

# Stability Analysis of an Agent-Based Smart Grid Control Marketplace

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**Abstract:** While coupling locally dispersed producers and consumers to large distributed networks comes with different socio-economic advantages as increase in production, higher market adaptability and higher resource efficiency, an immanent disadvantage is the accompanying raise in control complexity. Distributed agent-based control approaches are envisioned as a solution for managing distributed and complex production, supply, and infrastructure networks. Nevertheless they are difficult to be analyzed and hard to be handled. The major challenges coming with distributed control solutions may be found in the field of stability problems such as oscillatory network conditions potentially leading to network collapses. In this paper a general modeling and stability analysis approach for networked nodes is presented and applied on a marketplace of an agent-based smart grid system to distribute an energy demand between producers. Finally the analytical results are evaluated on a smart grid simulation.

**Keywords:** stability analysis, smart power applications, agents, distributed artificial intelligence, computer simulation

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## 1. INTRODUCTION

Following the political, environmental and finally economical parameters caused by diminishing resources and the projected impacts of climate change, the power generation and distribution systems in Europe and many other parts in the world face major changes. These changes include the rise of ever smaller electrical power plants like photovoltaic systems, household-size wind power plants, and small combined heat and power plants, which are distributed throughout the electricity grid. As these systems are furthermore dependent on external energy sources and factors like sun, wind or heat-demand, the raise in control complexity is remarkable. New control concepts have to be found to enable the linkage of this high number of dispersed electricity producers with an equally vast amount of electricity consumers. The future's challenges are to control and match the production and demand of all these hardly predictable, small scale entities. These challenges are not only constrained on the domain of smart grids but are also oppressing problems in other domains, such as traffic systems (Lämmer and Helbing, 2008, Helbing et al., 2013, Wior et al., 2013) and logistic networks (Dashkovskiy et al., 2012, Ouyang and Daganzo, 2011, Wior et al., 2012).

Traditional concepts with a centralized supervisory and control architecture do reach their limits when a high number of entities is involved. The amount of data and the effective degrees of freedom, given by the constantly changing restraints of the single nodes, is not manageable by one single controller. Thus distributed control solutions may provide means of tackling the upcoming problems (Lämmer and Helbing, 2008, Dashkovskiy et al., 2012). Due to their good scalability, Multi Agent Systems (MAS) are one viable option when deploying such a distributed architecture

(Wooldridge and Jennings, 1994). Following Wooldridge and Jennings (1994), MAS are constituted of a number of independent and autonomous software-artefacts or computer systems collaboratively following a higher goal. When implementing an MAS in the power domain, grid and power equipment management tasks may be shifted down to local decision processes performed by a multitude of active and independent agents. Solving local problems on the basis of local information enhances the performance of the entire system, making it possible to adapt to a dynamically changing environment.

Due to these advantages, agent based approaches have received for the last 15 years an ever growing attention in the research community, far beyond the borders of pure software engineering. Early implementations like *POWERMATCHER*, as presented by Kok et al. in 2005, have found their way into real life scenarios (see Kamphuis et al., 2010). Market based approaches like Wedde's *DEZENT* from 2006 and Linnenberg et al.'s *DEMAPOS* from 2011 may be regarded as successors of the early systems, featuring demand side management on the basis of local spot markets. On these markets every producer and consumer of electrical energy is regarded as a "prosumer", i.e. an entity which may produce or consume electrical energy and trade it by itself or in aggregated agent unions, e.g. a household or a factory. In contrast to the aforementioned systems, Richardot's (2006) approach focuses not on effective power but on reactive power control, featuring hierarchical agent communities, parting the grid into several control zones.

As depicted in Linnenberg et al. (2013) the utilization of decentralized control solutions involves a major increase in complexity. Problems are broken down to a large quantity of sub-problems, increasing the number of decision-makers and, thus, possible disturbances. Besides, coordination tasks are

introduced, which have not been necessary before. This growing number of problem-solving instances and coordination tasks leads to a major increase of possible sources of maloperations on the control side. In turn, this results in a less predictable behavior and in increasing difficulties in the analysis of the system and proof of its functionality.

As the regulatory and socio-economic demands in regard to power supply safety and system stability are defined on the base of nowadays state of technology, it is impossible to introduce a control system which is not proven to be at least as reliable as the current solutions. Regarding the fact that it is hard to prove the performance of an MAS, utilities and network operators tend to implement conservative control concept featuring centralized systems. This leads to a limited exploitation of the potentials of renewable energy sources, especially in the field of reserve energy provision. Thus the most imminent drawbacks of MAS have to be pointed out and approaches to solve these issues have to be found.

