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Investigating the Interaction Between Traffic Flow and Vehicle Platooning Using a Congestion Game ⋆

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Abstract: We consider a congestion game with two types of agents to describe the traffic flow on a road at various time intervals in each day. The first type of agents (cars) maximize a utility which is determined by a sum of a penalty for using the road at a time other than their preferred time interval, the average velocity of the traffic flow, and the congestion tax. The second type of agents (trucks or heavy-duty vehicles) can benefit from using the road together with other second-type agents. This is because the trucks can form platoons to save fuel through reducing the air drag force. We study a Nash equilibrium of this game to study the interaction between the traffic flow and the platooning incentives. We prove that the introduced congestion game does not admit a potential function unless we devise an appropriate congestion taxing policy. We use joint strategy fictitious play and average strategy fictitious play to learn a pure strategic Nash equilibrium of this congestion game. Lastly, we demonstrate the developed results on a numerical example using data from a highway segment in Stockholm.

Keywords: Transportation Systems; Game theory; Learning algorithms.

1. INTRODUCTION

Transportation of people and products is widely known to be a considerable source of air pollution (Mitra and Mazumdar, 2007; Fuglestvedt et al., 2008). For instance, a recent study (Fuglestvedt et al., 2008) shows that the transportation has contributed to approximately 15% of the total man-made carbon-dioxide since preindustrial era and also suggests that it will be responsible for roughly 16% of the carbon-emission over the next century. To overcome these problems, there have been many studies focused on proposing more efficient transportation methods. For instance, an experimental study (Alam et al., 2010) reports that two identical trucks can achieve 4.7%-7.7% reduction in the fuel consumption (depending on the distance between them) when platooning at 70 km/h. The phenomenon is primarily due to reduced air drag force when forming platoons. Therefore, in future, when most of the trucks are equipped with platooning devices, we can achieve a much higher fuel efficiency. However, there are many practical obstacles for platooning. For instance, a centralized decision-maker to coordinate the trucks would be very complex (and hence, difficult to implement in a large-scale setup). Additionally, the trucks are not on the road at the same time because they are owned by different strategic entities that are trying maximizing their profits or prefer not to share their costumers' private information. This motivates the use of a game theoretic framework for studying the traffic flow and its implications on the trucks' decision to use the road at the same time for increasing the possibility of forming platoons.

In this paper, we use an atomic congestion game with two types of agents to model the traffic flow on a road at certain time intervals. The term atomic is used here to emphasize the fact that we do not work with a continuum of players or fractional flows when modeling the traffic flow (Schmeidler, 1973). The utility of the first type of agents, which would not benefit significantly from moving together (e.g., ordinary cars and trucks without platooning equipment), is modeled by a sum of a penalty for deviating from the time interval on which they prefer to use the road, the average velocity of the traffic flow at that time, and the congestion tax. In addition to these terms, the second type of agents (e.g., trucks or other heavy-duty vehicles with platooning equipment) benefit from using the road at the same time as their peers. Note that this platooning incentive is indeed proportional to the average velocity of the traffic flow since these agents cannot benefit much at low velocities (Alam et al., 2010). We show that this congestion game is a potential game under appropriate congestion taxes for the first type of agents or platooning subsidies for the second type of agents. This would guarantee that the congestion game admits at least one pure strategy Nash equilibrium (Monderer and Shapley, 1996). Then, we use joint strategy fictitious play (Marden et al., 2009) and average strategy fictitious play (Xiao et al., 2013) to learn a pure strategy Nash equilibrium of this game. To prove the convergence of the average strategy fictitious play, we adapt parts of the proofs presented in (Xiao et al., 2013).

There have been many studies in traffic flow analysis and network routing using congestion games (Xiao et al., 2013; Levinson, 2005; Christodoulou and Koutsoupias, 2005; Correa et al., 2005; Rosenthal, 1973b,a). The authors in (Xiao et al., 2013) proposed a model that inspired the congestion game that we are considering in this paper. However, we study a congestion game, where a group of agents would benefit from using the road at the same time as each other, to study the interaction between the traffic flow and the platooning incentives. This platooning congestion game was considered from a practical perspective in (Farokhi and Johansson, 2013), to motivate the modeling assumptions and to extract appropriate simulation

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parameters using real traffic data. In this paper, we follow a theoretical approach to show the existence of a pure strategy Nash equilibrium and to prove the convergence of the learning algorithms.

The reminder of the paper is organized as follows. In Section 2, we introduce the described congestion game with two types of agents to model the traffic flow. We present conditions for the existence of a potential function for the introduced congestion game in Section 3. In Section 4, we introduce the joint strategy fictitious play and the average strategy fictitious play to learn a Nash equilibrium of the congestion game. Finally, we illustrate the developed results on a numerical example in Section 5 and conclude the paper in Section 6.

1.1 Notation

Let $\mathbb{R}$, $\mathbb{Z}$, and $\mathbb{N}$ denote the sets of real, integer, natural numbers, respectively. Furthermore, let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define $[\![N]\!] = \{1, \dots, N\}$ for any $N \in \mathbb{N}$. All the other sets are denoted by calligraphic letters such as $\mathcal{R}$. We use $|\mathcal{R}|$ to denote the cardinality of $\mathcal{R}$. Finally, we define the characteristic function $\mathbf{1}_{x=y}$ ($\mathbf{1}_{x\geq y}$) to be equal one whenever $x = y$ ($x \geq y$) holds true and zero otherwise.

2. PROBLEM SETUP

Consider an atomic congestion game composed of two type of agents, where each agent must choose from a finite action set $\mathcal{R} = \{r_1, r_2, \dots, r_R\}$ for some $R \in \mathbb{N}$. In this set, entries r_i, $i \in [\![R]\!]$, denote non-overlapping time intervals of the day that a vehicle can choose to use a road. Let $\{z_i\}_{i=1}^N$ and $\{x_i\}_{i=1}^M$ denote the actions of the agents of the first type and the second type, respectively. In the rest of the paper, for the sake of brevity, we name the agents of the first type cars and the agents of the second type trucks.

Car $i \in [\![N]\!]$ maximizes its utility described by

$$U_i(z_i, z_{-i}, x) = \xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) + v_{z_i}(z, x) + p_i^{\mathrm{c}}(z, x), \qquad (1)$$

where $\xi_i^{\mathrm{c}} : \mathcal{R} \times \mathcal{R} \to \mathbb{R}$ determines the penalty for using the road at time z_i instead of its preferred time interval $T_i^{\mathrm{c}} \in \mathcal{R}$, $p_i^{\mathrm{c}}(z, x)$ is a potential congestion taxing [1] policy for using the road at interval z_i, and $v_{z_i}(z, x)$ describes the average velocity of the traffic at that time. Following (Farokhi and Johansson, 2013; Xiao et al., 2013), in the rest of the paper, we assume that the average velocity at each interval is an affine [2] function of the number of vehicles (both cars and trucks) that are using the road at that time interval, that is, $v_r(z, x) = a n_r(z, x) + b$, where $n_r(z, x) = \sum_{\ell=1}^N \mathbf{1}_{\{z_\ell = r\}} + \sum_{\ell=1}^M \mathbf{1}_{\{x_\ell = r\}}$ for any $r \in \mathcal{R}$. Note that the choice of the penalty functions ξ_i^{c}, $i \in [\![N]\!]$, does not change the mathematical results presented in the paper (as the proofs do not rely on any special structure for them). However, various choices for this penalty can model the drivers' behavior. Following (Xiao et al., 2013), one possible choice for this function is $\xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) = \alpha_i^{\mathrm{c}}|z_i - T_i^{\mathrm{c}}|$ with scalar $\alpha_i^{\mathrm{c}} < 0$. This specific function shows that car i prefers to use the road on time and gets penalized symmetrically by deviating from it (i.e., it does not matter if the car uses the road earlier or later than T_i^{c}). Additionally, upon increasing $|\alpha_i^{\mathrm{c}}|$, the car becomes less flexible in changing its decision. Another example for this function could be $\xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) = \alpha_i^{\mathrm{c}} \max(z_i - T_i^{\mathrm{c}}, 0)$, where $\alpha_i^{\mathrm{c}} < 0$. Using this penalty function, car i can arrive earlier without incurring any additional cost but it gets penalized for using the road at a later time. For the simulation results in Section 5, we use the first mapping for all the vehicles. Similarly, truck $j \in [\![M]\!]$ maximizes its utility

