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Disturbance rejection through virtual extension of the system – geometric approach

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Abstract The article proposes a method that gives the ability to consider the mathematical model of the system as a symmetry-preserving one, even if the conditions of the symmetry for the plant model are not met or if the symmetry was lost during the system operation. The proposed approach is based here on an active disturbance rejection scheme, which assumes an augmentation of the system model, which further leads to an online reconstruction and attenuation of the effects of terms in the plant, that preclude its model from being a symmetry-preserving one. The effectiveness of the proposed technique is presented in the paper using an illustrative example, supported with simulation results.

Keywords: disturbance rejection, state observer, feedback systems, differential geometric methods, adaptive control, active compensation

1. INTRODUCTION

Finding the symmetry of a system allows to describe how it physically changes by the action of a transformation group. The symmetry group G of the system with state space X, maps solution to solution, i.e. one integral line of a vector field into another. This leads to a natural definition of a system that preserves its specified property, namely the system is invariant by the action of group G (or simply G-invariant).

Techniques exploiting symmetry are deeply studied in modern nonlinear control theory. They turn out to be a comprehensive and readable tools for observer and control design. In their article, Martin et al. (2004) consider rather simple system, however it shows the general idea in a clear way. Another attempt has been presented by Rudolph and Frohlich (2003).

In terms of observers that preserve symmetries, a considerable contribution has been made by Bonnabel et al. (2008). In his article, Bonnabel (2007) presented a modification of an invariant observer based on extended Kalman filter. Experimental results of the invariant control system for a mobile robot has been shown by Nowicki et al. (2013), where a combination of invariant controller with invariant observer was investigated.

The classical control theory techniques do not consider the geometry of the system, hence they can be seen from a mathematical point of view as local approximations only. Indeed, the classical methods consider the input, state, and output spaces as $\mathbb{R}^n$ manifolds. Therefore, the actions are expressed for an Euclidean space only, which is not true in most cases.

One can think about the state space as any manifold instead of the Euclidean space. The evolution of the system in time can push the solution off the space. Then, the local approximation to the nearest point on the manifold has to be found. On the contrary, when using the symmetry group, the solution always stays on the state manifold. Moreover, the fact that every component, even the most complicated one, has its geometrical representation, helps to understand how such system behaves.

There are many simple systems, whose invariance is obvious (one can think of a rigid body motion in free space). However, the consideration of some additional elements in the mathematical model can lead to breaking the symmetry. Generally, such phenomena appear as additional fields that are not invariant by the action of the symmetry group.

This paper proposes an active disturbance rejection control (ADRC) approach as a method of online estimation and rejection of particular effects that break the symmetry of the system. The resultant system in the ADRC scheme can be considered as a symmetry-preserving one, since the unwanted elements of the dynamics are reconstructed and mitigated in each time instant, hence does not influence the plant behavior in practice.

The ADRC methodology, recently summarized in a survey-type article by Gao (2013), assumes treating the mentioned terms collectively, as a time-varying parameter of the system. Such lumped components, often denoted in the literature as *total disturbance*, are considered in the

ADRC approach as an additional, virtual extension of the system [1].

The proposed control scheme poses both robust and adaptive features, since the unwanted effects occurring in the system are attenuated in each control cycle. The ADRC, in general, is a powerful tool, with numerous experimental case studies (Przybyla et al. (2012); Madonski et al. (2013)) and even industrial applications (Zheng and Gao (2010); Vincent (2011)).

The contribution of this paper is the proposition of the ADRC-based method that gives the ability to treat a given system, which does not hold the symmetry condition, as a symmetry-preserving one.

2. SYMMETRY-RECOVERY METHODOLOGY

2.1 System modeling

Assume that r-dimensional manifold X defines a state space of the system $\dot{x} = f(x,u)$ and G is a Lie group of dimension n. The Lie group maps the state X, input $U = \mathbb{R}^m$, and output Y of dimension p, via following transformations:

$$\begin{aligned}
(g,x) &\in G \times X \to \varphi_g(x) \in X,\\
(g,u) &\in G \times U \to \psi_g(u) \in U,\\
(g,y) &\in G \times Y \to \varrho_g(y) \in Y.
\end{aligned}$$

Assuming G is a symmetry group of the system, the following definition states that:

