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A Virtual Actuator Approach for Fault Tolerant Control of Switching LPV Systems ⋆

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Abstract: In this paper, virtual actuators are proposed as a Fault Tolerant Control (FTC) strategy for switching Linear Parameter Varying (LPV) systems subject to actuator faults. The overall solution relies on the addition of a virtual actuator block that keeps the stability and some desired performances without the need of retuning the nominal controller. It is shown that the design can be performed using polytopic techniques and Linear Matrix Inequalities (LMIs). Simulation results obtained with a four-wheeled omnidirectional mobile robot example are used to demonstrate the effectiveness of the proposed approach.

Keywords: Linear parametrically varying (LPV) methodologies, Fault Tolerant Control, Virtual Actuators, Model Reference

1. INTRODUCTION

In the last decades, the Linear Parameter Varying (LPV) paradigm has become a standard formalism in systems and control, for analysis, controller synthesis and system identification (Shamma, 2012). This class of systems is important because, by embedding the system nonlinearities in the varying parameters, gain-scheduling control of nonlinear systems can be performed using an extension of linear techniques. On the other hand, the *hybrid systems* paradigm is strongly related to LPV systems. Hybrid systems are dynamical systems that involve the interaction of continuous and discrete dynamics (Jiang et al., 2011). The study of hybrid systems is motivated by the fundamentally hybrid nature of many modern systems. As remarked by Shamma (2012), in the special case of discrete valued parameters, LPV systems constitute a specific case of hybrid dynamical system. When there are both continuous valued and discrete valued varying parameters, the resulting system is referred to as *switching LPV*. Recently, researchers have considered this class of system for improving the control system performance (Lu and Wu, 2004, Lu et al., 2006, He et al., 2010).

The objective of a Fault Tolerant Control (FTC) system (Blanke et al., 2006, Noura et al., 2009) is to maintain current performances close to desirable ones and preserve stability conditions in the presence of faults. The accommodation capability of a control system depends on many factors such as the severity of the fault, the robustness of the nominal system and the presence of mechanisms that introduce redundancy in the system components (Rodrigues et al., 2006). The existing design techniques mainly include the passive and the active approaches (Jiang and Yu, 2012, Rotondo et al., 2013). The *passive FTC techniques* are control laws that take into account the fault as a system perturbation. Thus, within certain margins, the control law has inherent fault tolerant capabilities, allowing the system to cope with the fault presence. On the other hand, the *active FTC techniques* compensate the faults either by selecting a precalculated control law or by synthesizing on-line a new control strategy. The adaptation of the control law is done by using some information about the fault so as to satisfy the control objectives with minimum performance degradation after the fault occurrence (see Zhang and Jiang (2008) or Benosman (2010) for a review). Research on hybrid system-based FTC (Yang et al., 2010) is a challenging issue for both theoretical and practical reasons, and has been recently investigated in a few papers, e.g. (Ji et al., 2003, Yang et al., 2009, Du et al., 2011, Ma and Yang, 2011, Yang et al., 2011, 2012).

The compensation for actuator faults causing severe performance deterioration of control systems has been an important and challenging research problem (Ma et al., 2013). One of the proposed solutions relies on the addition of a *virtual actuator* block that keeps the stability and some desired performances without the need of retuning the nominal controller. The main idea behind the virtual actuator method is to modify the plant with the faulty actuator adding the virtual actuator block that masks the fault, and allows the controller to see the same plant as before the fault. Initially proposed in a state-space formulation for LTI systems (Lunze and Steffen, 2006), this active FTC strategy has been extended successfully to LPV (Rotondo et al., 2014b), Takagi-Sugeno (Dziekan et al., 2011), piecewise affine (Richter et al., 2011), Lipschitz (Khosrowjerdi and Barzegary, 2013) and Hammerstein-Weiner (Richter, 2011) systems. An equivalent formulation in input-output form has been recently proposed in Blesa et al. (2014).

In this paper, the virtual actuator technique is proposed as a solution to the FTC problem for switching LPV systems

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subject to actuator faults. It is shown that the design can be performed using polytopic techniques, in particular solving a system of Linear Matrix Inequalities (LMIs), a problem for which efficient solvers are available (Löfberg, 2004, Sturm, 1999). The effectiveness of the proposed approach is shown through simulation results obtained with a four-wheeled omnidirectional mobile robot example.

The paper is structured as follows. Section 2 presents the proposed FTC strategy using virtual actuators in the context of switching LPV systems. The design using a single quadratic Lyapunov function and polytopic techniques is proposed in Section 3. The application example, a four-wheeled omnidirectional mobile robot simulator subject to actuator faults, is described in Section 4. Simulation results are shown in Section 5. Finally, the main conclusions are summarized in Section 6.

2. FTC STRATEGY USING SWITCHING LPV VIRTUAL ACTUATORS

Let us consider a continuous-time switching LPV system including actuator faults as follows:

$$\dot{x}(t) = A_\sigma(\vartheta(t))x(t) + B_{\sigma,f}(\vartheta(t),\phi(t))u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ and $u(t) \in \mathbb{R}^{n_u}$ are, respectively, the state and the input of the system, $A_\sigma(\vartheta(t))$ and $B_{\sigma,f}(\vartheta(t),\phi(t))$ are known matrices of appropriate sizes whose structure and dependence on the vector of varying parameters $\vartheta(t) \in \Theta \subset \mathbb{R}^{n_\vartheta}$ depend on the value of the switching signal $\sigma \in \{1,\ldots,S\} \subset \mathbb{N}^+$, that is assumed to be known. It is also assumed that the parameter set Θ is partitioned into a finite number of subsets $\{\Theta_i\}_{i\in\{1,\ldots,S\}}$ by means of a family of switching surfaces. The value of the switching signal σ determines which parameter subset is active, and thus determines the dynamic behavior of the system. The multiplicative actuator faults are embedded in the matrix $B_{\sigma,f}(\vartheta(t),\phi(t))$, as follows:

$$B_{\sigma,f}(\vartheta(t),\phi(t)) = B_\sigma(\vartheta(t))\,diag(\phi_1(t),\ldots,\phi_{n_u}(t)) \tag{2}$$

where $B_\sigma(\vartheta(t))$ denotes the faultless input matrix, and $\phi_i(t)$ represents the effectiveness of the i-th actuator, such that the extreme values $\phi_i = 0$ and $\phi_i = 1$ represent a total failure of the i-th actuator and the healthy i-th actuator, respectively.