In Linnenberg et al. (2013) several stability related disturbance sources have been discussed and specified. Besides the local optimization of global problems and inadequate reactions of single agents, latencies in communication and decision making processes were shown to be potential threads, leading to undesired oscillations and ruptures of network constraints or even network breakdowns. Sipahi et al. (2011) state that latencies may be found in distributed and interconnected control solutions due to the increased amount of communication needed to coordinate the control elements. Thus information received by a decision-maker may already be outdated at the time of taking a decision. Apart from general logistic systems, for which Hadel et al. have shown in 2006 that latencies may lead to an oscillatory behavior when agent-based control is applied, this type of error is particularly interesting for electricity grids where certain failure modes like short-circuits or line faults require immediate action due to the systems lack of buffer capacity. One or several of the following characteristics are thereby regarded as pre-requisites for oscillations (Linnenberg et al., 2013).

- *Several alternatives for a decision* – at one decision-node the choice between different alternatives has to be taken
- *Reaction is subject to time delays* – the information returned to the decision-point is subject to latencies and arrives therefore delayed in time
- *Different time delays regarding the different alternatives' reactions* – the reaction of the different options are matter to different latencies
- *Variable time delays in the reaction of each alternative* – the reaction of every single alternative is subject to a variable latency

Latencies may result from a variety of different components and processes in the system (e.g. from the decision making or communication process) and one or many of these effects may appear. In fact, it does not matter which single element leads to a delayed reaction of the system. A delayed reaction will easily result in oscillations and their negative effects.

In the following we will illustrate the occurrence of oscillations in a simple market based power control system using multi agent technology. A mathematical model will be derived from it, enabling the determination of a maximum permissible value for the proportional control gain  $K_p$  by means of a stability analysis approach. The structure of the paper is as follows: in Section 2 the multi agent based control system, its' link to the *MATLAB* based electricity grid simulation as well as the internals of the simulation are described. Afterwards an approximate mathematical description of the system is given and applied to calculate stability criteria for constant demand scenarios in Section 3. The approximate mathematical model is then adapted in Section 4, to facilitate the evaluation of more realistic scenarios considering fluctuating energy production and demand. Finally a short synopsis concludes the paper.

## 2. THE TEST SETUP

In order to incorporate all possible error sources of smart grid control systems, a tripartite system architecture consisting of a simulated electricity grid, the multi agent based control as well as an intermediate communication link was chosen, reflecting real world systems to a great extent. Nevertheless it has to be stated that the control algorithms in the form presented may not be found in the field yet. These are still subject to laboratory trials and small scale real-life implementations in well controlled environments.

### 2.1 Electricity Grid Simulation and Communication Link

To verify the control's actions an electricity grid simulation was implemented in *MATLAB*. The testbed represents a small single-phase grid in islanding mode. It includes *Simulink* models of consumers in form of a residential area and producers of electrical energy in form of combined heat and power plants (CHPP). Furthermore a dedicated grid model is implemented aggregating the power in- and outputs and calculating the grid frequency based on the attached inertias.

The residential area models are based on a 24h load profile and can be switched on or off by the control with no delay except the time needed for the communication act. It covers all non-controllable safety and comfort demands in a grid like lighting, food preparation or health critical systems which forcibly need a certain minimum amount of energy and shall only be switched off in the case of severe grid instability. The residential area models are represented by an energy saving lamp in the left part of Fig. 1.

The combined heat and power plant (CHPP) features a physical model of a group of real life combined heat and power plants. It represents a broad spectrum of gas and steam power stations. Within the system it is operated in a margin of 0-3900 kW (kilo Watts). Thereby, it is the most flexible power generator and allows for control in a wide power range. In order to reproduce a realistic start-up behavior, a ramp up function in form of a first order lag element was chosen to delay the effective power increase. In the left part of Fig. 1 the CHPP is represented by a block with a flash at its centre.

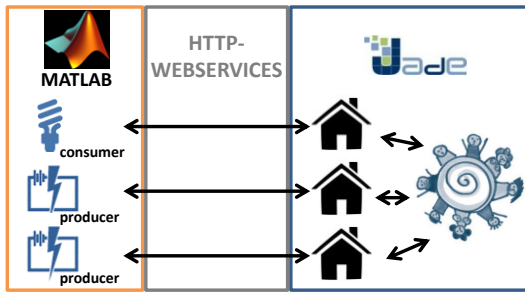


Fig. 1. Structure of the considered smart grid marketplace.

