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Wiener System Identification in Presence of Hysteresis Nonlinearity

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Abstract. The problem Wiener systems identification is addressed in presence of hysteresis nonlinearities, presently described by the Bouc-Wen model. The latter is nonlinear differential equation involving unknown parameters, some of which coming in nonlinearly. Except for stability, the linear subsystem is arbitrary and, in particular, it is not given a particular structure. By using sine excitations, the identification problem is reformulated as a set of nonlinear prediction-error optimization problems. The latter will be coped with using nonlinear least squares estimators which will formally be shown to be consistent.

1. INTRODUCTION

The standard Wiener model is constituted of a linear dynamic bloc and a memoryless nonlinearity connected in series as shown by Fig. 1. This model structure features a strong capability of approximating well nonlinear systems (e.g. Boyd and Chua, 1985). The problem of system identification based on different variants of the Wiener model has been given a great deal of interest, especially on the last decade, and several solutions are now available. In the case of fully parametric systems, the proposed identification methods include stochastic approaches (e.g. Wigren, 1993, 1994; Westwick and Verhaegen, 1996; Vanbeylen *et al.*, 2009; Lovera *et al.*, 2000; Wills and Ljung, 2010; Vanbeylen and Pintelon, 2010; Wills *et al.*, 2011) as well as deterministic approaches (e.g. Vörös, 1997, 2010; Bruls *et al.*, 1999). The available identification methods for nonparametric Wiener systems include stochastic methods (e.g. Greblicki and Pawlak, 2008; Mzyk, 2010) and frequency methods (e.g. Crama and Schoukens, 2001, 2005; Bai, 2003; Giri *et al.*, 2009; Schoukens and Rolain, 2012). Identification methods for semiparametric Wiener systems, where only the linear part is parameterized, have been proposed in (e.g. Hu and Chen, 2008; Bai and Reyland, 2009; Enqvist, 2010; Pelckmans, 2011). As a matter of fact, all proposed identification methods are based on several, more or less restrictive, assumptions concerning the system nonlinear part (invertible, monotone, odd), the linear subsystem (FIR, known structure), the input signals (Gaussian, PE). On recent year, the research scope concerning Wiener system identification has been widen including nonstandard Wiener system structures e.g. series-parallel Wiener systems (Lyzell and Enqvist, 2012; Lyzell et al., 2012; Schoukens and Rolain, 2012) and Wiener systems with memory nonlinearities (Dong *et al.*, 2009; Cerone *et al.*, 2009; Giri *et al.*, 2013a, 2013b). The present work is on Wiener system identification when memory nonlinearities are involved. So far, the emphasis has been restricted to memory nonlinearities of backlash and backlash-inverse types and the identification problem has been dealt with using predictive and frequency approaches. The problem of identifying Wiener systems containing general hysteresis nonlinearities has yet to be studied. Presently, this problem is addressed for a class of hysteresis nonlinearities described by the Bouc-Wen model. The latter is a widely used for describing technological components and devices, especially within the areas of civil and mechanical engineering (Ismail et al., 2009). It consists in a first-order nonlinear differential equation that relates the input displacement to the output restoring force in a hysteretic way. It involves a finite number of parameters making possible to accommodate the response of the model to the real hysteresis loops. From an identification viewpoint, the difficulty lies not only in the nonlinearity of the model dynamics but also in the fact that some of its unknown parameters come in nonlinearly. A deep analysis of the Bouc-Wen model can be found in (Ikhouane and Rodellar, 2007). Interestingly, it is formally shown that the steady-state response of a Bouc-Wen model, being excited with a so-called loading-unloading periodic input signal, is also a loading-unloading periodic signal. Furthermore, the input and output signals have the same period and are in phase. These features are presently exploited in the design of an semiparametric identification method for Wiener systems where the linear part assumes no particular structure. The method involves simple sine input excitations and an output signal averaging process reducing the effect of measurement noise. The filtered signals are based upon to estimate the nonlinearity parameters and the (linear subsystem) frequency response at a given set of frequencies. The first estimation problem is reformulated as a nonlinear prediction-error optimization problem and solved using a single separable least-squares estimator. The second estimation problem is solved using a set of least-squares estimators running in parallel. All estimators are formally shown to be consistent.

