracking controller scheme for uncertain nonlinear systems with input constraint, the following ''general pendulum'' system is considered. General pendulum is a nonlinear system which is a benchmark to show effectiveness of different control schemes for saturated systems (Goa and Selmic 2004; Labiod and Guerra 2008).

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -5x_1^3 - 2x_2 + \text{sat}(u)$$

The desired trajectories are taken as $x_{1d} = \text{Sin}(t)$ and $x_{2d} = \text{Cos}(t)$. Hidden and output layer of the NN are assumed to have 15 and 1 neurons, respectively and its weights are initialized randomly. Initial states and learning rates are considered $x(0) = [0.5 \quad 0.5]^T$, $\eta_i = 0.1$, $i = 1,2$, respectively. Other parameters are chosen as below:

$$A = -5I,\ K = -0.3I,\ h = [1.7 \quad 1.7]^T,$$
$$\rho_i = 0.001,\ 1 \le i \le 4,\ q = 100$$

Hence for comparisons two controllers are designed based on the proposed scheme (without a priori knowledge about nonlinear functions) and standard feedback linearization approach (we assumed $f(x)$, $g(x)$ are completely known). Figs. 1 and 2, depict that proposed scheme has a better tracking performance and desired trajectories are tracked rapidly. Moreover according to Fig. 3, it is obvious that by selecting parameters properly, proposed scheme can easily compensate saturation.

Note that comparing the results with the ones reported in Goa and Selmic (2004), show that a better tracking is achieved by the proposed approach; however the control signal was assumed smaller than what they have considered and also it should be mentioned that compare to their work a lot of assumptions such as availability of $g(x)$ and knowing approximation of $f(x)$ are relaxed.

It should be mentioned that by reducing bounds on the control signal the proposed scheme compensates saturation well, while tracking error slightly increased. In these cases existence of tracking error in the simulations is not a drawback of our scheme, because in the case of perfect tracking always a control signal with amplitude of greater than 4 is required. Therefore, no control scheme is able to guarantee perfect tracking with such amplitude.

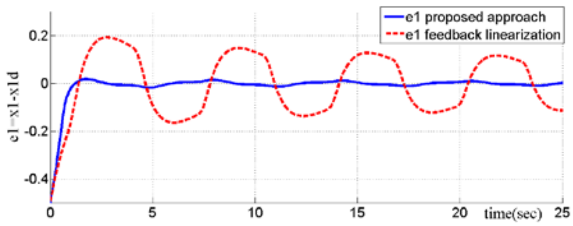

Fig. 1. Tracking errors with feedback linearization (dotted) and with proposed scheme (solid), $|u| \le 4$.

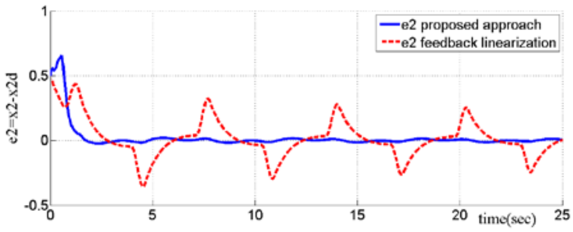

Fig. 2. Tracking errors with feedback linearization (dotted) and with proposed scheme (solid), $|u| \le 4$.

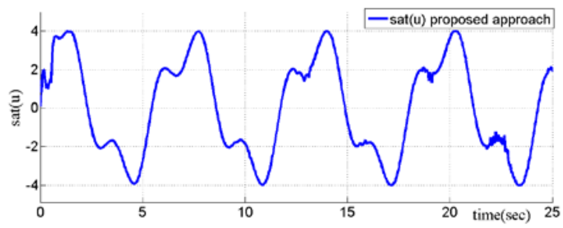

Fig. 3. Control signal with proposed scheme, $|u| \le 4$.

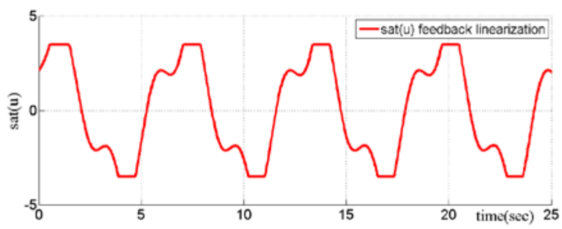

Fig. 4. Control signal with feedback linearization, $|u| \le 4$.

