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A New Cooperative Current-Sharing Control of Parallel Chargers for Energy Storage Type Light Rail Vehicles

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Abstract:
In order to address the output current imbalanced problem of the parallel chargers for energy storage type light rail vehicles, a current-sharing scheme is proposed based on the distributed cooperative control of multi-agent systems. If each charger is considered to be an agent, the current-sharing control design would resemble a consensus-tracking problem of multi-agent systems with bounded input constraints. The communication network among the chargers can be taken as an undirected graph. Each charger exchanges current information with its neighbors through the communication network. Due to the fact that the charging system has non-identical feature and the control input is bounded, input-output feedback linearization is adopted to transform the current-sharing control of the charging system to a first-order integrator consensus-tracking problem. A saturation function is introduced to design a bounded cooperative current-sharing control law based on the nearest neighborhood rule. The cooperative stability of closed-loop system under the fixed topology is rigorously proved by Lyapunov function integrating LaSalle invariant principle. The output current of parallel chargers can be balanced by adopting the proposed control methodology, which is verified by simulating a multi-charging test system.

1. INTRODUCTION

As a new type of electric traction light rail transportation system, the energy storage type light rail vehicle adopts super-capacitors as its power supply. With this energy storage technology, there is no need to construct a traction power grid and it is possible to recover the energy of regenerative braking. Energy storage type light rail vehicles need to be charged in seconds by the charging system when it parks at the platform. Therefore, the charging system should provide a large enough output power to shorten the charging process. An effective method to solve this problem would be by connecting several chargers in parallel to increase system capacity [Chen, 2009]. However, the challenge for the charging system is how to balance the charging current between the chargers.

If the charging current is not balanced, the charger with a higher output current has to bear a greater output power, which may lead to a large thermal stress and degrade the reliability and performance of the whole charging system. It is therefore necessary to design an effective current-sharing control strategy to balance the output current for charging system.

Several conventional approaches exist for current-sharing problems, such as central current-sharing control method, droop control method [Wang, 2012], master-slave method [Mazumder, 2008],etc. While such methods are undistributed, restrictive and have some drawbacks and limitations, which are not the appropriate approach for the charging system of energy storage type light rail vehicles when the chargers are parallel with each other. Over the last decades, consensus and cooperative control of multi-agent systems have received most of the attention[Cao, 2013],giving results in the areas such as flocking [Saber, 2007], formation control, distributed mobile sensor network, rendezvous in space, autonomous vehicles, smart grid [Xin, 2011][Bidram, 2013], shipboard power systems [Bidram, 2013].

Taking each charger in the charging system as an agent in multi-agent systems, the current-sharing control design resembles a current consensus-tracking problem, where the charger's current tracks the reference objective. Several chargers are connected in parallel with each other in the charging system for an energy storage type light rail vehicle. The adjacent chargers interact with each other and exchange the state information such as the charging current. This method is distributed, each charger only requires its own information and the information of its neighbors, and one faulty node cannot cause the collapse of the whole charging system, which is an appropriate

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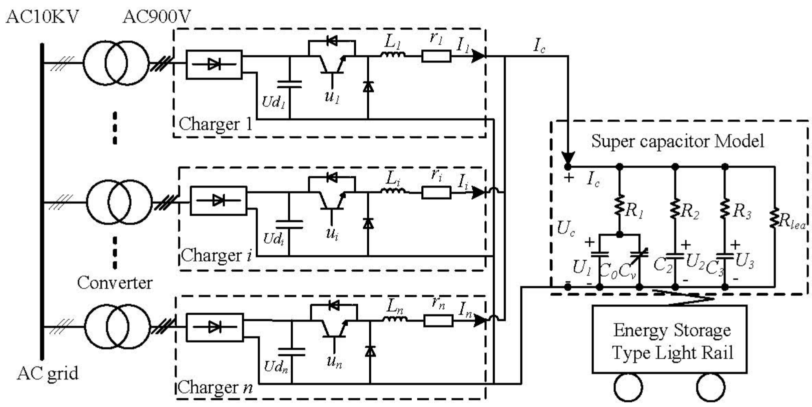

Fig. 1. The main circuit schematic diagram of charging system for an energy storage type light rail vehicle

method for the charging system to solve the current-sharing problem.

