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Decentralized Leader-Follower Flocking of Multiple Non-Holonomic Agents

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Abstract: In this paper, we study the cohesive flocking problem of multiple non-holonomic agents governed by extended second-order unicycle dynamics, such as robotic fish. Consider the system with only one leader. Combining the consensus algorithm and attraction/repulsion functions, a distributed flocking algorithm is presented for multiple robotic fish to complete the cohesive flocking task. According to LaSalle-Krasovskii invariance principle, the proposed algorithm enables followers to asymptotically track the leader's velocity and approach the equilibrium distances with their neighbors, provided that the initial interaction network among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time. During the evolutionary process, the connectivity of the communication network can be preserved due to the effect of the potential function. Finally, a simulation example is given to verify the proposed theoretical analysis.

Keywords: Decentralized control, flocking, leader-follower structure, non-holonomic agents, consensus algorithm, attraction/repulsion functions.

1. INTRODUCTION

In recent years, flocking has become one of the most important topics in the field of control theory Su et al. (2009); Dong (2011); Leonard et al. (2012); Wei et al. (2012); Zhan and Li (2013); Wang et al. (2013), due to its wide application in mobile sensor network, multi-robot systems, etc. Flocking is such a collective behavior that can be widely observed among social animals in nature, in which a group of scattered agents converge together based on neighbor information and simple rules.

Migrating flocking, known as leader-follower flocking, is a special flocking phenomenon where the flocking behaviors of the group are led by one or more experienced leaders. In nature, only one or a few individuals in a group could know the global information, such as the location of a food source or the route of a migration, and the others just adjust their behaviors according to the local information from their neighbors. It is obvious that leaders play an important role in guiding the behaviors of the group.

The existence of virtual leaders is a common assumption in the existing literatures on flocking Yu et al. (2010); Luo et al. (2012). Different from these virtual leaders, real leaders will participate in the energy distribution and formation construction of the whole system, which brings new challenge to the research on flocking. For example, Gu and Wang (2009) studied a leader-follower flocking problem concerning multiple leaders and tested the feasibility of the control algorithm on a group of wheeled mobile robots. They used consensus algorithm via local communication to estimate the position of flocking center for keeping the group connected, while we would like to use potential functions to solve the connectivity preservation problem.

Besides, Gu and Wang (2009) discussed the flocking problem of wheeled mobile robots, which are simplified as first-order unicycles. Similarly, Zhang et al. (2007), and Klein et al. (2008) also modeled robotic fish as first-order unicycles when they studied the coordination control of multiple robotic fish. Considering that the main propulsive force of the robotic fish comes from the latter part of its body, we make further efforts to draw the swimming robotic fish by a dynamic model named an extended second-order unicycle model, whose geometrical center and mass center don't coincide. So far, there is still no literature focusing on the flocking problem of multiple extended second-order unicycles. Thus, we will study the cohesive flocking problem for a network of extended second-order unicycles consisting of only one leader in this paper.

The main contributions of this paper is summarized. Firstly, we present a distributed flocking algorithm to solve the leader-follower flocking problem of multiple extended second-order unicycles considering only one leader. The algorithm is based on the combination of consensus and attraction/repulsion potential functions. Secondly, we analyze the stability of the closed-loop system by means of graph theory and LaSalle-Krasovskii invariance principle, provided that two initial conditions are satisfied: (1) the interaction network among the followers is an undirected connected graph; (2) there exists at least one follower having a leader neighbor. Therein, consensus algorithm is adopted for realizing the velocity alignment, while the problems of connectivity preservation and distance equilibrium are all solved by artificial potential field method. Finally, the simulation results are given to verify the effectiveness of the proposed algorithm.

The rest of this paper is organized as follows. Section II formulates the leader-follower flocking problem of multiple nonholonomic agents with only one leader. A control algorithm based on consensus and attractive/repulsion potential functions is presented, and the stability analysis of the closed-loop system is also provided in section III. The results of the numerical simulation are given in section IV. Finally, the conclusions are drawn in section V.

2. MODEL AND PROBLEM STATEMENT

2.1 Modeling of Robotic Fish

Let $\mathbb{N}$ denote the set of positive integers, $\mathbb{R}$ denote the set of real numbers, and $\mathbb{R}^+$ denote the set of positive real numbers. According to its kinematic characteristics, the motion of robotic

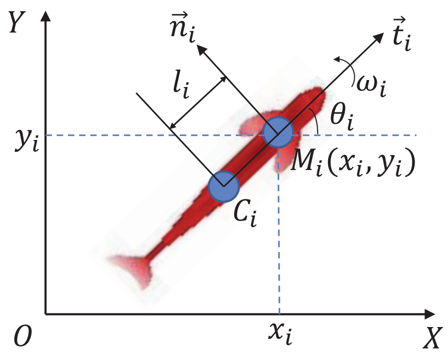


Fig. 1. The simplified model of robotic fish.