In this paper, the concept of virtual actuator introduced in Lunze and Steffen (2006) is extended to switching LPV systems. The main idea of this FTC method is to reconfigure the faulty plant such that the nominal controller could still be used without need of retuning it. The plant with the faulty actuators is modified adding the virtual actuator block that masks the fault and allows the controller to see the same plant as before the fault. The virtual actuator can be either a static or dynamic block, depending on the satisfaction of the following rank condition:

$$rank\left(B_{\sigma,f}(\vartheta(t),\phi(t))\right) = rank\left(B_\sigma(\vartheta(t))\right) \tag{3}$$

If (3) holds, e.g. in the case of multiplicative actuator faults, the reconfiguration structure is static and can be expressed as:

$$u(t) = N_{\sigma,v}(\vartheta(t),\phi(t))\,u_c(t) \tag{4}$$

where $u_c(t)$ is the controller output and:

$$N_{\sigma,v}(\vartheta(t),\phi(t)) = B^{\dagger}_{\sigma,f}(\vartheta(t),\phi(t))\,B_\sigma(\vartheta(t)) \tag{5}$$

Cases where (3) is not satisfied should be described through values of the matrix $B^*_\sigma(\vartheta(t))$ such that the following condition holds [2]:

$$B^*_\sigma(\vartheta(t)) = B_{\sigma,f}(\vartheta(t),\phi(t))\,N_{\sigma,v}(\vartheta(t),\phi(t)) \tag{6}$$

In such cases, the reconfiguration structure is expressed by:

$$u(t) = N_{\sigma,v}(\vartheta(t),\phi(t))\left(u_c(t) - M_{\sigma,v}(\vartheta(t))\,x_v(t)\right) \tag{7}$$

where $M_{\sigma,v}(\vartheta(t))$ is the gain of the switching LPV virtual actuator, while the virtual actuator state $x_v(t)$ is calculated as:

$$\begin{aligned}\dot{x}_v(t) = {} & \left(A_\sigma(\vartheta(t)) + B^*_\sigma(\vartheta(t))M_{\sigma,v}(\vartheta(t))\right)x_v(t) \\ & + \left(B_\sigma(\vartheta(t)) - B^*_\sigma(\vartheta(t))\right)u_c(t)\end{aligned} \tag{8}$$

When the actuator faults appear, the switching LPV virtual actuator reconstructs the vector $u(t)$ from the output of the nominal controller $u_c(t)$, taking into account the fault occurrence. The faulty plant and the switching LPV virtual actuator are called the *reconfigured switching LPV plant*, which is connected to the nominal switching LPV controller. If the reconfigured switching LPV plant behaves like the nominal plant, the loop consisting of the reconfigured plant and the switching LPV controller behaves like the nominal closed-loop system.

The switching LPV system (1) is controlled by a switching LPV state-feedback control law:

$$u_c(t) = K_\sigma(\vartheta(t))\,x(t) \tag{9}$$

where $K_\sigma(\vartheta(t))$ is the controller gain. Under faulty conditions, the controller (9) is slightly modified, as follows:

$$u_c(t) = K_\sigma(\vartheta(t))\left(x(t) + x_v(t)\right) \tag{10}$$

In the following, it is shown that thanks to the introduction of the virtual actuator block, the augmented system can be brought to a block-triangular form.

Theorem 1. Consider the augmented system made up by the faulty system (1), the virtual actuator (7)-(8) and the control law (10) [3]:

$$\begin{pmatrix}\dot{x}\\ \dot{x}_v\end{pmatrix} = \begin{pmatrix} A_\sigma + B^*_\sigma K_\sigma & B^*_\sigma(K_\sigma - M_{\sigma,v}) \\ (B_\sigma - B^*_\sigma)K_\sigma & A_\sigma + B^*_\sigma M_{\sigma,v} + (B_\sigma - B^*_\sigma)K_\sigma \end{pmatrix}\begin{pmatrix}x\\ x_v\end{pmatrix} \tag{11}$$

Then, there exists a similarity transformation such that the state matrix of the augmented system in the new state variables is block-triangular, as follows:

$$A_{aug}(\vartheta(t)) = \begin{pmatrix} A_\sigma + B_\sigma K_\sigma & 0 \\ (B_\sigma - B^*_\sigma)K_\sigma & A_\sigma + B^*_\sigma M_{\sigma,v}\end{pmatrix} \tag{12}$$

Proof: The proof is straightforward, and comes from introducing the new state variable $x_1(t) \triangleq x(t) + x_v(t)$ and considering the state $\left(x_1(t)\ \ x_v(t)\right)^T$.

Looking at (12), it can be seen that the state $x_1(t)$ is affected by $K_\sigma(\vartheta(t))$ through the matrix $A_\sigma(\vartheta(t)) + B_\sigma(\vartheta(t))K_\sigma(\vartheta(t))$, while the state $x_v(t)$ is affected by $M_{\sigma,v}(\vartheta(t))$ through the matrix $A_\sigma(\vartheta(t)) + B^*_\sigma(\vartheta(t))M_{\sigma,v}(\vartheta(t))$. Hence, the switching LPV controller and the switching LPV virtual actuator can be designed independently.