Definition 1. The system $\dot{x} = f\left(x,\, u\right)$ is G-invariant with G-equivariant output map $y = h\left(x,\, u\right)$, if for all g, x, u, y:

$$\begin{aligned}
f\left(\varphi_g(x), \psi_g(u)\right) &= \frac{\partial}{\partial x}\varphi_g(x)\, f\left(x,u\right),\\
h\left(\varphi_g(x), \psi_g(u)\right) &= \varrho_g\left(h\left(x,u\right)\right).
\end{aligned}$$

The G-invariance property can be interpreted such that changing of state, input, or output map of the system by the action of group G remains the dynamics of the system unchanged.

Now, consider a more specific dynamic SISO system, which does not hold the symmetry condition:

$$\dot{x} = f(x) + \kappa u + \zeta(t,x,u), \tag{1}$$

where x is the measurable plant state, u is the plant control signal, $f(x)$ stands for the known system dynamics, $\kappa \neq 0$ denotes the known input scaling factor and $\zeta(t,x,u)$ is the part of the dynamics that excludes the system from being a symmetry-preserving (SP) one.

If the unwanted term $\zeta(t,x,u)$ is time differentiable, then the system (1) can be artificially augmented with another state variable as seen below:

$$\begin{aligned}
\dot{x} &= f(x) + \kappa u + x_{r+1},\\
\dot{x}_{r+1} &= \dot{\zeta}\left(t,x,u\right),
\end{aligned} \tag{2}$$

[1] In this work, the *total disturbance* is defined as a combination of terms in the mathematical model of a given system that breaks its symmetry.

which now gives $dim(X) = r+1$ and x_{r+1} is the additional, fictitious state variable corresponding to the unknown term $\zeta(t,x,u)$.

If we assume that the information about the $\zeta(t,x,u)$ is however available at every time instant, one can choose a following modification of the control input:

$$u := \bar{u} - \frac{\hat{\zeta}(t,x,u)}{\kappa} = \bar{u} - \frac{\hat{x}_{r+1}}{\kappa}, \tag{3}$$

where $\hat{x}_{r+1} := \hat{\zeta}(t,x,u)$ is the estimate of the uncertain term $\zeta(t,x,u)$, $\bar{u}$ is the new control input, to be tuned to meet a certain system dynamic behavior, related with the given control task.

If $\hat{\zeta}(t,x,u) = \zeta(t,x,u)$, one can use (3) in (1), which results in a theoretical perfect compensation of the uncertain term, giving a simplified model of the system:

$$\dot{x} = f(x) + \kappa\bar{u},$$

which now holds the symmetry condition. Hence, the aim is to obtain a following symmetry-recovery scheme:

$$\dot{x} = \underbrace{f(x) + \kappa u + \zeta(t,x,u)}_{\text{not SP}} \overset{(3)}{\Longrightarrow} \dot{x} = \underbrace{f(x) + \kappa\bar{u}}_{\text{SP}}.$$

From (3), it is obvious that the effectiveness of the above simultaneous estimation and cancellation scheme is directly related to the quality of reconstruction of the uncertain *total disturbance* $\zeta(t,x,u)$. One approach is to use a state observer in order to estimate it (see Fig. 1).

2.2 Observer design

There is a precisely defined method of constructing a symmetry-preserving observer (further denoted as SP-ESO). Since the transformation group is a set, one can use a method, namely normalization, to find all the invariants. By several assumptions discussed by Bonnabel et al. (2008) and Nowicki et al. (2013), one can split $\varphi_g(\hat{x})$ into $\varphi_g^a(\hat{x})$ and $\varphi_g^b(\hat{x})$. The local solution of:

$$\varphi_g^a(\hat{x}) = c, \text{ for } c \in \varphi_g^a[\hat{X}],$$

i.e. $g = \gamma(\hat{x})$ maps $\hat{x}$ to the cross-section $\gamma : \hat{X} \to G$. This is known as a moving frame method. The above solution is necessary to define a new output error equation, namely invariant output error, given by:

$$E := \varrho_{\gamma(\hat{q})}(\hat{y}) - \varrho_{\gamma(\hat{q})}(y).$$

The motivation for such redefinition is the fact that the usual output error in the form $\hat{y} - y$ does not preserve the symmetry of the system.

