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Stabilization of Discrete-time Networked Fuzzy Systems ⋆

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Abstract: This paper is concerned with the stabilization problem for a class of discrete-time networked T-S fuzzy systems with bounded time delays and packet losses. By explicitly considering physical properties of networked control systems (NCSs), sufficient conditions for the existence of state feedback fuzzy controller is derived. Then a stabilization approach based on a parallel distributed compensation scheme is developed. The merit of the proposed method lies in its much less conservatism, which is achieved by guaranteeing the deceasement of Lyapunov functional at each control signal updating step. Illustrative examples are provided to show the advantage and effectiveness of the developed results.

1. INTRODUCTION

During the past decades, the fuzzy-logic theory has attracted great attention for its theoretical and practical significance in dealing with the analysis and synthesis problems of nonlinear systems. In particular, Takagi-Sugeno (T-S) fuzzy models have proven to be capable of approximating any smooth nonlinear systems to any specified accuracy, by smoothly blending a set of local linear models via fuzzy membership functions. Under this appealing framework, the well-developed linear system theory can be easily applied to the analysis and synthesis of nonlinear systems. Therefore, many nonlinear analysis problems have been studied based on the T-S fuzzy model, with a rich body of important results reported in the literature. To mention a few, the stability analysis and synthesis problems of T-S fuzzy systems have been investigated in Y. Y. Cao et al. [2001], H.H. Choi et al. [2007], H. Gao et al. [2007], J. Dong et al. [2009], G. Feng et al. [2010], Z. Xi et al. [2011], Y.-J. Chen et al. [2012], H. H. Choi et al. [2013], while the filtering or fault detection problems have been addressed in C. Lin et al. [2007], X. Li et al. [2013], M. Chadli et al. [2013]. Fore more details on this topic, we refer the readers to G. Feng et al. [2010], M. Chadli et al. [2013], and the references therein.

It should be noted that all the aforementioned literatures are based on an implicit assumption that the signals in T-S fuzzy systems are directly feedback to the destination nodes (i.e., the controller node or the actuator note) without any time delays or packet losses. Unfortunately, this assumption is difficult to be satisfied in a spatially distributed control system if data transmission is achieved via a communication network. In fact, it is appealing to employ communication network for data transmission in control systems from the practical engineering point of view, which brings a new active research front termed networked control systems (NCSs). Comparing with the traditional control systems, NCSs shows many significant advantages, such as low cost, ease of maintenance and high reliability. So far, NCSs have been widely used in a wide range of areas (e.g., remote surgery, industrial automation, unmanned vehicles, aerospace, and robots), with many interesting results reported in the literature, see e.g., Xiong et al. [2007], L. A. Montestruque et al. [2007], Xiong et al. [2009], Y. Shi et al. [2011], D. Nešić et al. [2012], X. He et al. [2013], and the reference therein.

Motivated by the advantages of T-S fuzzy models and NCSs, the networked control problem of T-S fuzzy systems has been an active field of research and there have been rich results on this topic in the literature. To mention a few, the state feedback robust control of networked T-S fuzzy systems with time delays and packet losses is addressed in H. G. Zhang et al. [2009]. The output feedback control of networked T-S fuzzy systems with multiple packet dropouts is studied in J. Qiu et al. [2011]. While in C. C. Hua et al. [2012], the decentralized memoryless state feedback controller design method is proposed for networked T-S fuzzy systems with time delays. The aforementioned research results have a significant impact on both the theoretical advances and practical applications of the stabilization problem for networked T-S fuzzy systems. Nevertheless, it is worth pointing out that, for the results obtained so far concerning this topic, especially for the results on state feedback stabilization problem of networked T-S fuzzy systems, there still leave much room for improvement in their conservatism.

Therefore, in this paper, we revisit the stabilization problem for a general class of discrete-time networked T-S fuzzy systems

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with time delays and packet losses, where our focus is to further reduce the conservatism of networked T-S fuzzy systems. By explicitly considering physical properties of NCSs, a NCSs model with some physical constraints is used to describe the closed-loop networked T-S fuzzy systems. Under this framework, an improved stability criterion dependent on both time delay bound and packet loss bound is derived, by guaranteeing the deceasement of Lyapunov functional at each control signal updating step rather than at each sampling step. Based on the obtained stability conditions, we further investigated the corresponding state feedback controller design problem. Illustrative examples are provided to show the advantage and effectiveness of the developed results.

The organization of this paper is as follows. The system description and problem formulation are provided in Section II. The stability conditions and controller design methodology for the nonlinear NCSs are presented in Section III. Two numerical examples are provided in Section IV. Finally we conclude this paper in Section V.

