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Fault Detection for a Class of Uncertain Linear Discrete-time Systems with Intermittent Measurements and Probabilistic Actuator Failures ⋆

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Abstract: This paper deals with fault detection (FD) problem for linear discrete-time systems subject to random intermittent measurements, probabilistic actuator failures, norm-bounded model uncertainty, and stochastic model uncertainty. By taking into account the probabilistic actuator failures, a new reference residual model is proposed to formulate the FD issue as an H_∞ model-matching problem. The corresponding reference residual is generated through maximizing a stochastic H_-/H_∞ or H_∞/H_∞ performance index via solving an algebraic Riccati equation (ARE). By the aid of the linear matrix inequality (LMI) techniques, a fault detection filter (FDF) is constructed such that the residual is sensitive to fault but insensitive to unknown inputs, mixed model uncertainties, random intermittent measurements and stochastic actuator failures. An illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords: Algebraic Riccati equation, discrete-time system, fault detection filter, intermittent measurement, model uncertainty, probabilistic actuator failure.

1. INTRODUCTION

Owing to the increasing demands for system safety and reliability, research on observer-based fault detection (FD) has received much attention during the past three decades, see Blanke et al. (2006); Ding (2008); Chadi et al. (2013); Alwi and Edwards (2013) and references therein. Generally speaking, the basic idea behind model-based FD is to design a fault detection filter (FDF) to generate a residual signal which is sensitive to fault but robust to unknown input. In reviewing of the development of FD, one efficient scheme is to formulate the design of FDF into an optimization problem which the so-called H_-/H_∞ or H_∞/H_∞ performance index is maximized, we refer to the unified approach in Ding (2008); Li and Zhou (2009); Zhong et al. (2010); Li and Zhong (2013) and related works therein. Another scheme is the H_∞ filtering formulation to minimize the error between fault and residual in the H_∞ norm sense, see e.g. Dong et al. (2011); Zhou et al. (2011); Du et al. (2012).

In real-world applications, it is of significance to study FD problems with model uncertainties since modeling errors always exist in engineering practise, which has led to considerable attention. In this literature, especially for discrete-time systems, the H_∞ filtering based FD scheme are widely utilized in the presence of norm-bounded uncertainty, polytopic uncertainty, or stochastic uncertainty, see e.g., Gao et al. (2008); Dong et al. (2011); Li and Zhong (2011); Zhou et al. (2011) and related references. In continuous-time domain, Zhong et al. (2003) proposes an H_∞ model-matching FD scheme for system with norm-bounded model uncertainty, which generates a residual to track an ideal residual obtained in the nominal case. Wang et al. (2008) and Li et at. (2009) extend this idea to systems with polytopic uncertainty.

On another front line, components' failures such as actuator or sensor gain reductions may emerge in chemical processes, distributed networks or when tracking a highly maneuvering target, which may result in substantial damage. It is worth pointing out that the phenomenon of intermittent measurements or missing measurements can be viewed as a special class of sensor gain failures, and the H_∞ filtering based FD approaches are addressed in Gao et al. (2008); He et al. (2008); Dong et al. (2011); Zhou et al. (2011); Du et al. (2012) and references therein. For systems with actuator failures, some results of passive fault tolerant control (FTC) methodology are published, e.g. Tian et al. (2011); Gu et al. (2013). However, there exist few results on designing a separate FD unit for potential application to active FTC in the presence of intermittent measurements, actuator failures, deterministic as well as stochastic model uncertainties, which motivates the present study.

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Inspired by Zhong et al. (2003), this paper investigates FD problem for linear discrete-time systems subject to intermittent measurements, random actuator failures, norm-bounded model uncertainty and stochastic model uncertainty via the H_∞ model-matching approach. The novelty of this article are two-folds: (i). A new reference residual model is established by taking random actuator failures into consideration, which infers an algebraic Riccati equation (ARE) based FD algorithm for systems affected by probabilistic actuator failures. (ii). An FDF is designed in terms of linear matrix inequality (LMI) formulation such that the generated residual tracks the reference residual in a stochastic H_∞ sense irrespective of intermittent measurements, actuator failures and mixed uncertainties.

Notations. For a matrix X, X^{T} and X^{-1} stand for the transpose and inverse of X, respectively. $X > 0$ ($X < 0$) denotes X is positive (negative) definite. R^n means the set of n-dimensional real vectors. I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively. $\mathrm{E}\{\vartheta(k)\}$ means the mathematical expectation of $\vartheta(k)$.$\|\zeta(k)\|_{2,\mathrm{E}}$ stands for the l_2-norm of $\zeta(k)$ with $\|\zeta(k)\|_{2,\mathrm{E}}^2 = \mathrm{E}\{\sum_{k=0}^{\infty}\zeta^{\mathrm{T}}(k)\zeta(k)\}$. $\langle(\mu_0, \mu(k)), (\varsigma_0, \varsigma(k))\rangle = \mathrm{E}\{\mu_0^{\mathrm{T}}\Pi\varsigma_0\} + \mathrm{E}\{\Sigma_{k=0}^{\infty}\mu^{\mathrm{T}}(k)\varsigma(k)\}$ gives the definition of the inner product on a Hilbert space for vector $\mu(k)$ and $\varsigma(k)$ with appropriate dimensions, where $\Pi > 0$ is a initial weighting matrix. The symbol $*$ within a matrix stands for the symmetric entries. $\delta_{i,j}$ represents the Kronecker delta function, which is equal to unity for $i = j$ and zero for $i \neq j$. $diag\{X_1, X_2, \ldots, X_n\}$ denotes a block diagonal matrix with diagonal blocks $X_1, X_2, \ldots, X_n$. $\mathrm{Prob}\{a\}$ represents the probability of an event 'a'.