In addition, the control input is bounded for the charging system due to some physical constraints, such as the duty ratio. And there are inevitably components error and manufacturing error in each charger system, the dynamics of the chargers in the charging system are non-identical. The charger has intrinsic nonlinear characteristics because of Buck DC/DC circuit's principle and super capacitor's feature. This paper seeks to address the challenge of how to design a bounded cooperative current-sharing control law and take into account both the charger's non-identical and nonlinear features.

2. PROBLEM FORMULATION

In this section, the problem to be solved for the charging system is set out in details.

As shown in Fig. 1, the charging system of an energy storage type light rail vehicle consists of 10kV AC supply grid, 10kV/900V AC converter, and several parallel charger subsystems. Each charger subsystem exchange state information with the aid of a communication network. The charger subsystem mainly consists of a three-phase bridge rectifier circuit and chopper BUCK DC / DC circuit.

Considering each charger as an agent, the nonlinear multi-charging system with non-identical nodes shown in Fig. 1 can also be uniformly described by

$$\begin{cases} \dot{x}_i = f_i(x_1, x_2, \cdots, x_n) + g_i(x_i)u_i \\ y_i = h_i(x_i) \end{cases} \tag{1}$$

where $i = 1, 2, \cdots n$, $x_i = \begin{bmatrix} I_i \\ U_c \end{bmatrix}$ is the i_{th} charger's state, $u_i(t) \in R$ is the control input, $f_i(x_1, x_2, \cdots, x_n), g_i(x_i)$ are bounded Lipschitz continuous function and given below

$$f_i(\cdot) = \begin{bmatrix} -\dfrac{r_i}{L_i} I_i - \dfrac{U_c}{L_i} \\ \dfrac{1}{C_0 + C_v U_c} \displaystyle\sum_{k=1}^{n} I_k \end{bmatrix}, g(x_i) = \begin{bmatrix} \dfrac{U_{d_i}}{L_i} \\ 0 \end{bmatrix} \tag{2}$$
$$h_i(x_i) = I_i$$

where I_i is the output current by each charging subsystem, U_c is super-capacitor's voltage, U_{d_i} is the DC input voltage which is obtained by three-phase bridge rectifier. L_i is the flux for the energy storage inductor, r_i is the equivalent resistance of the circuit. C_{sc} is the super capacitor's capacitance. The capacitance C_0 and voltage-dependent capacitance C_v are parameters values of the super capacitor, which are related with the super capacitor's terminal voltage U_c.

When the energy storage rail vehicle reaches the platform, it must be fully charged by the charging system in seconds so that it can run to the next station with enough power to be recharged again. Because manufacture error inevitably exists in the AC adapter, the DC input voltage U_{d_i} can not be exactly the same. There are also component error in each charger sub-system, such as L_i and r_i. Thus, each charger is non-identical and has intrinsic nonlinear feature due to the complex circuit and super-capacitor's principle. In addition, the line connection of the circuit can not be guaranteed to be completely symmetrical. Such reasons above will result in that the output current of each charger is not balanced, seriously affecting charging performance.

In this paper, in order to carry out current-sharing objective, we should design a bounded cooperative control $u_i(t)$ such that

$$\lim_{t \to +\infty} |y_i(t) - y_0| = 0 \tag{3}$$

where y_0 is the reference current. The IGBT duty ratio can be controlled directly to adjust the input voltage got by the DC-DC converter. The control objective y_0 is preconfigured in one of charging subsystem. Each controller communicates the current state information with its neighbors through RS-422 bus. Through information exchanges among the charging subsystems, the proposed cooperative control is designed to achieve the control objective defined in (3).

The output current balance problem i.e. current-sharing control design resembles a consensus-tracking problem of nonlinear and non-identical multi-agent systems with bounded input constraints. The current-sharing strategy based on distributed cooperative control will be proposed in the next section.

3. CURRENT-SHARING CONTROL DESIGN AND STABILITY ANALYSIS

In this section, a bounded cooperative controller is firstly designed under fixed topology based on nearest neighbor rule by using input-output feedback linearization. And then, the cooperative stability analysis of the multi-charging system under such fixed topology with the aid of the Lyapunov function integrating LaSalle Invariant Principle is given.