$$\begin{aligned} V_j(x_j, x_{-j}, z) = \xi_j^{\mathrm{t}}(x_j, T_j^{\mathrm{t}}) + v_{x_j}(z, x) + p_j^{\mathrm{t}}(z, x) \\ + \beta v_{x_j}(z, x) g(m_{x_j}(x)), \end{aligned} \qquad (2)$$

where $p_j^{\mathrm{t}}(z, x)$ denotes a potential congestion taxing policy for using road at interval x_j, $\xi_j^{\mathrm{t}}(x_j, T_j^{\mathrm{t}})$ determines the penalty for using the road at an interval other its preferred one, and $\beta v_{x_j}(z, x) g(m_{x_j}(x))$ characterizes the benefit for traveling at the same time as the other trucks. Let $g : [\![M]\!] \to \mathbb{R}$ be a non-decreasing mapping and $m_r(x) = \sum_{\ell=1}^M \mathbf{1}_{\{x_\ell = r\}}$ denote the number of trucks on interval r. This extra term can be motivated by the fact that whenever there are several trucks on the road at the same time interval, they can potentially form platoons to save fuel. Note that this term is a function of the average velocity of the flow as trucks cannot save a significant amount of fuel when platooning at low velocities (Alam et al., 2010). Hence, although the trucks prefer to travel at the same time, they also want to avoid the congested time intervals. The function $g : [\![M]\!] \to \mathbb{R}$ describes the dependency of the fuel saving to the number of trucks at a given time interval. In the rest of this paper, we assume that this function is identity; i.e., $g(m_{x_j}(x)) = m_{x_j}(x)$. Another example for this function could be $g(m_{x_j}(x)) = m_{x_j}(x)\mathbf{1}_{m_{x_j}(x) \geq \tau}$, which shows that the trucks do not benefit from traveling at the same time unless they reach a critical number τ.

Now, we are ready to define a congestion game with two types of players using normal-form representation of strategic games; see (Gibbons, 1992).

Definition 1. (Car–Truck Congestion Game): A car–truck congestion game is defined as a tuple $\mathcal{G} = ((\mathcal{R})_{i=1}^{N+M}; ((U_i)_{i=1}^N, (V_j)_{j=1}^M))$, that is, a combination of $N + M$ players with action space $(\mathcal{R})_{i=1}^{N+M}$ and utilities $((U_i)_{i=1}^N, (V_j)_{j=1}^M))$.

A pure strategy Nash equilibrium for a car–truck congestion game is a pair $(z, x) \in \mathcal{R}^N \times \mathcal{R}^M$ such that

$$\begin{aligned} U_i(z_i, z_{-i}, x) \geq U_i(z_i', z_{-i}, x), \quad \forall z_i' \in \mathcal{R}, \quad i \in [\![N]\!], \\ V_j(x_j, x_{-j}, z) \geq V_j(x_j', x_{-j}, z), \quad \forall x_j' \in \mathcal{R}, \quad j \in [\![M]\!]. \end{aligned}$$

To prove the existence of a pure strategy Nash equilibrium or to use various learning algorithms for finding an equilibrium, we focus on a subclass of games, namely, potential games (Monderer and Shapley, 1996). A car–truck congestion game is a potential game with potential function $\Phi : \mathcal{R}^N \times \mathcal{R}^M \to \mathbb{R}$ if

$$\begin{aligned} &\Phi(x, z_i, z_{-i}) - \Phi(x, z_i', z_{-i}) \\ &\qquad = U_i(z_i, z_{-i}, x) - U_i(z_i', z_{-i}, x), \qquad \forall i \in [\![N]\!], \\ &\Phi(x_j, x_{-j}, z) - \Phi(x_j', x_{-j}, z) \\ &\qquad = V_j(x_j, x_{-j}, z) - V_j(x_j', x_{-j}, z), \quad \forall j \in [\![M]\!]. \end{aligned}$$

With these definitions in hand, we are ready to present the results of the paper.

[1] Note that if $p_i^{\mathrm{c}}(z, x) < 0$, this term is a tax (since it reduces the utility of car i). However, if $p_i^{\mathrm{c}}(z, x) > 0$, this terms is a subsidy (since it increases its utility). In what follows, we use these terms to make sure that the overall game is a potential game. These taxes can also be used to enforce a socially optimal behavior. For instance, we can use mechanism design (see Jackson, 2003, for a survey) to optimize the combined fuel consumption as a socially preferable action.

[2] The affine relationship between the number of the vehicles on the road and the average velocity is explored and validated using real traffic data from Stockholm in (Farokhi and Johansson, 2013).

3. EXISTENCE OF POTENTIAL FUNCTION

Atomic congestion games with one type of agents (corresponding to the case where $M = 0$ or $N = 0$) are known to admit a potential functions even without imposing congestion taxes (Xiao et al., 2013; Roughgarden, 2007). In this section, however, we show that this property does not hold for car–truck congestion games unless we devise an appropriate taxing scheme.

3.1 Necessary Conditions

Let $\Phi : \mathbb{R}^N \times \mathbb{R}^M \to \mathbb{R}$ be a given mapping. We can define

$$\Delta_{x_j \to x'_j} \Phi(x, z) = \Phi(x, z) - \Phi(x', z)$$
$$\Delta_{z_i \to z'_i} \Phi(x, z) = \Phi(x, z) - \Phi(x, z'),$$

where $x' = (x'_j, x_{-j})$ and $z' = (z'_i, z_{-i})$. Using simple algebra, we can show that the operators commute, i.e.,

$$\Delta_{z_i \to z'_i} \Delta_{x_j \to x'_j} \Phi(x, z) = \Delta_{x_j \to x'_j} \Delta_{z_i \to z'_i} \Phi(x, z).$$

Now, we are ready to prove the following useful result.

PROPOSITION 1. A car–truck congestion game admits a potential function only if

$$\Delta_{x_i \to x'_j} \Delta_{z_i \to z'_i} V_j(z, x) = \Delta_{z_i \to z'_i} \Delta_{x_i \to x'_j} U_i(z, x),$$

for all $i \in [\![N]\!]$ and $j \in [\![M]\!]$.