The execution time of the simulation was optimized on a 4 Hz clocking. This is owed to the control's 2 Hz work cycle and the Nyquist-Shannon sampling theorem. Thus the simulation will calculate the grid conditions resulting from former control input parameters and precedent node states at least once before the control queries the according datasets.

To provide this information and maintain a high flexibility a modular concept as shown in Fig. 2 was chosen. Commands from the control are received through an *Apache Tomcat* webserver, from which they are transferred to *MATLAB* through the *Modelit Webserver Toolbox*. *MATLAB* is the core of the simulation, linking different sources of information with *MATLAB Simulink* modules, representing the devices connected to the grid. The characteristics and diurnal variations of those devices are based on real- world datasets which are stored in separate data files.

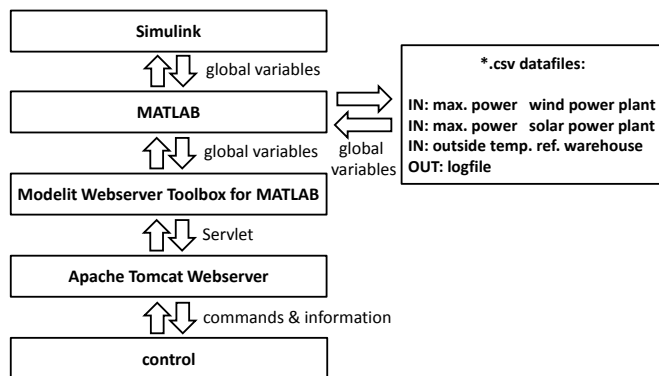


Fig. 2. Simulation setup (Linnenberg et al., 2011).

### 2.2 Multi agent based control

The multi agent based control utilized in this paper is an adapted version of the *DEcentralized MARKET based POWER control System* DEMAPOS (Linnenberg et al., 2011). In the present implementation it features three different agent types: *Prosumer*, *Housekeeper* and a central *Marketplace*, allowing the *Housekeepers* to trade the energy needed or provided by their underlying *Prosumers*.

The *Prosumers* can be seen as proxies to the real-world hardware or the testbed. They feature a plug-in architecture allowing for a flexible implementation of different communication channels and standards. The *Prosumer Agent* has in depth knowledge about the system under his control

and communicates its needs and capabilities together with its freedom of action to its superior *Housekeeper Agent*.

The *Housekeeper* or *Home Gateway* is located at the house service connection level. It monitors all underlying *Prosumer Agents* and tries to balance their energy offers and requests internally. If there is a surplus production or some unsatisfied needs it approaches the local marketplace and offers or requests the remaining energy. The prices requested for offered energy are equal to the amount of energy traded in the round before (the higher the demand the higher the price). To take into account the latencies occurring in networks featuring a high number of participants and thus making prolonged negotiations inevitable, the feedback of the price information is delayed by several seconds. An example for a typical latency during MAS negotiations is the time period given to bidders to answer to an announcement. See Section 3 and 4 for further details and the actual values used. The houses depicted in Fig. 1 represent the *Housekeeper Agents*.

The *Marketplace Agent* collects all requests and offers from its registered housekeepers. After receiving the commitments composed by an amount of energy to be traded, a corresponding price, as well as a flag indicating whether it is an offer or a request, the marketplace tries to match all obtained queries. To provide the power system with a faster response reserve capacity the marketplace aims to balance the utilization of the producers. Another control strategy would be to utilize always the currently cheapest producer, but here small price changes can lead to strong redistributions and hence unwanted oscillations or instabilities. After calculating the trading outcomes the results are communicated to the subordinate housekeepers and a new bidding round is started. The *Marketplace Agent* is represented in the right part of Fig. 1 by a round table with eight people around it.

Fig. 1 depicts the control and simulation setup used for all test runs. The arrows describe communication links between system components. The boxes show the execution environment of the individual elements. In Section 3 an approximated model of the agent-based smart grid control marketplace depicted in Fig. 1 is presented. Based on this approximate model the smart grid marketplace is then analyzed for stability.

### 3. AGENT-BASED SMART GRID CONTROL MARKETPLACE WITH CONSTANT DEMAND

In the following a linear time-invariant (LTI) model of an agent-based smart grid control marketplace with delayed feedback information and constant demand is presented and analyzed for stability. A modeling and stability analysis approach for decision nodes already applied for material handling systems in Wior et al. (2012) and for traffic systems in Wior et al. (2013) is utilized. Wior et al. (2012) presents an oscillatory problem in distributed material handling systems and analyzes the stability of decision nodes with constant and varying time delays. Wior et al. (2013) considers traffic systems and models and analyzes basic constellations of decision nodes with constant time delays. The same decision node modeling approach is applied in the next subsection to construct an approximate model of a smart grid marketplace.