Notation. Throughout this paper, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and the notation $\boldsymbol{P} > 0(\geq 0)$ means that $\boldsymbol{P}$ is real symmetric and positive definite (semidefinite). The superscript "^{T}" denotes matrix transposition; and for a matrix $\boldsymbol{A}$, $\mathrm{sym}(\boldsymbol{A})$ denotes $\boldsymbol{A}+\boldsymbol{A}^T$. I is the identity matrices with appropriate dimensions. In symmetric block matrices, we use "$*$" as an ellipsis for the terms introduced by symmetry. Sometimes, when no confusion would arise, the dimensions of a matrix will be omitted for convenience.

2. PROBLEM FORMULATION

The structure of NCSs considered in the paper is shown in Fig. 1, where the dynamics of the controlled plant is described by the T-S fuzzy model and it can be represented by the following form:

$$\begin{array}{l}\bullet\text{Plant rule } i\text{:}\\ \quad \text{IF } \theta_1(k) \text{ is } \mu_{i1}, \text{ and} \cdots, \theta_g(k) \text{ is } \mu_{ig},\\ \quad \text{THEN } \ \boldsymbol{x}(k+1)=\boldsymbol{F}_i\boldsymbol{x}(k)+\boldsymbol{G}_i\boldsymbol{u}(k)\\ \qquad\qquad (\text{for } i=1,2,\cdots,r)\end{array} \tag{1}$$

where $\mu_{i\varpi}$ ($\varpi=1,2,\cdots,g$) are the fuzzy sets, $\boldsymbol{x}(k)\in\mathbb{R}^n$ is the plant state, $\boldsymbol{u}(k)\in\mathbb{R}^m$ is the control input, $\boldsymbol{F}_i$ and $\boldsymbol{G}_i$ are matrices of compatible dimensions, r is the number of IF-THEN rules, $\theta=[\theta_1\ \ \theta_2\ \ \cdots\ \ \theta_g]$ are the premise variables. It is assumed that the premise variables do not depend on the input $\boldsymbol{u}(k)$. By using the fuzzy inference method with a center-average defuzzifier, product inference, and singleton fuzzifier, the controlled plant in (1) can be expressed as:

$$\boldsymbol{x}(k+1)=\sum_{i=1}^{r}\mu_i(k)[\boldsymbol{F}_i\boldsymbol{x}(k)+\boldsymbol{G}_i\boldsymbol{u}(k)], \tag{2}$$

where

$$\mu_i(k)=\frac{w_i(k)}{\sum\limits_{i=1}^{r}w_i(k)},\quad w_i(k)=\prod_{j=1}^{p}\mu_{ij}[\theta_j(k)] \tag{3}$$

It is assumed that $w_i(\theta(k))\geq 0$ for $i\in\mathcal{R}=1,2,\cdots,r$ and $\sum_{i=1}^{r}w_i(\theta(k))>0$ for k. Therefore, we can conclude that $\sum_{i=1}^{r}\mu_i(\theta(k))\geq 0$ for $i=1,2,\cdots,r$ and $\sum_{i=1}^{r}\mu_i(\theta(k))=1$ for all k.

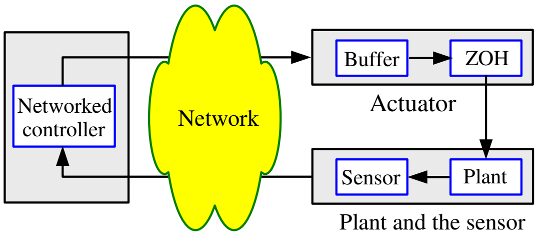

Fig. 1. The structure of the considered NCSs

In the considered NCSs, the sensor and actuator are time driven. The actuator employs logical zero-order holder (ZOH) with logical storage capacity, which enables NCSs to use the most recent control signal to control the plant. The networked controller is event driven and takes the following form:

$$\begin{array}{l}\bullet\text{Controller rule } i\text{:}\\ \quad \text{IF } \theta_1(k) \text{ is } \mu_{i1}, \text{ and} \cdots, \theta_g(k) \text{ is } \mu_{ig},\\ \quad \text{THEN } \ \boldsymbol{u}=\boldsymbol{L}_i\boldsymbol{x}\\ \qquad\qquad (\text{for } i=1,2,\cdots,r)\end{array} \tag{4}$$

where $\boldsymbol{L}_i$ are the feedback gains to be designed. Then the final output of the networked fuzzy controller is:

$$\boldsymbol{u}=\sum_{i=1}^{r}\mu_i(k)\boldsymbol{L}_i\boldsymbol{x} \tag{5}$$

Letting $\bar{\boldsymbol{F}}=\sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{F}$, $\bar{\boldsymbol{G}}=\sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{G}$, $\bar{\boldsymbol{L}}=\sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{L}_i$, and applying controller (5) to the plant (2), we can obtain the following compact closed-loop system:

$$\boldsymbol{x}(k+1)=\bar{\boldsymbol{F}}\boldsymbol{x}(k)+\bar{\boldsymbol{G}}\bar{\boldsymbol{L}}\boldsymbol{x}(k-\tau_k), \tag{6}$$

where τ_k is the input delay of control signal acting on the plant at time step k, which reflects the comprehensive effect of time delays, packet losses, and packet out-of-order. For τ_k in (6), a natural assumption can be made as follows:

$$\tau_1\leq\tau_k\leq\tau_2, \tag{7}$$

where τ_1 and τ_2 are the lower and upper bounds respectively.