Proof: Let $\Phi(x, z)$ be a potential function for the congestion game. Then, it must satisfy

$$\Delta_{x_j \to x'_j} V_j(x, z) = \Delta_{x_j \to x'_j} \Phi(x, z). \tag{3}$$

Let $x' = (x'_j, x_{-j})$ and $z' = (z'_i, z_{-i})$. Again, when noting that $\Phi(x, z)$ is a potential function, we get

$$\Phi(x, z) = \Phi(x, z') + \Delta_{z_i \to z'_i} U_i(z, x) \tag{4a}$$
$$\Phi(x', z) = \Phi(x', z') + \Delta_{z_i \to z'_i} U_i(z, x') \tag{4b}$$

Substituting (4) into (3) results in

$$\begin{aligned}\Delta_{x_j \to x'_j} V_j(x, z) =& \Phi(x, z) - \Phi(x', z)\\ =& \Delta_{x_j \to x'_j} \Phi(x, z')\\ &+ \Delta_{z_i \to z'_i} U_i(z, x) - \Delta_{z_i \to z'_i} U_i(z, x')\\ =& \Delta_{x_j \to x'_j} \Phi(x, z') + \Delta_{z_i \to z'_i} \Delta_{x_i \to x'_j} U_i(z, x)\\ =& \Delta_{x_j \to x'_j} V_j(x, z') + \Delta_{z_i \to z'_i} \Delta_{x_i \to x'_j} U_i(z, x),\end{aligned}$$

where the last equality follows from the definition of the potential function. Therefore, the identity in the statement of the proposition follows. ■

This proposition shows that it might not be possible to find a potential functions for car–truck congestion games.

COROLLARY 2. Let $p^c_i(z, x) = 0$ for $i \in [\![N]\!]$ and $p^t_j(z, x) = 0$ for $j \in [\![M]\!]$. A car–truck congestion game admits a potential function only if $\beta = 0$.

Proof: First, we prove the identity in (5) by simple algebraic manipulations. Similarly, we can show that

$$\Delta_{z_i \to z'_i} \Delta_{x_i \to x'_j} U_i(z, x) = a[\mathbf{1}_{x_j = z_i} + \mathbf{1}_{x'_j = z'_i} - \mathbf{1}_{x_j = z'_i} - \mathbf{1}_{x'_j = z_i}].$$

Therefore, following Proposition 1, the car–truck congestion game admits a potential function only if

$$\beta[\mathbf{1}_{x_j = z'_i} \mathbf{1}_{x'_j = z_i} - \mathbf{1}_{x_j = z_i} \mathbf{1}_{x'_j = z'_i}] [1 - \mathbf{1}_{z_j = z'_i}][g(m_{x_j}(x)) + g(m_{x'_j}(x'))] = 0$$

for all x, z and x'_j, z'_i. This is only possible if $\beta = 0$. ■

Potential games have many desirable attributes. For instance, these games always admit at least one pure strategy Nash equilibrium. In addition, many learning algorithms, such as, joint strategy fictitious play, are known to converge to a pure strategy Nash equilibrium for potential games. Given these important properties, a natural question that comes to mind is that whether it is possible to guarantee the existence of a potential function by imposing appropriate congestion taxes. We answer this question in the next subsection.

3.2 Imposing Congestion Taxes

In this subsection, we propose a taxing and a subsidy policy that guarantee the existence of a potential function.

THEOREM 3. Let each car $i \in [\![N]\!]$ pay the congestion tax

$$p^c_i(z, x) = a\beta \sum_{\ell=1}^{m_{z_i}(x)} g(\ell), \tag{6}$$

for using the road at time interval z_i. Then, the car–truck congestion game is a potential game with the potential function

$$\begin{aligned}\Phi(x, z) =& \sum_{i=1}^{N} \xi^c_i(z_i, T^c_i) + \sum_{j=1}^{M} \xi^t_j(x_j, T^t_j)\\ &+ \sum_{r=1}^{R} \sum_{k=1}^{n_r(x,z)} (ak + b) - a\beta \sum_{r=1}^{R} \sum_{\ell=1}^{m_r(x)} \sum_{k=1}^{\ell-1} g(k)\\ &+ \sum_{r=1}^{R} \beta(an_r(x, z) + b) \sum_{\ell=1}^{m_r(x)} g(\ell).\end{aligned}$$

Furthermore, this game admits at least one pure strategy Nash equilibrium.

Proof: See (Farokhi and Johansson, 2013). ■

REMARK 1. Note the tax $p^c_i(z, x)$ grows quadratically with the number of the trucks that are using the road at that time interval if the mapping $g : [\![M]\!] \to \mathbb{R}$ is a linear function. Therefore, the congestion tax policy $p^c_i(z, x)$ in Theorem 3 forces the cars to avoid the time intervals that the trucks use to travel together.

Instead of taxing the cars, we can also introduce a platooning subsidy for the trucks to get a potential game.

THEOREM 4. Let each truck $j \in [\![M]\!]$ receive the subsidy

$$p^t_j(x, z) = \beta(v_0 - (an_{x_j}(z, x) + b))m_{x_j}(x), \tag{7}$$

for a given $v_0 \in \mathbb{R}$. Then, the car–truck congestion game is a potential game with the potential function

$$\begin{aligned}\Psi(x, z) =& \sum_{i=1}^{N} \xi^c_i(z_i, T^c_i) + \sum_{j=1}^{M} \xi^t_j(x_j, T^t_j)\\ &+ \sum_{r=1}^{R} \sum_{k=1}^{n_r(x,z)} (ak + b) + \beta v_0 \sum_{r=1}^{R} \sum_{\ell=1}^{m_r(x)} g(\ell).\end{aligned}$$

Furthermore, this game admits at least one pure strategy Nash equilibrium.

Proof: Let us start with trucks. Note that with the introduced tax policy, the utility of truck j is equal

$$V_j(x_j, x_{-j}, z) = \xi^t_j(x_j, T^t_j) + v_{x_j}(z, x) + \beta v_0 g(m_{x_j}(x)).$$

Let us define $x' = (x'_j, x_{-j})$. If $x_j = x'_j$, the result trivially holds. Therefore, without loss of generality, we consider the case where $x_j \neq x'_j$. In what follows, we examine each term in the cost function separately. First, we define

$$
\begin{aligned}
\Delta_{x_i\to x'_j}\Delta_{z_i\to z'_i}V_j(z,x) &= \Delta_{x_i\to x'_j}\Delta_{z_i\to z'_i}\big(\xi^{\rm t}_j(x_j,T^{\rm t}_j)+v_{x_j}(z,x)+\beta v_{x_j}(z,x)g(m_{x_j}(x))\big)\\
&= \Delta_{x_i\to x'_j}\Delta_{z_i\to z'_i}\big(v_{x_j}(z,x)+\beta v_{x_j}(z,x)g(m_{x_j}(x))\big)\\
&= \Delta_{x_i\to x'_j}\big(v_{x_j}(z,x)-v_{x_j}(z',x)+\beta v_{x_j}(z,x)g(m_{x_j}(x))-\beta v_{x_j}(z',x)g(m_{x_j}(x))\big)\\
&= \Delta_{x_i\to x'_j}\big(a[\mathbf{1}_{x_j=z_i}-\mathbf{1}_{x_j=z'_i}][1-\beta g(m_{x_j}(x))]\big)\\
&= a[\mathbf{1}_{x_j=z_i}-\mathbf{1}_{x_j=z'_i}][1-\beta g(m_{x_j}(x))]-a[\mathbf{1}_{x'_j=z_i}-\mathbf{1}_{x'_j=z'_i}][1-\beta g(m_{x'_j}(x'))]\\
&= a[\mathbf{1}_{x_j=z_i}+\mathbf{1}_{x'_j=z'_i}-\mathbf{1}_{x_j=z'_i}-\mathbf{1}_{x'_j=z_i}]\\
&\quad -a\beta[\mathbf{1}_{x_j=z_i}-\mathbf{1}_{x_j=z'_i}]g(m_{x_j}(x))+a\beta[\mathbf{1}_{x'_j=z_i}-\mathbf{1}_{x'_j=z'_i}]g(m_{x'_j}(x'))\\
&= a[\mathbf{1}_{x_j=z_i}+\mathbf{1}_{x'_j=z'_i}-\mathbf{1}_{x_j=z'_i}-\mathbf{1}_{x'_j=z_i}]\\
&\quad +a\beta[\mathbf{1}_{x_j=z'_i}\mathbf{1}_{x'_j=z_i}-\mathbf{1}_{x_j=z_i}\mathbf{1}_{x'_j=z'_i}][1-\mathbf{1}_{z_j=z'_i}][g(m_{x_j}(x))+g(m_{x'_j}(x'))]
\end{aligned}
\tag{5}
$$

