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Building Temperature Control With Active Occupant Feedback

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Abstract: Most of the current studies and solutions developed for building temperature control have been designed independent of the occupant feedback. An acceptable temperature range for the occupancy level is estimated, and control input is designed to maintain temperature within that range during occupancy hours. In this work we have incorporated active user feedback so as to minimize the aggregate user discomfort taking into account the total energy cost to design the optimal control input. The focus is on a multi-zone building, with lumped heat transfer model based on thermal resistance and capacitance for system analysis. We provide a stability analysis and establish convergence of the proposed solution to a desired temperature that minimizes the sum of energy cost and aggregate user discomfort. Simulation results on a four-room example are presented to demonstrate the performance of the proposed approach and validate the model.

Keywords: Temperature control, occupant comfort feedback, human mediation.

1. INTRODUCTION

In recent years our quality of living has improved dramatically and so has the human comfort level expectations. All this has lead to an unprecedented rise in the consumption of energy resources. A large fraction of the energy consumption comes from its usage in buildings, both residential and commercial. It has been estimated that nearly 40% of the total energy consumption in US, which in turn accounts for 20% of total energy consumption worldwide, can be attributed to the residential and commercial building usage [Lombard et al. (2008)]. Building usage also accounts for more than 75% of current electricity consumption. Hence, one of the most important components of attaining global energy efficiency is that of attaining energy efficient operation of buildings. Heating, ventilation, and air conditioning (HVAC) system is one of the major energy consumers in buildings. So far, numerous design and solution approaches have been proposed for the control of HVAC systems. The approaches taken so far can be broadly classified into those focusing on optimal energy usage through variable electricity rates [Sane et al. (2006), Braun (1990), Henze (2005), Henze et al. (2004)], active and passive thermal energy storage [Henze (2005), Henze et al. (2004)], and more recently model predictive control (MPC) approach exploiting weather forecast information [Ma et al. (2012), Ma et al. (2011), Kelman et al. (2011), Ma and Borrelli (2012), Gatsis and Giannakis (2011)].

A building in its entirety is a complex network of heterogeneous and inter-connected subsystems. The occupants of a building constitute an important subsystem, whose comfort level must be accounted for, in optimizing energy usage in a building. However, existing approaches treat the building and its occupants as separate entities. Such a fragmented approach towards attaining energy efficiency in buildings will most likely be sub-optimal. An effective building energy control system must take into account the feedback of its occupants, and their individual comfort levels at the current temperature/heat input settings. This can be a very challenging task in large commercial buildings such as schools, libraries, offices etc. which have a very diverse collection of occupants with different ranges of preferred temperature. This calls for a control solution that does not rely on a priori knowledge of the comfortable temperature ranges/comfort functions of the individual occupants, but learns them through occupant feedback, and incorporates such feedback along with energy cost in controlling building temperature optimally. In this paper we present an optimal control solution for this problem, that is developed and analyzed using gradient optimization. In our model the feedback of the building occupants are taken as a periodic input and is used, along with energy usage considerations, in determining the adjustment to the control (energy) inputs, which in turn determines the evolution of the building temperature.

More specifically, in this paper we make the following contributions. We consider a multi-zone building, and use a lumped heat transfer model based on thermal resistance and capacitance for system analysis. We establish convergence of the proposed approach to a desired temperature and energy input equilibrium solution, such that it minimizes the sum of energy cost and aggregate user discomfort. Finally, we conduct simulations on a four-room model to demonstrate the performance of the proposed solution approach.

2. SYSTEM MODEL AND CONTROL ALGORITHM

In formulating our building energy control question, we first need to determine the choice of the building heat transfer model. In the past researchers have proposed different models such as the finite element method [Mebee (2011)], lumped mass and energy transfer model [Riederer et al. (2002), Wu et al. (2008)], and electrical circuit analogy with application of graph theory [Boyer et al. (1996), Fraisse et al. (2002), Xu et al. (2008), Athienitis et al. (1985)]. There is a tradeoff to be made between computational efficiency and accuracy of room representation when deciding upon the system model. The electrical analogy approach to modeling multiple interconnected zones reduces the heat transfer model to an equivalent electrical circuit network. The model can be further modified to include building occupancy, room and heating equipment dynamics [Athienitis et al. (1985), Chandan (2010)]. In this paper we take this electrical circuit analogy approach, and combine it with occupant feedback modeling.