Before proceeding, we introduce the following definitions. The time step at which new control signal arrives at ZOH is called *updating step*, while the one at which no new control signal arrives at ZOH is called *holding step*. Let $S\triangleq\{i_1,i_2,\cdots\}$ (a subsequence of $\{1,2,\cdots\}$) and $\delta_{i_k}\triangleq i_{k+1}-i_k$ denote the sequence of time indexes of *updating steps* and non-updating step number until time step i_k, respectively. As stated in our earlier work H. Li et al. [2013], one can easily infer that the following two properties always hold for NCSs:

$$\textbf{Pro. 1:}\ \begin{cases}\tau_{k+1}\leq\tau_k & \text{at updating step}\\ \tau_{k+1}=\tau_k+1 & \text{at holding step}\end{cases} \tag{8}$$

$$\textbf{Pro. 2:}\quad 0\leq\delta_{i_k}\leq\hat{\delta}\triangleq\max\nolimits_{i_k\in S}(\delta_{i_k}), \tag{9}$$

Property 2 means at least one time step is updating step every $\hat{\delta}$ time steps. Note that $\hat{\delta}$ reflects the comprehensive effect of time delay, maximum consecutive packet losses (MCPL) and packet out-of-order.

The objective of this paper is to design the networked controller (5) such that the NCS (6) is stable.

3. MAIN RESULTS.

In this section, we will first address the stability analysis problem for the considered the networked fuzzy system based on the Lyapunov functional method. By guaranteeing the deceasement of Lyapunov functional at each control signal updating step rather than at each sampling step, an improved stability criterion for NCS (6) is derived in the following Theorem.

Theorem 1: Given scalars $\hat{\delta}$, τ_1, and τ_2 ($\tau_2 \geq \tau_1 > 0$), the NCS (6) is asymptotically stable if there exist $\boldsymbol{P}_i > 0$, $\boldsymbol{Q}_i > 0$, $\boldsymbol{Z}_i > 0$, and $\boldsymbol{S}_i = [\boldsymbol{S}_{1i}^T \quad \boldsymbol{S}_{2i}^T]^T$, such that the following are feasible:

$$\Psi_{stlii} < 0, \qquad s,t,l,i \in \mathcal{R}, \tag{10}$$

$$\Psi_{stlij} + \Psi_{stlji} < 0, \quad s,t,l,i,j \in \mathcal{R}, 1 \leq i < j \leq r, \tag{11}$$

where

$$\Psi_{stlij} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \sqrt{\tau_2}\boldsymbol{S}_{1i} & \boldsymbol{A}_i^T\boldsymbol{P}_t & \Psi_{15} \\ * & \Psi_{22} & \sqrt{\tau_2}\boldsymbol{S}_{2i} & \boldsymbol{L}_j^T\boldsymbol{G}_i^T\boldsymbol{P}_t & \sqrt{\tau_2}\boldsymbol{L}_j^T\boldsymbol{G}_i^T\boldsymbol{Z}_j \\ * & * & -\boldsymbol{Z}_l & 0 & 0 \\ * & * & * & -\boldsymbol{P}_t & 0 \\ * & * & * & * & -\boldsymbol{Z}_j \end{bmatrix},$$

$$\Psi_{11} = -\boldsymbol{P}_i + \boldsymbol{Q}_i + \mathrm{sym}(\boldsymbol{S}_{1\mathrm{i}}), \quad \Psi_{12} = -\boldsymbol{S}_{1\mathrm{i}} + \boldsymbol{S}_{2\mathrm{i}}^{\mathrm{T}},$$

$$\Psi_{15} = \sqrt{\tau_2}(\boldsymbol{A}_i^T - \boldsymbol{I})\boldsymbol{Z}_j, \quad \Psi_{22} = -\frac{1}{\hat{\delta}}\boldsymbol{Q}_s - \mathrm{sym}(\boldsymbol{S}_{2\mathrm{i}}).$$