Once the solution is set, it can be used to get a complete set of invariants by applying $\gamma(\hat{x})$ to:

$$\mathcal{I}(\hat{x},u) := \left(\varphi^b_{\gamma(\hat{x})}(\hat{x}), \psi_{\gamma(\hat{x})}(u)\right).$$

Finally, take a set of invariant vector fields $W = \{\nu_1, ..., \nu_{r+1}\}$, where $r+1$ is the dimension of the manifold $\hat{X}$. The G-invariant frame W on $\hat{X}$ is defined for the set of G-invariant vector fields that forms a basis for the tangent space $T\hat{X}|_{\hat{x}}$. By Lemma 1 from Bonnabel et al. (2008), the vector fields are given by:

$$\nu_i(\hat{x}) = \left(\frac{\partial}{\partial \hat{x}_i}\phi_{\gamma(\hat{x})}(\hat{x})\right)^{-1} \frac{\partial}{\partial \hat{x}_i},$$

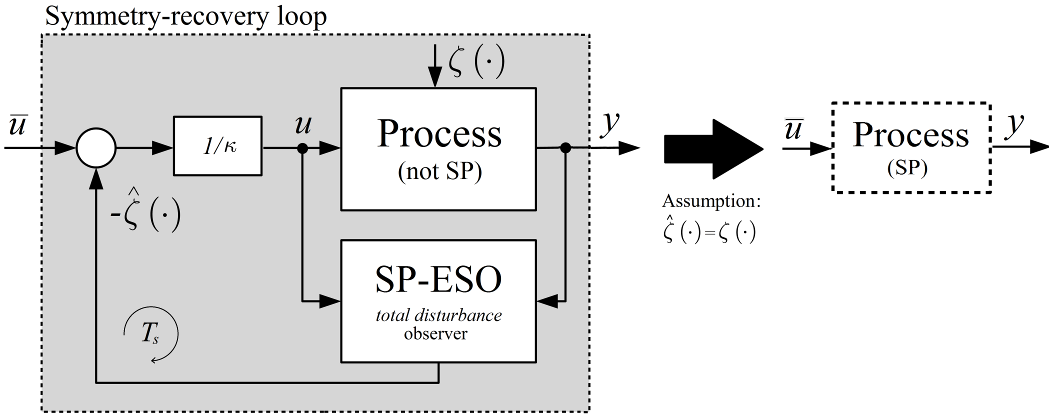

Figure 1. The idea of online estimation and mitigation of effects of the uncertain elements of the system dynamics.

for $i = 1, ..., r+1$ and the $\frac{\partial}{\partial \hat{x}}$ is a canonical base of $\hat{X}$.

Since the model function $f(\hat{x}, u)$ in the SP-ESO observer is G-invariant in the sense of Definition 1 and we replace the usual output error with an invariant one, the G-invariant observer is given by:

$$\dot{\hat{x}} = f(\hat{x}, u) + W(\hat{x})\mathcal{L}(\mathcal{I}, E)E,$$

where W is a G-invariant frame on $\hat{X}$, and $\mathcal{L}(\mathcal{I}, E)$ is a $(r+1)$-by-p "gain matrix" depending on a set of invariants and on an invariant output error. Due to the form of the observer, which is based on the nonlinear observer, the operations between terms are simply matrix multiplication.

Remark 2. For the augmented system model in (2), the $(r+1)$-th order SP-ESO is usually constructed. It results from the assumption that $\dot{\zeta}(t, x, u) = 0$ (which means that the *total disturbance*) is often considered to be a piecewise constant signal / degree-zero time polynomial. However, perturbation in real applications is rarely constant, but various examples (e.g. in Madonski and Herman (2013)) show that if $\hat{\zeta}(t, x, u) - \zeta(t, x, u)$ is small enough, this discrepancy can be practically acceptable, giving satisfactory quality [2] .