2. PROBLEM FORMULATION

Consider the following linear discrete-time system

$$\begin{cases} x(k+1) = (A + \Delta A + A_v v(k))x(k) + B\Xi(k)u(k) \\ \qquad\qquad\quad + B_d d(k) + B_f f(k) \\ y(k) \quad\;\;\; = \theta(k)Cx(k) + D_d d(k) \end{cases} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^q$, $y(k) \in R^{n_y}$, $d(k) \in R^{n_d}$, and $f(k) \in R^{n_f}$ denote the state, control input, measurement output, unknown input, and fault, respectively; $u(k)$, $f(k)$ and $d(k)$ are l_2-norm bounded. A, B, B_d, B_f, C and D_d are known constant matrices with appropriate dimensions. ΔA is the norm-bounded uncertain matrix described as

$$\Delta A = EHF$$

where E and F are known matrices and H satisfies

$$H^{\mathrm{T}}H \leq I$$

$A_v v(k)$ represents the stochastic uncertainty (multiplicative noise) where A_v is known and $v(k)$ is a standard stochastic scalar sequence with zero mean and satisfies $\mathrm{E}\{v(i)v(j)\} = \varepsilon\delta_{ij}$, where ε is a positive constant.

To describe intermittent measurements and multiple stochastic actuator failures respectively, two categories of random variables are introduced (Gao et al. (2008); Tian et al. (2011)). A Bernoulli distributed binary stochastic variable $\theta(k)$ depicts the measurement packet dropouts, which satisfies $\mathrm{Prob}\{\theta(k) = 1\} = \mathrm{E}\{\theta(k)\} = \varrho$ and $\mathrm{Prob}\{\theta(k) = 0\} = 1 - \mathrm{E}\{\theta(k)\} = 1 - \varrho$ with ϱ is a known constant. For actuator failures, define $\Xi(k) := diag\{\gamma_1(k), \ldots, \gamma_q(k)\}$ ($i = 1, \ldots, q$), where $\gamma_i(k)$ are scalar independent identical distributed random variables with the probability density function on the interval $[0, 1]$. Specifically, if $\gamma_i(k) = 1$ means the ith actuator is normal and $\gamma_i(k) \neq 1$ denotes the ith actuator fails with certain reduction rate. The mathematical expectation and variance of $\gamma_i(k)$ are ρ_i and σ_i^2 ($i = 1, \ldots, q$) respectively. The value of ϱ, ρ_i and σ_i can be obtained as prior knowledge by statistical test technique (Zhou et al. (2011)).

Before describing the underlying problem, without loss of generality, it is assumed that system (1) is exponentially stable in mean square sense (ESMS, refer to Dong et al. (2011) for details) for all deterministic and stochastic uncertainties, and (C, A) is uniformly detectable with (A, B_d) uniformly stabilizable. $v(k)$, $\theta(k)$ and $\gamma_i(k)$ ($i = 1, \ldots, q$) are assumed to be uncorrelated with each other.

One of the main task of FD in this paper is to generate a residual such that the sensitivity of residual to fault is enhanced with the robustness to disturbance and model uncertainties is increased. According to Zhong et al. (2003), the H_∞ model-matching FD scheme is adopted, which is to find an idealized reference model and minimize the l_2-induced gain from the exogenous inputs to the error between the generated residual and the reference residual. For this purpose, consider the following observer-based FDF as a residual generator

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + L(y(k) - \varrho C\hat{x}(k)) + B\bar{\Xi}u(k) \\ r(k) \quad\quad\;\; = V(y(k) - \varrho C\hat{x}(k)) \end{cases} \tag{2}$$

where $\hat{x}(k) \in R^n$ is an estimate for $x(k)$, $r(k) \in R^p$ is the generated residual, $\bar{\Xi} = \mathrm{E}\{\Xi(k)\}$, L is the observer gain matrix and V is the post-filter to be determined.