3.1 Controller design under fixed topology

We can build the direct relationship between the output y_i and the control input u_i by differentiating y_i with respect to time t in (1). The first-order derivative of the output y_i is

$$\begin{aligned}\dot{y}_i &= \frac{\partial h_i(x_i)}{\partial x_i}\dot{x}_i \\ &= \frac{\partial h_i(x_i)}{\partial x_i}\left(f_i(x_1, x_2, \cdots, x_n) + g_i(x_i)u_i\right) \\ &= L_{f_i}h_i + L_{g_i}h_i u_i\end{aligned} \tag{4}$$

where $L_{f_i}h_i = \nabla h_i f_i = \frac{\partial h_i(x_i)}{\partial x_i} f_i(x_1, x_2, \cdots, x_n)$, $L_{g_i}h_i = \nabla h_i g_i = \frac{\partial h_i(x_i)}{\partial x_i} g_i(x_i)$.

We can define an auxiliary control ϑ_i as follows:

$$\vartheta_i = L_{f_i}h_i + L_{g_i}h_i u_i \tag{5}$$

Substitute (5) into (4), we can get the first-order integrator linear system $\dot{y}_i = \vartheta_i$.

As the duty ratio of Insulated Gate Bipolar Transistor is bounded, this paper adopts a saturation function $\phi(\cdot)$ to construct cooperative control law, which distinguishes from the traditional saturation control law by introducing hyperbolic tangent function [Ren, 2010].

Definition 1. A general saturation function $\phi(\cdot)$ satisfies:

(1) $\phi(\cdot)$ is Lipschitz continuous,

(2) $\phi(z) = 0 \Leftrightarrow z = 0$,

(3) $z\phi(z) > 0, \forall z \neq 0$,

(4) $\phi_{\min} \leq \phi(z) \leq \phi_{\max}, \forall z \in R$.

The communication network between each charger is a bidirectional information flow by using RS-422 bus, which can be modeled by a fixed undirected graph $G(\nu, \varepsilon, \mathrm{A})$ with a nonempty finite set of n nodes $\nu = \{ v_1, v_2 \cdots v_n \}$, a set of edges or arcs $\varepsilon \subseteq \nu \times \nu$, and the associated adjacency matrix $\mathrm{A} = [a_{ij}] \in R^{n\times n}$.

The proposed auxiliary control ϑ_i can be designed as follows

$$\vartheta_i = c\Big(\sum_{j\in N_i} a_{ij}\phi(y_j - y_i) + \rho_i\phi(y_0 - y_i)\Big) \tag{6}$$

where $N_i = \{v_j \in \nu : (v_j, v_i) \in \varepsilon\}$ the set of neighbors of v_i, $a_{ij} > 0$ means that for the ith charger, it can receive the information of the jth charger. The control gain c is the coupling strength and $c > 0$, which can be chosen appropriately to improve the response time [Zhang, 2011]. The parameter ρ_i is the pinning gain and $\rho_i > 0$ for at least one i, which means that at least one charger knows the reference charging current.

Accordingly, the cooperative control input u_i can be carried out by ϑ_i as

$$u_i = \frac{\vartheta_i - L_{f_i}h_i}{L_{g_i}h_i} \tag{7}$$

Because the saturation function $\phi(x)$ is a bounded odd function with range $[\phi_{\min}, \phi_{\max}]$, the proposed control law is bounded.

When the communication topology is fixed, the distributed cooperative controller is formulated as follows by substituting (6) into (7)

$$u_i = \frac{c\big(\sum_{j\in N_i} a_{ij}\phi(y_j - y_i) + \rho_i\phi(y_0 - y_i)\big) - L_{f_i}h_i}{L_{g_i}h_i} \tag{8}$$

From (6) and (8), we know that the current-sharing control law based on the i_{th} charger's measured current information and the information of its neighbors.

By substituting (8) into (4), we have each sub-system's closed-loop dynamics:

$$\dot{y}_i = c\Big(\sum_{j\in N_i} a_{ij}\phi(y_j - y_i) + \rho_i\phi(y_0 - y_i)\Big) \tag{9}$$

To facilitate the analysis below, let us further define the new auxiliary state variable $\delta_i(t) = y_i(t) - y_0$. Furthermore, the each auxiliary subsystem's state equation can be written as:

$$\dot{\delta}_i = -c\Big(\sum_{j\in N_i} a_{ij}\phi(\delta_i - \delta_j) + \rho_i\phi(\delta_i)\Big) \tag{10}$$

3.2 Cooperative stability analysis under fixed undirected topology

The following Lemma 2 is given to facilitate the stability analysis in Theorem 3.

