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Vector Lyapunov Function based Stability of a Class of Applications Relevant 2D Nonlinear Systems ⋆

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Abstract: A substantial literature exists on the stability and control of 2D systems, including repetitive processes where iterative learning control algorithms designed in this setting have been experimentally verified. Most of this development has assumed that the dynamics can be adequately represented by linear state-space models, but applications exist where this assumption does not hold. This paper contributes to the development of a stability theory for nonlinear discrete-time 2D systems, where main results are on the use of vector Lyapunov functions to characterize exponential stability. The analysis includes systems where random failures modeled by a Markov chain with a finite set of states can arise in an iterative learning control application. An illustrative example is also given.

Keywords: Nonlinear 2D systems, vector Lyapunov functions, stability, iterative learning control.

1. INTRODUCTION

Vector Lyapunov functions (VLFs) introduced by Bellman (1962) and Matrosov (1962) are an extension of their classical counterparts and have been effectively used, see, for e.g., (Lakshmikantham et al., 1991) in many problem areas, such as complex standard, or 1D, large scale systems (Siljak, 1978). The published work on the use of VLFs for 2D systems includes (Kojima et al., 2011) where necessary and sufficient stability conditions were obtained in terms of such functions in a linear behavioral model setting. In this paper the VLF method is developed for stability analysis of nonlinear discrete 2D systems in form of repetitive process (Rogers et al., 2007).

Repetitive processes repeat the same finite duration operation over and over again. Each repetition is termed a pass and the duration the pass length. One industrial application is long-wall coal mining where the coal is cut by a machine that passes along the coal face and the objective is to maximize the volume of coal cut without penetrating the coal/stone interface. During each pass the machine rests on the pass profile cut during the previous pass, i.e., the height of the stone/coal interface above some datum line. Once a pass is complete, the machine is returned to the starting location and then pushed across to rest on the newly cut floor profile ready for the start of the next pass. The control problem is that the sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction and such oscillations cannot be removed by standard, or 1D, control laws (Rogers et al., 2007).

A major application area where repetitive process stability theory can be used is Iterative Learning Control (ILC), where linear model based designs have been experimentally verified (Hładowski et al., 2010). Other representations for 2D systems exist such as the Roesser (Roesser, 1975) and Fornasini Marchesini (Fornasini and Marchesini, 1985) state-space models. In common with repetitive processes, a very large part of the current literature on these systems is linear model based.

Cases will arise, however, where the assumption that the dynamics can be represented by a linear model is too weak. Hence there is a need to develop a stability theory for nonlinear 2D systems, where this paper focuses on repetitive processes given the recent progress to experimental verification in the ILC application. The eventual aim is to extend the previous linear model based results to the stage were nonlinear model based control law design is possible for applications.

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The major contribution of this paper is the use of VLFs to characterize pass profile exponential stability of nonlinear discrete repetitive processes, where the definition of this property includes stability along the pass for linear processes as a special case. Stability along the pass requires that a bounded initial pass profile produces a bounded sequence of pass profiles for all possible values of the pass length. This property is the basis of the ILC design with experimental verification. Pass profile exponential stability is an extension of this physically based property to nonlinear dynamics. Results on the stability of other classes of 3D nonlinear systems can be found in, e.g., (Kurek, 2012; Yeganefar et al., 2013) but no potential applications areas are given.

In the application of a control system, failures in operation can arise and this paper also gives the first results on the control of discrete nonlinear repetitive process where the failures are modeled as random switching. In particular, the system with failures is modeled by a state-space model with jumps in the parameter values and/or structure governed by a Markov chain with a finite set of states, often termed Markovian jump systems or systems with random structure, see, e.g., (Costa et al., 2004). Results on the development of control theory for Markovian jump systems, which address issues such as stability, optimal and robust control problems, in the 1D case can be found in, e.g., (Pakshin and Peaucelle, 2009) and for 2D linear systems in (Gao et al., 2004; Wu et al., 2008; Pakshin et al., 2011).

This paper begins in the next section with the development of new results on the stability of nonlinear repetitive processes using vector Lyapunov functions and then proceeds to develop new results for processes where failures in operation can occur, which are modeled as a Markov chain with a finite set of states. An application to ILC law design under sensor, or information, failures is also developed and an illustrative numerical example is given.