Proof: Let $\boldsymbol{\eta}(k) = \boldsymbol{x}(k+1) - \boldsymbol{x}(k)$. Then, for NCS (6), define a Lyapunov function as:

$$\boldsymbol{V}(i_k) = \sum_{j=1}^{3}\boldsymbol{V}_j(i_k), \ \boldsymbol{V}_1(i_k) = \boldsymbol{x}^T(i_k)\bar{\boldsymbol{P}}\boldsymbol{x}(i_k), \tag{12}$$

$$\boldsymbol{V}_2(i_k) = \sum_{\theta=-\tau_2}^{-1}\sum_{m=i_k+\theta}^{i_k}\boldsymbol{\eta}^T(m)\bar{\boldsymbol{Z}}\boldsymbol{\eta}(m), \tag{13}$$

$$\boldsymbol{V}_3(i_k) = \sum_{m=i_k-\tau_{i_k}}^{i_k-1}\boldsymbol{x}^T(m)\bar{\boldsymbol{Q}}\boldsymbol{x}(m), \tag{14}$$

where $\bar{\boldsymbol{P}} = \sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{P}_i > 0$, $\bar{\boldsymbol{Q}} = \sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{Q}_i > 0$, $\bar{\boldsymbol{Z}} = \sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{Z}_i > 0$ are to be determined.

To reduce unnecessary conservatism, we guarantee the deceasement of Lyapunov functional at each updating step in this paper. Following this idea, we have

$$\Delta\boldsymbol{V}(i_k) = \boldsymbol{V}(i_{k+1}) - \boldsymbol{V}(i_k) = \sum_{j=1}^{3}\Delta\boldsymbol{V}_j(i_k),$$

$$\Delta\boldsymbol{V}_j(i_k) = \boldsymbol{V}_j(i_{k+1}) - \boldsymbol{V}_j(i_k) = \sum_{l=0}^{\delta_{i_k}}\Delta\boldsymbol{V}_j(i_k+l).$$

where $\delta_{i_k} \leq \hat{\delta}$.

Then, along the solution of (6), $\Delta\boldsymbol{V}_j(i_k)$ ($j \in \{1,2\}$) takes the form of (15) and (16), shown at the bottom of the next page. For $\Delta\boldsymbol{V}_3(i_k) = \sum_{l=0}^{\delta_{i_k}}\Delta\boldsymbol{V}_3(i_k+l)$, two cases arise and they are discussed as follows.

Case 1: $0 \leq l < \delta_{i_k}$. In this case, the sampling steps are holding steps, and therefore we have $\tau_{i_k+l+1} = \tau_{i_k+l} + 1$ for NCSs (6). In view of this, $\Delta\boldsymbol{V}_4(i_k+l)$ ($0 \leq l < \delta_{i_k}$) can be obtained in the form of (17), shown at the bottom of the next page.

Case 2: $l = \delta_{i_k}$. In this case, the time step $i_k + \delta_{i_k} + 1$ is an updating step, and thus we have $\tau_{i_k+\delta_{i_k}+1} \leq \tau_{i_k+\delta_{i_k}}$. Then, $\Delta\boldsymbol{V}_4(i_k+\delta_{i_k})$ is expressed in (18), shown at the bottom of the next page.

Moreover, note that the control signal $\bar{\boldsymbol{L}}\boldsymbol{x}(i_k - \tau_{i_k})$ is used to control plant during time interval $[i_k h \quad i_{k+1}h)$. Then, for $0 \leq l \leq \delta_{i_k}$, it follows from $\tau_{i_k+l+1} = \tau_{i_k+l} + 1$ that $\boldsymbol{x}(i_k + l - \tau_{i_k+l}) = \boldsymbol{x}(i_k - \tau_{i_k})$. Therefore, from (17) and (18), we can obtain (19) shown at the bottom of the next page.

On the other hand, for any matrix $\bar{\boldsymbol{S}}(k) = [\bar{\boldsymbol{S}}_1^T(k) \quad \bar{\boldsymbol{S}}_2^T(k)]^T$ with appropriate dimensions, the following equation always hold:

$$\begin{aligned}\mathcal{A}(i_k+l,m) =& 2\frac{1}{\tau_{i_{k+l}}}\left[\,\boldsymbol{x}^T(i_k+l)\ \boldsymbol{x}^T(i_k+l-\tau_{i_k+l})\,\right]^T\bar{\boldsymbol{S}} \\ &\times\sum_{\theta=i_k+l-\tau_2}^{i_k+l-1}[\boldsymbol{x}(i_k+l)-\boldsymbol{x}(i_k+l-\tau_{i_k+l}) \\ &-\tau_{i_k+l}\boldsymbol{\eta}(m)] \\ =& 0\end{aligned} \tag{20}$$

where $0 \leq l \leq \delta_{i_k}$, $\bar{\boldsymbol{S}}_1 = \sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{S}_{1i}$, $\bar{\boldsymbol{S}}_2 = \sum_{i=1}^{r}\mu_i(\theta(k))\boldsymbol{S}_{2i}$.