We model a building as a collection of interconnected zones, and use a lumped heat transfer model for modeling its energy/temperature dynamics. In the lumped heat transfer model, the walls are modeled as an RC network resulting in the standard 3R2C model [Fraisse et al. (2002)] where each zone is modeled as a thermal capacitor. Heat flow modeling is based on temperature difference and thermal resistance: $Q = \Delta T/R$, where ΔT is the temperature difference, R is the thermal resistance and Q is the heat transferred across the resistance.

The heat flow and thermal capacitance model can be written for all the thermal capacitors in the system, with T_i as the temperature of the ith capacitor. Consider the system to have n thermal capacitors and l thermal resistors. With additional sources of heat input such as ambient environment, we can write the overall heat transfer model of the system with m zones as [Mukherjee et al. (2012)]:

$$C\dot{T} = -DR^{-1}D^T T + B_0 T_\infty + Bu + Bw, \tag{1}$$

where $T \in \mathbb{R}^n$ is the temperature vector (representing the temperature of the thermal capacitors from 3R2C model) and $u \in \mathbb{R}^m$ is the heat input vector for the model. Note that (T, u) are each functions of time $(T(t), u(t))$ and accordingly $\dot{T} = \frac{dT}{dt}$. Positive values of u correspond to heating the system and negative values correspond to cooling. In the above equation, $C \in \mathbb{R}^{n\times n}$ consists of the wall capacitances and is a diagonal positive definite matrix; $R \in \mathbb{R}^{l\times l}$ consists of the link thermal resistances and is a positive definite matrix. Also, $D \in \mathbb{R}^{n\times l}$ is the incidence matrix and is of full row rank [Lombard et al. (2008)], and $B_0 = -DR^{-1}d_0^T \in \mathbb{R}^n$ is a column vector with non-zero elements as the thermal conductances of nodes connected to the ambient. Further, T_∞ is the ambient temperature, $w \in \mathbb{R}^m$ is the environmental heat input into each zone, and $B \in \mathbb{R}^{n\times m}$ is the input matrix. In this study we neglect the environmental heat input and so our model equation (1) reduces to:

$$C\dot{T} = -DR^{-1}D^T T + B_0 T_\infty + Bu. \tag{2}$$

In our model, the zones are picked such that each of them has a heating/cooling unit, which in turn implies that B is of full row rank. Also, since matrix D is of full row rank the product $DR^{-1}D^T$ is a positive definite matrix. Further to model the active user feedback through human mediation, temperature regulation is done for the zones that are occupied and are also directly affected by heating/cooling devices. The vector of zone temperatures, denoted by y (which is a function of T) can be expressed as,

$$y = B^T T. \tag{3}$$

Our overall minimization objective (overall cost) is the sum of two terms: (i) energy cost (i.e, cost of heating/cooling), and (ii) aggregate discomfort cost of the occupants. The energy cost (i) is expressed as $\frac{1}{2}u^T\Gamma u$, where Γ is a positive definite matrix. Let S_j denote the set of all occupants in zone j, and $\rho = \sum_{j=1}^m |S_j|$ be the total number of occupants. Also let G_s denote the (convex) discomfort function of occupant s. Then the aggregate occupant discomfort cost (ii) is expressed as $\sum_{j=1}^m \sum_{s\in S_j} G_s(y_j(T))$, where $y_j(T) = [B^T T]_j$(from (3)) denotes the j^{th} element of y, or the temperature of zone j. Our minimization objective is thus expressed as,

$$U(u, T) = \frac{1}{2}u^T\Gamma u + \gamma \sum_{j=1}^m \sum_{s\in S_j} G_s(y_j(T)). \tag{4}$$

In (4), γ is a scalar constant that defines the relative weight provided to the aggregate occupant discomfort, as compared to the energy cost.

Assuming a constant ambient temperature T_∞, and using equilibrium condition (setting $\dot{T} = 0$ in (2)) we obtain:

$$T = h(u) = (DR^{-1}D^T)^{-1}(B_0 T_\infty + Bu). \tag{5}$$

Let the control input u be updated once every Δ time units, and be expressed as:

$$u_{k+1} = u_k - \eta\Theta(u, T), \tag{6}$$

where $\Theta(u, T)$ is based on the occupant comfort feedback, current system temperature, and energy cost (the exact form of $\Theta(u, T)$ will be defined later in this section). Also, η in the above equation is a scalar that can be loosely interpreted as the “feedback gain” of the system and represents the constant “step size” of a gradient descent algorithm (see the discussion below).