Remark 3. The procedure of online reconstruction of the unknown term (seen in (2)) and further cancellation of its effects (seen in (3)) can be also expanded with the ability to estimate the system modeling uncertainty as well as the external perturbations acting on the plant. Such approach assumes that the system can be expressed as:

$$\dot{x} = f(x) + \kappa u + \zeta(t, x, u) + \zeta_{int}(x) + \zeta_{ext}(\cdot), \qquad (4)$$

where $\zeta_{int}(x)$ is a function with unknown structure and/or parameters and $\zeta_{ext}(\cdot)$ represents the unknown and unmeasurable overall external disturbances acting on the system. One can rewrite (4) with a following simplified *local* model:

$$\dot{x} = f(x) + \kappa u + x_{r+1},$$
$$x_{r+1} = \zeta^*(t, x, u, \cdot),$$

where the generally unmeasurable time-varying *total disturbance* is now assumed to be:

$$\zeta^*(t, x, u, \cdot) = \zeta(t, x, \cdot) + \zeta_{int}(x) + \zeta_{ext}(\cdot).$$

which can also be treated as the system extra, virtual state variable x_{r+1}, as presented in (2). Remarks 2 and 3 also apply to the case when such *total disturbance* is considered.

3. ILLUSTRATIVE EXAMPLE

3.1 System modeling

Consider a following second order system, which describes the simplified dynamics of a 1DOF rigid-link manipulator arm:

$$\ddot{\theta} = \frac{1}{J}\tau - \frac{b}{J}\dot{\theta} \underbrace{- \frac{mglsin(\theta)}{J}}_{d(\theta)}, \qquad (5)$$

where $J > 0$ is the known inertia of the manipulator link, $b \neq 0$ is the known coefficient of the friction model, and $d(\theta) \in R$ is the generally unknown symmetry-breaking term related to the gravity effect (and also the assumed *total disturbance* in this case). The above system can also be expressed with the state representation as:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{1}{J}\tau - \frac{b}{J}x_2 + d(x_1), \end{cases} \qquad (6)$$

where $x_1 := \theta$, $x_2 := \dot{\theta}$ are the selected phase state variables.

3.2 Symmetry of the model

It is easy to verify that the system (6) does not admit symmetry under action of an unit circle group S^1. Indeed, by defining the group action as addition of angles, we have the following transformation maps:

$$\varphi_g(x) = \begin{pmatrix} x_1 + g \\ x_2 \end{pmatrix},$$

$$\psi_g(\tau) = \tau,$$

$$\varrho_g(y) = x_1 + g.$$

According to Definition 1, simple calculation shows:

[2] In a discrete-time application, this difference can be made neglectable if the sampling time of the system is appropriately decreased.

$$L = \begin{pmatrix} x_2 \\ \frac{1}{J}\tau - \frac{b}{J}x_2 + d(x_1+g) \end{pmatrix},$$
$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ \frac{1}{J}\tau - \frac{b}{J}x_2 + d(x_1) \end{pmatrix},$$
$$L \neq R.$$

Now, let us hypothetically assume that the term $d(x_1)$ does not exist, e.g. the considered system is placed in a gravity-free space. Analogous calculations shows that the symmetry is now held.

$$L = \begin{pmatrix} x_2 \\ \frac{1}{J}\tau - \frac{b}{J}x_2 \end{pmatrix},$$
$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ \frac{1}{J}\tau - \frac{b}{J}x_2 \end{pmatrix},$$
$$L = R.$$

3.3 Symmetry-preserving observer

If the unwanted term $d(x_1)$ is time differentiable, the system model (6) can be artificially extended with an extra state variable ($x_3 := d(x_1)$) and formulated as:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{1}{J}\tau - \frac{b}{J}x_2 + x_3, \\ \dot{x}_3 = \dot{d}(x_1). \end{cases} \tag{7}$$

For such augmented system model, the SP-ESO can be constructed. Taking (7), we can design the symmetry-preserving (or G-invariant) observer. Firstly, the action on the state has to be redefined:

$$\varphi_g(x) = \begin{pmatrix} x_1 + g \\ x_2 \\ x_3 \end{pmatrix},$$

recalling the fact that the state variable x_3 can be viewed locally as constant (see Remark 2).

