Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

An Evolutionary Game Theory Approach to Modeling VMI Policies

Fidel Torres * **Cesar Garcia-Diaz** ** **Naly Rakoto-Ravalontsalama** ***

* *Universidad de los Andes Bogota, Colombia (e-mail:* ftorres@uniandes.edu.co).
** *Universidad de los Andes Bogota, Colombia (e-mail:* ce.garcia392@uniandes.edu.co).
*** *Ecole des Mines de Nantes and IRCCyN Institute, France, (e-mail:* rakoto@mines-nantes.fr)

Abstract: The strategy of integration known as VMI (Vendor-Managed Inventory) allows the coordination of inventory policies between producers and buyers in supply chains. Based on a new proposed model for the implementation of VMI in a chain of two links composed of a producer and a buyer, this paper studies the evolution of individual strategies of the producer and the buyer by a formalism derived from the theory of evolutionary games. The conditions that determine the stability of evolutionarily stable strategies are derived and analyzed. Work results specify analytical conditions that favor the implementation of VMI on traditional chains without VMI.

Keywords: Supply chain management; game theory; stability analysis

1. INTRODUCTION

A full integration of the supply chain has become one of the industrys greatest dreams, thanks to the success achieved by different businesses working together with their suppliers and customers (Darwish and Odah, 2010). Initiatives like efficient customer response in the grocery industry and quick response in the garment industry (Waller et.al., 1999) are good examples of this concept. Vendor-Managed Inventory (VMI) is a supply chain initiative where the vendor decides on the appropriate inventory levels of each of the products and the appropriate inventory policies to maintain those levels. The retailer provides the vendor with access to its real-time inventory level. In this partnership program, the retailer may set certain service level and/or self-space requirements, which are then taken into consideration by the vendor. In recent years, there has been a growing interest in implementing VMI initiatives (Emigh, 1999), due to important recognition from different industrial leaders (Southard and Swenseth, 2008). This interest stems from the fact that there are benefits for the whole chain in terms of cost reduction, improved service levels and supplier performance (Choi et.al., 2004). VMI is a coordination mechanism that improves multi-firm supply chain efficiency (Waller et.al., 1999) between a supplier and its customers (Silver et.al., 1998). VMI can decrease inventory levels, increase fill rates in the supply chain (Yao et.al., 2007) and reduce lead times and inventory stock-outs (Daugherty et.al., 1999).

The models presented in this paper analyze a two-level supply chain in which external demand for a single item takes place at the purchaser. The paper proposes models with and without VMI, and establishes a cooperative VMI system by sharing demand and inventory information between agents (Lee et.al., 2000) (Cachon and Zipkin, 1999). Through a formalism based on evolutionary game theory, this paper characterizes the stability of individual strategies of the producer and the buyer and deduces the analytical conditions that favor the implementation of VMI for the two agents. The analysis allowed us to identify and characterize the evolutionary stable strategy (ESS) of the supply chain in implementing VMI. The dynamic equations show that both agents of the supply chain can adopt VMI as the preferred coordination strategy under certain parameter settings. A preliminary analysis shows that increasing the penalties for the non-adoption of VMI strategies, favors the implementation of the VMI strategy for both agents. The proposed dynamic model and stability analysis presented here serve to quantify the value of these penalties and predict the long-term behavior of individual agents based on system parameters and initial conditions. This article is divided as follows: Section 2 is a review of the literature. Section 3 describes the proposed model and develops the main results. Section 4 presents the evolutionary stability analysis of the supply chain and shows the results of four experiments using the model. Section 5 presents the main conclusions and future research avenues.

2. LITERATURE REVIEW

The first VMI models appeared in the late 1980s, when Wal-Mart, K-Mart and Procter & Gamble implemented major projects relating to supply-chain integration (Waller et.al., 1999), (Blatherwick, 1998). However, not until recently was this subject discussed in academic literature (Southard and Swenseth, 2008). To ensure proper classification of the published scientific literature on VMI, we established five categories: strategic, statistical characterization, simulation, deterministic modeling and game theoretical approaches.