We can now develop a continuous approximation to the evolution of the control input u, as

$$\dot{u} \approx \frac{u_{k+1} - u_k}{\Delta} = -\frac{\eta}{\Delta}\Theta(u, T). \tag{7}$$

Equations (2) and (7) govern how the system evolves. Time step Δ is the interval at which user feedback is solicited and the control input u is updated. A larger Δ implies a slower evolution of u. Then,

$$\Delta = \frac{\mathrm{d}t}{\mathrm{d}\tau} \implies \dot{u} = \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{\Delta}\left(\frac{\mathrm{d}u}{\mathrm{d}\tau}\right); \quad \dot{T} = \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\Delta}\left(\frac{\mathrm{d}T}{\mathrm{d}\tau}\right). \tag{8}$$

Control input equation (7) now becomes:

$$\frac{\mathrm{d}u}{\mathrm{d}\tau} = -\eta\Theta(u, T). \tag{9}$$

Similarly, equation (2) modeling the temperature evolution of the building can now be expressed as:

$$\frac{C}{\Delta}\frac{\mathrm{d}T}{\mathrm{d}\tau} = -DR^{-1}D^T T + B_0 T_\infty + Bu. \tag{10}$$

Define,

$$J(u) = U(u, h(u)), \tag{11}$$

i.e., $J(u)$ is obtained by plugging in $T = h(u)$ from (5) into (4). The energy cost term in (4) is strictly convex in u. The aggregate occupant discomfort term is convex in T, and therefore convex in u when T is set to $h(u)$, as $h(u)$ is linear in u. $J(u)$ is thus strictly convex in u and has a unique optimal solution u^*. Define

$$T^* = h(u^*), \tag{12}$$

which is also unique by definition. $\Theta(u,T)$, defined earlier, is expressed using Lyapunov functions $V(u)$ and $W(u,T)$, where

$$V(u) = J(u) - J(u^*); \tag{13}$$

and

$$W(u,T) = (T - h(u))^T P(T - h(u)), \tag{14}$$

P in the above equation is a symmetric positive definite matrix. We now define a combined Lyapunov function $L(u,T)$:

$$L(u,T) = (1-\alpha)V(u) + \alpha W(u,T), \tag{15}$$

where α satisfies $0 < \alpha < 1$. $\Theta(u,T)$ is then obtained as,

$$\Theta(u,T) = \nabla_u L(u,T). \tag{16}$$

The control input update equation in (9) can now be expressed as:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}\tau} &= -\eta(1-\alpha)\nabla_u[J(u) - J(u^*)] \\ &\quad -\eta\alpha\nabla_u[(T - h(u))^T P(T - h(u))].\end{aligned} \tag{17}$$

Simplifying further,

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}\tau} &= -\eta(1-\alpha)[\Gamma u + \gamma Y \Lambda F(u)] \\ &\quad -2\eta\alpha[((DR^{-1}D^T)^{-1}B)^T P(T - h(u))].\end{aligned} \tag{18}$$

In the above, $Y \in \mathbb{R}^{m\times m}$ is the Jacobian obtained using (3) and the equilibrium condition (5), expressed as

$$Y = (\frac{\partial y}{\partial u}) = B^T(DR^{-1}D^T)^{-1}B. \tag{19}$$

Also, $\Lambda \in \mathbb{R}^{m\times\rho}$ is the zone-occupant matrix that indicates which occupants are present in a zone ($\Lambda_{js} = 1$ if $s \in S_j$ else 0), and $F(u) \in \mathbb{R}^{\rho\times 1}$ is the "marginal discomfort" vector of the occupants, obtained by taking partial derivative of the occupant discomfort function with respect to y, evaluated at the corresponding equilibrium temperature $h(u)$ i.e. at $y = B^T h(u)$. In other words, the s^{th} element of $F(u)$, where $s \in S_j$, is obtained as

$$F_s(u) = (\frac{\partial G_s(y_j)}{\partial y_j})\Bigg|_{y_j=[B^T h(u)]_j}, \quad s \in S_j, \tag{20}$$

where $[B^T h(u)]_j$ is the j^{th} component of $B^T h(u)$.