The invariant output error is given by normalization procedure with $c = 0$:

$$\hat{x}_1 + g_\gamma = 0,$$

thereby, the solution states:

$$\gamma(\hat{x}_1) = -\hat{x}_1.$$

which gives:

$$E(\hat{x}, y) = (-\hat{x}_1 + \hat{x}_1) - (-\hat{x}_1 + x_1) = \hat{x}_1 - x_1.$$

Obviously, if one takes the canonical basis of prolonged S^1 the invariant frame is 3-by-3 identity matrix:

$$W = \frac{\partial}{\partial(x_1, x_2, x_3)}\varphi_g \begin{pmatrix} x_1 + g \\ x_2 \\ x_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In the end, the equation of G-invariant observer writes:

$$\begin{cases} \dot{\hat{x}}_1 = L_1(\hat{x}_1 - x_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 = L_2(\hat{x}_1 - x_1) + \frac{1}{J}\tau - \frac{b}{J}\hat{x}_2 + \hat{x}_3 \\ \dot{\hat{x}}_3 = L_3(\hat{x}_1 - x_1) \end{cases} \overset{aim}{\Rightarrow} \begin{cases} \hat{x}_1 \to \theta \\ \hat{x}_2 \to \dot{\theta} \\ \hat{x}_3 \to d(\theta) \end{cases}$$

where $\hat{x}_1, \hat{x}_2, \hat{x}_3$ are the estimates of the assumed state variables and $L_1, L_2, L_3 < 0$ are the observer gains (design parameters). Terms $\dot{\theta}$ and $d(\theta)$ are unknown, hence have to be reconstructed based on the I/O plant signals.

3.4 Symmetry-recovery loop

Once the information about the unknown term that breaks the symmetry in the system is reconstructed accurately (i.e. $\hat{d}(\theta) = d(\theta)$), using the SP-ESO from Sect. 3.3, a following modification of the control variable can be proposed for the system in (5):

$$\tau = \bar{\tau} - J\hat{d}(\theta), \tag{8}$$

where $\bar{\tau}$ is the new control input (to be designed later on). After substitution in (5) it further gives a new form of the system model, which can be locally (in small time intervals) treated as a symmetry-preserving one:

$$\ddot{\theta} = \underbrace{\frac{1}{J}\tau - \frac{b}{J}\dot{\theta} + d(\theta)}_{\text{not SP}} - \hat{d}(\theta) \overset{(8)}{\Longrightarrow} \ddot{\theta} = \underbrace{\frac{1}{J}\bar{\tau} - \frac{b}{J}\dot{\theta}}_{\text{SP}}. \tag{9}$$

Such procedure allows to treat the nominal plant, where the symmetry is not held (i.e. (5)), with a different model where the symmetry is held (i.e. (9)). The symmetry is recovered if the estimated term $\hat{d}(\theta)$ is being fed back to the system in real time and compensates for the effects of the time-varying function $d(\theta)$.

3.5 Results

The simulation study of the proposed symmetry-recovery scheme is performed for the exemplary 1DOF system from (5). The analysis is divided into two parts (E1: open-loop tests and E2: closed-loop tests) and conducted in the Matlab/Simulink environment. The fixed sampling time of the overall control system in each considered scenario is $T_s = 0.001$s with solver ODE5.

E1a: Symmetry-recovery test in the open-loop control The symmetry-recovery procedure was first tested in an open-loop scheme. The behavior of the system without symmetry but with the symmetry-preserving observer is compared with the results obtained for the symmetry-preserving system (i.e. without the symmetry-breaking term $d(\theta)$). The results, seen in Figs. 2 and 3, show that the proposed symmetry-preserving procedure works, since the outputs of the two systems are almost identical (cf. e_m[rad] representing the difference between the two systems). It is thanks to the estimation part that reconstructs the *total disturbance* with a small estimation error (denoted as $e_{td} = d(\theta) - \hat{d}(\theta)$).

E1b: Robustness analysis in the open-loop control The scenario from E1a is now tested with an extra, additive, and unknown sine-type disturbance in the control signal. From the results in Figs. 4 and 5 one can notice a significant influence of the extra perturbation on the SP system without symmetry-recovery. On the other hand, the system with symmetry-recovery manages to attenuate the additional disturbance (which was considered as an extra part of the *total disturbance*, cf. Remark 4) and behaved almost as the SP system without disturbance (from E1a).