Let $\Xi(i_k+l,m) = [\boldsymbol{x}^T(i_k+l)\ \boldsymbol{x}^T(i_k+l-\tau_{i_{k+l}})\ \boldsymbol{\eta}(m)]^T$. Then, from (15)-(16) and (19)-(20), we can readily have

$$\begin{aligned}\Delta\boldsymbol{V}(i_k) =& \Delta\boldsymbol{V}_1(i_k) + \Delta\boldsymbol{V}_2(i_k) + \Delta\boldsymbol{V}_3(i_k) + \mathcal{A}(i_k+l,m) \\ \leq& \frac{1}{\tau_{i_{k+l}}}\sum_{\theta=i_k+l-\tau_2}^{i_k+l-1}\Xi(i_k+l,m)^T \\ &\times\Bigg\{\sum_{s=1}^{r}\sum_{t=1}^{r}\sum_{l=1}^{r}h_s(\theta(i_k+l-\tau_{i_k+l})) \\ &\times h_t(\theta(i_k+l-\tau_{i_k+l+1}))h_l(\theta(i_k+l)) \\ &\times\Bigg[\sum_{i=1}^{r}h_i(\theta(i_k+l))\mathscr{S}_{stlii} \\ &+\sum_{i=1}^{r}\sum_{j=i+1}^{r}h_i(\theta(i_k+l))h_j(\theta(i_k+l)) \\ &\times(\mathscr{S}_{stlij}+\mathscr{S}_{stlji})\Bigg]\Bigg\}\,\Xi(i_k+l,m)\end{aligned} \tag{21}$$

where the expression $\mathscr{S}_{stlij}$ are shown at the bottom of the page.

Noting that $\tau_1 \leq \tau_{\delta_{i_k}} \leq \tau_2$ and by Schur complement, it is not difficult to get from (22) that

$$\begin{aligned}&\mathscr{S}_{stlii} < 0, \\ &(\mathscr{S}_{stlij} + \mathscr{S}_{stlji}) < 0.\end{aligned} \tag{23}$$

It implies $\Delta\boldsymbol{V}(i_k) < 0$ for any $\boldsymbol{x} \neq 0$, and therefore the asymptotic stability of NCS is established. The proof is completed. ■

The stability criteria of NCSs formulated earlier deserves some remarks.

Remark 1: Theorem 1 provides an improved stability condition dependent on both time delay bound and packet loss bound for networked fuzzy systems. The merit of the obtained stability condition lies in its much less conservatism, and it is achieved by guaranteeing the deceasement of Lyapunov functional at each control signal updating step rather than at each sampling step.

Remark 2: Please note that the computational complexity of stability criteria in Theorem 1 will increase if the number of fuzzy rules increase. To overcome this problem, let $\boldsymbol{P} = \boldsymbol{P}_i$, $\boldsymbol{Q} = \boldsymbol{Q}_i$, $\boldsymbol{Z} = \boldsymbol{Z}_i$, and follow the similar lines in the proof of Theorem 1, one can easily obtain the following stability criteria for networked fuzzy system.

Corollary 1: Given scalars $\hat{\delta}$, τ_1, and τ_2 ($\tau_2 \geq \tau_1 > 0$), the NCS (6) is asymptotically stable if there exist $\boldsymbol{P} > 0$, $\boldsymbol{Q} > 0$, $\boldsymbol{Z} > 0$, $\boldsymbol{S} = [\boldsymbol{S}_1^T \;\; \boldsymbol{S}_2^T]^T$, and $i \in \mathcal{R}$, such that the following are feasible:

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \sqrt{\tau_2}\boldsymbol{S}_1 & \boldsymbol{A}_i^T\boldsymbol{P} & \Theta_{15} \\ * & \Theta_{22} & \sqrt{\tau_2}\boldsymbol{S}_2 & \boldsymbol{L}_i^T\boldsymbol{G}_i^T\boldsymbol{P} & \sqrt{\tau_2}\boldsymbol{L}_i{}^T\boldsymbol{G}_i^T\boldsymbol{Z} \\ * & * & -\boldsymbol{Z} & 0 & 0 \\ * & * & * & -\boldsymbol{P} & 0 \\ * & * & * & * & -\boldsymbol{Z} \end{bmatrix} < 0, \quad (24)$$

$$\Theta_{11} = -\boldsymbol{P} + \boldsymbol{Q} + \mathrm{sym}(\boldsymbol{S}_1), \quad \Theta_{12} = -\boldsymbol{S}_1 + \boldsymbol{S}_2^{\mathrm{T}},$$
$$\Theta_{15} = \sqrt{\tau_2}(\boldsymbol{A}_i^T - \boldsymbol{I})\boldsymbol{Z}, \quad \Theta_{22} = -\frac{1}{\hat{\delta}}\boldsymbol{Q} - \mathrm{sym}(\boldsymbol{S}_2).$$

Corollary 1 provides an alternative stability criteria for networked T-S fuzzy system, and it can greatly reduce the computational complexity of stability condition, at a cost of higher a little more conservatism.