Let $e(k) = x(k) - \hat{x}(k)$, it follows from (1) and (2) that

$$\begin{cases} e(k+1) = (A - \varrho LC)e(k) + (B_d - LD_d)d(k) \\ \qquad\qquad\quad + B_f f(k) + B\tilde{\Xi}(k)u(k) + \nu(k)LCx(k) \\ \qquad\qquad\quad + A_v v(k)x(k) + \Delta Ax(k) \\ r(k) \quad\;\;\; = V(\varrho Ce(k) + \nu(k)Cx(k) + D_d d(k)) \end{cases} \tag{3}$$

where $\tilde{\Xi}(k) = \Xi(k) - \bar{\Xi}$ and $\nu(k) = \theta(k) - \varrho$.

To derive the reference residual model, we consider the following modified nominal system, i.e., the case that $\Delta A = 0$, $A_v = 0$, while taking the term $B\tilde{\Xi}(k)u(k)$ into account. The reference residual model is given as

$$\begin{cases} e_r(k+1) = (A - \varrho L_r C)e_r(k) + (B_d - L_r D_d)d(k) \\ \qquad\qquad\quad\; + B_f f(k) + B\tilde{\Xi}(k)u(k) \\ r_r(k) \quad\;\;\; = \varrho V_r Ce_r(k) + V_r D_d d(k) \end{cases} \tag{4}$$

where $e_r(k) \in R^n$ is the reference error state vector, $r_r \in R^p$ is the reference residual, L_r and V_r are the parameters of the reference model to be designed.

Let $\alpha_i(k) = \gamma_i(k) - \rho_i$ ($i = 1, \ldots, q$), by defining $\eta(k) = [e^{\mathrm{T}}(k)\; e_r^{\mathrm{T}}(k)\; x^{\mathrm{T}}(k)]^{\mathrm{T}}$, it follows from (1), (3) and (4) that the overall error dynamic is described as

$$\begin{cases} \eta(k+1) = A_\eta\eta(k) + B_w w(k) \\ r_e(k) \quad\;\; = C_\eta\eta(k) + D_w w(k) \end{cases} \tag{5}$$

where

$$
\begin{aligned}
w(k) &= [d^{\mathrm{T}}(k)\ \ u^{\mathrm{T}}(k)\ \ f(k)^{\mathrm{T}}]^{\mathrm{T}},\ r_e(k) = r(k) - r_r(k) \\
A_\eta &= A_0 + A_1 v(k) + \Delta A_2 + A_3 \nu(k),\ B_w = B_0 + B_1 \\
A_0 &= diag\{A - \varrho LC, A - \varrho L_r C, A\} \\
A_1 &= \begin{bmatrix} 0 & 0 & A_v \\ 0 & 0 & 0 \\ 0 & 0 & A_v \end{bmatrix},\ B_0 = \begin{bmatrix} B_d - LD_d & 0 & B_f \\ B_d - L_r D_d & 0 & B_f \\ B_d & B\bar{\Xi} & B_f \end{bmatrix} \\
\Delta A_2 &= \begin{bmatrix} 0 & 0 & \Delta A \\ 0 & 0 & 0 \\ 0 & 0 & \Delta A \end{bmatrix},\ A_3 = \begin{bmatrix} 0 & 0 & -LC \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\bar{B}_i &= Bdiag\{\underbrace{0, \dots, 0}_{i-1}, I, \underbrace{0, \dots, 0}_{q-i}\} \\
B_1 &= \sum_{i=1}^{q} \alpha_i(k) \begin{bmatrix} 0 & \bar{B}_i & 0 \\ 0 & \bar{B}_i & 0 \\ 0 & \bar{B}_i & 0 \end{bmatrix} \\
C_\eta &= C_1 + C_2 \nu(k),\ D_w(k) = [\,(V - V_r) D_d\ \ 0\ \ 0\,] \\
C_1 &= [\,\varrho VC\ \ -\varrho V_r C\ \ 0\,],\ C_2 = [\,0\ \ 0\ \ VC\,]
\end{aligned}
$$

Based on the preliminaries above, the FD problem can be mainly formulated as: under zero initial condition, find L and V such that (5) is ESMS for all uncertainties and the following performance index is minimized

$$
J_F = \sup_{w(k) \in l_2[0,\infty), w(k) \neq 0} \frac{\|r_e(k)\|_2^2}{\|w(k)\|_2^2} \tag{6}
$$

After designing the FDF, the remaining task is to evaluate the generated residual signal. In this paper, we choose $J(k) = (\sum_{k=0}^{k=k_T} r^{\mathrm{T}}(k) r(k))^{1/2}$ as the residual evaluation function, where k_T denotes the length of the evaluation time window (Ding (2008)). The corresponding threshold is adopted as $J_{th} = \sup_{f(k)=0} \mathrm{E}\{J(k)\}$ and hence the occurrence of faults can then be recognized based on the following logic

$$
\begin{cases} J(k) > J_{th}, \Rightarrow \text{ A fault is detected,} \\ J(k) \leq J_{th}, \Rightarrow \text{ No faults.} \end{cases}
$$

Remark 1. It should be pointed out that the choice of the reference residual model is not unique. By taking the probabilistic actuator failures into consideration, our proposed approach not only provides a reference model (4) to synthesis the FDF (2), but also addresses a direct FD way for systems with random actuator failures when the model uncertainties are neglected. This will be interpreted in the following section by extending the adjoint operator based optimization techniques in Li and Zhong (2013).