Afterwards, a frequency domain analysis to determine the stability of the marketplace is utilized.

### 3.1 Modeling a Marketplace with Constant Demand

The considered smart grid marketplace (see Fig. 1) contains one consumer with a constant demand  $v^{in}$  and two producers which produce the energy  $v_a^{out}(t)$  and  $v_b^{out}(t)$  to satisfy this demand. In Fig. 3 a block diagram of the approximate model of the marketplace is depicted. The distribution of the demand  $v^{in}$  among the producers is controlled through the distribution rate  $\alpha(t)$ , which is a continuously calculated fraction of the demand  $v^{in}$  to be supplied by producer  $a$ . The resulting demand is called the input demand  $v_a^{in}(t)$  for producer  $a$  [ $\rightarrow v_a^{in}(t)=v^{in}\cdot\alpha(t)$ ]. The remaining demand has to be supplied by producer  $b$ , hence its input demand  $v_b^{in}(t)$  is the difference between the consumer demand  $v^{in}$  minus the input demand  $v_a^{in}(t)$  [ $\rightarrow v_b^{in}(t)=v^{in}-v_a^{in}(t)=v^{in}\cdot(1-\alpha(t))$ ]. This ensures a complete division of  $v^{in}$  into  $v_a^{in}(t)$  and  $v_b^{in}(t)$  to cover continually the whole demand. The signals  $v^{in}$ ,  $v_a^{in}(t)$ ,  $v_b^{in}(t)$ ,  $v_a^{out}(t)$  and  $v_b^{out}(t)$  are given in kW.

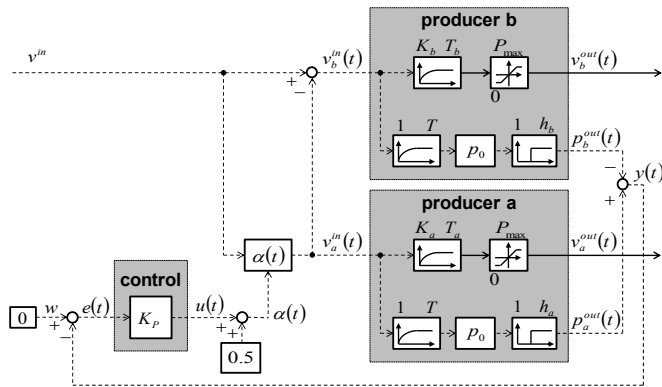


Fig. 3. Block diagram of the approximate model of a smart grid marketplace with two producers.

Each of the producers is a group of combined heat and power plants with each an arbitrary controllable energy production between zero and  $P_{max}=3900$  kW. As depicted in the block diagram in Fig. 3 both producers have the same structure. For better illustration, energy flows in Fig. 3 are represented by solid lines and information flows by dashed lines. The start-up dynamics of the combined heat and power plants are approximated by a first-order lag element behavior with  $K_a=K_b=1$  and  $T_a=T_b=0.1086$  (determined from the adapted DEMAPOS model). The offers of the producers  $p_a^{out}(t)$  and  $p_b^{out}(t)$  (given in monetary units) are negotiated by the group of combined heat and power plants within each producer. The negotiation process is represented by a price constant  $p_0$  and negotiation delays  $h_a$  or  $h_b$ . The producers' offers are equal to the amount of energy traded in the round before, hence, in the approximate model following this linear price calculation a price constant of  $p_0=1$  is used for both producers. As each producer represents a different group size of combined heat and power plants, the internal negotiations cause different negotiation delays. To account for negotiation delays, the latencies  $h_a$  and  $h_b$  are included. In this paper these latencies are assumed to be constant during the simulation period.

Furthermore, the intern processing times in the adapted DEMAPOS model are much smaller than the sampling time, hence to prevent algebraic loops in the system's feedback loop, a first-order lag element with  $T=0.5$  s was introduced in the adapted DEMAPOS model and the approximate model.

The difference between the production offers  $p_a^{out}(t)$  and  $p_b^{out}(t)$  is the feedback information  $y(t)$ , which is subtracted from the reference signal  $w$  and forms the control error  $e(t)$  as input for the controller with a proportional gain  $K_p$ . The reference signal  $w$  is set to zero, aiming to balance both offers  $p_a^{out}(t)$  and  $p_b^{out}(t)$  and hence the utilization of both producers. The distribution rate  $\alpha(t)$  is the sum of the output of the controller  $u(t)$  and a constant 0.5. If the error  $e(t)$  is zero, this 0.5 constant yields a distribution rate of 50% and hence an equipartition of the demand on both producers.