2. PASS PROFILE EXPONENTIAL STABILITY OF NONLINEAR REPETITIVE PROCESSES

The nonlinear discrete repetitive processes considered in this paper are described by the state-space model

$$\begin{aligned} x_{k+1}(t+1) &= f_1(x_{k+1}(t), y_k(t)), \\ y_{k+1}(t) &= f_2(x_{k+1}(t), y_k(t)), \\ &0 \le t \le T,\ k=0,1,2,... \end{aligned} \tag{1}$$

where the integer $T < \infty$ denotes the number of samples over the pass length and on pass k $x_k(t) \in \mathbb{R}^{n_x}$ is the current pass state vector, $y_k(t) \in \mathbb{R}^{n_y}$ is the pass profile vector and f_1 and f_2 are nonlinear functions such that $f_1(0,0,0)=0,\ f_2(0,0,0)=0$. The boundary conditions, i.e., the pass state initial vector sequence and the initial pass profile, are assumed to be of the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1},\ k \ge 0, \\ y_0(t) &= f(t),\ 0 \le t \le T, \end{aligned} \tag{2}$$

where the entries in the $n_x \times 1$ vector d_{k+1} are known constants and $f(t)$ is an $n_y \times 1$ vector whose entries are known functions of t. Moreover, if $|q|$ denotes the Euclidian norm of a vector q, it is assumed that the exists finite real numbers $M_f > 0$ and $0 < z_d < 1$ such that d_{k+1} and $f(t)$ satisfy

$$|f(t)|^2 \le M_f,\ |d_{k+1}|^2 \le \kappa_d z_d^{k+1},\ k=0,1,... \tag{3}$$

In particular, z_d represents the rate of convergence in k of the pass initial state vector sequence.

Note 1. All references to the boundary conditions from this point onwards will assume that they satisfy (3).

Note 2. The stability theory for repetitive processes is defined in terms of the pass profile and the model and results of this and the next section extend directly to the case when a current pass input is present.

In the control and systems theory developed for linear repetitive processes, the stability along the pass property has formed the basis for control law design and experimental verification (Rogers et al., 2007; Hładowski et al., 2010). This property demands that a bounded initial pass profile produces a bounded sequence of pass profiles for all possible values of the pass length and is based on linear operator theory in a Banach space setting. Hence it cannot be directly transferred to the nonlinear case. This paper introduces and characterizes a stability theory for nonlinear repetitive processes that includes stability along the pass of linear examples as a special case.

Define the norm of the pass profile vector in (1) as

$$||y_k|| = \sqrt{\sum_{t=0}^{T-1} |y_k(t)|^2} \tag{4}$$

and introduce the following stability definition.

Definition 1. A nonlinear repetitive process described by (1) and (2) is said to be pass profile exponentially stable if

$$||y_k|| \le \kappa z^k,\ \ 0 < z < 1, \tag{5}$$

where κ depends on the pass length T and z, in general, on z_d.

To obtain conditions for pass profile exponential stability, a vector Lyapunov approach is used with candidate function

$$V(x,y) = \begin{bmatrix} V_1(x_{k+1}(t)) \\ V_2(y_k(t)) \end{bmatrix}, \tag{6}$$

where $V_1(x) > 0,\ x \ne 0,\ \ V_2(y) > 0,\ y \ne 0,\ \ V_1(0) = 0,\ V_2(0) = 0$. The divergence operator of this function along the trajectories of system (1) is

$$\mathrm{div} V(x_{k+1}(t), y_k(t)) = \Delta_t V_1(x_{k+1}(t)) + \Delta_k V_2(y_k(t)), \tag{7}$$

where

$$\begin{aligned} \Delta_t V_1(x_{k+1}(t)) &= V_1(x_{k+1}(t+1)) - V_1(x_{k+1}(t)), \\ \Delta_k V_2(y_k(t)) &= V_2(y_{k+1}(t)) - V_2(y_k(t)). \end{aligned}$$

Theorem 2. Consider a nonlinear discrete repetitive process described by (1) and (2). Suppose also that there exist positive constants $c_1,\ c_2,\ c_3$ such that the vector Lyapunov function V and its divergence along the trajectories of (1) satisfy the inequalities

$$c_1|x_{k+1}(t)|^2 \le V_1(x_{k+1}(t)) \le c_2|x_{k+1}(t)|^2, \quad (8)$$
$$c_1|y_k(t)|^2 \le V_2(y_k(t)) \le c_2|y_k(t)|^2, \quad (9)$$
$$\text{div}V(x_{k+1}(t), y_k(t)) \le -c_3(|x_{k+1}(t)|^2 + |y_k(t)|^2). \quad (10)$$

Then this process is pass profile exponentially stable.

