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Automatic Generation of Feedforward Controllers using Dynamic Local Model Networks ⋆

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Abstract: A new approach for the automatic generation of a dynamic feedforward control law for nonlinear dynamic systems represented by discrete-time local model networks (LMN) is proposed. The generic model structure of LMNs offers the opportunity to apply such a general and automated approach for model inversion, even when the overall model complexity may be high. LMNs can represent nonlinear dynamic systems of almost arbitrary complexity. Their generic structure allows the generation of a feedback linearizing input transformation in a highly automated way. This paper proposes and discusses such an approach for the important class of LMNs with minimum-phase property. As a subclass of the class of minimum-phase LMNs, only those without numerator dynamics are considered in this manuscript. By using the input transformation, which results from feedback linearization, the feedforward control law is obtained. It can then be applied online for any reference trajectory without pre-planning. Thus, by representing a nonlinear dynamic system by the generic structure of an LMN and applying the proposed feedforward control law generation, a dynamic feedforward control for such a nonlinear system can be found automatically. Finally, the effectiveness of the method is shown on results for a Wiener model.

Keywords: Nonlinear Systems; Local Model Networks; Feedforward Control; Automatic Controllers; Feedback Linearization.

1. INTRODUCTION

The automatic generation of models from measured input-output data is nowadays an established approach in many engineering disciplines (e.g. Sjoeberg et al. (1995); Murray-Smith and Johansen (1997); Norgaard et al. (2000); Nelles (2001); Ljung (2010)). Commonly, such models are used to simulate the real process for various purposes, such as emission prediction in internal combustion engines (Maass et al., 2009), controlling heat exchangers (Novak and Bobal, 2009) or model predictive control in general (Townsend and Irwin, 2001), to name but a few. In recent years, significant research efforts have been made to also exploit the structure of nonlinear dynamic models in order to facilitate the design of control systems, Hametner et al. (2014); Gao (2004). When control tasks are considered, nonlinear model structures such as local model networks (LMN) can be used to determine control laws and their parameters, e.g. Hametner et al. (2013); Hafner et al. (2000). Often a feedback control strategy is combined with feedforward control (a so-called two-degree-of-freedom controller) to improve reference tracking performance. For reasons of simplicity, feedforward control is sometimes restricted to a static system inversion and only steady state input-output mapping is considered. To obtain a dynamic feedforward control law, usually some kind of model inversion has to be performed. Inspired by Silverman (1969), who investigated invertibility for time varying linear systems, Hirschorn (1979) extended the basic principles of system inversion to nonlinear systems. A historical perspective of this wide field as well as a detailed review of the strongly related feedback linearization technique is given in Isidori (1995) or Slotine and Li (1991). When the inversion is intended for output tracking, the reference trajectory is frequently assumed to be known in advance. This provides the opportunity to achieve at least approximate tracking even for non-minimum-phase systems (Devasia et al., 1996; Getz, 1995). Feedback linearization for discrete-time systems has been addressed for example by Lee et al. (1987); Monaco and Normand-Cyrot (1987) or Grizzle (1986).

In this manuscript the fully automated generation of a nonlinear dynamic feedforward control law is proposed for a discrete-time LMN. LMNs are a well-established multiple-model approach for data-driven modelling of nonlinear systems (e.g. Gregorcic (2004); Hametner and Jakubek (2011); Nelles (2001)). This model architecture

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interpolates between different local models, each valid in a certain operating regime which offers a versatile structure for the identification of nonlinear dynamic systems. Each operating regime represents a simple model, e.g. a linear regression model (Murray-Smith and Johansen, 1997), whose parameters are found by identification. Although the complexity of LMNs increases with the amount of local linear models to form a sophisticated nonlinear model, the model structure still remains generic. This fact is beneficially exploited when automatically generating a dynamic feedforward control law for arbitrarily complex LMNs. As a first step, the present contribution focuses on LMNs without numerator dynamics to evade the difficulty of internal dynamics or non-minimum-phase behavior. Neglecting the numerator dynamics completely is a justifiable choice for many physical processes. Even though explicit numerator dynamics are absent, the partitioning has to be chosen accordingly in order to avoid an additional excitation through the local affine term (see Section 2).