Following the strategic approach, Blatherwick (1998) analyzed some of VMIs benefits and disadvantages to the agents involved in the agreements. He also showed how supply chains have evolved to become co-managed inventories. Likewise, Holmstrm (1998) studied and characterized the adaptation of SAP R/3 in a partnership relationship within the context of VMI. Later, Emigh (1999) presented VMI cases in different industrial sectors and analyzed some technological requirements necessary to ensure successful implementation. A number of papers have addressed statistical characterization of VMI models, starting with Daugherty et.al. (1999), who presented the statistical results of a survey about the implementation of automatic replenishment programs in different industries.

Several studies make use of discrete event simulation techniques. The first work on this subject was published by Waller et.al. (1999) who compared order frequency in different scenarios, facing inventory reduction through experimentation with a VMI strategy. Additionally, Disney and Towill (2002) designed a VMI system with different cost levels and proposed a simulation method to determine the optimal parameters in the chain. We also identified a set of articles using mathematical modelling approaches, starting with Cachon and Zipkin (1999) who analyzed a two-level chain with stationary stochastic demand, fixed transport times and cooperative inventory policies. The same approach was developed by Lee et.al. (2000) who modelled a chain consisting of a manufacturer and a retailer in the presence of stochastic demand and information sharing between agents, which implied reducing inventories and costs. More recently mathematical modelling works have also been developed. Yao et.al. (2007) presented an analytical model applied to supply chains of two agents with and without VMI and found inventory cost reductions. Yao and Dresner (2008) proposed a model consisting of a manufacturer and a retailer with stochastic demand and examined management practices before and after information-sharing implementation, continuous replenishment and VMI.

A recent VMI approach has incorporated game theory approaches. In this category, the work of Yu et.al. (2009a) used evolutionary game theory to analyze a strategy of evolutionary stability in supply chains with VMI. An earlier work by Yu and Huang (2009b) formulated a model of a manufacturer and multiple retailers and proposed a computational algorithm based on an analysis of a response function and a generic demand function. Additional work by Yu et.al. (2009c) analyzed the interaction between a manufacturer and its retailers to optimize its marketing strategy for a product with VMI by using a game theoretical model between agents.

3. COORDINATION MODEL BETWEEN PRODUCER AND BUYER

The supply chain we study consists of a manufacturer and a buyer implementing VMI for a single product. This problem has been studied in (Yang et.al., 2003), (Choi et.al., 2004), and (Yao et.al., 2007). This approach proposes an implicit coordination strategy between supplier and buyer, but the studied models do not include explicit synchronization and coordination mechanisms between buyer and supplier. We propose a coordination scheme where the key difference with existent approaches involves a synchronization mechanism between the buyer and manufacturer replenishment cycles.

The notation used in our model is described as follows:

Parameters: $C, c, c', H, h, P, r, d, \delta, g, g'$
Variables: $T, t, Q, q, T_s, k, L, U, \tau_s, I_s$

where

C = Setup (ordering) costs for the manufacturer (in \$/setup)
c = Setup (ordering) costs for the buyer (in \$/setup)
c' = Setup (ordering) costs for the buyer with VMI (in \$/setup)
H = Holding cost of manufacturer inventory (in \$/unit/year)
h = Holding cost of buyer inventory (in\$/unit/year)
P = Manufacturer production rate (in units/year)
r = Demand rate (in units/year)
$d = H/h$ = Manuf. and buyer inventory holding cost ratio
$\delta = r/P$ = Demand and production rate
$g = C/c$ = Setup (ordering) cost ratio
$g' = C/c'$ = Setup (ordering) cost ratio with VMI

T = Manufacturer replenishment time (in years)
t = Buyer replenishment time (in years)
Q = Total quantity manufactured over T (in units)
q = Total quantity demanded over t (in units)
$T_s = q/P$ = Manufacturing time of buyer lot size q (in years)
k = Nb of buyer shipments placed during T (integer)
U = Manufacturer uptime (in years)
L = Nb of buyer shipments placed during U (integer)
$\tau_s = U - Lt$ = Fractional manufacturer up time (in years)
I_s = Manufacturer average inventory (in units)

The production plant manufactures and distributes a single product to the buyer, who has a known deterministic annual demand rate that is the same for the manufacturer and the buyer and is denoted by r. The system is studied before and after VMI implementation and is presented in Figure 1. In this article we adopted the convention, used in (Yao et.al., 2007) that uppercase parameters are for the manufacturer and lowercase parameters are for the buyer.