E2a: Symmetry-recovery test in the closed-loop control This test presents a symmetry-preserving control system, which combines the symmetry-recovery scheme in the inner loop with a symmetry controller in the outer loop. The main benefit of such structure is the fact that the whole control system preserves the same symmetries as the plant. Therefore, a G-invariant controller has to be designed. Take a simple PD controller in the form of:

$$\bar{\tau} = K_2 \begin{pmatrix} \epsilon \\ \dot{\epsilon} \end{pmatrix} = K_2 \begin{pmatrix} \theta_d - \theta \\ -\hat{x}_2 \end{pmatrix},$$

where θ_d[rad] is the reference angular position of the manipulator link, $K_i := \mathrm{diag}\{k_p, k_d\}$ with k_p and k_d being the proportional and derivative gains of the feedback controller, respectively. Simple calculations show, that it is invariant by the action of group G, since the transformed equation yields the same formula [3].

The results of the closed-loop system without any additional disturbances are seen in Figs. 6 and 7. The outcomes are again compared with the behavior of the SP system. It is seen that thanks to the symmetry-recovery tool, the two systems act similarly.

E2b: Robustness analysis in the closed-loop control The final test studies the closed-loop system with the influence of the extra unmodeled disturbance in the control signal. The results are gathered in Figs. 8 and 9. One can notice that the SP system (without symmetry-recovery loop) lacks robustness. The added perturbation generates a significant tracking error in this case. Contrary, the system with the symmetry-recovery scheme manages to estimate the unwanted discrepancy and reject its influence on the system output.

4. CONCLUSIONS AND FUTURE WORK

In this paper, the framework for symmetry-recovery has been presented. The ADRC-based approach allows the simultaneous estimation and decoupling of terms that break the symmetry of the system. The case study, supported with simulation results, shows that it is possible to use techniques exploiting invariance, for the system where the symmetry has been corrupted by considering some additional elements in the system dynamics.

Future work concerns the elaboration of the explicit formula for system prolongation, as well as an analysis of systems with complex geometric structures. Much work is going to be devoted to real-time experiments in laboratory conditions.

[3] For further readings about G-invariant controllers and advantages of this approach see Martin et al. (2004); Nowicki et al. (2013).

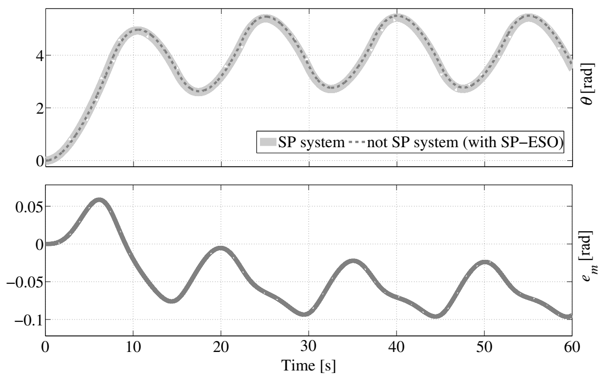

Figure 2. E1a: systems outputs (top) and comparison of the two systems (bottom).

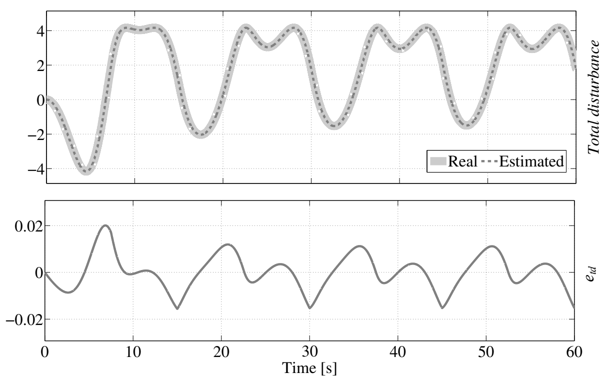

Figure 3. E1a: Estimated *total disturbance* (top) and its estimation error (bottom).

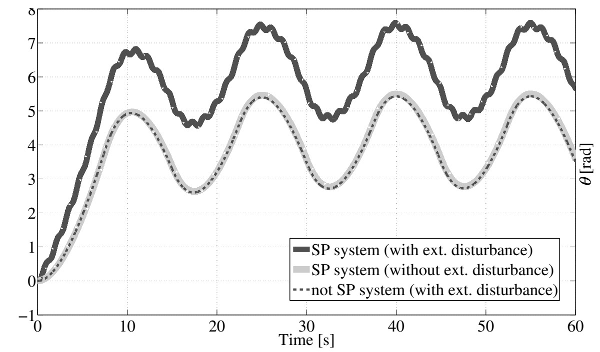

Figure 4. E1b: systems outputs, compared with the system without extra disturbance.