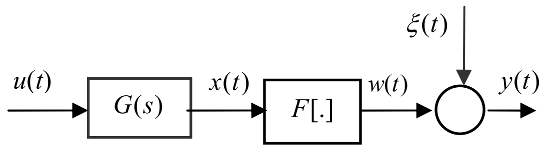

Fig. 1. Wiener model structure. $G(s)$ represents a linear subsystem, $F[.]$ represents a nonlinear operator

The paper is organized as follows: the identification problem is formulated in Section 2; the identification method is designed and analyzed in Section 3, A conclusion and a reference list end the paper.

2. IDENTIFICATION PROBLEM FORMULATION

2.1 System Modelling

Standard Wiener systems consist of a linear dynamic subsystem $G(s)$ followed in series by a memoryless nonlinear operator $F[.]$ (Fig. 1). Presently, hysteresis nonlinearities are considered. More specifically, the Wiener system is analytically described by the following equations:

$$x(t) = g(t) * u(t) \quad \text{with} \quad g(t) = \mathfrak{L}^{-1}(G(s)) \tag{1a}$$

$$w(t) = F[x](t) \tag{1b}$$

$$y(t) = w(t) + \xi(t) \tag{1c}$$

where $u(t)$ and $y(t)$ denote the control input and the measured output; $x(t)$ and $w(t)$ are inner signals not accessible to measurement. The extra input $\xi(t)$ accounts for measurement noise and other modelling effects; it is supposed to be zero-mean and uncorrelated with the control input $u(t)$. The symbol * in (1) refers to the convolution operator and $\mathfrak{L}^{1}$ to the Laplace transform-inverse. Accordingly, $g(t)$ is the impulse response of the linear subsystem and $G(s)$ its transfer function. It is just supposed that $g \in L_1$ so that the whole system is BIBO stable, making possible open-loop system identification. Interestingly, $G(s)$ assumes no particular structure and so it is allowed not to be finite order. The operator $F[.]$ undergoes the following differential equation referred to hysteresis Bouc-Wen model:

$$\dot{w} = \alpha\dot{x} - \beta|\dot{x}||w|^{\mu-1}w - \gamma\dot{x}|w|^{\mu} \tag{2a}$$

with $\alpha > 0$, $\beta + \gamma > 0$, $\beta - \gamma \geq 0$ and $\mu > 1$. The following result is formally established in (Ikhouane and Rodellar, 2007, p.18).

Proposition 1. The Bouc-Wen model output signal $w(t)$ is bounded, provided the input signal $x(t)$ is C^1 on $[0,\infty)$. Furthermore, an upper bound on $|w(t)|$ is $\max(|w(0)|, w_0)$ with $w_0 \overset{def}{=} \left(\frac{\alpha}{\beta+\gamma}\right)^{1/n}$ □

At this point, it is worth emphasizing the plurality of the couple $(G(s), F[x])$, defining the Wiener model. Accordingly, any couple $(\overline{G}(s), \overline{F}[\overline{x}])$, with $\overline{G}(s) = kG(s)$ and $\overline{F}[z] = F[z/k]$, is also a model whatever the scalar $k \neq 0$. From (2a) it turns out that one judicious choice is to let $k = 1/\alpha$ which leads to a model featuring $\alpha = 1$. Doing so, the number of unknown parameters in the corresponding hysteresis element reduces to 2. It is readily seen that the (β, γ)-parameters, in the model focused on, still satisfy the properties $\beta + \gamma > 0$, $\beta - \gamma \geq 0$, while the parameter μ remains unchanged. Now, to avoid additional notations, the particular model $(\overline{G}(s), \overline{F}[x])$ with the above features will continue to be denoted $(G(s), F[x])$ and the corresponding internal signals will still be denoted $x(t)$ and $w(t)$. Accordingly, the hysteresis element undergoes the following simpler version of equation (2):

$$\dot{w} = \dot{x} - \beta|\dot{x}||w|^{\mu-1}w - \gamma\dot{x}|w|^{\mu} \tag{2b}$$

with $\alpha > 0$, $\beta + \gamma > 0$, $\beta - \gamma \geq 0$ and $\mu > 1$. The identification problem at hand consists in estimating as accurately as possible, the nonlinear operator parameters (β, γ, μ) as well as the linear subsystem frequency gain $G(j\omega_i)$ $(i = 1 \ldots m)$, where the ω_i's and the number m are, to some extent, arbitrarily chosen by the user. Interestingly, the user needs not to a priori knows the transfer function $G(s)$.