3. MAIN RESULTS

3.1 Reference model design

To design the reference model, for (4), define

$$
r_f(k) = r(k)|_{\tilde{d}_k = 0},\ r_{\tilde{d}}(k) = r(k)|_{f_k = 0}
$$

where $\tilde{d}_k = [d^{\mathrm{T}}(0)\ \cdots\ d^{\mathrm{T}}(k);\ u^{\mathrm{T}}(0)\ \cdots\ u^{\mathrm{T}}(k)]^{\mathrm{T}}$ and $f_k = [f^{\mathrm{T}}(0)\ \cdots\ f^{\mathrm{T}}(k)]^{\mathrm{T}}$. Then, a linear operator that maps $f \mapsto r$ and a linear operator that maps $\tilde{d} \mapsto r$ can be defined as follows

$$
r_f(k) = \mathcal{G}_{rf} f(k),\ r_{\tilde{d}}(k) = \mathcal{G}_{r\tilde{d}} \tilde{d}(k)
$$

with the following induced norms (we refer to Wang et al. (2006, 2007) for more details about these measures)

$$
\begin{aligned}
\|\mathcal{G}_{rf}\|_\infty &= \sup_{f \in l_2,\ \|f\|_2 \neq 0} \frac{\|r_f(k)\|_{2,\mathrm{E}}^2}{\|f(k)\|_2^2} \\
\|\mathcal{G}_{r\tilde{d}}\|_\infty &= \sup_{\tilde{d} \in l_2,\ \|\tilde{d}\|_2 \neq 0} \frac{\|r_{\tilde{d}}(k)\|_{2,\mathrm{E}}^2}{\|\tilde{d}(k)\|_2^2} \\
\|\mathcal{G}_{rf}\|_- &= \inf_{f \in l_2,\ \|f\|_2 \neq 0} \frac{\|r_f(k)\|_{2,\mathrm{E}}^2}{\|f(k)\|_2^2}
\end{aligned}
$$

Thus, the reference model can be obtained through solving the following optimization problem: find L_r and V_r such that (4) is ESMS and satisfies the following performance

$$
\max_{L_r, V_r} \frac{\|\mathcal{G}_{rf}\|_\infty}{\|\mathcal{G}_{r\tilde{d}}\|_\infty} \ \text{ or } \max_{L_r, V_r} \frac{\|\mathcal{G}_{rf}\|_-}{\|\mathcal{G}_{r\tilde{d}}\|_\infty} \tag{7}
$$

For the purpose of deriving solution to (7), the following definitions and lemmas should be given first.

Definition 1. (Li and Zhou (2009)) Let $\mathcal{G}_s$ denotes an operator or a system mapping from l_2-norm bounded space S_1 to l_2-norm bounded space S_2. An operator $\mathcal{G}_s^\sim$ is called to be the adjoint operator of $\mathcal{G}_s$ from space S_2 to S_1 if $\langle \mathcal{G}_s(\mu_0, \mu), \varsigma \rangle = \langle (\mu_0, \mu), \mathcal{G}_s^\sim \varsigma \rangle$ for all $\mu \in S_1$ and $\varsigma \in S_2$, where μ_0 stands for the initial vector.

Definition 2. (Li and Zhong (2013)) Let $\mathcal{G}_s$ denote an operator or a system mapping from l_2-norm bounded input space S_1 to l_2-norm bounded output space S_2, then, $\mathcal{G}_s$ is said to be co-isometric if $\langle \mathcal{G}_s^\sim \varphi(k), \mathcal{G}_s^\sim \varphi(k) \rangle = \langle \varphi(k), \varphi(k) \rangle$ for all $\varphi(k) \in S_1$.

Lemma 1. (Li and Zhong (2013)) For two stochastic operator $A : y \mapsto z$ and $B : w \mapsto y$, where y, z, and w are l_2-norm bounded signals, we have

$$
\|AB\|_\infty \leq \|A\|_\infty \|B\|_\infty,\ \|AB\|_- \leq \|A\|_\infty \|B\|_-
$$

Lemma 2. Consider the following residual generators

$$
\begin{cases} \hat{x}^m(k+1) = A\hat{x}^m(k) + B\bar{\Xi}u(k) + L^m(y(k) - C\hat{x}^m(k)) \\ r^m(k) \quad\quad = V^m(y(k) - C(\theta(k)\hat{x}^m(k)),\ m = 1, 2 \end{cases}
$$

where L^m is the observer gain matrix such that $A - L^m C$ is ESMS and V^m is the post-filter. Then there exits an operator Q such that

$$
r^2(k) = Q r^1(k)
$$

Proof. The proof is in the same line as Lemma 1 in Zhong et al. (2010), and hence it is omitted here.