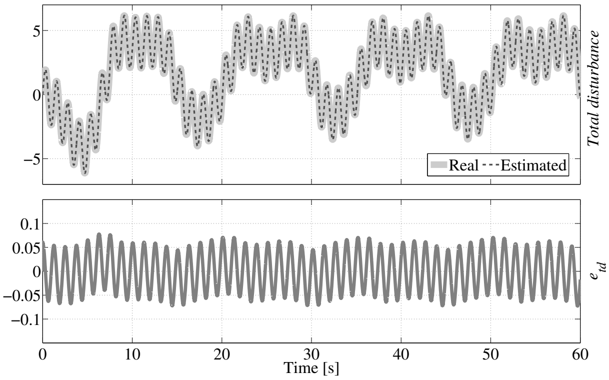

Figure 5. E1b: Estimated *total disturbance*.

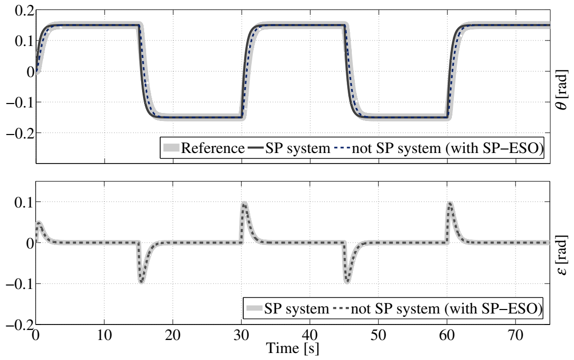

Figure 6. E2a: reference signal, obtained systems outputs (top) and tracking errors of the two systems (bottom).

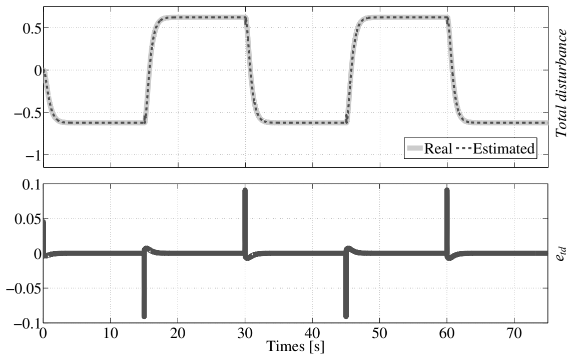

Figure 7. E2a: Estimated *total disturbance* (top) and its estimation error (bottom).

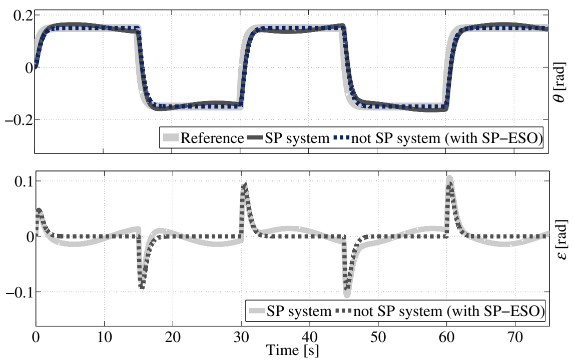

Figure 8. E2b: reference signal, obtained systems outputs (top) and tracking errors of the two systems (bottom).

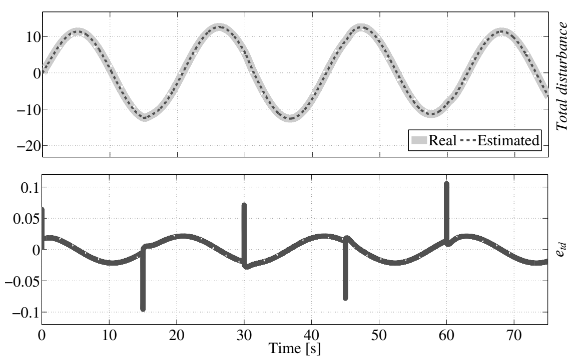

Figure 9. E2b: Estimated *total disturbance* (top) and its estimation error (bottom).

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