$\Psi_1(x,z)=\sum_{i=1}^{N}\xi^{\rm c}_i(z_i,T^{\rm c}_i)+\sum_{j=1}^{M}\xi^{\rm t}_j(x_j,T^{\rm t}_j)$. Now, it is easy to see that

$$\Psi_1(x,z)-\Psi_1(x',z)=\xi^{\rm t}_j(x_j,T^{\rm t}_j)-\xi^{\rm t}_j(x'_j,T^{\rm t}_j).$$

Second, we define $\Psi_2(x,z)=\sum_{r=1}^{R}\sum_{k=1}^{n_r(x,z)}(ak+b)$. For this term, we can show that

$$
\begin{aligned}
\Psi_2(x,z)-\Psi_2(x',z) &= \sum_{r=1}^{R}\sum_{k=1}^{n_r(x,z)}(ak+b)-\sum_{r=1}^{R}\sum_{k=1}^{n_r(x',z)}(ak+b)\\
&= \sum_{k=1}^{n_{x_j}(x,z)}(ak+b)+\sum_{k=1}^{n_{x'_j}(x,z)}(ak+b)\\
&\quad -\sum_{k=1}^{n_{x_j}(x',z)}(ak+b)-\sum_{k=1}^{n_{x'_j}(x',z)}(ak+b),
\end{aligned}
$$

where the second equality holds because of the fact that $n_r(x,z)=n_r(x',z)$ for all $r\neq x_j,x'_j$. Noticing that $n_{x_j}(x',z)=n_{x_j}(x,z)-1$ and $n_{x'_j}(x,z)=n_{x'_j}(x',z)-1$, we know that

$$\Psi_2(x,z)-\Psi_2(x',z)=(an_{x_j}(z,x)+b)-(an_{x'_j}(z,x')+b).$$

Finally, we define $\Psi_3(x,z)=\sum_{r=1}^{R}\sum_{\ell=1}^{m_r(x)}g(\ell)$. In this case, we can show that

$$
\begin{aligned}
\Psi_3(x,z)-\Psi_3(x',z) &= \sum_{r=1}^{R}\sum_{\ell=1}^{m_r(x)}g(\ell)-\sum_{r=1}^{R}\sum_{\ell=1}^{m_r(x')}g(\ell)\\
&= \sum_{\ell=1}^{m_{x_j}(x)}g(\ell)+\sum_{\ell=1}^{m_{x'_j}(x)}g(\ell)\\
&\quad -\sum_{\ell=1}^{m_{x_j}(x')}g(\ell)-\sum_{\ell=1}^{m_{x'_j}(x')}g(\ell)\\
&= g(m_{x_j}(x))-g(m_{x'_j}(x')).
\end{aligned}
$$

Therefore, we get

$$
\begin{aligned}
\Psi(x,z)-\Psi(x',z) &= \Psi_1(x,z)-\Psi_1(x',z)\\
&\quad +\Psi_2(x,z)-\Psi_2(x',z)\\
&\quad +\beta v_0(\Psi_3(x,z)-\Psi_3(x',z))\\
&= \xi^{\rm t}_j(x_j,T^{\rm t}_j)-\xi^{\rm t}_j(x'_j,T^{\rm t}_j)\\
&\quad +v_{x_j}(x,z)-v_{x'_j}(x',z)\\
&\quad +\beta v_0(g(m_{x_j}(x))-g(m_{x'_j}(x')))\\
&= V_j(x_j,x_{-j},z)-V_j(x'_j,x_{-j},z).
\end{aligned}
$$

The proof for cars follows the same line of reasoning. ∎

REMARK 2. Note that if v_0 is greater than the average velocity of the flow, the trucks get paid to use the road at the same time as their peers. This way the government incentivizes the trucks to form platoons. This subsidy is technically the difference of the fuel that the trucks would have saved if they formed a platoon at the velocity v_0 instead of the actual average velocity of the traffic flow $an_r(z,x)+b$. Therefore, the trucks would benefit from traveling together even at low velocities (which is a scenario where the trucks do not increase their fuel efficiency significantly through platooning). However, if v_0 is smaller than the average velocity of the flow, we reduce the extra utility that the trucks would receive from traveling together (and technically $p^{\rm t}_j(x,z)$ becomes a tax rather than a subsidy). Therefore, it becomes less likely for the trucks to stick together. To emphasize the fact that we are willing to pay the trucks rather than taxing them (and hence, dealing with the first scenario), we call $p^{\rm t}_j(x,z)$ a subsidy.

Algorithm 1 Joint strategy fictitious play for learning a Nash equilibrium.

Input: $p\in(0,1)$
Output: (x^*,z^*)
for $t=0,2,\dots$ **do**
  for $i=1,\dots,N$ **do**
    Calculate $z'_i\in\arg\max_{r\in\mathcal{R}}\hat{U}_i(r;t-1)$
    if $U_i(z'_i,z_{-i}(t-1),x(t-1))\leq U_i(z_i(t-1),z_{-i}(t-1),x(t-1))$ **then**
      $z_i(t)\leftarrow z_i(t-1)$
    else
      With probability $1-p$, $z_i(t)\leftarrow z_i(t-1)$, otherwise $z_i(t)\leftarrow z'_i$
    end if
    for $j=1,\dots,M$ **do**
      Calculate $x'_j\in\arg\max_{r\in\mathcal{R}}\hat{V}_j(r;t-1)$
      if $V_j(z(t-1),x'_j,x_{-j}(t-1))\leq V_j(z(t-1),x_j(t-1),x_{-j}(t-1))$ **then**
        $x_j(t)\leftarrow x_j(t-1)$
      else
        With probability $1-p$, $x_j(t)\leftarrow x_j(t-1)$, otherwise $x_j(t)\leftarrow x'_j$
      end if
    end for
  end for
end for

4. LEARNING A NASH EQUILIBRIUM

In this section, we study the convergence of two learning algorithms, namely, joint strategy fictitious play and average strategy fictitious play, when used in car-truck congestion games.

4.1 Joint Strategy Fictitious Play

We start by briefly introducing the learning algorithm and, then, analyze its convergence.