Based on the definitions and lemmas above, we are now ready to give the solution to (7), which is summarized in the following Theorem.

Theorem 3. For (4), the following matrix pair gives a solution to (7)

$$
\begin{cases} L_r = (\varrho APC^{\mathrm{T}} + B_d D_d^{\mathrm{T}})(\varrho^2 CPC^{\mathrm{T}} + D_d D_d^{\mathrm{T}})^{-1} \\ V_r = (\varrho^2 CPC^{\mathrm{T}} + D_d D_d^{\mathrm{T}})^{-1/2} \end{cases} \tag{8}
$$

where $P > 0$ satisfies the following ARE

$$
\begin{aligned}
P = {} & APA^{\mathrm{T}} - L_r(\varrho^2 CPC^{\mathrm{T}} + D_d D_d^{\mathrm{T}}) L_r^{\mathrm{T}} \\
& + B_d B_d^{\mathrm{T}} + \sum_{i=1}^{q} \sigma_i^2 B_i B_i^{\mathrm{T}}
\end{aligned} \tag{9}
$$

Proof. For (4), consider the operator $\mathcal{G}_{r\tilde{d}}$ which is realized by the following linear discrete-time system

$$\begin{cases} e(k+1) = A_e e(k) + B_e \tilde{d}(k) \\ r_{\tilde{d}}(k) \quad = C_e e(k) + D_e \tilde{d}(k) \end{cases}$$

where $A_e = A - \varrho LC$, $B_e = B_{\tilde{d}} - LD_{\tilde{d}}$, $C_e = \varrho VC$, $D_e = VD_{\tilde{d}}$ with $B_{\tilde{d}} = [B_d \ \sum_{i=1}^{q} \alpha_i(k)\bar{B}_i]$ and $D_{\tilde{d}} = [D_d \ 0]$. Let $\mathcal{G}_{r\tilde{d}}^{\sim}$ be the adjoint operator of $\mathcal{G}_{r\tilde{d}}$. According to Definition 1 and Li and Zhong (2011), the state-space realization of $\mathcal{G}_{r\tilde{d}}^{\sim}$ is

$$\begin{cases} \eta_a(k-1) = A_e^{\mathrm{T}}\eta_a(k) + C_e^{\mathrm{T}} r_{\tilde{d}}(k) \\ d_a(k) \quad = B_e^{\mathrm{T}}\eta_a(k) + D_e^{\mathrm{T}} r_{\tilde{d}}(k) \end{cases} \tag{10}$$

where $\eta(\infty) = 0$ and $\eta(-1) = 0$ under zero condition.

For (10), define

$$\mathbb{V}(\eta_a(k)) = \eta_a^{\mathrm{T}}(k) P \eta_a(k), \ P > 0$$

then, we have

$$\begin{aligned} &\mathrm{E}\{\sum_{k=0}^{\infty} d_a^{\mathrm{T}}(k) d_a(k) + \mathbb{V}(\eta_a(k-1)) \\ &-\mathbb{V}(\eta_a(k))\} + \eta_a(\infty)^{\mathrm{T}} P \eta_a(\infty) - \eta_a(-1)^{\mathrm{T}} P \eta_a(-1) \\ = \ &\mathrm{E}\{\sum_{k=0}^{\infty} \eta_a^{\mathrm{T}}(k)[B_e B_e^{\mathrm{T}} + A_e P A_e^{\mathrm{T}} - P]\eta_a(k) \\ &+2\eta_a^{\mathrm{T}}(k)[B_e D_e^{\mathrm{T}} + A_e P C_e^{\mathrm{T}}] r_{\tilde{d}}(k) \\ &+ r_{\tilde{d}}^{\mathrm{T}}[D_e D_e^{\mathrm{T}} + C_e P C_e^{\mathrm{T}}] r_{\tilde{d}}(k)\} = \mathrm{E}\{\sum_{k=0}^{\infty} d_a^{\mathrm{T}}(k) d_a(k)\} \end{aligned} \tag{11}$$

From (11) and Definition 2, if the following equation is satisfied

$$\|d_a\|_{2,\mathrm{E}}^2 = \|\mathcal{G}_{r\tilde{d}}^{\sim} r_{\tilde{d}}\|_{2,\mathrm{E}}^2 = \|r_{\tilde{d}}\|_{2,\mathrm{E}}^2$$

i.e. the following equations holds

$$\begin{cases} P = B_d B_d^{\mathrm{T}} + APA^{\mathrm{T}} + \varrho^2 LCPC^{\mathrm{T}} L^{\mathrm{T}} \\ \quad -\varrho LCPA^{\mathrm{T}} - \varrho APC^{\mathrm{T}} L^{\mathrm{T}} + \sum_{i=1}^{q} \sigma_i^2 B_i B_i^{\mathrm{T}} \\ 0 = (\varrho APC^{\mathrm{T}} - \varrho^2 LCPC^{\mathrm{T}} + B_d D_d^{\mathrm{T}} \\ \quad -LD_d D_d^{\mathrm{T}}) V^{\mathrm{T}} \\ I = V(D_d D_d^{\mathrm{T}} + \varrho^2 CPC^{\mathrm{T}}) V^{\mathrm{T}} \end{cases} \tag{12}$$

then $\mathcal{G}_{r\tilde{d}}$ is co-isometric.