Proof. It follows from (8), (9) and (10) that

$$V_1(x_{k+1}(t+1)) \le \lambda V_1(x_{k+1}(t)) + \lambda V_2(y_k(t)) - V_2(y_{k+1}(t)), \quad (11)$$

where $\lambda = 1 - \frac{c_3}{c_2}$. It is easy to see that $\lambda \in (0,1)$. Solving the inequality (11) with respect to $V_1(x_{k+1}(t))$ gives

$$V_1(x_{k+1}(t)) \le V_1(x_{k+1}(0))\lambda^t + \sum_{p=0}^{t-1}[\lambda V_2(y_k(p)) - V_2(y_{k+1}(p))]\lambda^{t-p-1}. \quad (12)$$

On introducing $H_k(t) = \sum_{p=0}^{t-1} V_2(y_k(t))\lambda^{t-p-1}$, it follows from (3), (8) and (12) that

$$H_{k+1}(t) \le \lambda H_k(t) + \lambda^t V_1(x_{k+1}(0)) \le \lambda H_k(t) + \lambda^t c_2\kappa_d z_d^{k+1}. \quad (13)$$

Solving the inequality (13) gives

$$H_k(t) \le \lambda^k H_0(t) + \lambda^t c_2\kappa_d z_d \sum_{i=0}^{k-1} z_d^i \lambda^{k-1-i} \quad (14)$$

and it follows from (14), by majorization of the right-hand side, that

$$H_k(t) \le z^k\left(H_0 + \frac{c_2\kappa_d\lambda^t}{1-\zeta}\right), \quad (15)$$

where $z = \max\{\lambda, \zeta\}$, $\zeta = \bar{z}^{\frac{1}{2}}$, $\bar{z} = \max\{z_d, \lambda\}$. Finally, given (15), setting $t = T$ and using (9) it follows that (5) holds.

3. EXTENSION TO SYSTEMS WITH FAILURES

An extension of the results in Section 2 to repetitive processes in the presence of failures is possible. The failures are modeled by a discrete-time Markov chain with a finite set of states and the process is described by the following state-space model

$$x_{k+1}(t+1) = \varphi_1(x_{k+1}(t), y_k(t), r(t)),$$
$$y_{k+1}(t) = \varphi_2(x_{k+1}(t), y_k(t), r(t)), \quad (16)$$

where $r(t)$ $(t \ge 0)$ is a discrete-time Markov chain with discrete state-space $\mathbb{N} = \{1, \ldots, \nu\}$ and transition probabilities given by

$$\text{P}(r(t+1) = j \mid r(t) = i) = \pi_{ij}, \ \sum_{j=1}^{\nu} \pi_{ij} = 1, \ i \in \mathbb{N}. \quad (17)$$

In this model φ_1 and φ_2 are nonlinear functions such that for all $r \in \mathbb{N}$ $\varphi_1(0,0,r) = 0$, $\varphi_2(0,0,r) = 0$ and the rest of the notation, including the boundary conditions, is the same as the state-space model in Section 2. The failure model (17) is motivated by the fact that the variable t is time along the pass and hence it is natural that the failures evolve over time.

Define the norm of the pass profile vector as

$$||y_k||_{\text{E}} = \sqrt{\text{E}[\sum_{t=0}^{T-1} |y_k(t)|^2]}, \quad (18)$$

where E denotes the expectation operator. Pass profile exponentially mean square stability is defined as follows.