4.1.1 Learning Algorithm Assume that the agents follow the joint strategy fictitious play algorithm (Marden et al., 2009). To do so, the agents must calculate the empirical average of their utility given the history of the decisions. Specifically, at each time step $t \in \mathbb{N}_0$, car $i \in [\![N]\!]$ should calculate $\hat{U}_i(r;t)$ using the recursive update law

$$\hat{U}_i(r;t) = (1-\lambda_t)\hat{U}_i(r;t-1) + \lambda_t U_i(r, z_{-i}(t), x(t)), \quad (8)$$

in which $\hat{U}_i(r;-1) = \xi_i^{\rm c}(r, T_i^{\rm c}), \forall r \in \mathcal{R}$. In (8), $z_{-i}(t)$ and $x(t)$ are the actions chosen by all the agents except car i at time step t. Furthermore, the forgetting factor $\lambda_t \in (0,1]$ shows the extent with which the agents forget the past in their decision making. In limiting cases, when $\lambda_t = 1$, the agents are myopic (and only remember the previous time steps) but, when $\lambda_t = 1/t$, the agents value the entire history of actions equally. Similarly, at each time step $t \in \mathbb{N}_0$, truck $j \in [\![M]\!]$ calculates $\hat{V}_j(r;t)$ using the recursive update law

$$\hat{V}_j(r;t) = (1-\lambda_t)\hat{V}_j(r;t-1) + \lambda_t V_j(r, x_{-j}(t), z(t)), \quad (9)$$

in which $\hat{V}_j(r;-1) = \xi_j^{\rm t}(r, T_j^{\rm t}), \forall r \in \mathcal{R}$. Now, by following Algorithm 1, one would expect to extract a Nash equilibrium.

4.1.2 Convergence Analysis Noting that with appropriate taxes the car–truck congestion game is a potential game, we can use the result of (Marden et al., 2009) to conclude the convergence of the learning algorithm.

Theorem 5. Let the action profile of the agents be generated by the joint strategy fictitious play in Algorithm 1. Assume that $\lambda_t = \lambda \in (0,1)$ or $\lambda_t = 1/t$ for all $t \in \mathbb{N}$. Then, this action profile almost surely converges to a pure strategy Nash equilibrium of the car–truck congestion game, if either the cars pay the congestion tax $p_i^{\rm c}(z,x)$ in (6) or the trucks receive the platooning subsidy $p_j^{\rm t}(x,z)$ in (7).

Proof: The proof is a consequence of combining Theorems 2.1 and 3.1 in (Marden et al., 2009) with Theorems 3 and 4. ■

Note that the joint strategy fictitious play might be restrictive in some aspects. For instance, all the agents must have access to all the individual decisions taken by the other agents to calculate their average cost function. In the next section, we adapt the average strategy fictitious play introduced in (Xiao et al., 2013) as an alternative. This learning algorithm requires a central node to broadcast the congestion prediction (i.e., an average of all the players actions) for all time intervals per day.

4.2 Average Strategy Fictitious Play

We introduce the average strategy fictitious play and study its convergence by extending parts of the proofs in (Xiao et al., 2013).

4.2.1 Learning Algorithm Before introducing the learning algorithm, we have to make the following standing assumption:

Assumption 1. The congestion tax policies satisfy
- $p_i^{\rm c}(z,x)$, $i \in [\![N]\!]$, is only a function of $n_{z_i}(x,z), m_{z_i}(x)$;
- $p_j^{\rm t}(x,z)$, $j \in [\![M]\!]$, is only a function of $n_{x_j}(x,z), m_{x_j}(x)$.

This assumption means that the congestion tax can only be a function of the traffic flow rather than the individual actions of the agents. The congestion taxing policy that we introduced in the previous section satisfies this assumption. To emphasize this fact, from now on, we write $p_i^{\rm c}(n_{z_i}(x,z), m_{z_i}(x))$ and $p_j^{\rm t}(n_{x_j}(x,z), m_{x_j}(x))$ with some abuse of notation.

Algorithm 2 Average strategy fictitious play for learning a Nash equilibrium.

Input: $p \in (0,1)$
Output: (x^*, z^*)
for $t = 1, 2, \dots$ **do**
  for $i = 1, \dots, N$ **do**
    Calculate $z_i' \in \arg\max_{r\in\mathcal{R}} \tilde{U}_i(r;t-1)$
    if $U_i(z_i', z_{-i}(t-1), x(t-1)) \le U_i(z_i(t-1), z_{-i}(t-1), x(t-1))$ **then**
      $z_i(t) \leftarrow z_i(t-1)$
    else
      With probability $1-p$, $z_i(t) \leftarrow z_i(t-1)$, otherwise $z_i(t) \leftarrow z_i'$
    end if
    for $j = 1, \dots, M$ **do**
      Calculate $x_j' \in \arg\max_{r\in\mathcal{R}} \tilde{V}_j(r;t-1)$
      if $V_j(z(t-1), x_j', x_{-j}(t-1)) \le V_j(z(t-1), x_j(t-1), x_{-j}(t-1))$ **then**
        $x_j(t) \leftarrow x_j(t-1)$
      else
        With probability $1-p$, $x_j(t) \leftarrow x_j(t-1)$, otherwise $x_j(t) \leftarrow x_j'$
      end if
    end for
  end for
end for

To initialize the algorithm, we let the agents to pick an arbitrary action from the set $\mathcal{R}$ at the first time step. We assume that there exists a central node [3] that can observe the traffic flow at each time interval. This central node uses the following recursive update laws to calculate the average number of the cars and trucks in each time interval

$$\bar{n}_r^{\rm c}(t) = (1-\lambda)\bar{n}_r^{\rm c}(t-1) + \lambda \sum_{\ell=1}^{N} \mathbf{1}_{\{z_\ell(t)=r\}},$$

$$\bar{n}_r^{\rm t}(t) = (1-\lambda)\bar{n}_r^{\rm t}(t-1) + \lambda \sum_{\ell=1}^{M} \mathbf{1}_{\{x_\ell(t)=r\}},$$

with $\bar{n}_r^{\rm c}(0) = \sum_{\ell=1}^{N} \mathbf{1}_{\{z_\ell(0)=r\}}$ and $\bar{n}_r^{\rm t}(0) = \sum_{\ell=1}^{M} \mathbf{1}_{\{x_\ell(0)=r\}}$ for all $r \in \mathcal{R}$. The superscripts c and t show that the aforementioned property is related to the cars or the trucks, respectively. In these recursive update laws, we should choose the forgetting factor $\lambda \in (0,1)$ to capture the extent with which we value the congestion information from the past. We can think of the numbers $\bar{n}_r^{\rm c}(t)$ and $\bar{n}_r^{\rm t}(t)$ as the forecasts that the central node (e.g., the department of transportation, the radio station, etc) announces on a day-to-day basis about the traffic flow for each time interval of the day. These values have a memory to remember the congestion in earlier days and get updated based on the actual observation of the traffic flow every midnight.