Annual holding inventory costs per unit are denoted as H for the manufacturer and h for the buyer, in monetary units per unit per year. Single-order costs are denoted with C for the manufacturer, c' for the buyer with VMI and c for the buyer without VMI. Production rate is constant and denoted with P and $P > r$. The buyer replenishment time is represented by t. The manufacturer replenishment time T is kt (with k integer) and contains L buyer replenishment cycles (with L integer). The time required to produce a lot size required for the buyer (q) is denoted by T_s. The lot size of the manufacturer is $Q = kq$. The explicit synchronization mechanism between the buyer and the manufacturer consists in sending q units to the buyer from the manufacturer during the buyer replenishment period t. These periodical replenishments are planned during the manufacturer replenishment period T. In our model we explicitly consider the uptime $U = Lt + \tau_s$. This uptime is not taken into consideration in other manufacturer-buyer VMI approaches (Yang et.al., 2003), (Choi et.al., 2004), and (Yao et.al., 2007)).

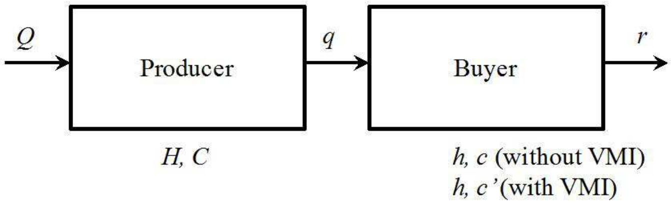

Fig. 1. The studied logistic system

The explicit replenishment coordination mechanism between the manufacturer and the buyer is represented in Figure 2. In this study we have deduced the mathematical conditions (Equations 4-12) needed to achieve the explicit manufacturer-buyer synchronization, represented with integer coordination constants k and L. In our model the replenishment cycle of the manufacturer T is exactly k buyer replenishment cycles and contains the uptime Lt + s. From a practical point of view, this mechanism makes logistical coordination between the manufacturer and the buyer much easier than in the other related VMI approaches (Yang et.al., 2003), (Dong and Chu, 2002), (Choi et.al., 2004), and (Yao et.al., 2007).

Without VMI, the manufacturer and the buyer relate to each other following a finite production rate model. Because the buyer average inventory level is driven by a simple EOQ model, his or her average inventory level is q/2. As a consequence, the buyers average total annual holding and setup cost is given by:

$$f(q) = c\ \frac{r}{q} + h\ \frac{q}{2} \tag{1}$$

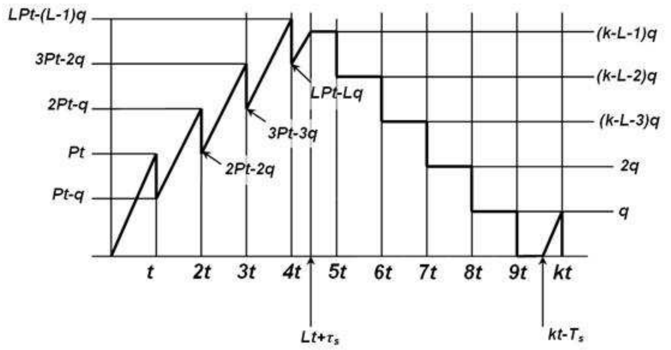

Fig. 2. Manufacturer's inventory level under VMI.