Definition 3. A nonlinear discrete repetitive process described by (16) and (2) is said to be pass profile exponentially mean square stable if there exist scalars $\kappa > 0$ and $0 < z < 1$ such that

$$||y_k||_{\text{E}} \le \kappa z^k. \quad (19)$$

To obtain conditions for pass profile exponentially mean square stability of processes described by (16), consider the candidate Lyapunov vector function

$$V(x_{k+1}(t), y_k(t), r(t)) = \begin{bmatrix} V_1(x_{k+1}(t), r(t)) \\ V_2(y_k(t), r(t)) \end{bmatrix}, \quad (20)$$

where $V_1(x,r) > 0, \ x \ne 0, \quad V_2(y,r) > 0, \ y \ne 0, \ V_1(0,r) = 0, \ V_2(0,r) = 0$ Introduce the operators $\mathcal{D}_t$ and $\mathcal{D}_k$ defined along the trajectories of (16):

$$\mathcal{D}_t V(\xi,\eta,i) = \text{E}[V_1(x_{k+1}(t+1), r(t+1)) - V_1(x_{k+1}(t), r(t)) \mid x_{k+1}(t) = \xi, y_k(t) = \eta, r(t) = i],$$
$$\mathcal{D}_k V(\xi,\eta,i) = \text{E}[V_2(y_{k+1}(t), r(t)) - V_2(\eta_k, i) \mid x_{k+1}(t) = \xi, y_k(t) = \eta, r(t) = i].$$

and define the operator $\mathcal{D}$ as the stochastic counterpart of divergence operator of Section 2:

$$\mathcal{D}V(\xi,\eta,i) = \mathcal{D}_t V(\xi,\eta,i) + \mathcal{D}_k V(\xi,\eta,i). \quad (21)$$

Theorem 4. Consider a discrete nonlinear repetitive process described by (16) and (17) and (2). Suppose also that there exist positive constants c_1, c_2, c_3 such that the function V and its operator $\mathcal{D}$ along the trajectories of the process described by (16) and (17) satisfies the inequalities

$$c_1|\xi|^2 \le V_1(\xi, i) \le c_2|\xi|^2, \quad (22)$$
$$c_1|\eta|^2) \le V_2(\eta, i) \le c_2|\eta|^2, \quad (23)$$
$$\mathcal{D}V(\xi,\eta,i) \le -c_3(|\xi|^2 + |\eta|^2), \quad (24)$$

$i \in \mathbb{N}$. Then this process is pass profile exponentially mean square stable.

Proof. It follows from (22), (23) and (24) that

$$\text{E}[V_1(x_{k+1}(t+1)] \le \lambda\text{E}[V_1(x_{k+1}(t))] + \text{E}[\lambda V_2(y_k(t)) - V_2(y_{k+1}(t))], \quad (25)$$

where $\lambda = 1 - \frac{c_3}{c_2}$. It is easy to see that $\lambda \in (0,1)$. Solving inequality (25) with respect to $V_1(x_{k+1}(t))$ gives

$$\text{E}[V_1(x_{k+1}(t))] \le \text{E}[V_1(x_{k+1}(0))]\lambda^t + \sum_{p=0}^{t-1}\text{E}[\lambda V_2(y_k(p)) - V_2(y_{k+1}(p))]\lambda^{t-p-1}. \quad (26)$$

Introduce the notation

$$H_k(t) = \mathrm{E}\left[\sum_{p=0}^{t-1} V_2(y_k(t))\right]\lambda^{t-p-1}$$

and then it follows from (26) and (22) that

$$\begin{aligned} H_{k+1}(t) &\leq \lambda H_k(t) + \lambda^t \mathrm{E}[V_1(x_{k+1}(0))] \\ &\leq \lambda H_k(t) + \lambda^t c_2 \kappa_d z_d^{k+1}. \end{aligned} \quad (27)$$

Solving the inequality (27) for the given boundary conditions (2) leads to (14) and the rest of the proof follows identical steps to that for Theorem 2.