Lemma 2. Suppose $\xi_i \in R^m, \zeta_j \in R^m, \forall i, j = 1, 2, \cdots n$ and $S = [s_{ij}]_{n\times n} \in R^{n\times n}$, if matrix S is symmetric, then

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}(\xi_i - \xi_j)^T\phi(\zeta_i - \zeta_j) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij}\xi_i^T\phi(\zeta_i - \zeta_j)\end{aligned} \tag{11}$$

Theorem 3. Consider the multi-charging system (1) with fixed and undirected communication topology $G(\nu, \varepsilon, \mathrm{A})$. Under the conditions that $c > 0$ and $\rho_i > 0$ for at least one charger, the cooperative current-sharing control objective below can be achieved by the control law (8)

$$\lim_{t\to+\infty} |y_i(t) - y_0| = 0 \tag{12}$$

i.e. the output current of the parallel chargers can ultimately be consensus and track the desired reference current. Furthermore, the overall closed-loop system is asymptotically cooperative stable.

Proof.

To prove the multi-charging system's cooperative stability, we turn to analyze the auxiliary system's (10) stability indirectly. For the auxiliary closed-loop system (10), the Lyapunov function candidate can be chosen as follows:

$$V = \frac{1}{2}\sum_{i=1}^{n} \delta_i^T \delta_i \tag{13}$$

Differentiate the Lyapunov function, we can have

$$\dot{V} = -c\sum_{i=1}^{n} {\delta_i}^T \sum_{j\in N_i} a_{ij}\phi(\delta_i - \delta_j) - c\sum_{i=1}^{n} {\delta_i}^T \rho_i \phi(\delta_i) \tag{14}$$

According to the undirected communication topology, we know $\sum\limits_{j\in N_i} a_{ij} = \sum\limits_{j=1}^{n} a_{ij}$. The derivative function $\dot{V}$ (14) can be transformed into

$$\dot{V} = -c\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}{\delta_i}^T \phi(\delta_i - \delta_j) - c\sum_{i=1}^{n} {\delta_i}^T \rho_i \phi(\delta_i) \tag{15}$$

Since the adjacent matrix A is symmetric for the undirected graph $G(\nu, \varepsilon, \mathrm{A})$ and based on Lemma 2, the above equation (15) is equal to:

$$\dot{V} = -\frac{c\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} a_{ij}(\delta_i - \delta_j)^T \phi(\delta_i - \delta_j)}{2} - c\sum_{i=1}^{n} \delta_i^T \rho_i \phi(\delta_i) \tag{16}$$

Since $\phi(\delta_i - \delta_j)$ and $(\delta_i - \delta_j)$, $\phi(\delta_i)$ and δ_i have the same sign component-wise, we get $\dot{V} \le 0$. To this end, the overall auxiliary system is stable.

Note that $\dot{V} \equiv 0$ implies that $\delta_i - \delta_j = 0$ and $\delta_i = 0$, which, in turn, implies that $\delta_i = \delta_j, \forall i \neq j$. From LaSalles Invariance principle, it follows that $\delta_i \to \delta_j, \forall i \neq j$ asymptotically as $t \to +\infty$, i.e. $\lim\limits_{t\to+\infty} |\delta_i - \delta_j| = 0$ and $\lim\limits_{t\to+\infty} |\delta_i| = 0$. That is to say, the overall auxiliary closed-loop system is asymptotically cooperative stable. Furthermore, we can obtain $\lim\limits_{t\to+\infty} |y_i(t) - y_0| = 0, \forall i \neq j$ since the fact is that $\delta_i = y_i - y_0$.

To this end, the current state of the all chargers can ultimately be consensus and track the desired reference current. Furthermore, the overall closed-loop system is asymptotically stable. This completes the proof.

4. SIMULATION RESULTS

In this section, we use the charging test systems shown in Fig. 1 to validate the feasibility of the proposed cooperative control scheme. Several different cases below are considered and compared in this section.

4.1 Simulation results for two different charging cases

Case I: The charging current is chosen only to be 600A in the whole charging process, i.e the control objective y_0 =600A.

Case II: The whole charging process includes two sequential phases, namely a fast charging phase and a trickle charging phase. At the fast charging phase, the charging current is chosen to be 600A. At the trickle charging phase, the charging current is chosen to be 133A, i.e. the control objective y_0 =133A at this phase.