We assume that $F_s(u)$, the "marginal discomfort" value of occupant s at the current input u, can be reasonably estimated from the discomfort feedback of occupant s at any time. In practice, the occupants may provide the feedback in some simple form describing their actual level of discomfort ("I am feeling hot", "I am feeling very cold" etc.). This feedback must be processed to estimate the marginal discomfort (derivative of the actual discomfort function) for the current input value u. Also note that an occupant $s \in S_j$ will provide a comfort feedback at the current temperature it experiences, which is $[B^T T]_j$, which may differ from the equilibriated temperature at which the feedback is desired $[B^T h(u)]_j$. Some adjustments may need to be made on that count as well, to estimate $F_s(u)$ appropriately based on the current discomfort feedback. Note however, that if the occupant feedback is collected after long intervals (i.e. Δ is large), allowing the temperature T to settle down to $h(u)$ or close to it before the occupant feedback is collected, this difference may be negligible. In the analysis and simulation section that follows (Sections 3 and 4, respectively), we assume that the $F_s(u)$ values are available (estimated perfectly).

3. STABILITY ANALYSIS

The system evolution is governed by the set of equations (10) and (18). The coefficient $DR^{-1}D^T$ in (10) is positive definite which makes the unforced system (with $u = 0$) exponentially stable.

Proposition 1. (u,T) given by equations (10) and (18) globally asymptotically converges to the equilibrium point $(u^*,\, T^*)$.

Proof: To prove that (10) and (18) drives the system to its equilibrium point, we show that the combined Lyapunov function $L(u,T)$ decreases with time, i.e. $\frac{\mathrm{d}L}{\mathrm{d}\tau} < 0$, unless $(u,\, T) = (u^*,\, T^*)$. We can express $\frac{\mathrm{d}L}{\mathrm{d}\tau}$ as

$$\frac{\mathrm{d}L}{\mathrm{d}\tau} = (\nabla_u L)^T \frac{\mathrm{d}u}{\mathrm{d}\tau} + (\nabla_T L)^T \frac{\mathrm{d}T}{\mathrm{d}\tau}. \tag{21}$$

where,

$$(\nabla_u L)^T \frac{\mathrm{d}u}{\mathrm{d}\tau} = (\nabla_u L)^T(-\eta\nabla_u L) = -\eta(\nabla_u L)^T(\nabla_u L), \tag{22}$$

which is < 0 unless $\nabla_u L(u,T) = 0$. Next from (10) and (13)-(15), we obtain:

$$\begin{aligned}(\nabla_T L)^T \frac{\mathrm{d}T}{\mathrm{d}\tau} &= 2\Delta(P(T - h(u)))^T \frac{\mathrm{d}T}{\mathrm{d}\tau} = 2\Delta(T - h(u))^T \\ &\quad P^T(-C^{-1}DR^{-1}D^T T + C^{-1}Bu + C^{-1}B_0 T_\infty).\end{aligned} \tag{23}$$

This expression can be further simplified to obtain,

$$(\nabla_T L)^T \frac{\mathrm{d}T}{\mathrm{d}\tau} = -2\Delta(T - h(u))^T(PC^{-1}DR^{-1}D^T)(T - h(u)). \tag{24}$$

We wish to choose the matrix P such that $PC^{-1}DR^{-1}D^T$ is positive definite. One selection is to have $P = C$, which gives:

$$(\nabla_T L)^T \frac{\mathrm{d}T}{\mathrm{d}\tau} = -2\Delta(T - h(u))^T(DR^{-1}D^T)(T - h(u)), \tag{25}$$

which is < 0 unless $T = h(u)$ since the matrix $(DR^{-1}D^T)$ is positive definite. The matrix C is related to system parameters (wall capacitance values) whose a priori knowledge is needed. Another choice could be $P = I$ ($n \times n$ identity matrix). Substituting $P = I$ we get,

$$(\nabla_T L)^T \frac{\mathrm{d}T}{\mathrm{d}\tau} = -2\Delta(T - h(u))^T(C^{-1}DR^{-1}D^T)(T - h(u)). \tag{26}$$

It is reasonable to assume that the symmetric part of the matrix $(C^{-1}DR^{-1}D^T)$ has positive eigenvalues implying that it is a positive definite matrix. Thus, in this case too, $(\nabla_T L)^T \frac{\mathrm{d}T}{\mathrm{d}\tau}$ is < 0 unless $T = h(u)$. Hence, $\frac{\mathrm{d}L}{\mathrm{d}\tau}$ given by (21) is < 0 unless $\nabla_u L(u,T) = 0$ and $T = h(u)$, which is only attained at (u^*, T^*). The result of Proposition 1 follows from standard Lyapunov stability criteria. □