3. DESIGN USING LINEAR MATRIX INEQUALITIES

The design problem to be solved consists in finding a matrix $K_\sigma(\vartheta(t))$ such that:

$$\dot{x}(t) = \left(A_\sigma(\vartheta(t)) + B_\sigma(\vartheta(t))K_\sigma(\vartheta(t))\right)x(t) \tag{13}$$

[2] Notice that the matrix $B^*_\sigma(\vartheta(t))$ does not depend on $\phi(t)$ because the matrix $N_{\sigma v}(\vartheta(t),\phi(t))$ eliminates the effects of actuator partial faults.

[3] The dependence of the matrices A_σ, B_σ, B^*_σ, K_σ and $M_{\sigma,v}$ on the varying parameter vector $\vartheta(t)$ has been omitted for lack of space.

is stable with poles in some desired region of the complex plane [4].

In this paper, both stability and pole clustering are analyzed within the quadratic Lyapunov framework, where the specifications are assured by the use of a single quadratic Lyapunov function. Despite the introduction of conservativeness with respect to other existing approaches, where the Lyapunov function is allowed to be parameter-varying, the quadratic approach has undeniable advantages in terms of computational complexity.

The switching LPV system (13) is quadratically stable if there exists $X_S = X_S^T > 0$ such that (He et al., 2010):

$$\left(A_\sigma(\vartheta)+B_\sigma(\vartheta)K_\sigma(\vartheta)\right)X_S + X_S\left(A_\sigma^T(\vartheta)+K_\sigma^T(\vartheta)B_\sigma^T(\vartheta)\right) < 0 \tag{14}$$

$\forall\sigma \in \{1,\dots,S\}$ and $\forall\vartheta\in\Theta$. On the other hand, given a subset $\mathscr{D}$ of the complex plane, defined by:

$$\mathscr{D} = \left\{z\in\mathbb{C} : \alpha + z\beta + \bar{z}\beta^T < 0\right\} \tag{15}$$

with $\alpha = \alpha^T \in \mathbb{R}^{m\times m}$ and $\beta \in \mathbb{R}^{m\times m}$, the switching LPV system (13) has its poles in $\mathscr{D}$ if there exists $X_{\mathscr{D}} = X_{\mathscr{D}}^T > 0$ such that:

$$\begin{aligned}\left[\alpha_{kl}X_{\mathscr{D}} + \beta_{kl}\left(A_\sigma(\vartheta)+B_\sigma(\vartheta)K_\sigma(\vartheta)\right)X_{\mathscr{D}}\right.\\ \left.+\beta_{lk}X_{\mathscr{D}}\left(A_\sigma^T(\vartheta)+K_\sigma^T(\vartheta)B_\sigma^T(\vartheta)\right)\right]_{k,l\in[1,m]} < 0\end{aligned} \tag{16}$$

$\forall\sigma \in \{1,\dots,S\}$ and $\forall\vartheta\in\Theta$, where α_{kl} and β_{kl} denote the generic entry of α and β, respectively. The main difficulty with using (14) and (16) is that they impose an infinite number of constraints. In order to reduce this number to finite, a polytopic approximation of (13) is considered, as follows:

$$A_\sigma(\vartheta(t)) = \begin{cases} \sum_{i=1}^{N_1}\gamma_i^{(1)}(\vartheta(t))A_i^{(1)},\ \gamma_i^{(1)}(\vartheta)\geq 0,\ \sum_{i=1}^{N_1}\gamma_i^{(1)}(\vartheta)=1\ if\ \sigma=1\\ \vdots\\ \sum_{i=1}^{N_s}\gamma_i^{(s)}(\vartheta(t))A_i^{(s)},\ \gamma_i^{(s)}(\vartheta)\geq 0,\ \sum_{i=1}^{N_s}\gamma_i^{(s)}(\vartheta)=1\ if\ \sigma=s\\ \vdots\\ \sum_{i=1}^{N_S}\gamma_i^{(S)}(\vartheta(t))A_i^{(S)},\ \gamma_i^{(S)}(\vartheta)\geq 0,\ \sum_{i=1}^{N_S}\gamma_i^{(S)}(\vartheta)=1\ if\ \sigma=S \end{cases} \tag{17}$$

$$B_\sigma(\vartheta(t)) = \begin{cases} \sum_{w=1}^{W_1}\delta_w^{(1)}(\vartheta(t))B_w^{(1)},\ \delta_w^{(1)}(\vartheta)\geq 0,\ \sum_{w=1}^{W_1}\delta_w^{(1)}(\vartheta)=1\ if\ \sigma=1\\ \vdots\\ \sum_{w=1}^{W_s}\delta_w^{(s)}(\vartheta(t))B_w^{(s)},\ \delta_w^{(s)}(\vartheta)\geq 0,\ \sum_{w=1}^{W_s}\delta_w^{(s)}(\vartheta)=1\ if\ \sigma=s\\ \vdots\\ \sum_{w=1}^{W_S}\delta_w^{(S)}(\vartheta(t))B_w^{(S)},\ \delta_w^{(S)}(\vartheta)\geq 0,\ \sum_{w=1}^{W_S}\delta_w^{(S)}(\vartheta)=1\ if\ \sigma=S \end{cases} \tag{18}$$

$$K_\sigma(\vartheta(t)) = \begin{cases} \sum_{i=1}^{N_1}\gamma_i^{(1)}(\vartheta(t))K_i^{(1)}, & if\ \sigma=1\\ \vdots\\ \sum_{i=1}^{N_s}\gamma_i^{(s)}(\vartheta(t))K_i^{(s)}, & if\ \sigma=s\\ \vdots\\ \sum_{i=1}^{N_S}\gamma_i^{(S)}(\vartheta(t))K_i^{(S)}, & if\ \sigma=S \end{cases} \tag{19}$$

[4] Following Ghersin and Sanchez-Peña (2002), and with a little abuse of language, the poles of an LPV system are defined as the set of all the poles of the LTI systems obtained by freezing $\vartheta(t)$ to all its possible values $\vartheta\in\Theta$.