Additionally, car $i \in [\![N]\!]$ and truck $j \in [\![M]\!]$ also keep track of the average number of times that they have chosen any $r \in \mathcal{R}$ following the recursive update laws

$$\bar{w}_{r,i}^{\rm c}(t) = (1-\lambda)\bar{w}_{r,i}^{\rm c}(t-1) + \lambda \mathbf{1}_{\{z_i(t)=r\}},$$

$$\bar{w}_{r,j}^{\rm t}(t) = (1-\lambda)\bar{w}_{r,j}^{\rm t}(t-1) + \lambda \mathbf{1}_{\{x_j(t)=r\}},$$

with $\bar{w}_{r,i}^{\rm c}(0) = \mathbf{1}_{\{z_i(0)=r\}}$ and $\bar{w}_{r,j}^{\rm t}(0) = \mathbf{1}_{\{x_j(0)=r\}}$ for all $r \in \mathcal{R}$. Finally, for all $i \in [\![N]\!]$ and $j \in [\![M]\!]$, we define the new "average" cost functions in (10a)-(10b). Now, if

[3] This central node is assumed to be a not-for-profit organization. Therefore, it is not trying to optimize its income or loss (i.e., the summation of the received taxes or the distributed subsidies) and, hence, it would not strategically deviate from the intended algorithm.

$$\tilde{V}_j(r;t) = [a(\bar{n}^{\rm c}_r(t)+\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_{r,j}(t)+1)+b]+\beta[a(\bar{n}^{\rm c}_r(t)+\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_{r,j}(t)+1)+b]g(\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_{r,j}(t)+1)$$
$$+\xi^{\rm t}_j(r,T^{\rm t}_j)+p^{\rm t}_j(\bar{n}^{\rm c}_r(t)+\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_{r,j}(t)+1,\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_{r,j}(t)+1), \quad (10a)$$
$$\tilde{U}_i(r;t) = \xi^{\rm c}_i(r,T^{\rm c}_i)+[a(\bar{n}^{\rm c}_r(t)+\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm c}_{r,i}(t)+1)+b]+p^{\rm c}_i(\bar{n}^{\rm c}_r(t)+\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm c}_{r,i}(t)+1,\bar{n}^{\rm t}_r(t)). \quad (10b)$$

we follow Algorithm 2, we expect to converge to a Nash equilibrium under some mild conditions.

4.2.2 Convergence Analysis First, we need to prove an intermediate lemma which shows that if Algorithm 2 reaches a Nash equilibrium, it stays there forever.

LEMMA 6. Let each truck $j \in [\![M]\!]$ receive the subsidy
$$p^{\rm t}_j(x,z)=\beta(v_0-(an_{x_j}(z,x)+b))m_{x_j}(x),$$
for a given $v_0 \in \mathbb{R}$. If $x(t)$ and $z(t)$, generated by Algorithm 2, is a pure strategy Nash equilibrium, and $z_i(t) \in \arg\max_{r\in\mathcal{R}} \tilde{U}_i(r;t-1)$ for all $i \in [\![N]\!]$ and $x_j(t) \in \arg\max_{r\in\mathcal{R}} \tilde{V}_j(r;t-1)$ for all $j \in [\![M]\!]$, then $x(t')=x(t)$ and $z(t')=z(t)$ for all $t' \geq t$.

Proof: The proof of this lemma follows the same line of reasoning as in the proof of Proposition 4.2 in (Xiao et al., 2013). Here, we only prove the results for the trucks as the proof for the cars is technically the same. First, note that for all $r \in \mathcal{R}$, we get (11a)-(11b). Now, using these update laws and the proposed subsidy policy, we get
$$\begin{aligned}\tilde{V}_j(r;t) &= \xi^{\rm t}_j(r,T^{\rm t}_j)+a(\bar{n}^{\rm c}_r(t)+\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_r(t)+1)+b\\ &\quad+\beta v_0(\bar{n}^{\rm t}_r(t)-\bar{w}^{\rm t}_{r,j}(t)+1)\\ &= \xi^{\rm t}_j(r,T^{\rm t}_j)\\ &\quad+a(1-\lambda)(\bar{n}^{\rm c}_r(t-1)+\bar{n}^{\rm t}_r(t-1)-\bar{w}^{\rm t}_r(t-1))\\ &\quad+a(\lambda(n_r(x(t),z(t))-\mathbf{1}_{\{x_j(t)=r\}})+1)+b\\ &\quad+\beta v_0(1-\lambda)(\bar{n}^{\rm t}_r(t-1)-\bar{w}^{\rm t}_{r,j}(t-1))\\ &\quad+\beta v_0(\lambda(m_r(x(t))-\mathbf{1}_{\{x_j(t)=r\}})+1)\\ &= (1-\lambda)\tilde{V}_j(r;t-1)+\lambda V_j(r,x_{-j}(t),z(t)).\end{aligned}$$
Therefore, we can prove that
$$\begin{aligned}\tilde{V}_j(x_j(t);t)&=(1-\lambda)\tilde{V}_j(x_j(t);t-1)+\lambda V_j(x_j(t),x_{-j}(t),z(t))\\ &\geq(1-\lambda)\tilde{V}_j(r;t-1)+\lambda V_j(r,x_{-j}(t),z(t))\\ &=\tilde{V}_j(r;t)\end{aligned}$$
for any $r \in \mathcal{R}$, where the inequality is direct consequence of the fact that the pair $x(t)$ and $z(t)$ is a pure strategy Nash equilibrium and $x_j(t) \in \arg\max_{r\in\mathcal{R}} \tilde{V}_j(r;t-1)$ for all $j \in [\![M]\!]$. Thus, $x_j(t) \in \arg\max_{r\in\mathcal{R}} \tilde{V}_j(r;t)$ and as a result, we get $x_j(t+1)=x_j(t)$ (following Algorithm 2). Now, using a simple mathematical induction, we can show $x_j(t+k)=x_j(t)$ for all $k \in \mathbb{N}$. ■

THEOREM 7. Let the action profile of the agents be generated by the average strategy fictitious play in Algorithm 2. Then, this action profile almost surely converges to a pure strategy Nash equilibrium of the car–truck congestion game, if the trucks receive the platooning subsidy $p^{\rm t}_j(x,z)$ in (7).

Proof: The proof follows from using Theorem 4 and Lemma 6 in the proof of Theorem 4.1 in (Xiao et al., 2013). ■

5. NUMERICAL EXAMPLE

In order to illustrate the developed results, we use a numerical example with $N = 10000$ cars and $M = 100$ trucks. In (Farokhi and Johansson, 2013), a comprehensive simulation study on the interactions of the traffic flow and the platooning incentives can be found. Following (Farokhi and Johansson, 2013), we know that the affine function $v_r(z,x)=an_r(z,x)+b$, with $a=-0.0110$ and $b=84.9696$, describes the relationship between the average velocity of the traffic and the number of the vehicles (both cars and trucks) for the northbound E4 highway from Lilla Essingen to the end of Fredhällstunneln in Stockholm, Sweden (see Figure 1). We divide the time horizon of 7:00am-9:00am into eight equal non-overlapping intervals of 15 min to construct the action set $\mathcal{R}=\{1,\dots,8\}$. Let $T^{\rm c}_i$, $i \in [\![N]\!]$, and $T^{\rm t}_j$, $j \in [\![M]\!]$, be randomly chosen from the set $\mathcal{R}$ using the discrete distribution
$$\mathbb{P}\{T=n\}=\begin{cases}1/6, & n=2,4,\\ 1/4, & n=3,\\ 1/12, & \text{otherwise.}\end{cases}$$
This way, we can model a situation in which the drivers prefer to use the road between 7:30am-7:45am (i.e., it corresponds to a rush hour). Finally, let $\alpha^{\rm c}_i$, $i \in [\![N]\!]$, and $\alpha^{\rm t}_j$, $j \in [\![M]\!]$, be randomly generated according to a uniform distribution over $[-7.5,-2.5]$. In the rest of this section with the exception of Subsection 5.4, we consider the case where the cars must pay the congestion tax described in Theorem 3.