4. APPLICATION TO ILC UNDER SENSOR FAILURES

In this section the stability results developed in Section 3 are applied to ILC design under sensor failures. To formulate the ILC problem, let the integer k denote the pass, termed trial in most of the ILC literature, number and $u_k(t) \in \mathbb{R}^l$, $x_k(t) \in \mathbb{R}^n$ and $y_k(t) \in \mathbb{R}^m$ the input, state and output vectors, respectively, at instant $0 \leq t \leq T < \infty$, where T denotes the pass length. Then the dynamics of the uncontrolled system are described by

$$\begin{aligned} x_k(t+1) &= Ax_k(t) + Bu_k(t), \\ y_k(t) &= C(r(t))x_k(t) \end{aligned} \quad (28)$$

with assumed boundary conditions

$$y_0(t) = 0,\ 0 \leq t \leq T,\ x_k(0) = x_0,\ k = 0, 1, ... \quad (29)$$

and $r(t)$ is the discrete-time Markov chain with a finite set of states $\mathbb{N} = \{1, \ldots, \nu\}$ corresponding to a number of possible failures and transition probabilities given by (17).

Let $y_{ref}(t)$ denote the supplied reference vector over $0 \leq t \leq T$, where each entry in $y_{ref}(t)$ is assumed to be differentiable. Then $e_k(t) = y_{ref}(t) - y_k(t)$ is the error on pass k and the objective of constructing a sequence of input functions such that the performance achieved is gradually improving with each successive pass can be expressed as a convergence condition on the input and error, i.e.,

$$\lim_{k\to\infty} |e_k(t)| = 0,\ \lim_{k\to\infty} |u_k(t) - u_\infty(t)| = 0. \quad (30)$$

A commonly used ILC law is to select the input on the current pass as that used on the previous pass plus a correction. In this work the ILC law on pass $k+1$ is of the form

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t), \quad (31)$$

where $\Delta u_{k+1}(t)$ is the correction term to be designed. The novel feature of ILC is all information generated on a completed pass is available for use in the computation of $\Delta u_{k+1}(t)$. This allows the use of temporal information that is non-causal in the standard sense provided it is generated and stored on a previous pass.

Definition 5. A system described by (28) is said to be convergent if for all $0 \leq t \leq T$

$$\mathrm{E}[|e_k(t)|^2] = \mathrm{E}[|y_{ref}(t) - y_k(t)|^2] \to 0,\ k \to \infty \quad (32)$$

and

$$\mathrm{E}[|u_k(t) - u_\infty(t)|^2] \to 0,\ k \to \infty. \quad (33)$$

To write the ILC dynamics as a discrete linear repetitive process, introduce, for analysis purposes only, the vector

$$\upsilon_{k+1}(t+1) = x_{k+1}(t) - x_k(t). \quad (34)$$

Then the controlled dynamics can be written in the form

$$\begin{aligned} \upsilon_{k+1}(t+1) &= A\upsilon_{k+1}(t) + B\Delta u_{k+1}(t-1), \\ e_{k+1}(t) &= -C(r(t))A\upsilon_{k+1}(t) + e_k(t) \\ &\quad - C(r(t))B\Delta u_{k+1}(t-1). \end{aligned} \quad (35)$$

Suppose also that in addition to output vector $(y(t))$ some other state variables are available for measurement as represented by the vector $z(t) \in \mathbb{R}^q$, where

$$z_k(t) = D(r(t))x_k(t) \quad (36)$$

and $D(r)$, $r \in \mathbb{N}$, is a full rank matrix of compatible dimensions. Also introduce $\Delta z_{k+1}(t+1) = z_{k+1}(t) - z_k(t)$.

Consider the case when

$$\begin{aligned} \Delta u_{k+1}(t) &= F_1(i)\Delta z_{k+1}(t) \\ &\quad + F_2(i)e_k(t+1),\ \text{if } r(t) = i. \end{aligned} \quad (37)$$

Then if (37) guarantees pass profile exponential mean square stability of (35) it follows from Theorem 4 that the ILC law is convergent. To construct stabilizing control law matrices $F_1(i)$ and $F_2(i)$, $i \in \mathbb{N}$, the conditions of Theorem 4 are employed.