2.2. System Response to Sinusoidal Excitation

The frequency identification approach is based on the investigation of the system response to a sinusoidal excitation $u(t) = U\cos(\omega t)$. Let the Wiener system (1-2) be excited with a sinusoidal input $u(t) = U\cos(\omega t)$ for some amplitude/frequency couple (U, ω). It follows from (1a-c) that the steady-state behaviour is described by the following equalities:

$$x_{U,\omega}(t) = U|G(j\omega)|\cos(\omega t - \varphi(\omega)) \tag{3a}$$

$$w_{U,\omega}(t) = F[x_{U,\omega}](t) \tag{3b}$$

$$y_{U,\omega}(t) = w_{U,\omega}(t) + \xi(t) \tag{3c}$$

with $\varphi(\omega) = -\angle G(j\omega)$. An interesting feature of the periodic signal $x_{U,\omega}(t)$ is that, on any interval of the form $\left[\frac{2k\pi+\varphi}{\omega}, \frac{2k\pi+\varphi}{\omega} + \frac{2\pi}{\omega}\right]$ $(k \in \mathbf{N})$, the signal is decreasing on the subinterval $\left[\frac{2k\pi+\varphi}{\omega}, \frac{2k\pi+\varphi}{\omega} + \frac{\pi}{\omega}\right]$ and is increasing on $\left[\frac{2k\pi+\varphi}{\omega} + \frac{\pi}{\omega}, \frac{2k\pi+\varphi}{\omega} + \frac{2\pi}{\omega}\right]$. Periodic signals of this type are referred to loading-unloading in (Ikhouane and Rodellar, 2007) and a key property the Bouc-Wen hysteresis model, excited with these signals, is described in the following proposition:

Proposition 2. Consider the Bouc-Wen hysteresis operator (2b), being excited with the sinusoidal signal $x_{U,\omega}(t)$ defined by (3a). Then, the resulting steady-state output $w_{U,\omega}(t)$ enjoys the following properties:

1) $w_{U,\omega}(t)$ is a class C^1, $2\pi/\omega$-periodic, and loading-unloading signal on $[0,\infty)$.
2) $w_{U,\omega}(t)$ and $x_{U,\omega}(t)$ are in phase in the sense that one has, for all (sufficiently large) $k \in \mathbf{N}$:
   a) the derivatives $\dot{w}_{U,\omega}(t)$ and $\dot{x}_{U,\omega}(t)$ are nonpositive

on $\left[\frac{2k\pi+\varphi}{\omega},\frac{2k\pi+\varphi}{\omega}+\frac{\pi}{\omega}\right]$ and are negative on $\left(\frac{2k\pi+\varphi}{\omega},\frac{2k\pi+\varphi}{\omega}+\delta\right)$, for some $0<\delta<\frac{\pi}{\omega}$.

b) $\dot{w}_{U,\omega}(t)$ and $\dot{x}_{U,\omega}(t)$ are nonnegative on $\left[\frac{2k\pi+\varphi}{\omega}+\frac{\pi}{\omega},\frac{2k\pi+\varphi}{\omega}+\frac{2\pi}{\omega}\right]$ and are positive on $\left(\frac{2k\pi+\varphi}{\omega}+\frac{\pi}{\omega},\frac{2k\pi+\varphi}{\omega}+\frac{\pi}{\omega}+\delta\right)$, for some $0<\delta<\frac{\pi}{\omega}$ □

The proof of the above results can be found in (Ikhouane and Rodellar, 2007, Chapter 3). Note that the fact that $w_{U,\omega}(t)$ is a class C^1 is an immediate consequence of Proposition 1, which ensures the boundedness of $w_{U,\omega}(t)$, and of equation (2b) which ensures the boundedness of $\dot{w}_{U,\omega}(t)$, using the fact that $\dot{x}_{U,\omega}(t)$ is bounded, due to (3a).