Obviously, neither LMNs nor feedback linearization are a conceptual novelty. Both concepts have already been introduced several decades ago and are well established ever since. However, combining both ideas offers the opportunity to provide a substantial tool for dynamic feedforward control. Numerous applications in various branches of the industry benefit of the presented approach as merely adequate measured input-output data are required to identify a model (i.e. an LMN) of almost any arbitrary nonlinear dynamic process. To automatically obtain a feedforward control law for this process, the LMN is represented in discrete-time state-space form, which is then transformed into a feedback linearized normal representation. To determine the required feedforward input value for the desired reference trajectory, an input transformation is utilized. Therein the current and past model outputs are replaced by the desired reference values. Additionally, the resulting feedforward control law can be implemented online, thus no trajectory pre-planning is required. The application of the presented approach generally leads to good results concerning tracking performance. In the results section, the effectiveness of the approach is demonstrated on a Wiener model.

Feedforward control of LMNs has been considered in literature before. Karer et al. (2011) applied feedforward control to a dynamic hybrid fuzzy model of a batch reactor with both discrete and continuous states. Therein the partitioning considers the output only and the validity functions are triangular. In the present contribution also the input can be used as a dimension of the partition space, which is an important prerequisite for the partitioning of many nonlinearities. In addition, a hierarchical discriminant tree yields the validity functions instead of utilizing fuzzy rules. Nentwig and Mercorelli (2008) proposed an algorithm for a combined analytical/numerical inversion of a static fuzzy neural network applied to a throttle valve control. In contrast, the presented approach in this paper also holds for dynamic LMNs and in addition directly incorporates nonlinear validity functions into the automatic feedforward control law generation.

Subsequently, the generic model structure of LMNs is shortly reviewed in Section 2. Feedback linearization in general, its application to the LMN and the resulting feedforward control law is demonstrated in Section 3. Results for the Wiener model are given in Section 4.

2. LOCAL MODEL NETWORKS

The architecture of dynamic local model networks is depicted in Fig. 1. First, an ordered set for the indices

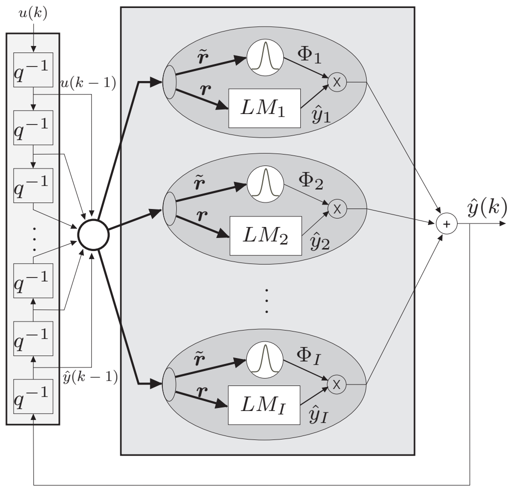

Fig. 1. Architecture of a local model network

of the local models is defined:

$$\mathcal{I} = (i \in \mathbb{N} \,|\, 1 \leq i \leq I) \tag{1}$$

where I denotes the number of local linear models. LMNs with external dynamics have an input vector $\boldsymbol{r}(k)$, usually called regressor, with past inputs and outputs according to Fig. 1:

$$\boldsymbol{r}(k) = [\boldsymbol{u}(k-\mathcal{M}) \ \hat{\boldsymbol{y}}(k-\mathcal{N}) \ 1], \quad \boldsymbol{r}(k) \in \mathbb{R}^{1\times O} \tag{2}$$

where O denotes the dimension of the regressor vector. The sets for the orders $\mathcal{M}$ of the used time delays of the inputs and for the feedback output orders $\mathcal{N}$ may be user defined. The system order is denoted by $N = \max(\mathcal{N})$ and the maximum time shift of the input by $M = \max(\mathcal{M})$. The number of elements of the sets $\mathcal{M}$ and $\mathcal{N}$ is referred to as $|\mathcal{M}|$ and $|\mathcal{N}|$, respectively. Thus, $O = |\mathcal{M}| + |\mathcal{N}| + 1$ holds.

From Fig. 1 it becomes obvious that the input vector $\tilde{\boldsymbol{r}}(k)$ of the validity functions $\Phi_{\mathcal{I}}(\tilde{\boldsymbol{r}}(k))$, which spans the so-called partition space, can be chosen differently from the input vector $\boldsymbol{r}(k)$ of the local models:

$$\tilde{\boldsymbol{r}}(k) = \left[\boldsymbol{u}(k-\tilde{\mathcal{M}}) \ \hat{\boldsymbol{y}}(k-\tilde{\mathcal{N}})\right], \quad \tilde{\boldsymbol{r}}(k) \in \mathbb{R}^{1\times\tilde{O}}. \tag{3}$$

The sets $\tilde{\mathcal{M}}$ and $\tilde{\mathcal{N}}$ are usually subsets of $\mathcal{M}$ and $\mathcal{N}$. They are also user defined.