The matrix $K_\sigma(\vartheta(t))$, with vertex gains (19) assures that the switching LPV system (13) with state and input matrix as in (17) and (18), respectively, is quadratically stable and has its poles in $\mathscr{D}$ if there exist $X = X^T > 0$ and $\Gamma_i^{(s)}$, $i=1,\dots,N_s$, $s=1,\dots,S$ such that:

$$\left(A_i^{(s)}X + B_w^{(s)}\Gamma_i^{(s)}\right) + \left(A_i^{(s)}X + B_w^{(s)}\Gamma_i^{(s)}\right)^T < 0 \tag{20}$$

$$\left[\alpha_{kl}X + \beta_{kl}\left(A_i^{(s)}X + B_w^{(s)}\Gamma_i^{(s)}\right) + \beta_{lk}\left(A_i^{(s)}X + B_w^{(s)}\Gamma_i^{(s)}\right)^T\right]_{k,l\in[1,m]} < 0 \tag{21}$$

with $i=1,\dots,N_s$, $w=1,\dots,W_s$ and $s=1,\dots,S$. Then, the vertex gains can be easily obtained as:

$$K_i^{(s)} = \Gamma_i^{(s)}X^{-1} \tag{22}$$

The proposed design conditions using LMIs can be applied both to the case of controller design and to the case of virtual actuator design, by making the changes $B_\sigma(\vartheta(t)) \rightarrow B_\sigma^*(\vartheta(t))$ and $K_\sigma(\vartheta(t)) \rightarrow M_{\sigma,v}(\vartheta(t))$.

4. APPLICATION EXAMPLE

The application example used in this paper is a four wheeled omnidirectional robot in simulation. Its dynamic model, including actuator faults, is given by the following set of differential equations (Oliveira et al., 2009):

$$\dot{x} = v_x \tag{23}$$

$$\begin{aligned}\dot{v}_x = &\left(A_{11}c_\theta^2 + A_{22}s_\theta^2\right)v_x + \left[(A_{11}-A_{22})s_\theta c_\theta - \omega\right]v_y\\ &+K_{11}c_\theta sign(v_x c_\theta + v_y s_\theta) - B_{21}s_\theta(\phi_0 u_0 + f_0)\\ &-K_{22}s_\theta sign(-v_x s_\theta + v_y c_\theta) + B_{12}c_\theta(\phi_1 u_1 + f_1)\\ &-B_{23}s_\theta(\phi_2 u_2 + f_2) + B_{14}c_\theta(\phi_3 u_3 + f_3)\end{aligned} \tag{24}$$

$$\dot{y} = v_y \tag{25}$$

$$\begin{aligned}\dot{v}_y = &\left[(A_{11}-A_{22})s_\theta c_\theta + \omega\right]v_x + \left(A_{11}s_\theta^2 + A_{22}c_\theta^2\right)v_y\\ &+K_{11}s_\theta sign(v_x c_\theta + v_y s_\theta) + B_{21}c_\theta(\phi_0 u_0 + f_0)\\ &+K_{22}c_\theta sign(-v_x s_\theta + v_y c_\theta) + B_{12}s_\theta(\phi_1 u_1 + f_1)\\ &+B_{23}c_\theta(\phi_2 u_2 + f_2) + B_{14}s_\theta(\phi_3 u_3 + f_3)\end{aligned} \tag{26}$$

$$\dot{\theta} = \omega \tag{27}$$

$$\begin{aligned}\dot{\omega} = &A_{33}\omega + B_{31}(\phi_0 u_0 + f_0) + B_{32}(\phi_1 u_1 + f_1)\\ &+K_{33}sign(\omega) + B_{33}(\phi_2 u_2 + f_2) + B_{34}(\phi_3 u_3 + f_3)\end{aligned} \tag{28}$$

where (x,y) is the robot position, θ is the angle with respect to the defined front of robot ($s_\theta \triangleq \sin\theta$ and $c_\theta \triangleq \cos\theta$), v_x, v_y and ω are the corresponding linear/angular velocities, and u_0, u_1, u_2 and u_3 the motor voltage applied to the wheel 1, 2, 3 and 4, respectively. The additive actuator fault in the i-th motor is denoted by f_i, while ϕ_i denotes the multiplicative fault. The values used for the coefficients A_{ii}, B_{ij}, K_{ii} are listed in Table 1.

Table 1. Values of the coefficients A_{ii}, B_{ij}, K_{ii}

Coefficient	Value	Coefficient	Value
A_{11}	-3.3605	B_{31}	3.6079
A_{22}	-3.4368	B_{32}	3.6079
A_{33}	-5.7363	B_{33}	3.6079
B_{12}	-0.3950	B_{34}	3.6079
B_{14}	0.3950	K_{11}	-0.8008
B_{21}	0.3950	K_{22}	-0.9486
B_{23}	-0.3950	K_{33}	-6.0746

By introducing the following reference model:

$$\dot{x}_r = v_x^r \tag{29}$$

$$\begin{aligned}\dot{v}_x^r =& \left(A_{11}c_\theta^2 + A_{22}s_\theta^2\right) v_x^r + \left[(A_{11}-A_{22}) s_\theta c_\theta - \omega\right] v_y^r \\ &+ K_{11} c_\theta sign\left(v_x c_\theta + v_y s_\theta\right) - B_{21} s_\theta \left(\hat{\phi}_0 u_0^r + \hat{f}_0\right) \\ &- K_{22} s_\theta sign\left(-v_x s_\theta + v_y c_\theta\right) + B_{12} c_\theta \left(\hat{\phi}_1 u_1^r + \hat{f}_1\right) \\ &- B_{23} s_\theta \left(\hat{\phi}_2 u_2^r + \hat{f}_2\right) + B_{14}\left(\hat{\phi}_3 u_3^r + \hat{f}_3\right)\end{aligned} \tag{30}$$