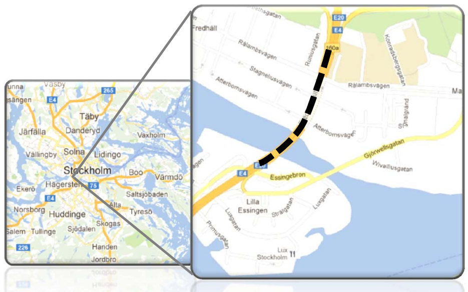

Fig. 1. The dashed black curve shows the northbound E4 highway between Lilla Essingen and Fredhällstunneln in Stockholm, Sweden.

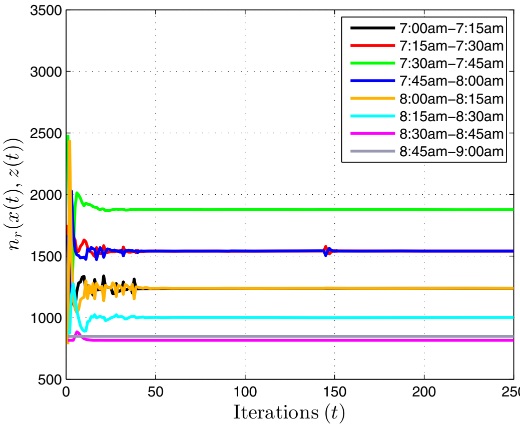

Fig. 2. $n_r(x(t),z(t))$, $r \in \mathcal{R}$, versus the iteration number for $\beta = 10^{-3}$ when using the joint strategy fictitious play in Algorithm 1.

$$
\begin{aligned}
\bar{n}_r^{\mathrm{c}}(t) + \bar{n}_r^{\mathrm{t}}(t) - \bar{w}_r^{\mathrm{t}}(t) &= (1-\lambda)\bar{n}_r^{\mathrm{c}}(t-1) + \lambda \sum_{\ell=1}^{N} \mathbf{1}_{\{z_\ell(t)=r\}} + (1-\lambda)\bar{n}_r^{\mathrm{t}}(t-1) + \lambda \sum_{\ell=1}^{M} \mathbf{1}_{\{x_\ell(t)=r\}} \\
&\quad - (1-\lambda)\bar{w}_{r,j}^{\mathrm{t}}(t-1) - \lambda \mathbf{1}_{\{x_j(t)=r\}} \\
&= (1-\lambda)(\bar{n}_r^{\mathrm{c}}(t-1) + \bar{n}_r^{\mathrm{t}}(t-1) - \bar{w}_r^{\mathrm{t}}(t-1)) + \lambda(n_r(x(t), z(t)) - \mathbf{1}_{\{x_j(t)=r\}}), \qquad \text{(11a)} \\
\bar{n}_r^{\mathrm{t}}(t) - \bar{w}_{r,j}^{\mathrm{t}}(t) &= (1-\lambda)\bar{n}_r^{\mathrm{t}}(t-1) + \lambda \sum_{\ell=1}^{M} \mathbf{1}_{\{x_\ell(t)=r\}} - (1-\lambda)\bar{w}_{r,j}^{\mathrm{t}}(t-1) - \lambda \mathbf{1}_{\{x_j(t)=r\}} \\
&= (1-\lambda)(\bar{n}_r^{\mathrm{t}}(t-1) - \bar{w}_{r,j}^{\mathrm{t}}(t-1)) + \lambda(m_r(x(t)) - \mathbf{1}_{\{x_j(t)=r\}}). \qquad \text{(11b)}
\end{aligned}
$$

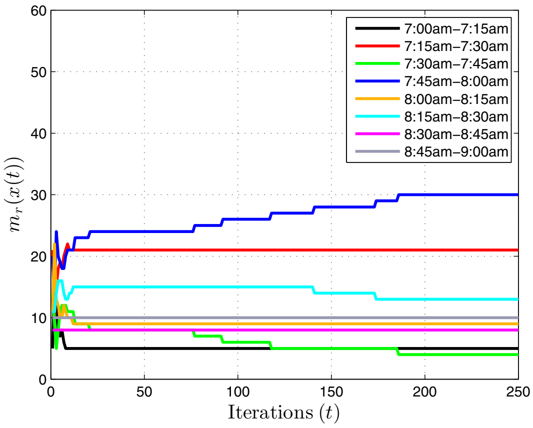

Fig. 3. $m_r(x(t))$, $r \in \mathcal{R}$, versus the iteration number for $\beta = 10^{-3}$ when using the joint strategy fictitious play in Algorithm 1.

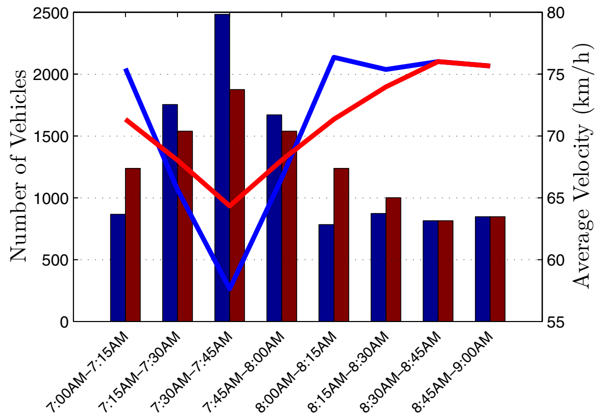

Fig. 4. Number of the vehicles and the average velocity in each time interval for the case where the drivers neglect the congestion in their decision making (blue) and for the case where they implement the learned Nash equilibrium (red).

5.1 *Joint Strategy Fictitious Play*

Let us start by considering the joint strategy fictitious play in Algorithm 1 with parameters $\beta = 10^{-3}$, $p = 0.4$, and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{N}_0$. Figure 2 shows the number of vehicles in each interval versus the iteration number. Considering the fact that there are $|\mathcal{R}|^{N+M} \simeq 10^{9100}$ various action combinations [4] in this example, the learning algorithm converges to a pure Nash equilibrium relatively fast in terms of the number of the iterations. Figure 3 shows the number of trucks in each interval as a function of the iteration number. As we can clearly see, at the learned equilibrium, thirty trucks use the same time interval to commute together. Figure 4 shows the number of the vehicles and the corresponding average velocity in each time interval. The blue color illustrates the case where the drivers do not consider the congestion in their decision making; i.e., $z_i = T_i^{\mathrm{c}}$ for all $i \in [\![N]\!]$ and $x_j = T_j^{\mathrm{t}}$ for all $j \in [\![M]\!]$. The red color denotes the case where the drivers implement the learned pure strategy Nash equilibrium. Evidently, the proposed congestion game increases the worse-case average velocity of the traffic flow by 12%.

[4] To put this number into perspective, recall that there are only around 10^{80} atoms in the visible universe.

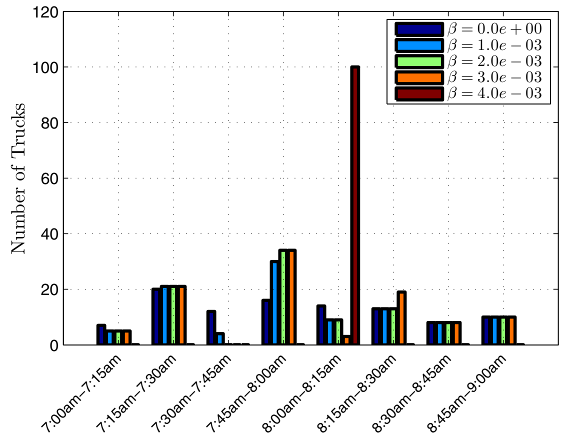

Fig. 5. Number of the trucks in each time interval for various choices of the coefficient β.