The scalar local model outputs

$$\hat{y}_i(k) = \boldsymbol{r}(k)\boldsymbol{\theta}_i \quad \forall \mathcal{I} \tag{4}$$

with the local parameter vector $\boldsymbol{\theta}_i \in \mathbb{R}^{O\times 1}$ containing the local parameters $\boldsymbol{b}_{\mathcal{M}}^{(i)}$ of the input, the local parameters $\boldsymbol{a}_{\mathcal{N}}^{(i)}$ of the autoregressive part and the local affine term $c^{(i)}$ of model i

$$\boldsymbol{\theta}_i^{\mathrm{T}} = \left[\boldsymbol{b}_{\mathcal{M}}^{(i)} \ \boldsymbol{a}_{\mathcal{N}}^{(i)} \ c^{(i)}\right] \forall \mathcal{I} \tag{5}$$

are used subsequently to form the global model output $\hat{y}(k)$ by weighted aggregation, see Fig. 1:

$$\hat{y}(k) = \sum_{\mathcal{I}} \Phi_i(\tilde{\boldsymbol{r}}(k))\hat{y}_i(k). \tag{6}$$

The validity functions, which are found by a hierarchical discriminant tree (Jakubek and Hametner, 2009), are constrained to form a partition of unity

$$\sum_{\mathcal{I}} \Phi_i = 1 \tag{7}$$

$$0 \leq \Phi_i \leq 1, \quad \forall \mathcal{I}. \tag{8}$$

To ensure its minimum-phase property, two restrictions are imposed on the LMN. First, no numerator dynamics are considered, thus the limit $|\mathcal{M}| = 1$ holds. Additionally, either no partitioning using the input is taken into account (i.e. $\tilde{\mathcal{M}}$ is the empty set) or the order of the input used in the partition space must be chosen equally to the input order in the regressor (i.e. $\tilde{\mathcal{M}} = \mathcal{M}$). Otherwise two effects could occur. On the one hand, when $\tilde{\mathcal{M}} \neq \mathcal{M}$, a nonlinear term containing different orders of the input appears in the numerator. On the other hand, an excitation by the local affine term in combination with the validity functions would be possible, which acts as an additional input $\sum_{\mathcal{I}} \Phi_i(\tilde{\boldsymbol{r}}(k))c^{(i)}$ containing a different order of the input in $\tilde{\boldsymbol{r}}(k)$ than in the regressor itself. As the minimum-phase behavior would not be guaranteed anymore in either of these cases, only systems which fulfill $|\mathcal{M}| = 1$ and $\tilde{\mathcal{M}} = \mathcal{M}$ if input partitioning is used, are considered subsequently.

2.1 State-Space Formulation of LMNs

To apply the feedback linearization technique, the LMN is described by a (non-minimum-realisation) state-space system of the form

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(\boldsymbol{\Phi}_k)\boldsymbol{x}(k) + \boldsymbol{B}(\boldsymbol{\Phi}_k)u(k) + \boldsymbol{f}(\boldsymbol{\Phi}_k) \\ \hat{y}(k) &= \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(k). \end{aligned} \tag{9}$$

The state vector $\boldsymbol{x} \in \mathbb{R}^{(M-1+N)\times 1}$ is defined as

$$\boldsymbol{x}(k) = \begin{bmatrix} u(k-M+1) \\ \vdots \\ u(k-1) \\ \hline \hat{y}(k-N+1) \\ \vdots \\ \hat{y}(k) \end{bmatrix}. \tag{10}$$

Note, that due to the time shifted evaluation of the update equation for $\boldsymbol{x}(k+1)$ in (9) (i.e. for $\hat{y}(k+1)$ in (6) respectively), the validity functions are determined using $\tilde{\boldsymbol{x}}(k) = \tilde{\boldsymbol{r}}(k+1)$. Their notation in vector form is

$$\boldsymbol{\Phi}_k = \boldsymbol{\Phi}(\tilde{\boldsymbol{x}}(k)) = [\Phi_1 \cdots \Phi_i \cdots \Phi_I]^{\mathrm{T}}. \tag{11}$$