Following a similar path in Li and Zhong (2013), by applying Lemma 1 and Lemma 2, if $\mathcal{G}_{r\tilde{d}}$ is co-isometric, we then have that

$$\frac{\|\mathcal{G}_{rf}\|_\infty}{\|\mathcal{G}_{r\tilde{d}}\|_\infty} \le \|\mathcal{G}_{rf,r}\|_\infty, \ \frac{\|\mathcal{G}_{rf}\|_-}{\|\mathcal{G}_{r\tilde{d}}\|_\infty} \le \|\mathcal{G}_{rf,r}\|_-$$

with $\mathcal{G}_{rf,r} = \mathcal{G}_{rf}|_{L=L_r, V=V_r}$, which gives the optimal value of maximizing the performance index (7). Furthermore, by solving the associated equations in (12), we can obtain (8) and (9), respectively.

From Dragan et al. (2009), if (C, A) is uniformly detectable with (A, B_d) uniformly stabilizable, the solution of the ARE (9) is bounded, and the proposed reference model is ESMS. This completes the proof.

Remark 2. It is worth mentioning that, by setting $\mathbb{V}(\eta_a(k)) = \eta_a^{\mathrm{T}}(k) P(k+1) \eta_a(k)$, Theorem 3 can be easily extended to linear discrete time-varying systems suffering from multiple random actuator failures, which is a generalized form of Theorem 1 in Zhong et al. (2010). Detailed computation of $\mathcal{G}_{rf}$ or $\mathcal{G}_{r\tilde{d}}$ is unnecessary since the solution to the optimization problem (7) is independent of the specific form of these operators.

3.2 Synthesis of the FDF

We are now in position to design L and V of the FDF. To proceed, the following two lemmas are given first.

Lemma 4. (Boyd et al. (1994)) Consider the following discrete-time system with multiplicative noise

$$\begin{cases} x(k+1) = (A + \sum_{i=1}^{m} A_i p_i(k)) x(k) + (B + \sum_{i=1}^{m} B_i p_i) d(k) \\ z(k) \quad = (C + \sum_{i=1}^{m} C_i p_i(k)) x(k) + (D + \sum_{i=1}^{m} D_i p_i) d(k) \end{cases}$$

where p_i $(i = 1, \ldots, m)$ are scalar standard stochastic scalar sequences with zero mean and satisfies $\mathrm{E}\{p(i)p(j)\} = \epsilon_i^2 \delta_{ij}$ with ϵ_i $(i = 1, \cdots, m)$ are positive constants. For a given scalar $\beta > 0$, (13) is ESMS and satisfies

$$\sup_{d \in l_2, \ \|d\|_2 \neq 0} \frac{\|z(k)\|_{2,\mathrm{E}}^2}{\|d(k)\|_2^2} \le \beta^2$$

if and only if there exists $Q > 0$ such that

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} P & 0 \\ 0 & \beta^2 I \end{bmatrix} + \sum_{i=1}^{m} \epsilon_i^2 \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} < 0$$

Lemma 5. (Dong et al. (2011)) Let $W = W^{\mathrm{T}} < 0$, E and F be real matrices with appropriate dimensions, then the following inequality

$$W + EHF + F^{\mathrm{T}} H^{\mathrm{T}} E H^{\mathrm{T}} < 0$$

holds for all H that satisfies $H^{\mathrm{T}} H \le I$ if and only if there exists a scalar $\mu > 0$, such that

$$W + \mu E E^{\mathrm{T}} + \mu^{-1} F^{\mathrm{T}} F < 0$$

Based on Lemma 4 and Lemma 5, the design of L and V is presented in the following Theorem 6.

Theorem 6. Given a scalar $\beta > 0$. If there exist matrices $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, a matrix X and a scalar $\mu > 0$, satisfying the following inequality

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & 0 & R_{17} \\ * & R_{22} & 0 & 0 & 0 & R_{26} & 0 \\ * & * & R_{33} & 0 & 0 & 0 & 0 \\ * & * & * & R_{44} & 0 & 0 & 0 \\ * & * & * & * & R_{55} & 0 & 0 \\ * & * & * & * & * & R_{66} & 0 \\ * & * & * & * & * & * & R_{77} \end{bmatrix} < 0$$

where

$$R_{11} = diag\{-Q_1, -Q_2, -Q_3, -\beta^2 I, -\beta^2 I, -\beta^2 I\}$$

$$R_{22} = R_{33} = R_{44} = diag\{-Q_1, -Q_2, -Q_3, -I\}$$

$$R_{55} = diag\{\underbrace{R_{22}, \ldots, R_{22}}_{q}\}$$

$$R_{66} = R_{77} = diag\{-\mu I, -\mu I, -\mu I\}, R_{15} = [R_{15}^1, \ldots, R_{15}^q]$$