2.3. Signal Pre-Processing

One difficulty of the identification problem under study is that the system output y is infected by the disturbance ξ whose stochastic law is not known. Then, a suitable output signal preprocessing is required to reduce the disturbance effect on the quality of the system parameter estimates. As stated in Subsection 2.2, all system signals are $2\pi/\omega$-periodic (in steady-state), except for ξ and y. Then, just as suggested in (Ljung, 1997), a judicious signal filtering is to perform the following $2\pi/\omega$-periodic averaging:

$$\bar{s}_N(t)=\frac{1}{N}\sum_{k=0}^{N-1}s\left(t+k\frac{2\pi}{\omega}\right),\ \text{for } 0\le t\le\frac{2\pi}{\omega} \tag{4a}$$

$$\bar{s}_N(t)=\bar{s}_N\left(t-\frac{2\pi}{\omega}\right),\ \text{otherwise} \tag{4b}$$

with $N>>1$, where $s(t)$ is any signal and $\bar{s}_N(t)$ its $2\pi/\omega$-periodic average signal. It readily follows that $\bar{s}_N(t)=s(t)$ whenever $s(t)$ is $2\pi/\omega$-periodic. Presently, this is the case (in steady-state) of the signals (u,x,w) and, consequently, all these signals are equal (in steady-state) to their $2\pi/\omega$-periodic average versions obtained by (4a-b). Then, operating the averaging (4) on all terms of (3c) one gets in steady-state:

$$\bar{y}_{U,\omega,N}(t)=w_{U,\omega}(t)+\bar{\xi}_N(t) \tag{5}$$

Accordingly, the noise ξ is presently supposed to be a zero-mean ergodic stochastic process featuring the $2\pi/\omega$-periodic stationarity on the set of ω's of interests. The periodic stationarity means that $E\left(\xi(t+k\frac{2\pi}{\omega})\right)=E(\xi(t))$, for all t,k. This implies that:

$$\bar{\xi}_N(t)\underset{N\to\infty}{\to}E(\xi(t+kT))\quad\text{(w.p. 1)},\quad\text{whatever } k\in\mathbf{N} \tag{6}$$

Then, it immediately follows from (5) that, in steady-state (i.e. when $w_{U,\omega}(t)$ becomes periodic):

$$\bar{y}_{U,\omega,N}(t)-w_{U,\omega}(t)\underset{N\to\infty}{\to}0\quad\text{(w.p. 1)} \tag{7}$$

3. SYSTEM PARAMETER IDENTIFICATION

Throughout this section, the Wiener system is submitted to sinusoidal excitations $u(t)=U\cos(\omega t)$ so that the signal expressions (3a-c) hold along with Proposition 1 and property (7).

3.1 Phase Estimator

Using Proposition 1 (Part 2), one gets:

$$\varphi(\omega)=\omega t_k\ \text{(modulo } 2\pi\text{)},\ k\in\mathbf{N} \tag{8}$$

with $t_k=\frac{2(k+1)\pi+\varphi}{\omega}$ denote the periodic time instants where the (undisturbed) output $w_{U,\omega}(t)$ achieves its maxima. As $w_{U,\omega}(t)$ is not accessible to measurement, one can only use its estimate $\bar{y}_{U,\omega,N}(t)$, obtained by operating (4a-b) on $y_{U,\omega}(t)$. Letting $0\le\bar{t}_{U,\omega,N}\le\frac{2\pi}{\omega}$ be the time instant where $\bar{y}_{U,\omega,N}(t)$ achieves its maximum, (8) suggests the following phase estimator:

$$\hat{\varphi}_N(\omega)=\omega\bar{t}_{U,\omega,N} \tag{9}$$

It readily follows from (7) that $\bar{t}_{U,\omega,N}$ converges, w.p.1 as $N\to\infty$, to some $t_k=\frac{2(k+1)\pi+\varphi}{\omega}$. Then, it follows comparing (8) and (9) that:

$$\hat{\varphi}_N(\omega)\underset{N\to\infty}{\to}\varphi(\omega),\ \text{modulo } 2\pi\quad\text{(w.p.1)} \tag{10}$$