In our case studies, there exist 3 nonidentical chargers parallelling with each other in the charging system. The main parameters for the charging systems model are given in Table 1. The simulation parameters for the super-capacitor are $R_1 = 5.6m\Omega$, $C_0 = 92.3F$, $C_v = 0.0747F/V$. The initial current of the chargers are 0A. The super-capacitor's initial voltage U_c is 500V, objective voltage is 900V.

Table 1 The main parameters for the charging systems model

Charger i	$L_i(mH)$	$r_i(\Omega)$	$U_{d_i}(V)$
1	3.05	0.0035	1335
2	3.12	0.0031	1272
3	2.95	0.0029	1295

The given communication topology between each charging sub-system is shown in Fig. 2, which means that only the first charger can know the current-sharing objective and the other chargers exchange the current state information via this communication network.

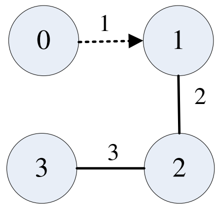

Fig. 2. The given communication topology between each charging sub-system

When the coupling strength c is chosen to be 5, the current sharing curve for Case I and Case II under the communication topology are plotted in Fig. 3 and Fig. 4 respectively, which illustrates that the current has been balanced and the consensus and cooperative objective has been accomplished. As shown in Fig. 4, the charging current ascends to be 600A from 0A within 5s, and then it goes to be constant current charging stage until the super-capacitor is charged to be about 870V. Since the super capacitor's voltage will decline since that there exists electric charge redistribution process at the end of the charging stage when we always choose the large charging current in Case I, the charging current is then reduced to 133A at time instant $t = 27.5s$ in Case II.

Comparing Case I with Case II, the super capacitor's voltage will decline about a little in Case I when the charging current is chosen only to be 600A. On the contrary, if we choose 600A as the charging current at the first stage and 133A at the last stage, the super-capacitor can be charged fully to be 900V as shown in Fig. 5, which illustrates that voltage objective has been achieved by adopting the proposed control strategy. As shown in Fig. 6, the convergence performance will be better when the coupling strength is chosen to be larger, while the ripple peak value will become larger as shown in the subgraph of Fig. 6. In a word, we can conclude that when we choose a

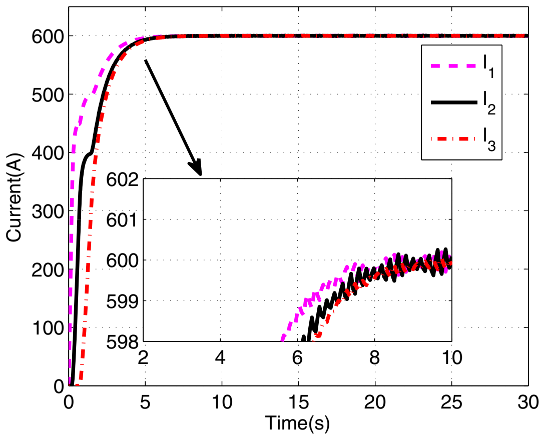

Fig. 3. The current-sharing curve for Case I under the given communication topology,the coupling strength $c = 5$

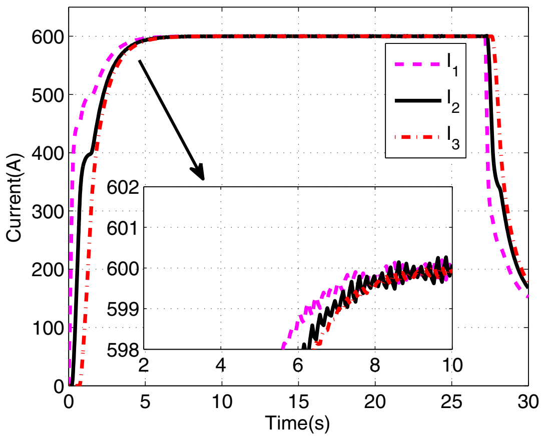

Fig. 4. The current-sharing curve for Case II under the given communication topology, the coupling strength $c = 5$