fish can be simply decomposed into translational motion and rotational motion. As shown in Fig. 1, considering its body shape and swimming mode, the robotic fish is modeled by the following dynamic equation

$$\begin{aligned} \dot{x}_i(t) &= \upsilon_i(t)\cos\theta_i(t) - \omega_i(t)l_i\sin\theta_i(t) \\ \dot{y}_i(t) &= \upsilon_i(t)\sin\theta_i(t) + \omega_i(t)l_i\cos\theta_i(t) \\ \dot{\theta}_i(t) &= \omega_i(t) \\ \dot{\upsilon}_i(t) &= a_i(t) \\ \dot{\omega}_i(t) &= b_i(t)/l_i \end{aligned} \tag{1}$$

where $p_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$ is the position vector of agent i, $\theta_i(t) \in \mathbb{R}$ is the heading angle of agent i, $\upsilon_i(t) \in \mathbb{R}$ is the thrusting speed of agent i, $\omega_i(t) \in \mathbb{R}$ is the rotational speed of agent i, $l_i \in \mathbb{R}^+$ is the distance between the geometrical center C_i and the mass center M_i of agent i, $\vartheta_i(t) = \omega_i(t)l_i \in \mathbb{R}$ is the tangential speed of agent i, $a_i(t) \in \mathbb{R}$ is the thrusting acceleration of agent i, and $b_i(t) \in \mathbb{R}$ is the rotational acceleration of agent i. Here, $\theta_i(t) \in [0, 2\pi)$ is measured from the x-axis in the anticlockwise rotation. It should be noted that $l_i = l_d$, $i = 1, \cdots, N$, where l_d is a constant. Unless specified otherwise, all variables in this paper are time-variant. For example, we may use p_i instead of $p_i(t)$ for short in the following sections.

Let $q_i(t) = [\upsilon_i(t), \vartheta_i(t)]^T$ be the velocity vector of agent i, we can obtain the following matrix equations from (1)

$$\begin{aligned} \dot{p}_i(t) &= H_i^T(t)q_i(t) \\ \dot{q}_i(t) &= u_i(t) \end{aligned} \tag{2}$$

where $H_i(t) = \begin{bmatrix} \cos\theta_i(t) & \sin\theta_i(t) \\ -\sin\theta_i(t) & \cos\theta_i(t) \end{bmatrix}$, $\dot{\theta}_i = \omega_i$, and $u_i(t) = [a_i(t), b_i(t)]^T$, $i = 1, \cdots, N$.

2.2 Problem Statement

The system under consideration consists of N agents traveling in a two-dimensional Euclidean space. Each agent only communicates with its neighbor due to its limited interaction capability. Let $N_i(t)$ denote the neighbor set of the follower $i \in \mathbb{F}$ at time t, and the initial neighbor set of the follower i is defined as

$$N_i(0) = \{j \,|\, \|p_i(0) - p_j(0)\| < D, j = 1, \cdots, N, j \neq i\} \tag{3}$$

where $D > 0$ is a constant, and $\|\bullet\|$ is the Euclidean norm.

We consider the robotic fish system with only one leader. If an agent has external control input, we call it a leader; else, we call it a follower. Without loss of generality, let leader set be $\mathbb{L} = \{1\}$ and follower set be $\mathbb{F} = \{2, \cdots, N\}$. We suppose that interconnection with the leader is unidirectional, while interconnection with the follower is bidirectional Tanner (2004). Thus, the interaction network for the multi-agent system including the leader and the followers is a directed graph $G(t)$ with vertex set $\mathcal{V} = \{1, \cdots, N\}$ and edge set $\varepsilon(t) = \{(i, j) | (i, j) \in \mathbb{F} \times \mathcal{V}, j \in N_i(t)\}$. At the same time, the followers' interaction network $\hat{G}(t)$ with vertex set $\mathbb{F}$ and edge set $\hat{\varepsilon}(t) = \{(i, j) | (i, j) \in \mathbb{F} \times \mathbb{F}, j \in N_i(t)\}$ is an undirected graph. In order to clarify the neighbor relationship, we introduce the adjacency matrix $A_N(t)$ of the graph $G(t)$ and the Laplacian matrix $L_{N-1}(t)$ of the graph $\hat{G}(t)$. $A_N(t) = [w_{ij}(t)]_{N \times N}$ is defined as

$$w_{ij}(t) = \begin{cases} 1, & i = 1, j = 1 \\ 1, & i \in \mathbb{F}, j \in N_i \\ 0, & otherwise \end{cases} \tag{4}$$

and $L_{N-1}(t) = [l_{ij}(t)]_{(N-1)\times(N-1)}$ is given by

$$l_{ij}(t) = \begin{cases} -w_{ij}(t), & i \neq j \\ \sum_{k=2, k\neq i}^{N} w_{ik}(t), & i = j. \end{cases} \tag{5}$$

For $\hat{G}(t)$ is an undirected graph, the Laplacian matrix $L_{N-1}(t)$ is symmetric and positive semi-definite.