Similarly, without VMI the manufacturer is guided by a finite production rate model. The change in manufacturer inventory level over time is shown in Figure 3. Feasibility requires that $P > r$. Average inventory level can be deduced as $Q/2(1 - r/P)$, according to the economic production quantity (EPQ) model (Silver et.al., 1998). In consequence, the manufacturers average total annual holding and setup cost is:

$$F(Q) = C\ \frac{r}{Q} + h\ \frac{Q}{2}(1 - \frac{r}{P}) \tag{2}$$

It follows that with optimal order quantities q and Q for the buyer and manufacturer, the optimal total costs of the system without VMI are:

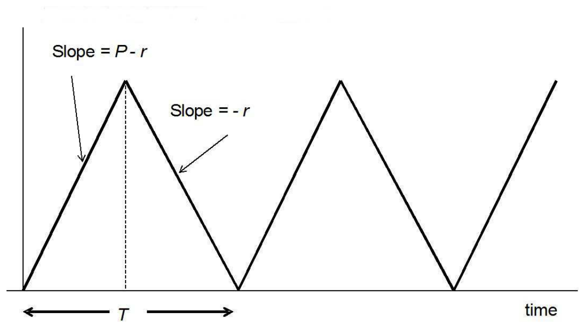

Fig. 3. Manufacturer's inventory level without VMI.

$$TC^* = \sqrt{2r}\left[\sqrt{CH(1 - \frac{r}{P})} + \sqrt{ch}\right] \tag{3}$$

Considering our manufacturer-buyer coordinated and synchronized VMI system shown in Figure 2, manufacturer and buyer replenishment times are related through an integer coordination constant called k. The synchronization scheme implies that the manufacturer sends the buyer the lot size q each replenishment time t. In this sense, manufacturer lot size Q is equal to kq. If I_s is the manufacturers average inventory, the total cost of the manufacturer-buyer coordinated VMI system is given by:

$$TC_{VMI} = c'\ \frac{r}{q} + h\ \frac{q}{2} + C\ \frac{r}{Q} + HI_s \tag{4}$$

In our VMI approach, we can calculate the area under the curve for the manufacturers inventory over his/her replenishment time $T = kt$. Dividing this value by T, we get the manufacturers average inventory (denoted by I_s), given by Equation 5.

$$I_s = \frac{q}{2}\left[k(1 - \frac{r}{p}) + 2\ \frac{r}{p} - 1\right] \tag{5}$$

Optimizing the expression in Equation 4 and relaxing the integrality condition on k, the optimal supply chain total ordering and inventory holding cost is calculated in Equation 6.

$$TC^*_{VMI} = \sqrt{2r}\left[\sqrt{k}(1 - \frac{r}{p}) + 2\ \frac{r}{p} - 1\right] \tag{6}$$

The new optimal order quantities (lot sizes) for the buyer with and without VMI are given by Equations 7 and 8, respectively:

$$q^*_{VMI} = \sqrt{\frac{2c'r}{\left[h + H\left(2\frac{r}{p} - 1\right)\right]}} \tag{7}$$

$$q^* = \sqrt{\frac{2c'r}{h}} \tag{8}$$

Starting from these results, the expressions for the optimal total cost for the buyer and the producer with and without VMI are given in Equations 9, 10, 11 and 12 respectively.

$TC_{man,VMI} =$

$$\sqrt{2CHr\left(1-\frac{r}{p}\right)}+\frac{H}{2}\left(2\frac{r}{p}-1\right)\sqrt{\frac{2c'r}{\left[h+H\left(2\frac{r}{p}-1\right)\right]}} \tag{9}$$

$$TC_{buyer,VMI} = \frac{1}{2}\sqrt{\frac{2c'r}{\left[h+H\left(2\frac{r}{p}-1\right)\right]}}\left[2h+H\left(2\frac{r}{p}-1\right)\right] \tag{10}$$

$$TC_{man} = \sqrt{2CHr\left(1-\frac{r}{p}\right)} \tag{11}$$

$$TC_{buyer} = \sqrt{2crh} \tag{12}$$

Following the analysis, two cost matrices associated with the implementation of VMI for the producer and the buyer (Yu et.al., 2009a) are proposed in Table 1. Each of the actors in the chain can take these individual decisions: implement or not VMI. According to individual decisions taken by each agent, they assume different costs in the supply chain. In the proposed cost matrices, m and n are the investment amounts made by the manufacturer and the buyer, respectively, when they adopt the VMI strategy. Parameters p_1, p_2 are the goodwill losses of the manufacturer and the buyer respectively when they violate the VMI contract.