5.2 *Effect of the Fuel-Saving Coefficient*

Here, we demonstrate the effect of the fuel-saving coefficient β on the behavior of trucks. We perform all the simulations using the joint strategy fictitious play with $p = 0.4$ and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{N}_0$. Figure 5 illustrates the number of trucks in each time interval at the learned equilibrium for various choices of β. As we expect, when $\beta = 0$, the trucks are reluctant to commute at the same interval. However, as we increase the coefficient β, a higher number of trucks stick together. For $\beta = 4 \times 10^{-3}$, all hundred trucks commute during one time interval.

5.3 *Robustness of the Joint Strategy Fictitious Play*

Let us consider a scenario in which at iteration $t = 50$, an unexpected problem, like an accident, drastically decreases the average velocity during 7:15am-8:00am. To model this phenomenon, we assume that at $t = 50$, the average velocity is given by $(an_r(x(t), z(t)) + b)/10$ at $r = 2, 3, 4$. Figure 6 shows the number of the vehicles in each interval versus the iteration number. Note that the number of the vehicles that use $r = 2, 3, 4$ suddenly decreases after the disruption for a short while but the learning algorithm recovers fairly fast.

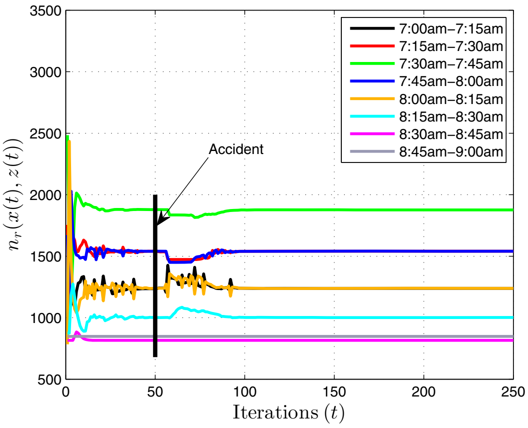

Fig. 6. $n_r(x(t), z(t))$, $r \in \mathcal{R}$, versus the iteration number when an unexpected behavior (e.g., an accident) disrupt the traffic flow on the fiftieth iteration.

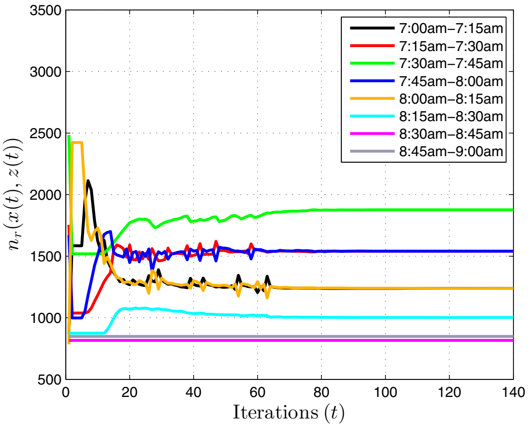

Fig. 7. $n_r(x(t), z(t))$, $r \in \mathcal{R}$, versus the iteration number for $\beta = 10^{-3}$ and $v_0 = 85$ when using the average strategy fictitious play in Algorithm 2.

5.4 Average Strategy Fictitious Play

In this subsection, we use the average strategy fictitious play in Algorithm 2 with $\beta = 10^{-3}$, $\lambda = 3 \times 10^{-2}$, and $p = 0.4$. We also implement the platooning subsidy in (7) with $v_0 = 85$. Figure 7 illustrates $n_r(x(t), z(t))$, $r \in \mathcal{R}$, versus the iteration number. The proposed algorithm clearly converges to a Nash equilibrium relatively fast.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we introduced an atomic congestion game with two types of agents to model the traffic flow on a road at certain time intervals. The first type of agents (cars) optimized its utility that was composed of a penalty for deviating from its preferred time interval, the average velocity of the traffic at the time of using the road, and the congestion tax (or subsidy). However, in addition to the aforementioned terms, the utility of the second type of agents (trucks) contained a term for modeling the potential platooning incentives (because of using the road at the same time as their peers). We showed that the introduced congestion game cannot be potential game unless we devise an appropriate congestion taxing policy. We used joint strategy fictitious play and average strategy fictitious play to learn a pure strategy Nash equilibrium of the introduced congestion game. Finally, we illustrated the results on a numerical example. As a future work, we can study the price of anarchy (or the price of stability) to study the inefficiency of the acquired pure strategy Nash equilibrium.

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REFERENCES

Alam, A., Gattami, A., and Johansson, K.H. (2010). An experimental study on the fuel reduction potential of heavy duty vehicle platooning. In *Proceedings of the 13th International IEEE Conference on Intelligent Transportation Systems*, 306–311.

Christodoulou, G. and Koutsoupias, E. (2005). The price of anarchy of finite congestion games. In *Proceedings of the 37th Annual ACM Symposium on Theory of Computing*, 67–73. ACM.

Correa, J., Schulz, A., and Stier-Moses, N. (2005). On the inefficiency of equilibria in congestion games. *Integer Programming and Combinatorial Optimization*, 171–177.

Farokhi, F. and Johansson, K.H. (2013). A game-theoretic framework for studying truck platooning incentives. In *Proceedings of the 16th International IEEE Conference on Intelligent Transportation Systems*, 1253–1260.

Fuglestvedt, J., Berntsen, T., Myhre, G., Rypdal, K., and Skeie, R. (2008). Climate forcing from the transport sectors. *Proceedings of the National Academy of Sciences*, 105(2), 454–458.

Gibbons, R. (1992). *Game Theory for Applied Economists*. Princeton University Press.

Jackson, M.O. (2003). Mechanism theory. In U. Derigs (ed.), *Optimization and Operations Research*, Encyclopedia of Life Support Systems. EOLSS Publishers, Oxford, UK.

Levinson, D. (2005). Micro-foundations of congestion and pricing: A game theory perspective. *Transportation Research Part A: Policy and Practice*, 39(7–9), 691–704.

Marden, J.R., Arslan, G., and Shamma, J.S. (2009). Joint strategy fictitious play with inertia for potential games. *IEEE Transactions on Automatic Control*, 54(2), 208–220.

Mitra, D. and Mazumdar, A. (2007). Pollution control by reduction of drag on cars and buses through platooning. *International Journal of Environment and Pollution*, 30(1), 90–96.

Monderer, D. and Shapley, L. (1996). Potential games. *Games and Economic Behavior*, 14(1), 124–143.

Rosenthal, R.W. (1973a). The network equilibrium problem in integers. *Networks*, 3(1), 53–59.

Rosenthal, R.W. (1973b). A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory*, 2(1), 65–67.

Roughgarden, T. (2007). Routing games. In N. Nisan, T. Roughgarden, E. Tardos, and V.V. Vazirani (eds.), *Algorithmic game theory*. Cambridge University Press.

Schmeidler, D. (1973). Equilibrium points of nonatomic games. *Journal of Statistical Physics*, 7(4), 295–300.

Xiao, N., Wang, X., Wongpiromsarn, T., You, K., Xie, L., Frazzoli, E., and Rus, D. (2013). Average strategy fictitious play with application to road pricing. In *Proceedings of the American Control Conference*, 1923–1928.