3.2 Nonlinearity Parameter Estimation

In this subsection, the hysteresis parameters (β,γ,μ) will be estimated using the data collected on the Wiener system being excited by the input $u(t)=U_1\cos(\omega_1 t)$ for a given amplitude/frequency couple (U_1,ω_1). The latter is arbitrary but one must have $|G(j\omega_1)|\neq 0$. Then, (11) writes:

$$\dot{w}_{U_1,\omega_1}=\dot{x}_{U_1,\omega_1}-\beta|\dot{x}_{U_1,\omega_1}||w_{U_1,\omega_1}|^{\mu-1}w_{U_1,\omega_1}-\gamma\dot{x}_{U_1,\omega_1}|w_{U_1,\omega_1}|^{\mu} \tag{12}$$

with:

$$x_{U_1,\omega_1}(t)=U_1|G(j\omega_1)|\cos(\omega_1 t-\varphi(\omega_1)); \tag{13a}$$

$$\dot{x}_{U_1,\omega_1}(t)=-U_1|G(j\omega_1)|\,\omega_1\sin(\omega_1 t-\varphi(\omega_1)) \tag{13b}$$

due to (3a). From Proposition 1 (Part 2), one has $\text{sgn}(\dot{x}_{U_1,\omega_1})=\text{sgn}(\dot{w}_{U_1,\omega_1})$. Accordingly, (12) becomes:

$$\dot{w}_{U_1,\omega_1}=\dot{x}_{U_1,\omega_1}-\beta\dot{x}_{U_1,\omega_1}\,\text{sgn}(\dot{w}_{U_1,\omega_1})|w_{U_1,\omega_1}|^{\mu-1}w_{U_1,\omega_1}-\gamma\,\dot{x}_{U_1,\omega_1}|w_{U_1,\omega_1}|^{\mu} \tag{14}$$

Substituting to $\dot{x}_{U_1,\omega_1}(t)$ its expression given by (13), equation (14) rewrites:

$$\dot{w}_{U_1,\omega_1} = -\left|G(j\omega_1)\right| U_1\omega_1 \sin(\omega_1 t - \varphi(\omega_1))$$
$$+ \beta\left|G(j\omega_1)\right| U_1\omega_1 \sin(\omega_1 t - \varphi(\omega_1))\mathrm{sgn}(\dot{w}_{U_1,\omega_1})\left|w_{U_1,\omega_1}\right|^{\mu-1} w_{U_1,\omega_1}$$
$$+ \gamma\left|G(j\omega_1)\right| U_1\omega_1 \sin(\omega_1 t - \varphi(\omega_1))\left|w_{U_1,\omega_1}\right|^{\mu} \quad (15a)$$
$$= \psi^T(\mu,t)\theta^* \quad (15b)$$

where the parameter vector $\theta^* \in \mathbf{R}^{+3}$ and the regressor $\psi(\mu,t) \in \mathbf{R}^3$ are defined by:

$$\theta^* = \left[\left|G(j\omega_1)\right| \quad \beta\left|G(j\omega_1)\right| \quad \gamma\left|G(j\omega_1)\right|\right]^T \quad (16a)$$
$$\psi^T(\mu,t) = \left[-U_1\omega_1 \sin(\omega_1 t - \varphi(\omega_1)) \ldots\right.$$
$$\ldots U_1\omega_1 \sin(\omega_1 t - \varphi(\omega_1))\mathrm{sgn}(\dot{w}_{U_1,\omega_1})\left|w_{U_1,\omega_1}\right|^{\mu-1} w_{U_1,\omega_1} \ldots$$
$$\left.\ldots U_1\omega_1 \sin(\omega_1 t - \varphi(\omega_1))\left|w_{U_1,\omega_1}\right|^{\mu}\right] \in \mathbf{R}^3 \quad (16b)$$