Choose the candidate vector Lyapunov function as (20) with $V_1(\upsilon_{k+1}(t), r(t))) = \upsilon_{k+1}^T(t)P_1(r(t))\upsilon_{k+1}(t)$, $V_2(e_k(t), r(t)) = e_k^T(t)P_2(r(t))e_k(t)$, where P_1 and P_2 are compatibly dimensioned symmetric positive definite matrices, denoted by > 0 from this point onwards. Also the stochastic divergence operator $\mathcal{D}$ of the function (20) in this case must satisfy (24). Calculating this operator along the trajectories of the system described by(35)–(37) gives the following sufficient conditions for pass profile exponential mean square stability

$$\begin{aligned} P(i) &= \mathrm{diag}[P_1(i)\ P_2(i)] > 0, \\ A_c^T(i)\sum_{j=1}^{\nu}\pi_{ij}H_i(j)A_c(i) &- P(i) + Q(i) < 0,\ i \in \mathbb{N}, \end{aligned} \quad (38)$$

where $Q(i) = Q^T(i) > 0$, $H_i(j) = \mathrm{diag}[P_1(j)\ P_2(i)]$, $F_{1D}(i) = F_1(i)D(i)$,
$A_c(i) = \begin{bmatrix} A + BF_{1D}(i) & BF_2(i) \\ -C(i)A - C(i)BF_{1D}(i) & I - C(i)BF_2(i) \end{bmatrix}$. Setting $X(i) = P^{-1}(i)$ and introducing the additional variables $Y_1(i), Y_2(i), Z_1(i)$, routine calculations give the following coupled set of Linear Matrix Inequality (LMI) and linear matrix equalities with respect to these variables

$$\begin{gathered} \begin{bmatrix} S_{11}(i) & S_{12}(i) & S_{13}(i) \\ S_{12}^T(i) & S_{22}(i) & 0 \\ S_{13}^T(i) & 0 & S_{33}(i) \end{bmatrix} > 0, \\ X(i) > 0,\ i \in \mathbb{N}, \end{gathered} \quad (39)$$

$$D(i)X_1(i) = Z_1(i)X_1(i), \quad (40)$$

where $S_{11}(i) = \mathrm{diag}[X_1(i)\ X_2(i)]$, $S_{12}(i) = [S_{121}(i) \ldots S_{12\nu}(i)]$,
$S_{12j}(i) = \pi_{ij}^{\frac{1}{2}}\begin{bmatrix} \bar{A}(j) & BY_2(j) \\ -C(j)\bar{A}(j) & X_2(j) - C(j)BY_2(j) \end{bmatrix}$, $\bar{A}(j)$

$= AX_1(j) + BY_{1D}(j)$, $S_{13}(i) = \text{diag}[X_1(i)\ X_2(i)]$, $S_{22}(i) = \text{diag}[X_1(1)\ X_2(i)\ X_1(2)\ X_2(i) \dots X_1(\nu)\ X_2(i)]$, $S_{33}(i) = Q^{-1}(i)$, $Y_1(i) = F_1(i)Z_1(i)$, $Y_2(i) = F_2(i)X_1$, $Y_{1D}(i) = Y_1(i)D(i)$, π_{ij} is defined by (17) and the following result has been established.

Theorem 6. Consider the ILC dynamics described by (35) and (37) and suppose that the conditions of (39) and (40) are feasible and set $F_1(i) = Y_1(i)X_1^{-1}(i)$ and $F_2(i) = Y_2(i)X_2^{-1}(i)$, $i \in \mathbb{N}$. Then ILC convergence occurs.

In the case without failures the inequality (38) reduces to the 2D Lyapunov inequality that guarantees stability along the pass (Rogers et al., 2007) of discrete linear repetitive processes.

Note 3. If $D(r(t)) \equiv 0$ then the update law takes the form

$$\Delta u_{k+1}(t) = F_2(i)e_k(t) \text{ if } r(t) = i.$$

In this case $F_1(i) = 0, i \in \mathbb{N}$ and matrix $F_2(i)$ is found from the LMI's (39) with obvious simplifications.