6. CONCLUSIONS

This paper deals with the tracking problem for uncertain nonlinear systems subjected to input saturation and external disturbances. To compensate saturation an auxiliary system has been designed and error has been modified based on auxiliary states. By utilizing feedback linearization approach, nonlinear in parameter neural networks, BP algorithm, a novel adaptive controller is designed to achieve desired trajectories. UUB of all signals has been ensured by decomposing the closed loop system in to two subsystems and using Lyapunov direct method and bounds of tracking error is given in terms of design parameters. Finally to illustrate effectiveness of the proposed ''general pendulum'' system has been controlled.

REFERENCES

Abdollahi, F., Talebi, H.A., and Patel, R.V. (2006). Stable Identification of Nonlinear Systems Using Neural Networks: Theory and Experiments. *IEEE/ASME Transactions on Mechatronics*, 11 (4), 488-495.

Chen, M., Ge, S.S., and Ren, B. (2011). Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints. *Automatica*, 47 (3), 452-465.

Cybenko, G. (1989). Approximation by superposition of sigmoidal functions. *Mathematics of Control, Signals and Systems*, Springer, 303- 314.

Goa, W., and Selmic, R.R. (2004). Neural Network Control of a Class of Nonlinear Systems with Actuator Saturation. *In Proceedings of the American control conference*, 147-156, USA, June.

Kanellakopoulos, I., Kokotovic, P., and Morse, A. S. (1991). Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Transactions on Automatic. Control*, 36 (11), 1241-1253.

Kiirason, S.P., and Annaswamy, A.M. (1994). Adaptive Control in the Presence of Input Constraints. *IEEE Transactions on Automatic Control,* 39 (11), 2325 – 2330.

Kim, Y. H., and Ha, I. J. (2000). Asymptotic state tracking in a class of nonlinear systems via learning-based inversion. *IEEE Transactions on Automatic Control*, 45 (11), 2011–2027.

Labiod, S., and Guerra, T.M. (2008). Direct Adaptive Fuzzy Control for Nonlinear Systems with Input Amplitude and Rate Saturation Constraints. *In Proceedings of the 16th Mediterranean Conference on Control and Automation*, 394-399, Ajaccio, June.

Li, Y., Qiang, Sh., Zhuang, X., and Kaynak, O. (2004). Robust and Adaptive Backstepping Control for Nonlinear Systems Using RBF Neural Networks. *Neurocomputing*, 15 (3), 693-701.

Li, T., Li, R., and Li, G. (2011a). Decentralized adaptive neural control of nonlinear interconnected large scale systems with unknown time delays and input saturation. *IEEE Transactions on Systems, Man, and Cybernetics,* Part B, 74 (14-15), 2277-2283.

Li, J., Li, T., and Li, Y. (2011b). NN-Based Adaptive Dynamic Surface Control for a Class of Nonlinear Systems with Input Saturation. *In Proceedings of the 7th IEEE Conference on Industrial Electronics and Applications (ICIEA),* 570-575, Singapore, July.

Slotine, J.J.E., and Li, W. (1991). *Applied nonlinear control.* Prentice Hall.

Talebi, H. A., Patel, R.V., and Asmer, H. (2000). Neural network based dynamic modeling of flexible-link manipulators with application to the SSRMS. *Joumal of Robotic systems*, 17 (7), 385-401

Wen, C., Zhou, J., Liu, Z., and Su, H. (2011). Robust Adaptive Control of Uncertain Nonlinear Systems in the Presence of Input Saturation and External Disturbance. *IEEE Transactions on Automatic Control,* 56 (7), 1672-1678.

Wen, Y., and Ren, X. (2012). Neural observer-based adaptive compensation control for nonlinear time varying delays systems with input constraints. *Expert Systems with Applications*, 39 (2), 1944-1955.

Ye, J. (2008). Adaptive control of nonlinear PID-based analog neural networks for a nonholonomic mobile robot. *Journal of Neurocomputing*, 71 (7), 1561-1565.

Yuan, R., Yi, J., Yu, W., and Fan, G. (2011). Adaptive Controller Design for Uncertain Nonlinear Systems with Input Magnitude and Rate Limitations. *In Proceedings of the American control conference*, 3536-3541, June.