Theorem 1 gives necessary stability conditions on the existence of stabilizing state feedback gains for networked fuzzy systems. However, it is difficult to calculate stabilizing gains directly from Theorem 1, since the obtained stability conditions are nonlinear in controller gains. To circumvent the synthesis problem, let $\bar{\boldsymbol{P}}_t = \boldsymbol{P}_t^{-1}$, $\bar{\boldsymbol{Z}}_j = \bar{\boldsymbol{Z}}_j^{-1}$, and pre- and post- multiplying (10) and (11) by $\mathrm{diag}\{\boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \bar{\boldsymbol{P}}_t, \bar{\boldsymbol{Z}}_j\}$. Then we can readily obtain the following theorem.

Theorem 2: Given scalars $\hat{\delta}$, τ_1, and τ_2 ($\tau_2 \geq \tau_1 > 0$), the NCS (6) is asymptotically stable if there exist $\boldsymbol{P}_i > 0$, $\bar{\boldsymbol{P}}_i > 0$, $\boldsymbol{Q}_i > 0$, $\boldsymbol{Z}_i > 0$, $\bar{\boldsymbol{Z}}_i > 0$, and $\boldsymbol{S}_i = [\boldsymbol{S}_{1i}^T \;\; \boldsymbol{S}_{2i}^T]^T$, such that the following are feasible:

$$\Upsilon_{stlii} < 0, \qquad s,t,l,i \in \mathcal{R}, \quad (25)$$
$$\Upsilon_{stlij} + \Psi_{stlji} < 0, \quad s,t,l,i,j \in \mathcal{R}, 1 \leq i < j \leq r, \quad (26)$$
$$\boldsymbol{P}_i\bar{\boldsymbol{P}}_i = \boldsymbol{I}, \quad \boldsymbol{Z}_i\bar{\boldsymbol{Z}}_i = \boldsymbol{I}, \quad (27)$$

where

$$\Delta \boldsymbol{V}_1(i_k) = \sum_{l=0}^{\delta_{i_k}} [\bar{\boldsymbol{F}}\boldsymbol{x}(i_k+l) + \bar{\boldsymbol{G}}\bar{\boldsymbol{L}}\boldsymbol{x}(i_k+l-\tau_{i_k+l})]^T \bar{\boldsymbol{P}} [\bar{\boldsymbol{F}}\boldsymbol{x}(i_k+l) + \bar{\boldsymbol{G}}\bar{\boldsymbol{L}}\boldsymbol{x}(i_k+l-\tau_{i_k+l})] - \sum_{l=0}^{\delta_{i_k}} \boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{P}}\boldsymbol{x}(i_k+l), \quad (15)$$

$$\Delta \boldsymbol{V}_2(i_k) = \sum_{l=0}^{\delta_{i_k}} \left[\tau_2 \boldsymbol{\eta}^T(i_k+l)\bar{\boldsymbol{Z}}\boldsymbol{\eta}(i_k+l) - \sum_{\theta=i_k+l-\tau_2}^{i_k+l-1} \boldsymbol{\eta}^T(\theta)\bar{\boldsymbol{Z}}\boldsymbol{\eta}(\theta) \right] \quad (16)$$

$$\Delta \boldsymbol{V}_3(i_k+l) = \sum_{l=i_k+l+1-\tau_{i_k+l+1}}^{i_k+l+1-1} \boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+l) - \sum_{l=i_k+l-\tau_{i_k+l}}^{i_k+l-1} \boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+l) = \boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{Q}}x(i_k+l), \quad (17)$$

$$\begin{aligned} \Delta \boldsymbol{V}_3(i_k+\delta_{i_k}) &= \sum_{l=i_k+\delta_{i_k}+1-\tau_{i_k+\delta_{i_k}+1}}^{i_k+\delta_{i_k}+1-1} \boldsymbol{x}^T(i_k+\delta_{i_k})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+\delta_{i_k}) - \sum_{l=i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}}}^{i_k+\delta_{i_k}-1} \boldsymbol{x}^T(i_k+\delta_{i_k})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+\delta_{i_k}) \\ &\leq \sum_{l=i_k+\delta_{i_k}+1-\tau_{i_k+\delta_{i_k}}}^{i_k+\delta_{i_k}+1-1} \boldsymbol{x}^T(i_k+\delta_{i_k})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+\delta_{i_k}) - \sum_{l=i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}}}^{i_k+\delta_{i_k}-1} \boldsymbol{x}^T(i_k+\delta_{i_k})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+\delta_{i_k}) \\ &= \boldsymbol{x}^T(i_k+\delta_{i_k})\bar{\boldsymbol{Q}}x(i_k+\delta_{i_k}) - \boldsymbol{x}^T(i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}})\bar{\boldsymbol{Q}}x(i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}}). \end{aligned} \quad (18)$$