Note, with a very high  $K_p$ -value the cheapest producer would be asked to provide its full production capacity. Slight cost changes would then cause a complete changeover from one producer to another. This would result in network oscillations and should therefore be avoided. Instead the  $K_p$ -value has to be adjusted to smoothly distribute an energy demand between producers.

The approximate model of a marketplace shown in Fig. 3 has been implemented as a Matlab/Simulink simulation. In Fig. 4 a comparison of the results of a nearly marginal stable behavior from the adapted DEMAPOS and the approximate model is shown (for the case  $v^{in} = 1300$  kW,  $h_a = 5$  s,  $h_b = 6$  s,  $p_0 = 1$ , and  $T = 0.5$  s). Although the approximate model's results show a slightly smaller frequency, the approximate model is close enough to the adapted DEMAPOS model to form the basis for a stability analysis.

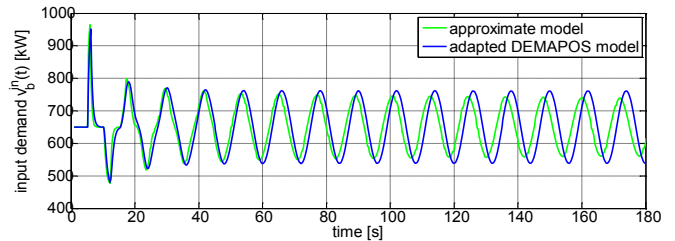


Fig. 4. Comparison of simulation results from the adapted DEMAPOS model and the approximate model.

In the following, an approximate representation of the described smart grid marketplace as a transfer function is derived from the block diagram presented in Fig. 3.

$$\begin{aligned}
 Y &= P_a^{out} - P_b^{out} \\
 &= e^{-s \cdot h_a} \cdot 1 / (T \cdot s + 1) \cdot p_0 \cdot v^{in} \cdot (0.5 + K_p \cdot (W - Y)) \\
 &\quad - e^{-s \cdot h_b} \cdot 1 / (T \cdot s + 1) \cdot p_0 \cdot (v^{in} - v^{in} \cdot (0.5 + K_p \cdot (W - Y))) \\
 &= p_0 \cdot v^{in} \cdot 0.5 \cdot (e^{-s \cdot h_a} - e^{-s \cdot h_b}) / (T \cdot s + 1) \\
 &\quad + p_0 \cdot v^{in} \cdot K_p \cdot (W - Y) \cdot (e^{-s \cdot h_a} + e^{-s \cdot h_b}) / (T \cdot s + 1)
 \end{aligned} \tag{1}$$

Note that the dependency on  $s$ , like in  $Y(s)$ , is omitted to make the equations more compact. The term  $0.5 \cdot (e^{-s \cdot h_a} - e^{-s \cdot h_b})$  is a difference between the constant value of 0.5 delayed by  $h_a$  minus another constant value of 0.5

delayed by  $h_b$ . If both delays  $h_a$  and  $h_b$  are close to each other, this difference can be treated as a relatively small disturbance and is hence neglected within the approximate model. Note, this simplification introduces an error into the model which grows if the difference between both time delays grows (in a later adapted approximate model presented in Section 4, this simplification will be avoided). The resulting transfer function of the approximate model is shown in (2).

$$\frac{Y}{W} = \frac{p_0 \cdot v^{in} \cdot K_p \cdot (e^{-s \cdot h_a} + e^{-s \cdot h_b})}{T \cdot s + 1 + p_0 \cdot v^{in} \cdot K_p \cdot (e^{-s \cdot h_a} + e^{-s \cdot h_b})} \quad (2)$$

Therefore, the marketplace with constant negotiation delays and a constant demand can be approximated as a linear time-invariant (LTI) system. Although, for simplicity, only a model for a proportional controller is derived, it is straightforward to extend it for other linear controllers.

### 3.2 Analysis of a Marketplace with Constant Demand

To determine the stability of the marketplace with constant negotiation delays and a constant demand, as modeled in (2), any standard frequency domain stability analysis methods for LTI systems can be utilized. First, the constant negotiation delays are approximated by a Padé approximant of eighth order. With that approximation, the method root locus analysis is used to determine the maximum proportional gain  $K_p$  which still yields stability of the marketplace. In Table 1 for different combinations of latencies the root locus analysis results are compared with the adapted DEMAPOS simulation results as well as the approximate model simulation results. In Table 1 a maximum  $K_p$  limit for stable behavior and a minimum  $K_p$  limit for unstable behavior are given, due to the difficulty to determine the  $K_p$  value at marginal stability.