$$\dot{x}_y = v_y^r \tag{31}$$

$$\begin{aligned}\dot{y}_r =& \left[(A_{11}-A_{22}) s_\theta c_\theta + \omega\right] v_x^r + \left(A_{11}s_\theta^2 + A_{22}c_\theta^2\right) v_y^r \\ &+ K_{11} s_\theta sign\left(v_x c_\theta + v_y s_\theta\right) + B_{21} c_\theta \left(\hat{\phi}_0 u_0^r + \hat{f}_0\right) \\ &+ K_{22} c_\theta sign\left(-v_x s_\theta + v_y c_\theta\right) + B_{12} s_\theta \left(\hat{\phi}_1 u_1^r + \hat{f}_1\right) \\ &+ B_{23} c_\theta \left(\hat{\phi}_2 u_2^r + \hat{f}_2\right) + B_{14} s_\theta \left(\hat{\phi}_3 u_3^r + \hat{f}_3\right)\end{aligned} \tag{32}$$

$$\dot{\theta}_r = \omega_r \tag{33}$$

$$\begin{aligned}\dot{\omega}_r =& A_{33}\omega_r + B_{31}\left(\hat{\phi}_0 u_0^r + \hat{f}_0\right) + B_{32}\left(\hat{\phi}_1 u_1^r + \hat{f}_1\right) \\ &+ K_{33} sign(\omega) + B_{33}\left(\hat{\phi}_2 u_2^r + \hat{f}_2\right) + B_{34}\left(\hat{\phi}_3 u_3^r + \hat{f}_3\right)\end{aligned} \tag{34}$$

where (x_r, y_r) is the reference vehicle position, θ_r is its angle, v_x^r, v_y^r and ω_r are the corresponding linear/angular velocities and u_0^r, u_1^r, u_2^r, u_3^r are the reference inputs (feedforward actions), then, if the tracking errors $e_1 \triangleq x_r - x$, $e_2 \triangleq v_x^r - v_x$, $e_3 \triangleq y_r - y$, $e_4 \triangleq v_y^r - v_y$, $e_5 \triangleq \theta_r - \theta$, $e_6 \triangleq \omega_r - \omega$ and the new inputs $\Delta u_i \triangleq u_i^r - u_i$, $i = 0,1,2,3$, are defined, under the assumption of perfect fault estimation ($\hat{\phi}_i = \phi_i$ and $\hat{f}_i = f_i$ $\forall i = 0,1,2,3$), the error model for the faulty four wheeled omnidirectional mobile robot can be obtained from (23)-(34) and be brought to a quasi-LPV representation, as follows:

$$\begin{aligned}\begin{pmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\\ \dot{e}_5\\ \dot{e}_6\end{pmatrix} =& \begin{pmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & \vartheta_1 & 0 & \vartheta_2 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & \vartheta_3 & 0 & A_{11}+A_{22}-\vartheta_1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & A_{33}\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\\ e_4\\ e_5\\ e_6\end{pmatrix} \\ &+ \begin{pmatrix}0 & 0 & 0 & 0\\ -B_{21}\vartheta_4 & B_{12}\vartheta_5 & -B_{23}\vartheta_4 & B_{14}\vartheta_5\\ 0 & 0 & 0 & 0\\ B_{21}\vartheta_5 & B_{12}\vartheta_4 & B_{23}\vartheta_5 & B_{14}\vartheta_4\\ 0 & 0 & 0 & 0\\ B_{31} & B_{32} & B_{33} & B_{34}\end{pmatrix}\begin{pmatrix}\phi_0 & 0 & 0 & 0\\ 0 & \phi_1 & 0 & 0\\ 0 & 0 & \phi_2 & 0\\ 0 & 0 & 0 & \phi_3\end{pmatrix}\begin{pmatrix}\Delta u_0\\ \Delta u_1\\ \Delta u_2\\ \Delta u_3\end{pmatrix}\end{aligned} \tag{35}$$

where the vector of varying parameters is:

$$\vartheta(t) = \begin{pmatrix}\vartheta_1(t)\\ \vartheta_2(t)\\ \vartheta_3(t)\\ \vartheta_4(t)\\ \vartheta_5(t)\end{pmatrix} = \begin{pmatrix}A_{11}\cos^2\theta + A_{22}\sin^2\theta\\ (A_{11}-A_{22})\sin\theta\cos\theta - \omega\\ (A_{11}-A_{22})\sin\theta\cos\theta + \omega\\ \sin\theta\\ \cos\theta\end{pmatrix}$$

As shown in Rotondo et al. (2014a), in order to find a polytopic controller for (35), it is useful to split the subset of the varying parameter space generated by ϑ_4 and ϑ_5 in more regions, such that (35) becomes a switching LPV system. In particular, in this work, the quadrants have been considered as regions, with $\theta = k\pi/2$, $k \in \mathbb{Z}$ being the switching condition, such that:

$$\sigma = \begin{cases}1 & if \quad \cos\theta \geq 0 \; AND \; \sin\theta \geq 0\\ 2 & if \quad \cos\theta \geq 0 \; AND \; \sin\theta < 0\\ 3 & if \quad \cos\theta < 0 \; AND \; \sin\theta < 0\\ 4 & if \quad \cos\theta < 0 \; AND \; \sin\theta \geq 0\end{cases}$$

To make the robot tracking a desired trajectory, proper values of u_0^r, u_1^r, u_2^r and u_3^r should be fed to the reference model, such that its state equals the one corresponding to the desired trajectory, chosen to be circular, as follows:

$$x_r(t) = \rho\cos\left(\theta_r(t)\right) \tag{36}$$

$$y_r(t) = \rho\sin\left(\theta_r(t)\right) \tag{37}$$

$$\theta_r(t) = 2\pi t/T \tag{38}$$

where ρ is the circle radius and T is the desired revolution period around the circle center.