Table 1: Cost matrices

	Buyer VMI	Buyer non-VMI
Man VMI	$TC_{man\ VMI}$ $TC_{buyer\ VMI}$	$TC_{man}+m$ $TC_{buyer}+p_2$
Man non-VMI	$TC_{man}+p_1$ $TC_{buyer}+n$	TC_{man} $TC_{buyer}+p_2$

4. EVOLUTIONARY STABILITY ANALYSIS OF THE VMI-DRIVEN SUPPLY CHAIN

Evolutionary game theory can be used to study the stability of strategies followed by buyer and producer in the game depicted in Table 1. Evolutionary game theory combines static feature of an evolutionary stable strategy (ESS) (Yu et.al., 2009a) with dynamic nature of Replicator dynamics by Taylor and Jonker (1978). The concept of ESS was initially proposed by Maynard-Smith (1973, 1974, and 1982). The definition of ESS in a single population is as follows. Given a $N \times N$ matrix A, representing the fitness function $f(r,s) = r^t As$, a state s is an ESS if and only if for all other states x, either i) $f(s,s) > f(x,s)$ or ii) $f(s,s) = f(x,s) and f(s,x) > f(x,x)$.

The idea is that an ESS state s resists to all possible mutations x because they are i) less adapted, or ii) equally adapted to the current state, but less adapted if they invade the whole population.

Taylor and Junker (1978) proposed a dynamic equation, called *Replicator Dynamics*" which reflects the dynamics and interaction among the agents involved in the game (Yu et.al., 2009a). For the game we are analyzing, the cost matrices of Table 1 should be multiplied by -1 to obtain fitness matrices A and B for producer and buyer, respectively. Assuming that α is the probability that the producer selects the VMI strategy and β is the probability that the buyer selects the VMI strategy. Replicator dynamics equations are:

$$\frac{d\alpha}{dt} = \alpha\left[U_{man,VMI}(t) - U_{man}(t)\right] \tag{13}$$

$$\frac{d\beta}{dt} = \beta\left[U_{buyer,VMI}(t) - U_{buyer}(t)\right] \tag{14}$$

Where $U_{man,VMI}(t)$ and $U_{buyer,VMI}(t)$ correspond to the expected payoff of the producer and the buyer at time t when they implement VMI; $U_{man,VMI}(t)$ is the producer average payoff at time t and $U_{man,VMI}(t)$ is the buyer average payoff at time t. From a dynamic point of view, the probability that each agent selects VMI strategy increases (decreases) if its payoff is bigger (smaller) than the average payoff in the population. Thus, the following equations are derived:

$$\begin{aligned} U_{man,VMI}(t) &= A_{1,1}\beta + A_{1,2}(1-\beta) \\ U_{man}(t) &= A_{2,1}\beta + A_{2,2}(1-\beta) \end{aligned} \tag{15}$$

$$U_{man}(t) = \alpha U_{man,VMI}(t) + (1-\alpha)U_{man}(t)$$

and similarly:

$$\begin{aligned} U_{buyer,VMI}(t) &= B_{1,1}\beta + B_{1,2}(1-\beta) \\ U_{buyer}(t) &= B_{2,1}\beta + B_{2,2}(1-\beta) \end{aligned} \tag{16}$$

$$U_{buyer}(t) = \alpha U_{buyer,VMI}(t) + (1-\alpha)U_{buyer}(t)$$

We have also the following equations:

$$\frac{d\alpha}{dt} = \alpha(1-\alpha)\left[U_{man,VMI}(t) - U_{man}(t)\right] \tag{17}$$

$$\frac{d\beta}{dt} = \beta(1-\beta)\left[U_{buyer,VMI}(t) - U_{buyer}(t)\right] \tag{18}$$

4.1 Stability analysis of the producer-buyer VMI behavior

The stable states of the Replicator dynamic equation are the Nash equilibrium (NE). They are often referred to as the evolutionary equilibriums (EE) (Ellison and Fudenburg, 2000)). When $\frac{d\alpha}{dt} = 0$ and $\frac{d\beta}{dt} = 0$, then the EE of Equations 17 and 18 are: $E_1 = (0,0), E_2 = (0,1), E_3 = (1,0), E_4 = (1,1)$ and $E_5 = (e_{5a}, e_{5b})$ where