The system matrix $\boldsymbol{A}(\boldsymbol{\Phi}_k) \in \mathbb{R}^{(M-1+N)\times(M-1+N)}$ is time-variant and contains the time shifts of the input (above the dashed line) as well as those of the output

$$\boldsymbol{A}(\boldsymbol{\Phi}_k) = \begin{bmatrix} \underline{\boldsymbol{I}}_{M-1} & \boldsymbol{0}_{M-1\times N} \\ \hline \boldsymbol{0}_{N-1\times M-1} & \overline{\boldsymbol{I}}_N \\ \boldsymbol{b}^{\mathrm{T}}(\boldsymbol{\Phi}_k) & \boldsymbol{a}^{\mathrm{T}}(\boldsymbol{\Phi}_k) \end{bmatrix}, \tag{12}$$

where abbreviations are defined as

$$\overline{\boldsymbol{I}}_j = [\boldsymbol{0}_{j-1\times 1} \ \mathbf{I}_{j-1}], \qquad \overline{\boldsymbol{I}}_j \in \mathbb{R}^{(j-1)\times j} \tag{13}$$

$$\underline{\boldsymbol{I}}_j = \begin{bmatrix} [\boldsymbol{0}_{j-1\times 1} \ \mathbf{I}_{j-1}] \\ \boldsymbol{0}_{1\times j} \end{bmatrix}, \ \underline{\boldsymbol{I}}_j \in \mathbb{R}^{j\times j} \tag{14}$$

for an arbitrary index j with $\mathbf{I}$ denoting the identity matrix.

The last row of (12) contains the parameters $\boldsymbol{b}^{\mathrm{T}}(\boldsymbol{\Phi}_k) \in \mathbb{R}^{1\times(M-1)}$ of the input and of the autoregressive part $\boldsymbol{a}^{\mathrm{T}}(\boldsymbol{\Phi}_k) \in \mathbb{R}^{1\times N}$ in the form

$$\boldsymbol{b}^{\mathrm{T}}(\boldsymbol{\Phi}_k) = [b_M(\boldsymbol{\Phi}_k) \ \cdots \ b_3(\boldsymbol{\Phi}_k) \ b_2(\boldsymbol{\Phi}_k)] \tag{15}$$

$$\boldsymbol{a}^{\mathrm{T}}(\boldsymbol{\Phi}_k) = [a_N(\boldsymbol{\Phi}_k) \ \ldots \ a_1(\boldsymbol{\Phi}_k)]. \tag{16}$$

Each term in (15) and (16) is found by weighted aggregation of the corresponding local parameter. In addition, the parameters of those orders, which do not appear in the original regressor $\boldsymbol{r}(k)$, are set to zero. Thus, the individual entry $b_m(\boldsymbol{\Phi}_k)$ with index $\{m \in \mathbb{N} \,|\, 2 \leq m \leq M\}$ in (15) and $a_n(\boldsymbol{\Phi}_k)$ with index $\{n \in \mathbb{N} \,|\, 1 \leq n \leq N\}$ in (16) are found according to

$$b_m(\boldsymbol{\Phi}_k) = \begin{cases} \sum_{\mathcal{I}} \Phi_i(\tilde{\boldsymbol{x}}(k))\, b^{(i)}_{M+1-m} & m \in \mathcal{M} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$a_n(\boldsymbol{\Phi}_k) = \begin{cases} \sum_{\mathcal{I}} \Phi_i(\tilde{\boldsymbol{x}}(k))\, a^{(i)}_{N+1-n} & n \in \mathcal{N} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

The input matrix $\boldsymbol{B}(\boldsymbol{\Phi}_k) \in \mathbb{R}^{(M-1+N)\times 1}$ is defined as

$$\boldsymbol{B}(\boldsymbol{\Phi}_k) = \begin{bmatrix} \boldsymbol{0}_{M-2\times 1} \\ 1 \\ \hline \boldsymbol{0}_{N-1\times 1} \\ b_1(\boldsymbol{\Phi}_k) \end{bmatrix}. \tag{19}$$

The term $\boldsymbol{f}(\boldsymbol{\Phi}_k) \in \mathbb{R}^{(M-1+N)\times 1}$ introduces a validity function dependent offset term

$$\boldsymbol{f}(\boldsymbol{\Phi}_k) = \begin{bmatrix} \boldsymbol{0}_{M-2+N\times 1} \\ \sum_{\mathcal{I}} \Phi_i(\tilde{\boldsymbol{x}}(k))c^{(i)} \end{bmatrix}. \tag{20}$$

The output matrix $\boldsymbol{c}^{\mathrm{T}} \in \mathbb{R}^{1\times(M-1+N)}$ is constant

$$\boldsymbol{c}^{\mathrm{T}} = [\boldsymbol{0}_{1\times M-2+N} \ \ 1]. \tag{21}$$

3. FEEDFORWARD CONTROL

3.1 Feedback Linearization of Discrete-Time Systems

The input-output linearization problem for a general discrete-time nonlinear non-affine system