Adapting the reference input calculation made in Rotondo et al. (2014a) to the faulty case, the following is obtained:

$$\begin{pmatrix}u_0^r(t)\\ u_1^r(t)\\ u_2^r(t)\\ u_3^r(t)\end{pmatrix} = A_{ref}^{\dagger}(t) B_{ref}(t) \tag{39}$$

where $\dagger$ denotes the pseudo-inverse and:

$$A_{ref}(t) = \begin{pmatrix}-B_{21}s_\theta\hat{\phi}_0 & B_{12}c_\theta\hat{\phi}_1 & -B_{23}s_\theta\hat{\phi}_2 & B_{14}c_\theta\hat{\phi}_3\\ B_{21}c_\theta\hat{\phi}_0 & B_{12}s_\theta\hat{\phi}_1 & B_{23}c_\theta\hat{\phi}_2 & B_{14}s_\theta\hat{\phi}_3\\ B_{31}\hat{\phi}_0 & B_{32}\hat{\phi}_1 & B_{33}\hat{\phi}_2 & B_{34}\hat{\phi}_3\end{pmatrix} \tag{40}$$

$$B_{ref}(t) = \left(\; \beta_{ref1}(t) \;\; \beta_{ref2}(t) \;\; \beta_{ref3}(t) \;\right)^T \tag{41}$$

$$\begin{aligned}\beta_{ref1}(t) =& \rho\frac{2\pi}{T}\left(\sin\frac{2\pi t}{T}\vartheta_1(t) - \cos\frac{2\pi t}{T}\left(\vartheta_2(t) + \frac{2\pi}{T}\right)\right)\\ &- K_{11}\vartheta_5(t) sign\left(v_x\vartheta_5(t) + v_y\vartheta_4(t)\right)\\ &- K_{22}\vartheta_4(t) sign\left(v_y\vartheta_5(t) - v_x\vartheta_4(t)\right)\\ &+ B_{21}s_\theta\hat{f}_0 - B_{12}c_\theta\hat{f}_1 + B_{23}s_\theta\hat{f}_2 - B_{14}c_\theta\hat{f}_3\end{aligned}$$

$$\begin{aligned}\beta_{ref2}(t) =& \rho\frac{2\pi}{T}\left(\sin\frac{2\pi t}{T}\left(\vartheta_3(t) - \frac{2\pi}{T}\right) - \cos\frac{2\pi t}{T}\vartheta_1(t)\right)\\ &- K_{11}\vartheta_4(t) sign\left(v_x\vartheta_5(t) + v_y\vartheta_4(t)\right)\\ &- K_{22}\vartheta_5(t) sign\left(v_y\vartheta_5(t) - v_x\vartheta_4(t)\right)\\ &- B_{21}c_\theta\hat{f}_0 - B_{12}s_\theta\hat{f}_1 - B_{23}c_\theta\hat{f}_2 - B_{14}s_\theta\hat{f}_3\end{aligned}$$

$$\begin{aligned}\beta_{ref3}(t) =& -A_{33}\frac{2\pi}{T} - K_{33} sign\left(\omega(t)\right)\\ &- B_{31}\hat{f}_0 - B_{32}\hat{f}_1 - B_{33}\hat{f}_2 - B_{34}\hat{f}_3\end{aligned}$$

5. SIMULATION RESULTS

The overall polytopic approximation (17)-(18) of the four wheeled omnidirectional mobile robot quasi-LPV model (35) has been obtained by considering:

$$\vartheta_1 \in \left[\underline{\vartheta}_1, \overline{\vartheta}_1\right] = \left[\min\left(A_{11}, A_{22}\right), \max\left(A_{11}, A_{22}\right)\right]$$

$$\vartheta_2 \in \left[\underline{\vartheta}_2, \overline{\vartheta}_2\right] = \left[\min\left((A_{11}-A_{22})s_\theta c_\theta\right) - \overline{\omega}, \max\left((A_{11}-A_{22})s_\theta c_\theta\right) - \underline{\omega}\right]$$

$$\vartheta_3 \in \left[\underline{\vartheta}_3, \overline{\vartheta}_3\right] = \left[\min\left((A_{11}-A_{22})s_\theta c_\theta\right) + \underline{\omega}, \max\left((A_{11}-A_{22})s_\theta c_\theta\right) + \overline{\omega}\right]$$

with:

$$\underline{\omega} = -\overline{\omega} = \frac{\left(B_{31}u_0^{\max} + B_{32}u_1^{\max} + B_{33}u_2^{\max} + B_{34}u_3^{\max} + K_{33}\right)}{A_{33}}$$

where $u_i^{max} = 12V$, $i = 0, \ldots, 3$ denotes the maximum input voltage that can be applied to the i^{th} motor, that is assumed to be limited by symmetric constant saturation limits, $u_i \in [-u_i^{max}, u_i^{max}]$.

The controller and the four virtual actuators (one for each wheel) have been designed using (20) and (21), to assure stability [5] and pole clustering in:

$$\mathscr{D} = \{z \in \mathbb{C} : \mathrm{Re}(z) < -0.1\}$$

The results shown in this paper refer to a simulation which lasts $30s$, where the four wheeled mobile robot is driven from the initial state:

$$\left(x(0), v_x(0), y(0), v_y(0), \theta(0), \omega(0)\right)^T = 0_{6\times 1}$$

[5] In the case of quasi-LPV systems obtained from a nonlinear system, the closed loop system could be unstable for some operating conditions despite the feasibility of the design conditions. A rigorous analysis of the stability should also take into account the region of attraction estimates as in Bruzelius et al. (2003).

to the desired trajectory, defined as in (36)-(38) with $\rho = 2$ and $T = 20\,s$. The desired trajectory has been generated by the reference model (29)-(34) using the reference inputs calculated as (39)-(41), starting from the initial reference state:

$$(x_r(0), v_x^r(0), y_r(0), v_y^r(0), \theta_r(0), \omega_r(0))^T = (\rho, 0, 0, 2\pi\rho/T, 0, 2\pi/T)^T$$

The fault scenario considered in this paper is a total loss of the first wheel motor starting from time $t = 15\,s$:

$$\phi_0(t) = \begin{cases} 1 & if\ t < 15\,s \\ 0 & if\ t \geq 15\,s \end{cases} \tag{42}$$

It is assumed that, in the simulation where the proposed FTC is applied (referred to as *with FTC*), the virtual actuator is activated at time $t = 20\,s$.