$$R_{12} = \begin{bmatrix} A^{\mathrm{T}}Q_1 - \varrho C^{\mathrm{T}}X & 0 & 0 & \varrho C^{\mathrm{T}}V^{\mathrm{T}} \\ 0 & (A-\varrho L_r C)^{\mathrm{T}}Q_2 & 0 & -\varrho C^{\mathrm{T}}V_r^{\mathrm{T}} \\ 0 & 0 & A^{\mathrm{T}}Q_3 & 0 \\ B_d^{\mathrm{T}} - D_d^{\mathrm{T}}X & (B_d - L_r D_d)^{\mathrm{T}}Q_2 & B_d Q_3 & D_d^{\mathrm{T}}(V^{\mathrm{T}} - V_r^{\mathrm{T}}) \\ 0 & 0 & B^{\mathrm{T}}\Xi Q_3 & 0 \\ B_f^{\mathrm{T}}Q_1 & B_f^{\mathrm{T}}Q_2 & B_f^{\mathrm{T}}Q_3 & 0 \end{bmatrix}$$

$$R_{13} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa C^{\mathrm{T}}V^{\mathrm{T}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; R_{14} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \varepsilon A_v^{\mathrm{T}}Q_1 & 0 & \varepsilon A_v^{\mathrm{T}}Q_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_{16} = \mu \begin{bmatrix} F^{\mathrm{T}} & 0 & 0 \\ 0 & F^{\mathrm{T}} & 0 \\ 0 & 0 & F^{\mathrm{T}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; R_{25} = \mu \begin{bmatrix} Q_1 E & 0 & 0 \\ 0 & Q_2 E & 0 \\ 0 & 0 & Q_3 E \end{bmatrix}$$

$$R_{15}^{i} = \begin{bmatrix} 0 \; 0 \; 0 \; (\sigma_i B_i Q_i)^{\mathrm{T}} \; 0 \; 0 \; 0 \end{bmatrix}^{\mathrm{T}}, \; (i = 1, \ldots, q)$$

with $\kappa = \sqrt{\varrho - \varrho^2}$. Then, (5) is ESMS and the following H_∞ performance is fulfilled

$$\sup_{w \in l_2, \; \|w\|_2 \neq 0} \frac{\|r_e(k)\|_{2,\mathrm{E}}^2}{\|w(k)\|_2^2} \leq \beta^2 \tag{13}$$

with the filter gain matrix $L = Q^{-1}X^{\mathrm{T}}$.

Proof. Remembering that for $\nu(k)$ and $\alpha_i(k)$ $(i = 1, \ldots, q)$, we have

$$\mathrm{E}\{\nu(k)\} = 0, \; \mathrm{E}\{\nu^2(k)\} = \varrho - \varrho^2$$
$$\mathrm{E}\{\alpha_i(k)\} = 0, \; \mathrm{E}\{\alpha_i^2(k)\} = \sigma_i^2$$

thus, Theorem 6 can be derived by applying Lemma 4, Lemma 5 and Schur Complement Lemma (Boyd et al. (1994)) with $Q = diag\{Q_1, Q_2, Q_3\}$. Due to limitation of space, the detailed proof is omitted here.

Remark 3. For system (5), Theorem 6 provides a feasible solution to the FDF design problem that satisfies an auxiliary H_∞ performance (13). Notice that our original target is to minimize the performance index (6). This can be realized by a repeated application of Theorem 6 for deriving the minimum of β. Furthermore, the conservatism brought by the diagonalization of matrix Q can also be reduced via this simple iterative optimization algorithm.

4. AN ILLUSTRATIVE EXAMPLE

To illustrate the effectiveness of the achieved result, consider an industrial continuous-stirred tank reactor (CSTR) system in Gao et al. (2008), which investigates chemical species A reacts to form species B. Fig. 1 depicts the physical structure of the system, where C_{Ai} is the input concentration of a key reactant A; C_A is the output concentration of A; T is the reaction temperature; T_c is the cooling medium temperature. Define the state variable as follows:

$$x = [C_A \; T]^{\mathrm{T}}, \; u = [T_c \; C_{Ai}]^{\mathrm{T}}$$

and thus, a discrete-time model in the form (1) can be obtained as follows (Gao et al. (2008))