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{F}(\boldsymbol{x}(k), u(k)) \\ y(k) &= h(\boldsymbol{x}(k)) \end{aligned} \tag{22}$$

is shortly reviewed (adapted from Henson and Seborg (1997)). The composition of the scalar function $h(\boldsymbol{x}) : \mathbb{R}^{d\times 1} \to \mathbb{R}$ and the vector function $\boldsymbol{F}(\boldsymbol{x}) : \mathbb{R}^{d\times 1} \to \mathbb{R}^{d\times 1}$ with d describing the system dimension, is defined as: $h \circ \boldsymbol{F}(\boldsymbol{x}) = h(\boldsymbol{F}(\boldsymbol{x}))$. Higher order compositions are defined recursively: $h \circ \boldsymbol{F}^j(\boldsymbol{x}) = h \circ \boldsymbol{F}^{j-1}(\boldsymbol{F}(\boldsymbol{x}))$, where $h \circ \boldsymbol{F}^0(\boldsymbol{x}) = h(\boldsymbol{x})$. The composition operator plays the same role as does the Lie derivative in the continuous-time case.

The discrete-time system (22) is said to have relative degree δ at the point $(\boldsymbol{x}_0, u_0)$ if:

- $\frac{\partial}{\partial u(k)} h \circ \boldsymbol{F}^j(\boldsymbol{x}(k), u(k)) = 0$ for all $(\boldsymbol{x}, u)$ in a neighborhood of $(\boldsymbol{x}_0, u_0)$ and all $j \leq \delta - 1$.
- $\frac{\partial}{\partial u(k)} h \circ \boldsymbol{F}^\delta(\boldsymbol{x}_0, u_0) \neq 0$.

Thus, by definition of the relative degree all compositions fulfilling $j \leq \delta - 1$ are independent of the current input $u(k)$ and can be written as

$$h \circ \boldsymbol{F}^j(\boldsymbol{x}(k), u(k)) = h \circ \boldsymbol{F}_0^j(\boldsymbol{x}(k)), \quad 1 \leq j \leq \delta - 1. \quad (23)$$

To represent the system (22) in normal form, a diffeomorphism $[\boldsymbol{\xi}^{\mathrm{T}}(k), \boldsymbol{\eta}^{\mathrm{T}}(k)]^{\mathrm{T}} = \boldsymbol{\Gamma}(\boldsymbol{x}(k))$ defining the new coordinates $\boldsymbol{\xi}(\cdot)$ and $\boldsymbol{\eta}(\cdot)$ is constructed as follows. The $\boldsymbol{\xi}(\cdot)$ coordinates are chosen as

$$\xi_j(k) = h \circ \boldsymbol{F}_0^{j-1}(\boldsymbol{x}(k)), \quad 1 \leq j \leq \delta. \quad (24)$$

The remaining $d - \delta$ variables $\eta_j(k) = \Gamma_{\delta+j}(\boldsymbol{x}(k)),\ 1 \leq j \leq d-\delta$ can be chosen arbitrarily such that $\boldsymbol{\Gamma}$ is invertible and $\frac{\partial}{\partial u(k)}\Gamma_j \circ \boldsymbol{F}(\boldsymbol{x}(k), u(k)) = 0$. As a result, the system in normal form is

$$\begin{aligned}
\xi_1(k+1) &= \xi_2(k) \\
\xi_2(k+1) &= \xi_3(k) \\
&\vdots \\
\xi_\delta(k+1) &= h \circ \boldsymbol{F}^\delta(\boldsymbol{\Gamma}^{-1}(\boldsymbol{\xi}(k), \boldsymbol{\eta}(k), u(k))) = v(k) \\
\boldsymbol{\eta}(k+1) &= \boldsymbol{q}(\boldsymbol{\xi}(k), \boldsymbol{\eta}(k)) \\
y(k) &= \xi_1(k)
\end{aligned} \quad (25)$$

where $\boldsymbol{q}(\boldsymbol{\xi}(k), \boldsymbol{\eta}(k))$ represents the unobservable internal dynamics, which is independent of $u(\cdot)$ by construction:

$$q_j(\boldsymbol{\xi}(k), \boldsymbol{\eta}(k)) = \Gamma_{\delta+j} \circ \boldsymbol{F}(\boldsymbol{\Gamma}^{-1}(\boldsymbol{\xi}(k), \boldsymbol{\eta}(k))), \quad 1 \leq j \leq d-\delta. \quad (26)$$

The system (25) can be considered as a chain of δ time-shifts with output $y(k)$ and input $v(k)$. The latter is found by a nonlinear algebraic equation representing the input transformation

$$v(k) = h \circ \boldsymbol{F}^\delta(\boldsymbol{x}(k), u(k)). \quad (27)$$

As in the continuous-time case, for any control law the unobservable internal dynamics need to be asymptotically stable to achieve closed-loop stability. The original system (22) has full relative degree if the relative degree equals the system order, i.e. $\delta = d$ holds, and no internal dynamics exists.