**Table 1. Comparison of the stability limits of different models of a marketplace with constant demand**

models	parameters					
	$v^{in} = 1300 \text{ kW}, p_0 = 1, \text{ and } T = 0.5 \text{ s}$					
	$h_a = 5 \text{ s}$ $h_b = 6 \text{ s}$		$h_a = 5 \text{ s}$ $h_b = 7.5 \text{ s}$		$h_a = 5 \text{ s}$ $h_b = 12.5 \text{ s}$	
	$K_p$ - stable [ $\cdot 10^{-4}$ ]	$K_p$ - not stable [ $\cdot 10^{-4}$ ]	$K_p$ - stable [ $\cdot 10^{-4}$ ]	$K_p$ - not stable [ $\cdot 10^{-4}$ ]	$K_p$ - stable [ $\cdot 10^{-4}$ ]	$K_p$ - not stable [ $\cdot 10^{-4}$ ]
<b>adapted DEMAPOS model</b> [see Section 2]						
simulation	4.01	4.04	4.59	4.61	4.47	4.53
<b>approximate model</b> [see Fig. 3 and (2)]						
simulation	4.12	4.13	4.73	4.74	4.89	4.90
root locus analysis	4.11	4.12	4.72	4.73	4.88	4.89
<b>adapted approximate model</b> [see Fig. 5 and (4)]						
simulation	4.12x	4.13x	4.73x	4.74x	4.89x	4.90x
	1300	1300	1300	1300	1300	1300
root locus analysis	4.11x	4.12x	4.72x	4.73x	4.88x	4.89x
	1300	1300	1300	1300	1300	1300

As shown in Table 1, the results of the root locus analysis of the approximate model are very close to the approximate model simulation results. Furthermore, the approximate model is close to but does not exactly represent a marketplace in the adapted DEMAPOS model. A modeling error results in

a deviation in the stability limits between around 2.5% (in case of the latencies  $h_a = 5 \text{ s}$  and  $h_b = 6 \text{ s}$ ) and around 9.5% (in case of the latencies  $h_a = 5 \text{ s}$  and  $h_b = 12.5 \text{ s}$ ). Finally, despite of a modeling error, with the presented analysis approach based on the approximate model (2) it is possible to analyze an agent controlled smart grid marketplace with constant demand.

## 4. AGENT-BASED SMART GRID CONTROL MARKETPLACE WITH VARIABLE DEMAND

In Section 3 a smart grid marketplace with two producers with each a constant negotiation delay as well as one consumer with a constant demand was successfully modeled as an LTI model and analyzed with a root locus approach. Dealing with more realistic marketplaces, a variable demand  $v^{in}(t)$  has to be considered. Unfortunately, with a variable demand the approximate model used in Section 3 and shown in Fig. 3 becomes nonlinear because of a multiplication of the two variable signals  $v^{in}(t)$  and  $\alpha(t)$ . To analyze such a nonlinear model a frequency domain analysis as the root locus approach is not applicable anymore. To be able to analyze a decision node with variable demand, in Wior et al. (2013) the variable demand  $v^{in}(t)$  has been considered as an uncertain demand  $v^{in}$  and a robust stability criterion from Wu et al. (2010) has been applied. This criterion is Lyapunov based and formulated as an LMI (linear matrix inequality), but leads to quite conservative results (conservatism means that the analysis approach states instability at a control gain lower than the system's real value). To avoid this conservatism the approximate model shown in Fig. 3 will be reformulated in the following subsection to result, despite a variable demand, in an LTI model. Afterwards this adapted approximate model is analyzed with a root locus approach.

### 4.1 Modeling a Marketplace with Variable Demand

To provide an LTI model of a smart grid marketplace with variable demand, the previous approximate model (2) presented in Fig. 3 is reformulated to eliminate the dependency of the stability of the system on the variable demand  $v^{in}(t)$ . A block diagram of the adapted approximate model is presented in Fig. 5. Comparing the new adapted approximate modeling approach with the previous approximate modeling approach the structure of a producer is