Fig. 1 shows the tracking of the desired circular trajectory in the $(x - y)$ plane. It can be seen that, if no faults occur, the robot trajectory (blue line) reaches asymptotically the reference (black line). On the other hand, under fault occurrence, the robot trajectory deviates from the desired one in the case without FTC (green line). The activation of the proposed FTC strategy allows to recover the asymptotic stability of the error system (red line). Thanks to the introduction of the virtual actuator block, all the tracking errors go to zero, as depicted in Fig. 2. Finally, the effect of the proposed FTC strategy on the control inputs is shown in Fig. 3.

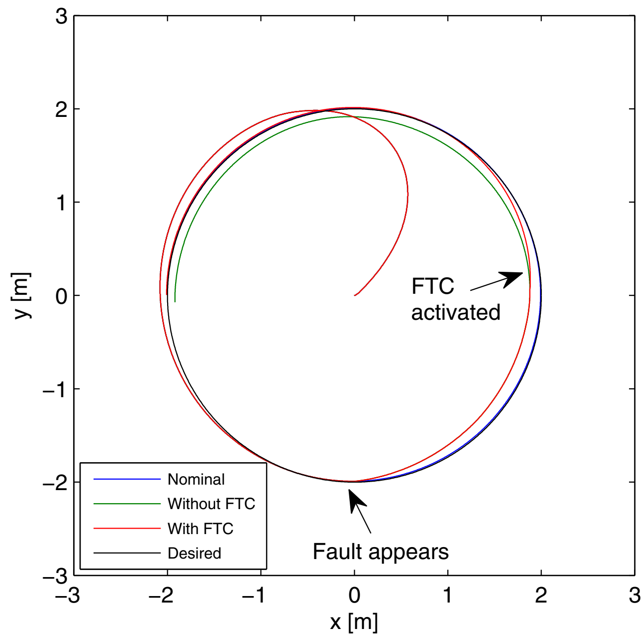

Fig. 1. Tracking of the desired circular trajectory.

6. CONCLUSIONS

This paper has proposed an FTC strategy using *switching LPV virtual actuators* for switching LPV systems. This FTC method adapts the faulty plant to the nominal switching LPV controller instead of adapting the switching LPV controller to the faulty plant. In this way, the faulty plant together with the switching LPV virtual actuator block allows the switching LPV controller to see the same plant as before the fault. The addition of the

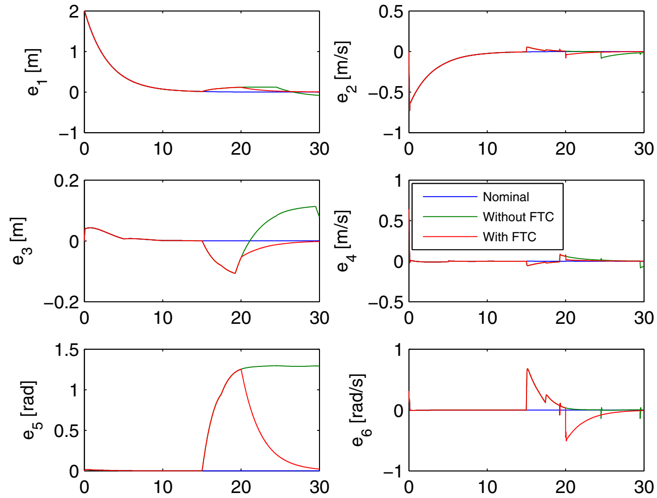

Fig. 2. Tracking errors.

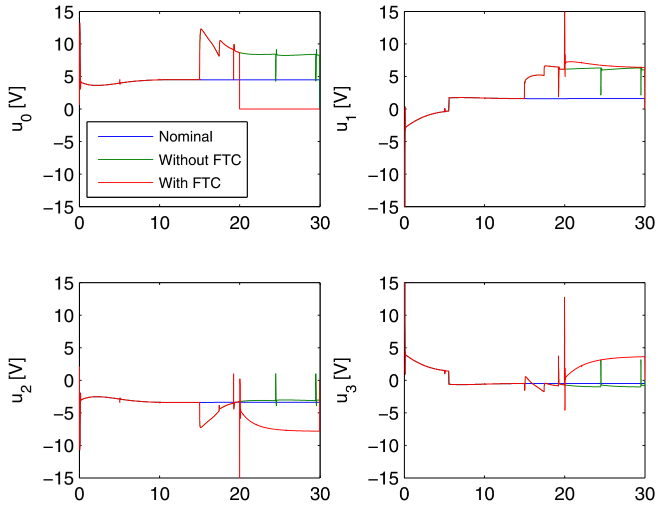

Fig. 3. Control inputs.

virtual actuator block keeps the stability and some desired performances under fault occurrence.

The overall loop consists of the nominal switching LPV controller and the switching LPV virtual actuators. Both are designed using polytopic techniques, solving a system of LMIs, so as to achieve stability and pole clustering in a desired region. Both specifications have been analyzed within the quadratic Lyapunov framework, through the use of a single quadratic Lyapunov function.

The potential and performances of the proposed approach have been demonstrated in an illustrative application to a four-wheeled omni-directional mobile robot simulator subject to actuator faults.

REFERENCES

M. Benosman. A survey of some recent results on nonlinear fault tolerant control. *Mathematical Problems in Engineering*, pages 1–25, 2010.

M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. *Diagnosis and Fault-Tolerant Control*. Springer-Verlag Berlin Heidelberg, 2006.

J. Blesa, D. Rotondo, V. Puig, and F. Nejjari. FDI and FTC of wind turbines using the interval observer approach and virtual actuators/sensors. *Control Engineering Practice*, 24: 138–155, 2014.