4. SIMULATION

For simulation study we consider a four-room building, borrowing from an example in [Moore et al. (2011)] and including heat transfer to the ambient for all rooms. It is illustrated in Figure 1 below. In the figure, each double headed arrow represents a thermal connection between the two corresponding sides. The connection between two rooms through an open door is represented by a single resistance, and the same through the wall is represented using 3R2C wall model. We place two users (occupants) in room 4, and one user each in the other three rooms. This model of four rooms and eight walls gives us 20 capacitive elements and 27 resistive elements, resulting in the incidence matrix, D as 20×27. With the dimensions of the model as in Figure 1, volumetric heat capacity values and thermal resistance values as per [Mukherjee et al. (2012)], we can obtain values for the matrices in equation (2). Using this information we simulate the model with ambient temperature at $T_\infty = 15°C$ and Δ as five minutes.

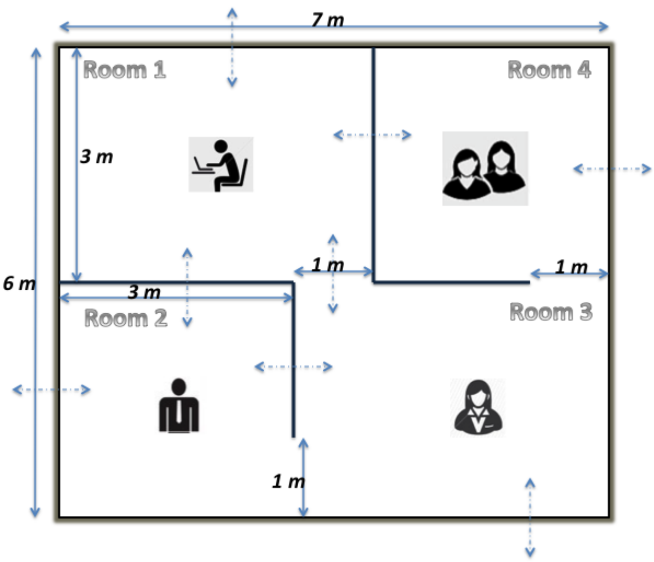

Fig. 1. Four room example model used for simulation. Each room has occupancy as illustrated in the figure.

Figures 2 and 3 demonstrates convergence of our model. The temperature preference of each user (occupant) is depicted in Table 1. The simulation is run over a 5 hour period from 7am to 12pm. The results show that the temperature and control (heat) input converges to the desired values.

Table 1. Each user's temperature preference with users 4 and 5 in room 4 having a common preference range.

	Low Temperature Limit (deg C)	High Temperature Limit (deg C)
User 1	19	23
User 2	18	23
User 3	19	22
User 4	20	24
User 5	20	23

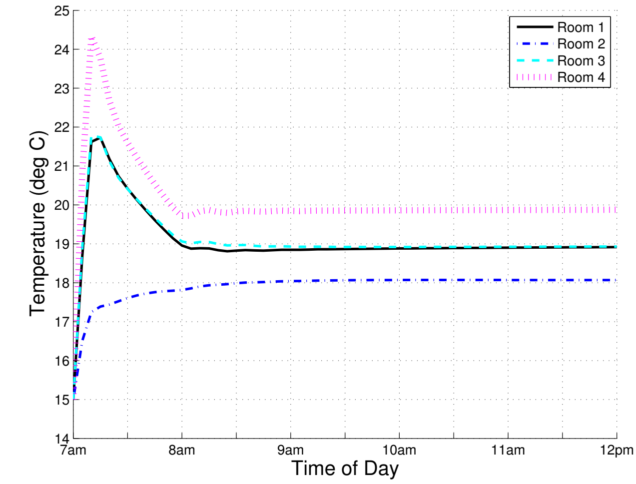

Fig. 2. Temperature evolution over a 5 hour period for occupancy as per Figure 1 and user temperature preference as per Table 1.