Note that, consistent estimators of the phase $\varphi(\omega_1)$ and the signal w_{U_1,ω_1} are available. Accordingly, one can (temporarily) suppose that $\varphi(\omega_1)$, w_{U_1,ω_1} and $\dot{w}_{U_1,\omega_1}$ are perfectly known. It turns out that equation (15) involves two unknown quantities, the vector $\theta^* \in \mathbf{R}^{+3}$, on the one hand, and the scalar $\mu \in \mathbf{R}^+$ on the other. This observation motivates the introduction of the following optimization problem:

$$\min_{\nu>1,\theta\in\mathbf{R}^3} J(\nu,\theta) \quad (17a)$$

with

$$J(\nu,\theta) = \int_{2k\pi/\omega_1}^{2(k+1)\pi/\omega_1} \left(\dot{w}_{U_1,\omega_1}(t) - \theta^T\psi(\nu,t)\right)^2 dt \quad (17b)$$

where $k \in \mathbf{N}$ is any sufficiently large integer (so that $w_{U_1,\omega_1}(t)$ can be supposed to be periodic for $t \ge 2k\pi/\omega$). It readily follows from (15b) and (17b) that $J(\mu,\theta^*) = 0$ i.e. $J(\nu,\theta)$ does achieve its global minimum at (μ,θ^*). But, the question is whether this global minimum is unique. This will be proved to be the case in Proposition 3. Then, it just remains to design a search method to explicitly determine the unique solution (μ,θ^*) to this optimisation problem. The main difficulty is that the function $J(\nu,\theta)$ is quadratic in θ but not in ν (see (17b)). Then, the optimization problem (17a-b) can be coped with using the separable least-squares technique, which is a form of relaxation. Accordingly, one temporarily assumes that ν is known in (17b). Then, (17a) becomes a least-squares problem and its solution is the following:

$$\Theta(\nu) = \left(\int_{2k\pi/\omega_1}^{2(k+1)\pi/\omega_1} \psi(\nu,t)\psi^T(\nu,t)dt\right)^{-1} \times \int_{2k\pi/\omega_1}^{2(k+1)\pi/\omega_1} \dot{w}_{U_1,\omega_1}(t)\psi(\nu,t)dt \quad (18)$$

where $k \in \mathbf{N}$ is as in (17b). At this point, it is worth noticing that, if μ is substituted to ν in (18) then one gets θ^*, using (15b). Specifically, one has:

$$\theta^* = \Theta(\mu) = \left(\int_{2k\pi/\omega_1}^{2(k+1)\pi/\omega_1} \psi(\mu,t)\psi^T(\mu,t)dt\right)^{-1} \times \int_{2k\pi/\omega_1}^{2(k+1)\pi/\omega_1} \dot{w}_{U_1,\omega_1}(t)\psi(\mu,t)dt \quad (19)$$

provided that the matrix to be inversed on the right side of (19) is invertible. Now, let us go back to (17b) and let the right side of (18) be substituted there to θ. Doing so, one gets the following nonlinear function which only involves the variable ν:

$$I(\nu) \overset{def}{=} J(\nu,\Theta(\nu)) = \int_{2k\pi/\omega}^{2(k+1)\pi/\omega} \left(\dot{w}_{U_1,\omega_1}(t) - \Theta(\nu)^T\psi(\nu,t)\right)^2 dt \quad (20)$$

Note that for this problem to be well posed, $\Theta(\nu)$ must exist. In view of (18), this will be the case if the matrix (to be inversed) on the right side of (18) is invertible, whatever $\nu > 1$. Under this condition, the separable least-squares technique operates this way: first, minimize $I(\nu)$ to obtain an estimate $\hat{\mu}$ of μ; then replace the latter by its estimate in (18) to get an estimate $\hat{\theta}$ of θ.

Proposition 3. Let the Wiener system (1-2) be excited by $u(t) = U_1\cos(\omega_1 t)$, where the amplitude $U_1 > 0$ and the frequency $\omega_1 > 0$ are arbitrary but $\left|G(j\omega_1)\right| \ne 0$. Then, one has the following properties:

1) The optimization problem (17a-b), involving the cost function $J(\nu,\theta)$ and the equality constraint (15b), has a unique solution, namely: $\arg\min_{\nu>1,\theta\in\mathbf{R}^3} J(\nu,\theta) = (\mu,\theta^*)$.