F. Bruzelius, S. Pettersson, and C. Breitholtz. Region of attraction estimates for LPV-gain scheduled control systems. In *Proceedings of the 7th European Control Conference*, 2003.

D. Du, B. Jiang, and P. Shi. Active fault-tolerant control for switched systems with time delay. *International Journal of Adaptive Control and Signal Processing*, 25:466–480, 2011.

Ł. Dziekan, M. Witczak, and J. Korbicz. Active fault-tolerant control design for Takagi-Sugeno fuzzy systems. *Bulletin of the Polish Academy of Sciences*, 59(1):93–102, 2011.

A. Ghersin and R. Sanchez-Peña. LPV control of a 6-DOF vehicle. *IEEE Transactions on Control Systems Technology*, 10(6):883–887, 2002.

X. He, G. M. Dimirovski, and J. Zhao. Control of switched LPV systems using common Lyapunov function method and F-16 aircraft application. In *Proceedings of the 2010 IEEE International Conference on Systems, Man and Cybernetics*, pages 386–392, 2010.

M. Ji, Z. Zhang, G. Biswas, and N. Sarkar. Hybrid fault adaptive control of a wheeled mobile robot. *IEEE/ASME Transactions on Mechatronics*, 8(2):226–233, 2003.

B. Jiang, H. Yang, and V. Cocquempot. Results and perspectives on fault tolerant control for a class of hybrid systems. *International Journal of Control*, 84(2):396–411, 2011.

J. Jiang and X. Yu. Fault-tolerant control systems: a comparative study between active and passive approaches. *Annual Reviews in Control*, 36(1):60–72, 2012.

M. J. Khosrowjerdi and S. Barzegary. Fault tolerant control using virtual actuator for continuous-time Lipschitz nonlinear systems. *International Journal of Robust and Nonlinear Control*, 2013.

J. Löfberg. YALMIP: A toolbox for modeling and optimization in MATLAB. In *Proceedings of the CACSD Conference*, 2004.

B. Lu and F. Wu. Switching LPV control designs using multiple parameter-dependent Lyapunov functions. *Automatica*, pages 1973–1980, 2004.

B. Lu, F. Wu, and S.W. Kim. Switching LPV control of an F-16 aircraft via controller state reset. *IEEE Transactions on Control Systems Technology*, 14(2):267–277, 2006.

J. Lunze and T. Steffen. Control reconfiguration after actuator failures using disturbance decoupling methods. *IEEE Transactions on Automatic Control*, 51(10):1590–1601, 2006.

H.-J. Ma and G.-H. Yang. Adaptive logic-based switching fault-tolerant controller design for nonlinear uncertain systems. *International Journal of Robust and Nonlinear Control*, 21: 404–428, 2011.

H.-J. Ma, G.-H. Yang, and D. Ye. Switching fault-tolerant control for a class of nonlinear systems with actuator faults. In *Proceedings of the 25th Chinese Control and Decision Conference (CCDC)*, pages 4797–4802, 2013.

H. Noura, D. Theilliol, J.-C. Ponsart, and A. Chamseddine. *Fault-tolerant control systems: Design and Practical Applications*. Springer-Verlag London Limited, 2009.

H. P. Oliveira, A. J. Sousa, A. P. Moreira, and P. J. Costa. Modeling and assessing of omni-directional robots with three and four wheels. In A. D. Rodić, editor, *Contemporary robotics - challenges and solutions*. InTech, 2009.

J. H. Richter. *Reconfigurable control of nonlinear dynamical systems - A fault-hiding approach*. Springer-Verlag, Heidelberg, 2011. ISBN 978-3-642-17627-2 (Print) 978-3-642-17628-9 (Online).

J. H. Richter, W. P. M. H. Heemels, N. van de Wouw, and J. Lunze. Reconfigurable control of piecewise affine systems with actuator and sensor faults: stability and tracking. *Automatica*, 47:678–691, 2011.

M. Rodrigues, D. Theilliol, and D. Sauter. Fault tolerant control design for switched systems. In *2nd IFAC Symposium on Analysis and Design of Hybrid Systems (ADHS)*, pages 223–228, 2006.

D. Rotondo, F. Nejjari, and V. Puig. Passive and active FTC comparison for polytopic LPV systems. In *Proceedings of the European Control Conference (ECC)*, pages 2951–2956, 2013.

D. Rotondo, F. Nejjari, and V. Puig. Model reference switching quasi-LPV control of a four wheeled omnidirectional robot. In *19th IFAC World Congress*, 2014a.

D. Rotondo, F. Nejjari, and V. Puig. A virtual actuator and sensor approach for fault tolerant control of LPV systems. *Journal of Process Control*, 24(3):203–222, 2014b.

J. S. Shamma. An overview of LPV systems. In J. Mohammadpour and C. Scherer, editors, *Control of Linear Parameter Varying Systems with Applications*, pages 3–26. Springer, 2012.

J. F. Sturm. Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones. *Optimization methods and software*, 11-12:625–653, 1999.

H. Yang, V. Cocquempot, and B. Jiang. Robust fault tolerant tracking control with application to hybrid nonlinear systems. *IET Control Theory and Applications*, 3(2):211–224, 2009.

H. Yang, B. Jiang, and V. Cocquempot. *Fault tolerant control design for hybrid systems*. Springer-Verlag Berlin Heidelberg, 2010.

H. Yang, B. Jiang, and V. Cocquempot. Supervisory fault-tolerant regulation for nonlinear systems. *Nonlinear Analysis: Real World Applications*, 12:789–798, 2011.

H. Yang, B. Jiang, and Y. Zhang. Tolerance of intermittent faults in spacecraft attitude control: switched system approach. *IET Control Theory and Applications*, 6(13):2049–2056, 2012.

Y. Zhang and J. Jiang. Bibliographical review on reconfigurable fault-tolerant control systems. *Annual reviews in control*, 32: 229–252, 2008.