- $e_{5a} = \left(\frac{n}{n+p_2+TC_{buyer}-TC_{buyer,VMI}}\right)$
- $e_{5b} = \left(\frac{m}{m+p_1+TC_{man}-TC_{man,VMI}}\right)$

The stability of the EE (Friedman (1991)) can be analyzed by the Jacobi matrix J, which is such that:

$$J = \begin{bmatrix} \frac{\partial}{\partial\alpha}\left(\frac{d\alpha}{dt}\right) & \frac{\partial}{\partial\beta}\left(\frac{d\alpha}{dt}\right) \\ \frac{\partial}{\partial\alpha}\left(\frac{d\beta}{dt}\right) & \frac{\partial}{\partial\beta}\left(\frac{d\beta}{dt}\right) \end{bmatrix} \tag{19}$$

with

$$\frac{\partial}{\partial \alpha}\left(\frac{d\alpha}{dt}\right) = (1-2\alpha)\left[\beta(p_1 + m + TC_{man} - TC_{man,VMI}) - m\right]$$
$$\frac{\partial}{\partial \beta}\left(\frac{d\alpha}{dt}\right) = \alpha(1-\alpha)\left[p_1 + m + TC_{man} - TC_{man,VMI}\right]$$
$$\frac{\partial}{\partial \alpha}\left(\frac{d\beta}{dt}\right) = \beta(1-\beta)\left[p_2 + n + TC_{buyer} - TC_{buyer,VMI}\right]$$
$$\frac{\partial}{\partial \beta}\left(\frac{d\beta}{dt}\right) = (1-2\beta)\left[\alpha(p_2 + n + TC_{buyer} - TC_{buyer,VMI}) - n\right]$$

The stability of the EE depends on the sign of $tr(J)$ and $det(J)$ in E_1, E_2, E_3, E_4 and E_5. The results of this analysis are represented in Table 2, which indicates that the only permanent evolutionary stable strategy (ESS) is $E_1 = (0, 0)$.

Table 2: The local stability of the EE

EE	$tr(J)$	$det(J)$	Type of stability
E_1	$-m-n<0$	$mn>0$	ESS
E_2	$n+p_1+TC_{man}$ $-TC_{man,vmi}$	$n(p_1+TC_{man}$ $-TC_{man,vmi})$	unstable or saddle
E_3	$m+p_2+TC_{buyer}$ $-TC_{buyer,vmi}$	$m(p_2+TC_{buyer}$ $-TC_{buyer,vmi})$	unstable or saddle
E_4	$-p_1-TC_{man}$ $+TC_{man,vmi}$ $-p_2-TC_{buyer}$ $+TC_{buyer,vmi}$	$(p_1+TC_{man}$ $-TC_{man,vmi})$ $(p_2+TC_{buyer}$ $-TC_{buyer,vmi})$	ESS or saddle or unstable
E_5	0	<0	saddle

The stability analysis shows that when penalties p_1 and p_2 become large, the equilibrium E_4 which is associated with producer and buyer both selecting as their preferred strategy to implement VMI, becomes an ESS. In this case, the equilibrium E_5 is located inside the square of probabilities and merges with E_1. Under these circumstances it is more favorable than when both agents support the implementation of VMI.

4.2 Experiments for evolutionary dynamics

The simulation software Dynamo (Sandholm and Dokumaci, 2007) was used to study the evolutionary dynamics of producer and buyer strategies in the supply chain. Varying the system parameters, we simulated 4 different logistics systems. With the help of the software, phase diagrams of each experiment were obtained. The selected parameter sets are shown in Table 3.

Table 3: Selected parameter sets for experiments

Parameters	Experiment 1	Experiment 2	Experiment 3	Experiment 4
C	300	300	300	300
c	100	100	100	100
c'	80	80	80	99
H	19	19	19	19
h	20	20	20	20
P	2000	2000	4000	4000
r	200	1800	320	3990
m	300	300	300	300
p_1	100	100	100	10
p_2	200	200	200	200
n	500	500	500	500

The phase diagrams obtained in each experiment are shown in Figure 4.