$$\begin{aligned} \Delta \boldsymbol{V}_3(i_k) &= \sum_{l=0}^{\delta_{i_k}} [\boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+l)] - \boldsymbol{x}^T(i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}}) \\ &= \sum_{l=0}^{\delta_{i_k}} [\boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+l)] - \frac{1}{\delta_{i_k}} \sum_{l=0}^{\delta_{i_k}} [\boldsymbol{x}^T(i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+\delta_{i_k}-\tau_{i_k+\delta_{i_k}})] \\ &= \sum_{l=0}^{\delta_{i_k}} [\boldsymbol{x}^T(i_k+l)\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+l) - \frac{1}{\delta_{i_k}} \boldsymbol{x}^T(i_k+l-\tau_{i_k+l})\bar{\boldsymbol{Q}}\boldsymbol{x}(i_k+l-\tau_{i_k+l})] \end{aligned} \quad (19)$$

$$\mathscr{S}_{stlij} = \begin{bmatrix} -\boldsymbol{P}_i + \boldsymbol{Q}_i + \mathrm{sym}(\boldsymbol{S}_{1\mathrm{i}}) & -\boldsymbol{S}_{1i} + \boldsymbol{S}_{2i}^T & \sqrt{\tau_{\delta_{i_k}}}\boldsymbol{S}_{1i} & \boldsymbol{A}_i^T\boldsymbol{P}_t & \sqrt{\tau_2}(\boldsymbol{A}_i^T - \boldsymbol{I})\boldsymbol{Z}_j \\ * & -\frac{1}{\hat{\delta}}\boldsymbol{Q}_s - \mathrm{sym}(\boldsymbol{S}_{2\mathrm{i}}) & \sqrt{\tau_{\delta_{i_k}}}\boldsymbol{S}_{2i} & \boldsymbol{L}_j^T\boldsymbol{G}_i^T\boldsymbol{P}_t & \sqrt{\tau_2}\boldsymbol{L}_j^T\boldsymbol{G}_i^T\boldsymbol{Z}_j \\ * & * & -\boldsymbol{Z}_l & 0 & 0 \\ * & * & * & -\boldsymbol{P}_t & 0 \\ * & * & * & * & -\boldsymbol{Z}_j \end{bmatrix}, \quad (22)$$

$$\Upsilon_{stlij} = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \sqrt{\tau_2}\boldsymbol{S}_{1i} & \boldsymbol{A}_i^T & \Upsilon_{15} \\ * & \Upsilon_{22} & \sqrt{\tau_2}\boldsymbol{S}_{2i} & \boldsymbol{L}_j^T\boldsymbol{G}_i^T & \sqrt{\tau_2}\boldsymbol{L}_j^T\boldsymbol{G}_i^T \\ * & * & -\boldsymbol{Z}_l & 0 & 0 \\ * & * & * & -\bar{\boldsymbol{P}}_t & 0 \\ * & * & * & * & -\bar{\boldsymbol{Z}}_j \end{bmatrix}, \quad (28)$$

$$\Upsilon_{11} = -\boldsymbol{P}_i + \boldsymbol{Q}_i + \mathrm{sym}(\boldsymbol{S}_{1\mathrm{i}}),\ \Upsilon_{12} = -\boldsymbol{S}_{1\mathrm{i}} + \boldsymbol{S}_{2\mathrm{i}}^{\mathrm{T}}, \quad (29)$$

$$\Upsilon_{15} = \sqrt{\tau_2}(\boldsymbol{A}_i^T - \boldsymbol{I}),\ \Upsilon_{22} = -\frac{1}{\hat{\delta}}\boldsymbol{Q}_s - \mathrm{sym}(\boldsymbol{S}_{2\mathrm{i}}). \quad (30)$$

Then, *cone complementarity linearization* (CCL) approach can be employed in this paper to calculate the stabilizing gains from Theorem 2. It is worth mentioning that the CCL based controller design procedure is quite standard, and therefore we omit it in this paper to avoid unnecessary repetition. For more details on the CCL approach, we refer the readers to our earlier work H. Li et al. [2009] and the reference therein.

4. ILLUSTRATIVE EXAMPLES

In this section, two examples are provided to illustrate the advantage and effectiveness of the proposed approach.