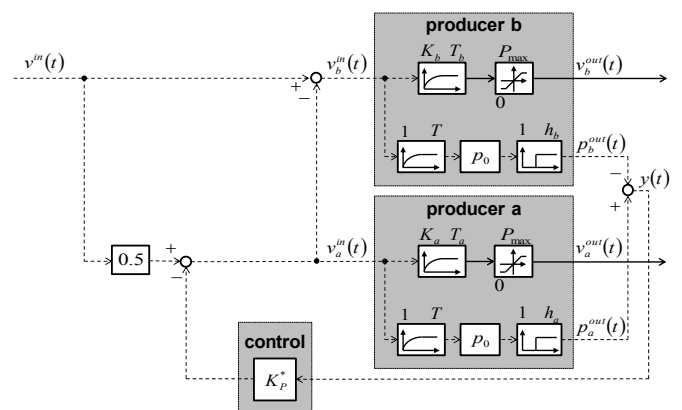


Fig. 5. Block diagram of the adapted approximate model.



still the same but a new control structure is utilized. In the previous modeling approach the control loop determines the distribution rate  $\alpha(t)$  which in turn distributes the demand, but in the adapted approximate modeling approach the control loop directly controls the input demands  $v_a^{in}(t)$  and  $v_b^{in}(t)$ . The feedback information  $y(t)$  is the input for the controller with gain  $K_p^*$ . To determine the input demand  $v_a^{in}(t)$  the consumer demand  $v^{in}(t)$  is first scaled by the constant 0.5 to be then reduced or increased by the controller's output. If the controller's output is zero, this 0.5 constant yields an equipartition of the demand on both producers.

A transfer function representation of the described adapted approximate model of a smart grid marketplace with variable demand is given in the following.

$$\begin{aligned} Y &= P_a^{out} - P_b^{out} \\ &= e^{-s \cdot h_a} \cdot 1/(T \cdot s + 1) \cdot p_0 \cdot (V^{in} \cdot 0.5 - K_p^* \cdot Y) \\ &\quad - e^{-s \cdot h_b} \cdot 1/(T \cdot s + 1) \cdot p_0 \cdot (V^{in} - V^{in} \cdot 0.5 + K_p^* \cdot Y) \\ &= p_0 \cdot 0.5 \cdot (e^{-s \cdot h_a} - e^{-s \cdot h_b}) \cdot V^{in}/(T \cdot s + 1) \\ &\quad - p_0 \cdot K_p^* \cdot (e^{-s \cdot h_a} + e^{-s \cdot h_b}) \cdot Y/(T \cdot s + 1) \end{aligned} \quad (3)$$

The resulting transfer function is shown in (4).

$$\frac{Y}{V^{in}} = \frac{p_0 \cdot 0.5 \cdot (e^{-s \cdot h_a} - e^{-s \cdot h_b})}{T \cdot s + 1 + p_0 \cdot K_p^* \cdot (e^{-s \cdot h_a} + e^{-s \cdot h_b})} \quad (4)$$

Therefore, the marketplace with constant negotiation delays and a variable demand can be represented as an LTI system. Comparing the transfer functions (2) and (4) the numerator and hence the stability of (4) is not anymore dependent on the demand  $v^{in}$ . Therefore, the marketplace can be adjusted stable independently of the demand which is an advantage of the adapted approximate model (4). Note that in the derivation of the adapted model (4) no simplification was taken as it was necessary for (2). Furthermore, the adapted DEMAPOS control structure was slightly changed to fit with the new control approach shown in Fig. 5.

#### 4.2 Analysis of a Marketplace with Variable Demand

Before in the following the adapted approximate model (4) will be utilized to analyze a marketplace with variable demand, at first, its suitability for marketplaces with constant demands is evaluated. At the end of Table 1 simulation results as well as results from a root locus analysis, both based on the adapted approximate model, are presented. These results correspond exactly with the results of the approximate model considering that the difference between the denominators of (2) and (4) is  $K_p \cdot v^{in} \rightarrow K_p^*$ . Therefore, the stability analysis of smart grid marketplaces with constant demands can equally be based on an approximate model or on an adapted approximate model.

The LTI adapted approximate model (4) of a marketplace with constant negotiation delays and a variable demand is analyzed for stability with the root locus approach, as already discussed in Subsection 3.2. The variable demand is a recorded 24h demand profile of a residential area. To avoid prolonged simulation times with the adapted DEMAPOS

model the 24h demand profile was compressed to fit within a 20 min period. The compressed demand profile together with marginal stable simulation results from the adapted DEMAPOS model (for the case  $h_a = 5$  s,  $h_b = 6$  s,  $p_0 = 1$ , and  $T = 0.5$  s) are depicted in Fig. 6. A comparison of the found stability limits is shown in Table 2.