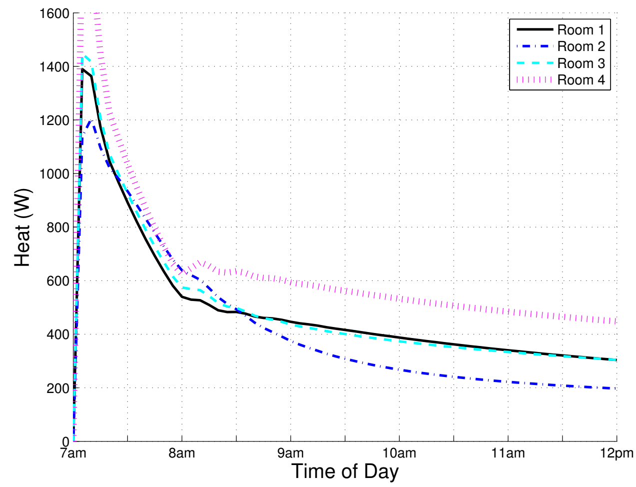

Fig. 3. Heat (control) input for the 5 hour period simulation with uninterrupted occupancy, and user temperature preference as in Table 1.

The evolution and convergence of the simulated system is affected by four major model parameters, feedback gain (step size) parameter (η), user feedback weight parameter

Table 2. User schedule over a 48 hour period. + marks the entry of corresponding user and − indicates user leaving.

	7 am	8 am	9 am	12 to 1 pm	4 pm	5 pm	6 pm
Day 1	(Sim Start) U1+	U2+	U3+, U4+, U5+	Lunch Hour	U1−	U2−	U3−, U4−, U5−
Day 2		U3+, U4+, U5+	U1+, U2+	Lunch Hour		U3−, U4−, U5−	U1−, U2−
Day 3	(Sim End)						

(γ), trade-off parameter (α) as defined in (15), and heat cost parameter (Γ). η controls the size of change in u in each step, and γ signifies the weight given to the penalty associated with the building occupant discomfort. A higher γ would result in a high value of control input for a given user comfort feedback vector. Both η and γ were tuned to obtain a reasonable trade-off between heat (control) input and user discomfort level. The scaling parameter α determines how much weight is provided in moving the control input u in the gradient direction of $J(u)$; the results presented in this paper are with low value of α. In this study we use a time varying heat cost Γ. Once the last user leaves the building we increase the value of Γ, so that heat input (energy) is minimized during non-occupant hours.

The results in Figures 2 and 3 with uninterrupted occupancy, do not reflect a typical real world scenario. We next present simulation results over a 48 hour period with the occupants moving in and out as per Table 2 schedule. Room 4 is occupied by two users U4 and U5. Two different cases are possible in general with multiple users in the same room: users (U4 and U5) have a common range as in Table 1, or the users pose a conflicting comfort range as per Table 3. The temperature and control input for common range is shown in Figures 4 and 5.

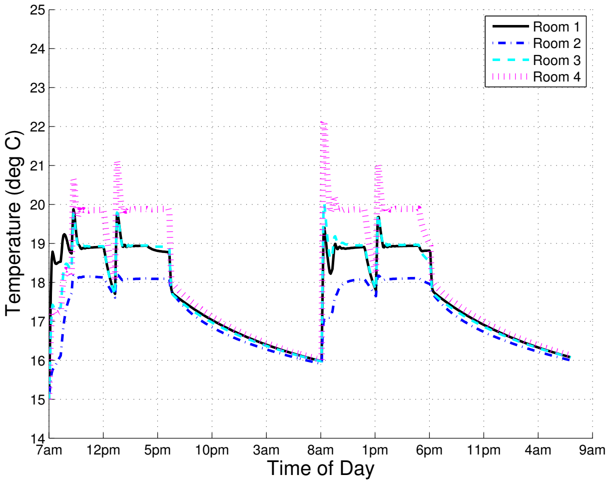

Fig. 4. Temperature profile for the model in Figure 1 over a 48 hour period with user schedule as per Table 2 and temperature preference as per Table 1

Results for conflicting temperature preference as per Table 3 is captured in Figures 6 and 7. In this scenario since there is no common comfortable range for both the users of room 4, its not possible to satisfy both the users simultaneously. Hence, the temperature of room 4 settles between 22°C and 23°C. Since the temperature settles at a much higher range the overall heat consumed is higher when compared to the earlier case.