5. NUMERICAL EXAMPLE

Consider the case when the process state, input and output matrices are

$$A = \begin{bmatrix} -0.002961 & 1 & 0 \\ -0.0008363 & -0.002961 & 0.3035 \\ 0 & 0 & 1 \end{bmatrix},$$
$$B = [\,0\ \ 0\ \ 0.1563\,]^T,$$
$$C = [0.0003718\ 0.007077\ 0.02335].$$

Suppose also that the failures are modeled as a discrete-time Markov chain $r(t)$ with two states corresponding to two possible modes. In the first mode $D(1) = I$ and in the second $D(2) = 0$. Also let $\pi_{11} = 0.95$ and $\pi_{22} = 0.05$, i.e., the measured vector $z(t)$ is lost for a short period of time. In this case the control law matrices obtained from Theorem 6 and Note 3 are

$$\begin{aligned} F_1(1) &= [-0.0096\ -0.2814\ -51.9978], \\ F_2(1) &= 922.88, \\ F_1(2) &= [0\ \ 0\ 0],\ F_2(2) = 7.3787. \end{aligned} \tag{41}$$

Suppose that the reference signal shown in Fig. 1 is used. Then Fig. 2. confirms that without failures the error (e_k) converges rapidly in k. The information failures break the monotonicity of error convergence, as confirmed in Fig. 3. However, effects will be limited in the case when the failures are fixed and feedback control law is switched using (41), see Fig. 3. a), in comparison to the use of the same $F2$ without failures Fig. 3. b).

6. CONCLUSIONS

This paper has used VLFs to characterize a physically based definition of stability for nonlinear discrete repetitive processes, a class of 2D systems with physical applications. Also new results on the practically relevant case where sensor failures occur have been developed. These results provide the basis for future research on the development of an applicable control systems theory for repetitive processes.

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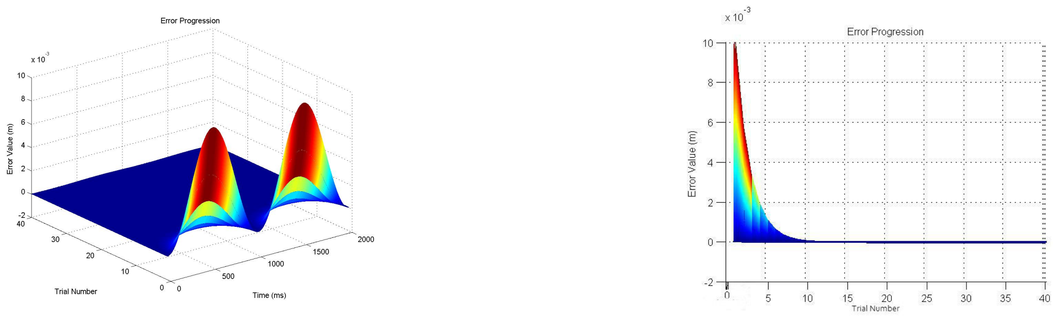
a) Reference signal

b) Output signal without failures

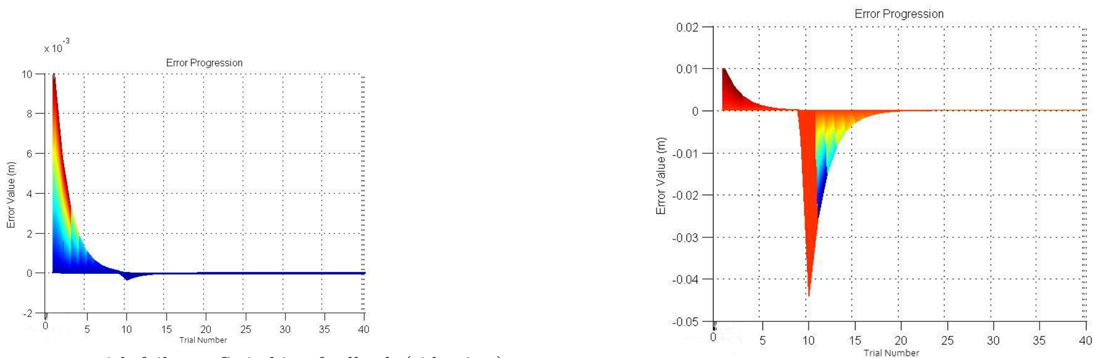
Fig. 1. Input and output signals without failures



a) Error convergence without failures

b) Error convergence without failures(side view)

Fig. 2. Error values without failures



a) Error convergence with failures. Switching feedback (side view)

b) Error convergence with failures. Nonswitching feedback (side view)

Fig. 3. Error values under failures