3.2 Feedback Linearization of Local Model Networks

To determine the relative degree δ of the LMN, its state-space representation (9) is considered such that the general system (22) becomes

$$\boldsymbol{F}(\boldsymbol{x}(k), u(k)) = \boldsymbol{A}(\boldsymbol{\Phi}_k)\boldsymbol{x}(k) + \boldsymbol{B}(\boldsymbol{\Phi}_k)u(k) + \boldsymbol{f}(\boldsymbol{\Phi}_k) \quad (28)$$

$$h(\boldsymbol{x}(k)) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(k) = x_{M-1+N}(k). \quad (29)$$

According to the definition of the relative degree

$$\frac{\partial}{\partial u(k)} h \circ \boldsymbol{F}^j(\boldsymbol{x}(k), u(k)) = 0, \quad \forall j \leq \delta - 1 \quad (30)$$

must hold. This is fulfilled under two conditions

- $b_j(\boldsymbol{\Phi}_k) = 0, \quad \forall \boldsymbol{\Phi}_k, \quad \forall j \leq \delta - 1$
- $\dfrac{\partial \boldsymbol{\Phi}_{k+j-1}}{\partial u(k)} = 0 \rightarrow \dfrac{\partial}{\partial u(k)}\tilde{\boldsymbol{x}}(k+j-1) = 0, \quad \forall j \leq \delta - 1$

With the definitions $\tilde{\boldsymbol{x}}(k) = \tilde{\boldsymbol{r}}(k+1)$ and (3) the second condition can be reformulated as

$$j - \min(\tilde{\mathcal{M}}) < 0, \quad \forall j \leq \delta - 1 \quad (31)$$

Thus, the relative degree δ is determined primarily by the index of the first non-zero numerator parameter b_j (i.e. $b_j = 0,\ \forall j < \delta$) such that $\delta = \min(\mathcal{M})$. If input partitioning is used, additionally $\delta = \min(\tilde{\mathcal{M}})$ must hold as the minimum-phase assumption was introduced, which required $\tilde{\mathcal{M}} = \mathcal{M}$.

3.3 Feedforward Control Law Generation

Due to its generic model structure, the normal form representation (25) of a minimum-phase LMN is readily and automatically available by applying the system transformation described in the previous section. For such a normal system, the implementation of dynamic feedforward control is straightforward. If the transformed input (27) is chosen such that $v(k) = w(k+\delta)$, where $w(\cdot)$ describes the desired output trajectory, exact tracking $\hat{y}(k+\delta) = w(k+\delta)$ can be achieved. The original input $u(k)$ is found by solving

$$w(k+\delta) - h \circ \boldsymbol{F}^\delta(\boldsymbol{w}(k), u(k)) = 0, \quad (32)$$

where $\boldsymbol{w}(\cdot)$ denotes the state vector $\boldsymbol{x}(\cdot)$ with the values of $\hat{y}(\cdot)$ being replaced by the reference values $w(\cdot)$. In case of input partitioning $\tilde{\mathcal{M}} = \mathcal{M}$ is required and (32) is an implicit equation. It is then solved numerically, whereas in case of output partitioning only, an explicit solution is possible.

The feedforward control law (32) can be applied online without the need for an offline trajectory pre-planning, although for big changes of the reference (e.g. a step function with a high amplitude compared to the identified output interval), a high control input may result, which possibly violates input constraints. The easiest way to avoid such violations is to incorporate a low-pass filter for the reference value with appropriate time constant ahead of the feedforward control.

4. RESULTS

To demonstrate the effectiveness of the proposed automatic dynamic feedforward control law generation, a third order stable Wiener process is considered. It consists of a dynamic linear transfer function $G(z) = P(z)/U(z)$ in cascade with a static nonlinearity $f(p(k))$ at the output with $p(k)$ as the intermediate variable at the output of the linear part. In the present simulation, $G(z)$ and $f(p(k))$ have been chosen as

$$G(z) = \frac{0.6z^{-3}}{1 - 1.3z^{-1} + 0.8825z^{-2} - 0.1325z^{-3}} \quad (33)$$

$$y(k) = f(p(k)) = \arctan(p(k)). \quad (34)$$

As the linear transfer function has full relative degree, the LMN numerator and denumerator orders are chosen as $\mathcal{M} = \{3\}$ and $\mathcal{N} = \{1,2,3,4\}$. The partition space is spanned by $\tilde{\boldsymbol{r}}(k) = [u(k-3),\ \hat{y}(k-1)]$, thus $\tilde{\mathcal{M}} = \{3\}$ and $\tilde{\mathcal{N}} = \{1\}$. Identification of an LMN with six local linear models using an APRB-signal for the excitation in $u(\cdot)$ yields a model fit of $R^2 = 98.81\%$ in validation. In Fig. 2 the identification data are shown and local models are represented by contour lines of their validity functions. Those models that are intersected by the equilibrium line are equilibrium models whereas models number five and six are off-equilibrium models with unstable local dynamics (Jakubek et al., 2008). As it is the minimum-phase property only, which has to be guaranteed, feedforward control can be applied.