2) The optimization problem (20), involving the cost function $I(\nu)$ and the equality constraints (15b), has a unique solution, namely: $\arg\min_{\nu>1} I(\nu) = \mu$ □

Proof. Appendix A.

The above result is quite important. But, it still is not practical because the function $I(\nu)$ involves unavailable signals, i.e. $\dot{w}_{U_1,\omega_1}(t)$ and $\psi(\nu,t)$ (see (20)). In turn $\psi(\nu,t)$ involves $\varphi(\omega_1)$ (see (16b)) which is unknown. A practical cost function to estimate ν is the following:

$$I_N(\nu) = \int_0^{2\pi/\omega} \left(\dot{\bar{y}}_{U_1,\omega_1,N}(t) - \Theta_N(\nu)^T\psi_N(\nu,t)\right)^2 dt \quad (21)$$

with:

$$\Theta_N(\nu) = \left(\int_0^{2\pi/\omega} \psi_N(\nu,t)\psi_N^T(\nu,t)dt\right)^{-1} \times \int_0^{2\pi/\omega} \dot{\bar{y}}_{U_1,\omega_1,N}(t)\psi_N(\nu,t)dt \quad (22)$$

$$\psi_N^T(\nu,t) = \left[-U_1\omega_1 \sin(\omega_1 t - \hat{\varphi}_N(\omega_1)) \ldots\right.$$
$$\ldots U_1\omega_1 \sin(\omega_1 t - \hat{\varphi}_N(\omega_1))\mathrm{sgn}(\dot{\bar{y}}_{U_1,\omega_1,N})\left|\bar{y}_{U_1,\omega_1,N}\right|^{\mu-1} \bar{y}_{U_1,\omega_1,N}$$
$$\left. U_1\omega_1 \sin(\omega_1 t - \hat{\varphi}_N(\omega_1))\left|\bar{y}_{U_1,\omega_1,N}\right|^{\mu}\right] \in \mathbf{R}^3 \quad (23)$$

where we have used the fact that $\dot{\bar{y}}_{U_1,\omega_1,N}(t)$ is constructively $2\pi/\omega$-periodic (by definition (4a-b)). With the above notations, the (μ,θ^*)-estimator writes as follows:

$$\hat{\mu}_N = \arg\min_{\nu>1} I_N(\nu) \tag{24a}$$

$$\hat{\theta}_N = \Theta_N(\hat{\mu}_N) \tag{24b}$$

It is readily seen from (16a) that estimates of $|G(j\omega_1)|$, β and γ are easily obtained from $\hat{\theta}_N$.

Proposition 4. Let the Wiener system (1-2) be excited by $u(t)=U_1\cos(\omega_1 t)$, where the amplitude $U_1>0$ and the frequency $\omega_1>0$ are arbitrary but $|G(j\omega_1)|\neq 0$. Then, the estimator $(\hat{\mu}_N,\hat{\theta}_N)$ converges to (μ,θ^*), w.p.1, as $N\to\infty$ □

Proof: provided upon request by the authors •

3.3 Frequency Gain Estimator

We have already got an accurate estimate of the particular gain modulus $|G(j\omega_1)|$ and the phase $\varphi(\omega_1)=\angle G(j\omega_1)$. Now, the aim is to estimate the frequency gain $G(j\omega_i)$ for $(m-1)$ further frequencies ω_i $(i=2\ldots m)$, arbitrarily chosen by the user. Again, we will separately estimate the gain modulus $|G(j\omega_i)|$ and the phases $\varphi(\omega_i)=\angle G(j\omega_i)$. The latter can be estimated using the consistent estimator defined by (9). Then, it remains to design an estimator for the gain modulus. To this end, let us rewrite equation (15a) for an arbitrary amplitude/frequency couple (U_i,ω_i):

$$\begin{aligned}\dot{w}_{U_i,\omega_i} &= -|G(j\omega_i)|\,U_i\omega_i\sin(\omega_i t-\varphi(\omega_i))\\ &+\beta|G(j\omega_i)|\,U_i\omega_i\sin(\omega_i t-\varphi(\omega_i))\operatorname{sgn}(\dot{w}_{U_i,\omega_i})\left|w_{U_i,\omega_i}\right|^{\mu-1}w_{U_i,\omega_i}\\ &+\gamma|G(j\omega_i)|\,U_i\omega_i\sin(\omega_i t-\varphi(\omega_i))\left|w_{U_i,\omega_i}\right|^{\mu}\\ &=\chi(\eta,t)|G(j\omega_i)|\end{aligned} \tag{25}$$