5. CONCLUSION AND FUTURE WORK

In this work we have shown that the building temperature and energy usage can be controlled effectively through dynamic feedback from the users (occupants) based on their comfort levels. Our simulation study showed that

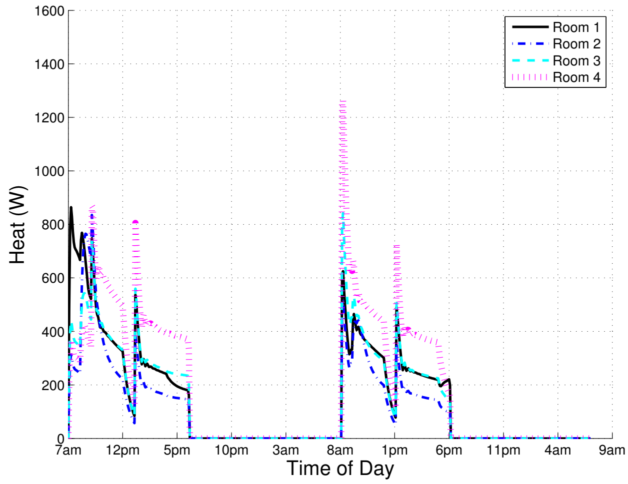

Fig. 5. Heat input over a 48 hour period with user schedule as per Table 2 and temperature preferences as shown in Table 1.

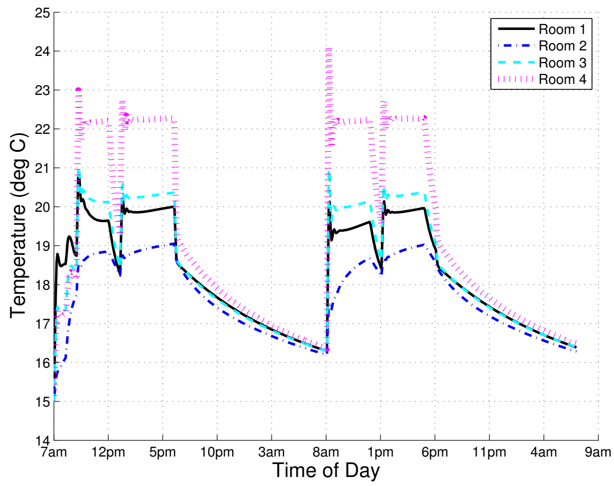

Fig. 6. Temperature profile with user schedule in Table 2 and temperature preference from Table 3.

with effective tuning of a few parameters, the control algorithm attains a fast temperature convergence rate with reasonable energy expenditure.

As discussed at the end of Section 2, our analysis and simulation study assumes that the derivatives of the occupant discomfort functions at the equilibrated temperature ($h(u)$, for the current control input u) can be estimated

Table 3. Each user's range of comfortable temperature. Users 4 and 5 occupying room 4 now have conflicting temperature preferences.

	Low Temperature Limit (deg C)	High Temperature Limit (deg C)
User 1	19	23
User 2	18	23
User 3	19	22
User 4	20	22
User 5	23	25

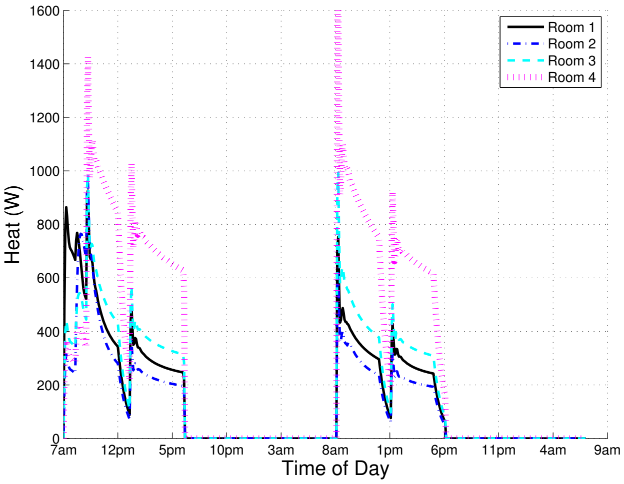

Fig. 7. Heat input with user schedule following Table 2 and temperature preference in Table 3.

accurately, to be used in the proposed control algorithm. In practice, occupant feedback will correspond to non-equilibrated temperatures leading to errors in the implementation of the proposed control policy. The effect of such errors on the stability of the control policy could possibly be studied in the framework of singular perturbation theory [Kokotovic et al. (1986)]. The effect of inaccuracies due to factors such as discreteness of the occupant's feedback or other estimation errors also need to be evaluated. These research issues are currently under investigation.

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