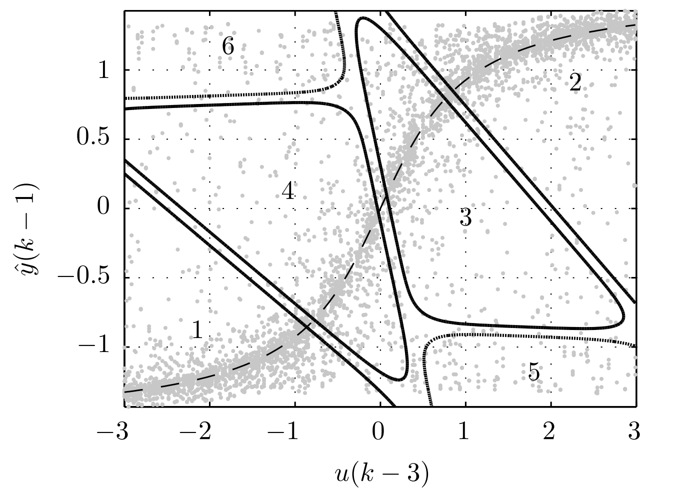

Fig. 2. Contour plot of validity functions, the identification data (grey dots) and the equilibrium line (dashed line)

For the chosen orders of the LMN the following implicit feedforward control law results

$$\begin{aligned} -w(k+3) + \boldsymbol{c}^{\mathrm{T}} \boldsymbol{A}(\boldsymbol{\Phi}_{k+2})\boldsymbol{A}(\boldsymbol{\Phi}_{k+1})\boldsymbol{A}(\boldsymbol{\Phi}_{k})\boldsymbol{w}(k) + \\ \left(a_2(\boldsymbol{\Phi}_{k+2}) + a_1(\boldsymbol{\Phi}_{k+2})a_1(\boldsymbol{\Phi}_{k+1})\right) c(\boldsymbol{\Phi}_{k}) + \\ a_1(\boldsymbol{\Phi}_{k+2})c(\boldsymbol{\Phi}_{k+1}) + c(\boldsymbol{\Phi}_{k+2}) + b_3(\boldsymbol{\Phi}_{k+2})u(k) = 0. \end{aligned} \tag{35}$$

Note that besides the explicit appearance of $u(k)$ in (35) also the validity function $\boldsymbol{\Phi}_{k+2}$ is a function of $u(k)$. Thus, in case of input partitioning, a numerical solution to (35) has to be found, whereas for output partitioning only, an explicit feedforward control law would result. Subsequently, all results are found with feedforward control only, thus no feedback information is used at all. As reference trajectory $w(\cdot)$, which is shown as dots in the upper panel of Fig. 3, a sequence of low-pass filtered steps is considered. The time constant of the filter is chosen faster than that of the Wiener model to excite the system sufficiently, but yet slow enough to ensure an adequately smooth signal. The reference is congruent to the feedforward controlled simulation result $\hat{y}(\cdot)$ of the LMN, which is represented by a solid line in the upper panel. In the middle panel of Fig. 3 the feedforward control input signal can be seen. Because of the low-pass filtered appearance of the reference, it remains within the interval of $[-3, 3]$ used for identification, thus no model extrapolation occurs. Applying this input signal to the actual Wiener model leads to the system response $y_{\mathrm{Wiener}}(\cdot)$ depicted as solid line in the lower panel of Fig. 3 together with the desired reference as dotted line. Note, that no delay in the reference tracking occurs. Slight oscillations appearing around sample 50 and 70 are caused by model discrepancies only and can be reduced by using an LMN with a higher fit to represent the non-linear process more precisely.