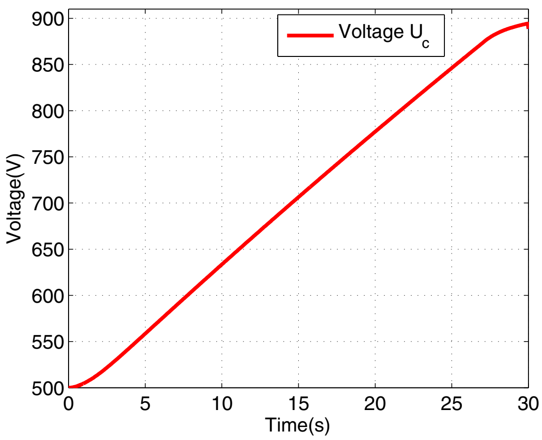

Fig. 5. The voltage curve of the whole charging system for Case II under the given communication topology

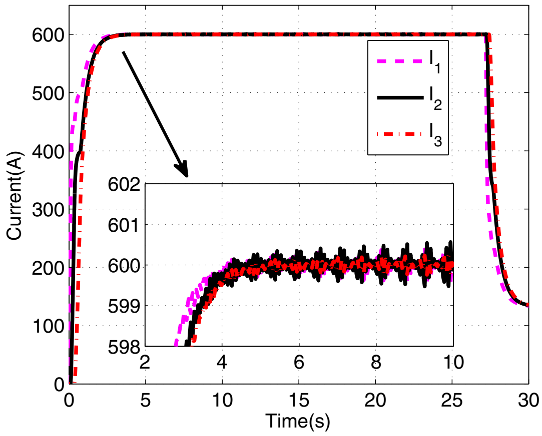

Fig. 6. The current sharing curve for Case II under the given communication topology, the coupling strength $c = 10$

larger coupling strength, the convergence performance will be improved, which will unfortunately result in a larger ripple peak value at the same time.

From Fig. 3 and Fig. 4, we can see that the charging current of the first charger is always larger than that of the other chargers within the first 5 seconds. The reason is that we assume that only the first charger can directly have access to the current-sharing object. The other chargers can only be indirectly affected via communication in a cooperative way. In the end, all chargers can converge to the current-sharing object via local interaction, which is the key idea of cooperative current-sharing control approach.

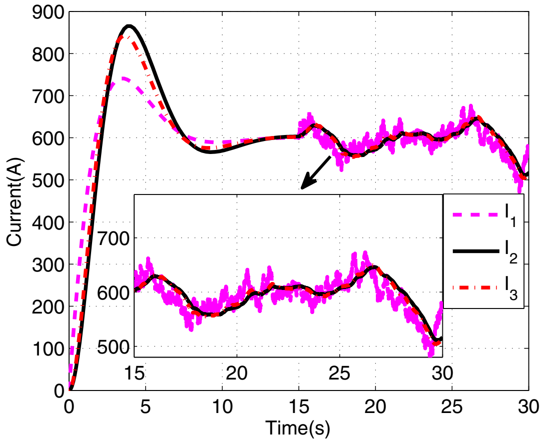

Fig. 7. The current-sharing curve for Case I by adopting master-slave current-sharing approach with respect to failure

4.2 Comparisons for different current-sharing approaches

In this subsection, we compare master-slave and central current-sharing approach with the proposed cooperative current-sharing approach when there exists fault in the charging system. When we use master-slave current-sharing approach, the first charger is assumed as the main module and the remaining two chargers follow the main

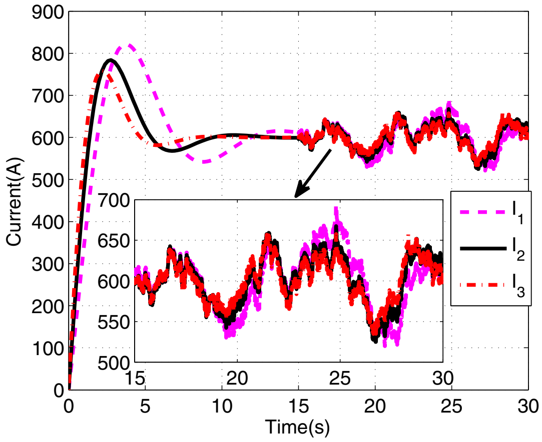

Fig. 8. The current-sharing curve for Case I by adopting central current-sharing approach with respect to failure

module's current. For central current-sharing approach, a central controller sends a unified command to the three chargers. As shown in Fig. 7 and Fig. 8, the whole charging current curve cannot converge and the whole system's reliability cannot be maintained when the main module and the central controller are faulty at time instant $t = 15s$ during the charging procedure. In addition, there exist overshoot in Fig. 7 and Fig. 8 to some extent. Too large overshoot is not allowed in the charging system, which may harm the charger and endanger the human operator.