with:

$$\eta=[\beta\ \gamma\ \mu]^T \tag{26a}$$

$$\begin{aligned}\chi(\eta,t) &= -U_i\omega_i\sin(\omega_i t-\varphi(\omega_i))\\ &+\beta U_i\omega_i\sin(\omega_i t-\varphi(\omega_i))\operatorname{sgn}(\dot{w}_{U_i,\omega_i})\left|w_{U_i,\omega_i}\right|^{\mu-1}w_{U_i,\omega_i}\\ &+\gamma\, U_i\omega_i\sin(\omega_i t-\varphi(\omega_i))\left|w_{U_i,\omega_i}\right|^{\mu}\end{aligned} \tag{26b}$$

Since at this stage an accurate estimate of η is available, $|G(j\omega_i)|$ turns out to be the only real unknown quantity in (25). The latter immediately yields the following least-squares solution:

$$|G(j\omega_i)|=\left(\int_{2k\pi/\omega}^{2(k+1)\pi/\omega}(\chi(\eta,t))^2\,dt\right)^{-1}\times\int_{2k\pi/\omega}^{2(k+1)\pi/\omega}\dot{w}_{U_i,\omega_i}(t)\chi(\eta,t)dt \tag{27a}$$

for any integer k. This suggests the following estimator:

$$\left|\hat{G}_N(j\omega_i)\right|=\left(\int_0^{2\pi/\omega}(\chi_N(\hat{\eta}_N,t))^2\,dt\right)^{-1}\times\int_0^{2\pi/\omega}\dot{\bar{y}}_{U_i,\omega_i}(t)\chi_N(\hat{\eta}_N,t)dt \tag{27b}$$

with:

$$\hat{\eta}_N=[\hat{\beta}_N\ \hat{\gamma}_N\ \hat{\mu}_N]^T \tag{28a}$$

$$\begin{aligned}\chi(\hat{\eta}_N,t) &= -U_i\omega_i\sin(\omega_i t-\hat{\varphi}_N(\omega_i))\\ &+\hat{\beta}_N U_i\omega_i\sin(\omega_i t-\hat{\varphi}_N(\omega_i))\operatorname{sgn}(\dot{\bar{y}}_{U_i,\omega_i,N})\left|\bar{y}_{U_i,\omega_i,N}\right|^{\hat{\mu}-1}\bar{y}_{U_i,\omega_i,N}\\ &+\hat{\gamma}_N\, U_i\omega_i\sin(\omega_i t-\hat{\varphi}_N(\omega_i))\left|\bar{y}_{U_i,\omega_i,N}\right|^{\hat{\mu}}\end{aligned} \tag{28b}$$

where we have used the fact that (by definition (4a-b)) $\dot{\bar{y}}_{U_i,\omega_i,N}$ is $2\pi/\omega$-periodic.

Proposition 5. Let the Wiener system (1-2) be excited by the input $u(t)=U_i\cos(\omega_i t)$ $(i=2\cdots m)$, where the amplitude $U_i>0$ and the frequency $\omega_i>0$ are arbitrarily chosen but $|G(j\omega_i)|\neq 0$. Then, the estimator $\left|\hat{G}_N(j\omega_i)\right|$ converges to $|G(j\omega_i)|$, w.p.1, as $N\to\infty$ □

The proof is provided upon request by the authors.

4. CONCLUSION

The problem of system identification is addressed for Wiener systems where nonlinear element is a hysteresis operator described by the Bouc-Wen model. The latter is a nonlinear differential equation containing unknown parameters, some of which come in nonlinearly. The identification method involves sine input signals and the output filtering (4a-b). Based on these signals, the phase estimator (9), the hysteresis parameters estimator (24a-b), and the frequency gain estimator (27b) are successively designed and formally shown to be consistent. To the author's knowledge no previous study has dealt with the identification problem for Wiener systems containing hysteresis nonlinearities.

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