Example 1: Consider a networked nonlinear system in (6) with the following parameters:

$$\boldsymbol{F}_1 = \begin{bmatrix} -0.291 & 1 \\ 0 & 0.95 \end{bmatrix}, \quad \boldsymbol{G}_1\boldsymbol{L}_1 = \begin{bmatrix} 0.012 & 0.014 \\ 0 & 0.015 \end{bmatrix},$$
$$\boldsymbol{F}_2 = \begin{bmatrix} -0.1 & 0 \\ 1 & -0.2 \end{bmatrix}, \quad \boldsymbol{G}_2\boldsymbol{L}_2 = \begin{bmatrix} 0.01 & 0 \\ 0.01 & 0.015 \end{bmatrix}. \quad (31)$$

The basic idea of this example is to keep the lower delay bound τ_1 constant first, and then we are interested in the admissible upper delay bound τ_2, below which the NCS is stable for all $\tau_1 \le \tau_k \le \tau_2$.

A comprehensive study is carried out, with the comparative results listed in Table 1. It has been shown that the proposed method obtains much larger admissible upper delay bound than the existing one in H. Gao et al. [2009], especially when the MCPL bound $\hat{\delta}$ in NCSs is small. Therefore, the result in this paper is less conservative than that in H. Gao et al. [2009] for this example.

Table 1. The admissible τ_2 for given τ_1.

	$\tau_1 = 5$	$\tau_1 = 8$	$\tau_1 = 10$
Theorem 1 in H. Gao et al. [2009]	16	18	20
Theorem 1 with $\hat{\delta} = 10$	33	33	33
Theorem 1 with $\hat{\delta} = 5$	> 100	> 100	> 100
Theorem 1 with $\hat{\delta} = 3$	> 100	> 100	> 100

Example 2: Consider a NCS shown in Fig. 1, where the controlled plant can be expressed by (1) and its parameters are given as follows:

$$\boldsymbol{F}_1 = \begin{bmatrix} -0.291 & 1 \\ 0 & 0.95 \end{bmatrix}, \boldsymbol{F}_2 = \begin{bmatrix} -0.1 & 0 \\ 1 & -0.2 \end{bmatrix},$$
$$\boldsymbol{G}_1 = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad \boldsymbol{G}_2 = \begin{bmatrix} 0.01 \\ 0.015 \end{bmatrix}, \quad (32)$$

The membership functions for Plant Rule 1 and 2 are of the following form:

$$\mu_1[\boldsymbol{x}_1(k)] = \left\{1 - \frac{1}{1+e^{-7[\boldsymbol{x}_1(k)-\frac{\pi}{6}]}}\right\} \times \frac{1}{1+e^{-7[\boldsymbol{x}_1(k)-\frac{\pi}{6}]}},$$
$$\mu_2[\boldsymbol{x}_1(k)] = 1 - \mu_1[\boldsymbol{x}_1(k)]. \quad (33)$$

In the studied scenario , the network condition are set to $\tau_k \in \{1, 2, 3, 4, 5\}$ and $\hat{\delta} = 2$, By the proposed method, we obtain a stabilizing controller of the form (4), with the following gains:

$$\boldsymbol{L}_1 = [2.4830\ \ -2.7007], \quad \boldsymbol{L}_2 = [-0.3566\ 2.5660]. \quad (34)$$

With the initial state $\boldsymbol{x}_0 = [10, -10]^T$, typical simulation result of the above networked T-S fuzzy system is depicted in Fig. 2. It can be seen that the above networked system is asymptotically stable and shows satisfactory control performance, which illustrates the effectiveness of the proposed method.

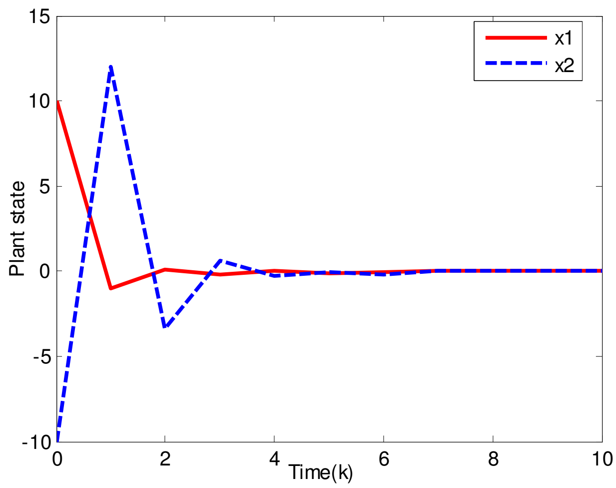

Fig. 2. Typical simulation results using the proposed method.

5. CONCLUSIONS

In this paper, the stabilization problem has been investigated for a general class of networked T-S fuzzy systems with time delays and packet losses. To reduce the conservatism of NCSs, an improved stability criterion dependent on both time delay bound and packet loss bound is derived, by guaranteeing the deceasement of Lyapunov functional at each control signal updating step rather than at each sampling step. Then, a stabilizing controller design method based on the PDC scheme has also been provided. Finally, two illustrative examples are provided to demonstrate the effectiveness of the approaches proposed in this paper. Further research topics include extending the main results of this paper to the H_∞ case.

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