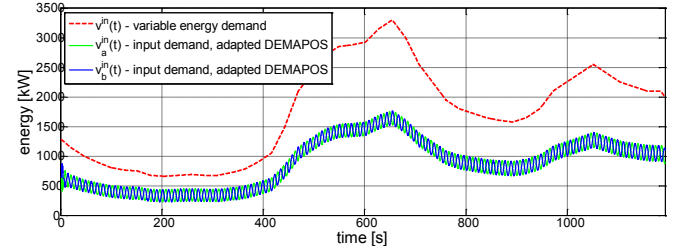


Fig. 6. Variable energy demand and marginal stable simulation results from the adapted DEMAPOS model.

**Table 2. Comparison of the stability limits of different models of a marketplace with variable demand**

models	parameters			
	$v^{in}(t) \triangleq 24h$ demand profile, $p_0=1$ , $T=0.5$ s			
	$h_a = 5$ s and $h_b = 6$ s	$h_a = 5$ s and $h_b = 12.5$ s		
	$K_p$ -stable	$K_p$ -not stable	$K_p$ -stable	$K_p$ -not stable
<b>adapted DEMAPOS model</b> [see Section 2]				
simulation	0.522	0.526	0.582	0.590
<b>adapted approximate model</b> [see Fig. 5 and (4)]				
simulation	0.534	0.537	0.634	0.640
root locus analysis	0.535	0.536	0.635	0.636

As shown in Table 2, the stability limits found by utilizing the adapted approximate model are higher than the ones from the adapted DEMAPOS model. The  $K_p$ -stable values deviate by around 2.5% in case of the latency combination  $h_a = 5$  s and  $h_b = 6$  s as well as by around 9% in case of  $h_a = 5$  s and  $h_b = 12.5$  s. The size of the deviations matches with the one found in the case of a marketplace with constant demand (see Table 1). Both results based on the adapted approximate model, the simulation results as well as the results from the root locus analysis, match well. Note, the new modeling and analysis approach allows for clearly less conservative results as previously found in Wior et al. (2013) in the analysis of traffic nodes. Finally, with the presented analysis approach based on the adapted approximate model (4) it is possible to analyze agent controlled smart grid marketplaces with constant as well as variable energy demands.

## 5. CONCLUSION

Multi agent systems are distributed control approaches that are able to manage the high complexity of large distributed systems; like production networks, traffic systems, and smart grids; but are difficult to be analyzed for stability, especially if feedback information latencies are included. In this paper, an agent controlled smart grid marketplace with constant latencies, which distributes an energy demand between producers, was successfully modeled as well as analyzed for stability. The analytical results obtained were evaluated on a smart grid simulation. The applied modeling and analysis

approaches are not limited to the smart grid domain but can also be applied for production and traffic systems.

In this paper, only single smart grid decision nodes, i.e. single marketplaces, were considered. But the modeling and analysis approaches utilized for marketplaces with variable demands (see Section 4) can also be extended to more complex smart grid networks. Smart grids have a hierarchical structure with different decision levels: a marketplace on the one hand controls the production and demand on its own decision level but also the surplus production or unsatisfied needs from lower decision levels, e.g. from the connected housekeepers, and on the other hand offers its own surplus production or unsatisfied demand on the next higher decision level, e.g. a higher marketplace on a higher voltage level. Each of these decision entities, i.e. the housekeepers, the marketplaces and the higher marketplaces, are decision nodes facing the same distribution problem as presented in this paper. For example, if at a decision node the sum of offered production is higher than the sum of stated demand, it can be assumed that the demand will be fully satisfied and exactly the same situation as discussed in this paper is given. But if at a decision node the sum of stated demand should be higher than the sum of offered production, the different energy purchasers are set to be the different alternatives between which the production has to be distributed and hence the roles change. Comparing with the situation presented in this paper the roles change but the same modeling and analysis approach can still be utilized for this new problem situation. Therefore, in a cascaded manner following the smart grid hierarchy from bottom to top, each decision node on the lower decision level is first adjusted stable with the approaches presented in this paper. Hence its propagated demand or surplus will be stable, as well. Then, the decision node on the next higher level can be adjusted stable and so forth. To extend the analysis to complex smart grid networks becomes possible, because each decision node is autonomous and only coupled to other decision nodes through the demanded or offered energy. For the decision node it is not important on which decision level it resides or how complex the other decision nodes are but only the demanded or offered energy. Applying the modeling and stability analysis presented in this paper to smart grid networks will be scope of future work.

Furthermore, considering more realistic smart grid marketplaces with time-varying latencies will significantly increase the complexity for a stability analysis and will be tackled by the authors in the future.

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