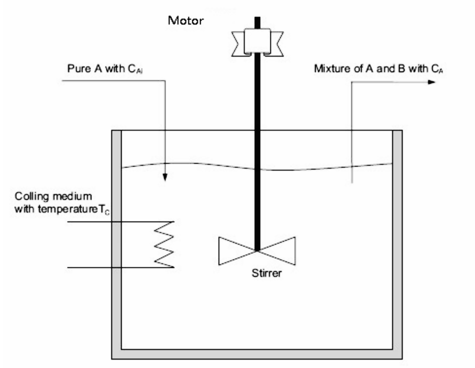

Fig. 1. A CSTR model

$$A = \begin{bmatrix} 0.9719 & -0.0013 \\ -0.0340 & 0.8628 \end{bmatrix}, \; B = \begin{bmatrix} -0.0839 & 0.0232 \\ 0.0761 & 0.4144 \end{bmatrix}$$
$$B_d = diag\{0.1, 0.3\}, \; B_f = [\, -0.0839 \; 0.0761 \,]^{\mathrm{T}}$$
$$C = [\, 1 \; 0.1 \,], \; D_d = [0 \; 0.1]$$

The unknown input $d(k)$ is set up by stochastic sequences that are uniformly distributed in the interval $[-0.5, 0.5]$. A step-wise fault signal is assumed as follows

$$f(k) = \begin{cases} 1, & k \in [20, 40] \\ -1, & k \in [40, 60] \\ 0, & \text{otherwise} \end{cases}$$

The input signal $u(k)$ is selected as

$$u(k) = \begin{cases} [0.15 \; 0.15]^{\mathrm{T}}, & k \in [20, 60] \\ 0, & \text{otherwise} \end{cases}$$

In what follows, two cases are considered :

Case 1: there is no model uncertainties and missing packet dropouts in (1), while random actuator faults occur with the following probabilistic density functions of $\gamma_1(k)$ and $\gamma_2(k)$

$$p(\gamma_1) = \begin{cases} 0 & \gamma_1 = 0 \\ 0.1 & \gamma_1 = 0.5, \\ 0.9 & \gamma_1 = 1 \end{cases} \; p(\gamma_2) = \begin{cases} 0 & \gamma_2 = 0 \\ 0.2 & \gamma_2 = 0.5 \\ 0.8 & \gamma_2 = 1 \end{cases}$$

which infers $\rho_1 = 0.95$, $\rho_2 = 0.9$, $\sigma_1^2 = 0.15$ and $\sigma_2^2 = 0.2$. By applying Theorem 3, we have

$$L = [\, 0.6538 \; 0.4545 \,]^{\mathrm{T}}, \; V = 6.8211$$

Fig. 2. displays the generated residual r(k) for *Case 1.*

Case 2: there exists mixed model uncertainties, missing measurement and random actuator failures simultaneously with the following parameter matrices

$$A_v = diag\{0.01, 0.01\}, \; E = [0.01 \; 0.05]^{\mathrm{T}}, \; F = [0.1 \; 0.1]$$

The probabilistic density functions of $\gamma_1(k)$ and $\gamma_2(k)$ are the same as *Case 1.* $v(k)$ is a zero mean white noise with $\varepsilon = 0.5$. Set $\mu = 0.1$ and apply Theorem 6, we have

$$L = [\, 1.6775 \; 1.1042 \,]^{\mathrm{T}}, \; V = -0.1283$$

The minimum of β is $\beta_{\min} = 1.9351$. Fig. 3 shows the residual $r(k)$ for *Case 2.* After 300 times Monte Carlo simulations without fault, the threshold is obtained as J_{th} =0.0199. Figure 4 gives the residual evaluation function $J(k)$ for both faulty and fault free case. It can be seen from the simulation results that the residual can deliver fault alarms in 1 second after the fault occurs.

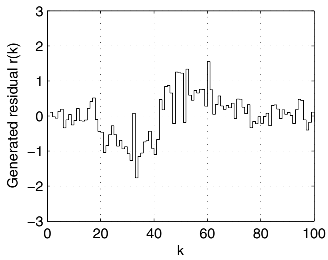

Fig. 2. Generated residual $r(k)$ (*Case 1*)

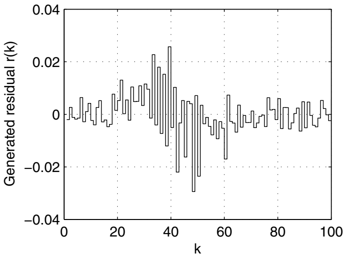

Fig. 3. Generated residual $r(k)$ (*Case 2*)

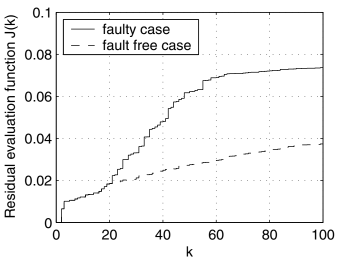

Fig. 4. Residual evaluation function $J(k)$

5. CONCLUSION

In this paper, the FD problem for linear discrete-time systems with intermittent measurements, actuator failures and mixed model uncertainties has been investigated via the H_∞ model-matching approach. A novel reference model has been addressed in the presence of the probabilistic actuator faults. An FDF has been designed in the LMI formulation such that the residual is sensitive to fault but robust to unknown inputs, mixed model uncertainties, random intermittent measurements and probabilistic actuator failures. The achieved result has been illustrated by a practical example.

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