However, as shown in Fig. 9, the current-sharing result is excellent although some failure exists in charger 2 at time instant $t = 15s$ due to some external disturbance. The control objective can be achieved through a adjusting procedure plotted in the subgraph of Fig. 9, which is carried out by communication between each charger. The charging performance has been maintained because of the robustness of the proposed cooperative approach. And there isn't any overshoot in Fig. 9. Compared with master-slave and central current-sharing approach, the proposed cooperative current-sharing approach is much more robust with respect to failures.

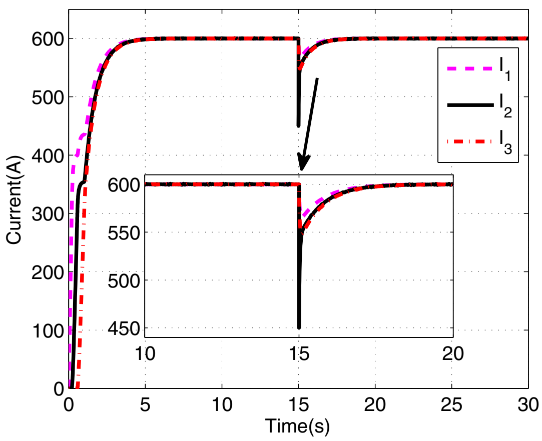

Fig. 9. The current-sharing curve for Case I by adopting the proposed cooperative current-sharing approach with respect to failure

5. CONCLUSIONS

In this paper the bounded distributed cooperative control of the nonlinear and non-identical multi-agent systems is adopted to carry out a current-sharing strategy for the charging system of energy storage type light rail vehicles, which successfully addresses the current imbalance problem. Input-output feedback linearization is introduced to convert the distributed cooperative current-sharing control of multi-charging with non-identical subsystem to a first-order integrator consensus problem with bounded input constraints. A bounded cooperative current-sharing control law is put forward by introducing a novel saturation function. The proposed cooperative current-sharing strategy is distributed and bounded, each charging sub-system only needs its own information and the information of its neighbors. The whole closed-loop charging system is proved to be cooperative stable and the response speed can be improved by tuning current-sharing control parameters. Simulation results show that our current-sharing approach is effective and feasible.

REFERENCES

W. Chen, X. Ruan, H. Yan,C.K.Tse. DC/DC conversion systems consisting of multiple converter modules: stability, control, and experimental verifications. *IEEE Trans. on Power Electron.*24(6):1463-1474, 2009.

J. B. Wang. Study of cable resistance and remote-sensing scheme in parallel DC/DC converter system via primary droop current-sharing control. *IET on Power Electron.* 5(6):885-898, 2012.

S. K. Mazumder, M. Tahir, K. Acharya. Master-slave current-sharing control of a parallel DC/DC converter system over an RF communication interface. *IEEE Trans. on Ind. Electron.* 55(1):59-66, 2008.

Y. Cao, W. Yu, W. Ren, G. Chen. An Overview of Recent Progress in the Study of Distributed Multi-agent Coordination. *IEEE Transactions on Ind. Inform.* 9(1):427-438, 2013.

R. Olfati-Saber, J. Alex Faxand, and R. M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE.* 95(1):215-233, 2007.

H. Xin, Z. Qu, J. Seuss, and A. Maknouninejad. A self organizing strategy for power flow control of photovoltaic generators in a distribution network. *IEEE Trans. on Power Sys.*26(3): 1462-1473, 2011.

A. Bidram, A. Davoudi, F. L. Lewis, J. M. Guerrero. Distributed Cooperative Secondary Control of Microgrids Using Feedback Linearization. *IEEE Trans. on Power Sys.* 28(3): 3462-3470, 2013.

A. Bidram, A. Davoudi, F. L. Lewis. Distributed control for AC shipboard power systems. *2013 IEEE Electric Ship Technologies Symposium*,282-286, 2013.

W. Ren. Consensus Tracking Under Directed Interaction Topologies Algorithms and Experiments. *IEEE Transactions on Control Sys. Tech.* 18(1):1290-1298, 2010.

H. Zhang, F. L. Lewis, A. Das. Optimal design for synchronization of cooperative systems: State feedback, observer and output feedback. *IEEE Transactions on Autom. Control*, 56(8):1948-1952,2011.

R. Diestel. Graph theory. *NY: Springer-Verlag.*1997.