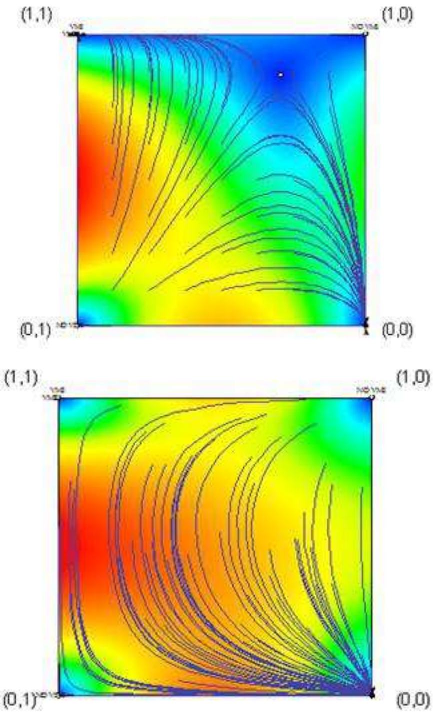

Fig. 4. Experiment 1 and Experiment 2

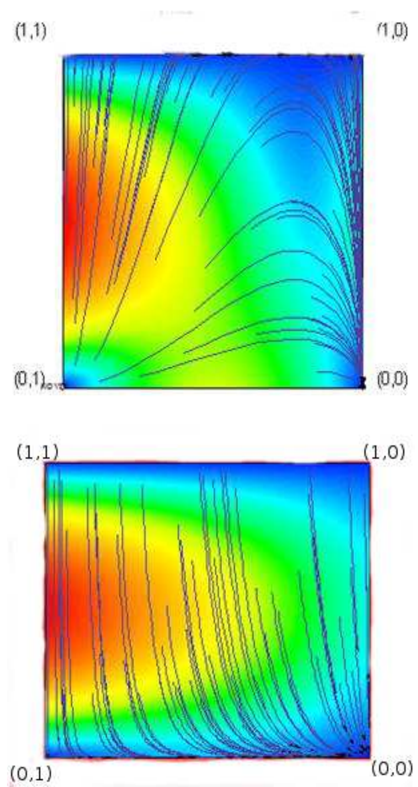

Fig. 5. Experiment 3 and Experiment 4

The logistic system presented in Experiment 1 shows a saddle equilibrium in E_5. In this case, the supply chain can evolve into stable equilibriums characterized in that either both agents implement or do not implement VMI. The convergence into one of these two ESS states E_1 or E_4 depends on the initial conditions. Experiments 2, 3 and 4 show that demand and capacity are critical parameters because they affect the stability type of each EE. These experiments also illustrate that the equilibrium E_4 which corresponds to the case where both producer and buyer are selecting to implement VMI, can easily evolve to any type of equilibrium depending of parameter settings. The equilibrium E_5 does not always exist inside the feasible square of probabilities $[0, 1]^2$. The analytical condition that ensures that E_4 is an ESS that corresponds to $tr(J) < 0$ and $det(J) > 0$, where $Tr(J)$ and $det(J)$ is the trace and the determinant of J, respectively. In consequence, the agents must carefully select the supply chain parameters to ensure that the optimal strategy for both is to implement VMI.

5. CONCLUSION

This paper presents an analysis of a supply chain between a producer and a buyer driven by VMI as coordination strategy. The analysis is based on evolutionary game theory and the study of evolutionary stability of the producer and buyer strategies according to the replicator dynamics. The analysis allowed us to identify and characterize the ESS in implementing VMI. It is shown that both agents of the supply chain can adopt VMI as the preferred coordination strategy under certain parameter settings. Obviously increasing the penalties for non-adoption of VMI strategies results in favoring the implementation of the VMI strategy for both agents. The proposed dynamic model and stability analysis presented serve to quantify the value of these penalties and predict long-term behavior of individual agents. As a future work a deeper sensitivity analysis will study the effect of all other parameters on the stability of VMI-conduced strategies. On the other hand discretizing the system evolution to obtain sequential game dynamics may also be another option.

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