To further illustrate the effective operation of the automatically generated feedforward control law, Fig. 4 shows the result for a different reference trajectory. The reference $w(\cdot)$ is depicted as dotted line and the feedforward controlled output $y_{\mathrm{Wiener}}(\cdot)$ of the Wiener model as solid line. Those intervals where one of the off-equilibrium models becomes active by more than 1‰ are shaded in grey. In the first 225 samples the transitions in the reference are chosen very smooth. As a result, only slight steady state deviations occur, which could be eliminated by feedback control very easily. Starting from sample 226 the transitions are chosen fast as compared to the time constant of the linear part of the Wiener model. Merely around sample 390 and 430 the nonlinearity is not covered by the LMN exactly, thus resulting in slight but acceptable oscillations of the output of the Wiener model.

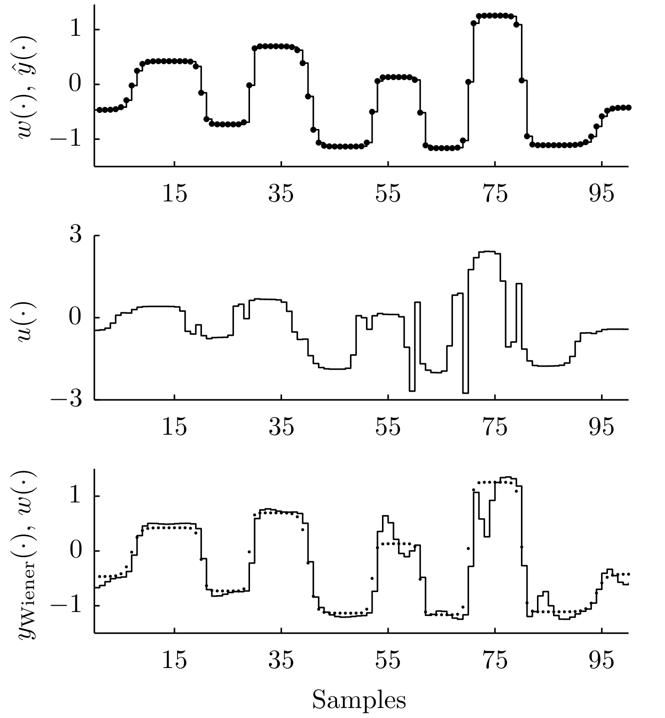

Fig. 3. Simulation of a sequence of low-pass filtered reference steps; Upper panel: reference $w(\cdot)$ and LMN simulation output $\hat{y}(\cdot)$; middle panel: feedforward input signal $u(\cdot)$; lower panel: feedforward controlled actual Wiener model output $y_{\mathrm{Wiener}}(\cdot)$ in solid and reference $w(\cdot)$ dotted

5. OUTLOOK

In this manuscript an effective but yet simple approach has been introduced to automatically attain a dynamic feedforward control law for the generic model structure of an LMN. In contrast to a static model inversion, a dynamic feedforward control with an online reference trajectory generation will improve the closed-loop performance. Currently the application is restricted to LMNs without numerator dynamics (i.e. $|\mathcal{M}| = 1$) to evade non-minimum-phase behavior. In future work, this restriction will be eliminated by assessing the internal stability beforehand to indicate whether the LMN is globally minimum-phase or not. Additionally, input constraints have only been addressed implicitly by low-pass filtering the desired reference to avoid excessive reference steps.

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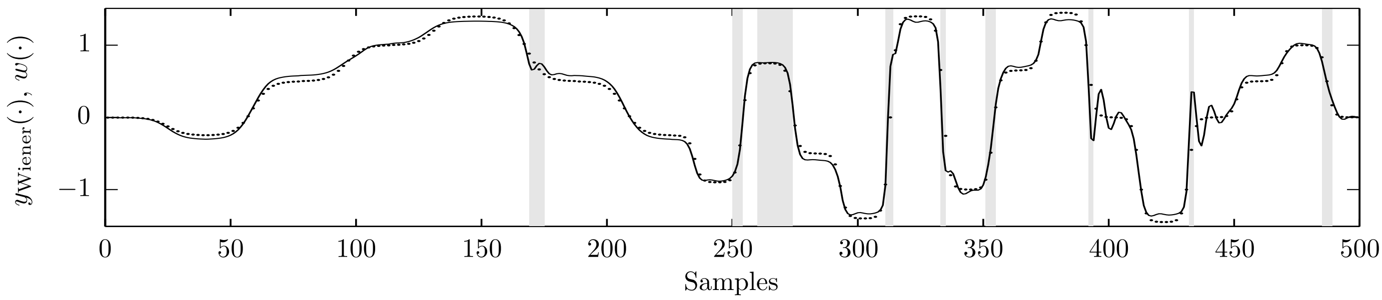

Fig. 4. Feedforward controlled Wiener model output $y_{\text{Wiener}}(\cdot)$ with reference $w(\cdot)$ as dotted line and intervals, where an unstable local model is active, being shaded in grey.

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