Suppose that the interaction network $G(t)$ switches at t_p, $p = 1, 2, \cdots$. $G(t)$ is a fixed graph in each nonempty, bounded, and contiguous time-interval $[t_r, t_{r+1})$, $r = 0, 1, \cdots$. Here, $t_0 = 0$. Given that $\hat{G}(0)$ is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time $t_0 = 0$. In order to preserve the connectivity of the interaction network $G(t)$, the hysteresis adding new edges to the network is introduced Zavlanos et al. (2007); Su et al. (2010), such that

(1) if $(i, j) \in \varepsilon(t^-)$ and $\|\hat{p}_i(t) - \hat{p}_j(t)\| < 2D$, where $\hat{p}_i(t) = p_i(t) - p_i^*(t)$ and $p_i^*(t) = \int_0^t w_{i1}(t)H_i^T(t)q_1 d\tau$ $(i = 1, \cdots, N)$, then $(i, j) \in \varepsilon(t)$, for $t > 0$;
(2) if $(i, j) \notin \varepsilon(t^-)$ and $\|p_i(t) - p_j(t)\| < D$, then $(i, j) \in \varepsilon(t)$, for $t > 0$.

Let the leader swim at a constant thrusting speed and a constant rotational speed. Then, the problem is how to design control algorithms for followers to asymptotically track the leader's velocity and approach the equilibrium distances with their neighbors.

3. FLOCKING ALGORITHM

We assume synchronous motion and no time delays. We just consider the leader to do such a curve motion with a constant thrusting speed υ_1 and a constant rotational speed ω_1. Thus, the control protocol for leader is given by

$$\dot{q}_1 = 0 \tag{6}$$

where $q_1 = [\upsilon_1, l_d\omega_1]$, and $\omega_1 \neq 0$. Then, the control protocol for follower i is designed by

$$\begin{aligned} a_i &= -\sum_{j\in N_i}(\hat{\upsilon}_i - \hat{\upsilon}_j) - \sum_{j\in N_i}(\dot{\hat{p}}_i^T - \dot{\hat{p}}_j^T)t_i - \sum_{j\in N_i}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|)^T t_i \\ b_i &= -l_d\sum_{j\in N_i}(\hat{\theta}_i - \hat{\theta}_j) - l_d\sum_{j\in N_i}(\hat{\omega}_i - \hat{\omega}_j) - \sum_{j\in N_i}(\dot{\hat{p}}_i^T - \dot{\hat{p}}_j^T)n_i \\ &\quad - \sum_{j\in N_i}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|)^T n_i \end{aligned} \tag{7}$$

where $\hat{\theta}_i = \theta_i - w_{i1}\theta_1$, $\hat{\upsilon}_i = \upsilon_i - w_{i1}\upsilon_1$, $\hat{\omega}_i = \omega_i - w_{i1}\omega_1$, and $\hat{p}_i = p_i - p_i^*$, $i = 1,\cdots,N$. Here, $p_i^* = \int_0^t w_{i1}H_i^T q_1 d\tau$. Thus, we have $\dot{\hat{p}}_i = \dot{p}_i - \dot{p}_i^* = H_i^T q_i - w_{i1}H_i^T q_1 = H_i^T\hat{q}_i$, where $\hat{q}_i = q_i - w_{i1}q_1$. Besides, $\hat{p}_{ij} = \hat{p}_i - \hat{p}_j$, $t_i = \begin{bmatrix}\cos\theta_i \\ \sin\theta_i\end{bmatrix}$ and $n_i = \begin{bmatrix}-\sin\theta_i \\ \cos\theta_i\end{bmatrix}$ are two unit vectors orthogonal to each other, and $\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|)$ is the gradient of an artificial potential function $V(\|\hat{p}_{ij}\|)$ with the following definition:

Definition 1 (Potential function): Potential $V(\|\hat{p}_{ij}\|)$ is a differentiable, nonnegative, radially unbounded function of the Euclidean norm $\|\hat{p}_{ij}\|$ between agent i and j, such that
(1) $V(\|\hat{p}_{ij}\|) \to \infty$ as $\|\hat{p}_{ij}\| \to 0$;
(2) $V(\|\hat{p}_{ij}\|) \to \infty$ as $\|\hat{p}_{ij}\| \to 2D$;
(3) $V(\|\hat{p}_{ij}\|)$ attains its unique minimum when the Euclidean norm $\|\hat{p}_{ij}\|$ equals to a certain value between 0 and $2D$.

Then, we have Theorem 1 to solve the leader-follower cohesive flocking problem.

Theorem 1 (Leader-follower cohesive flocking). Consider a system of N agents with dynamics (1). The leader and the followers are respectively steered by control algorithms (6) and (7). Suppose that the initial interaction topology among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time. Then the following statements hold:
1. The thrusting speed, the rotational speed, and the heading angle of each follower asymptotically become the same as those of the leader.
2. The system approaches a cohesive configuration that minimizes all agent potentials.

Proof. Consider the system (1) with control input (6) and (7) on time interval $[t_r, t_{r+1})$, where the interaction network of the robotic fish system is fixed. Let $q_i^* = [0, l_d\theta_i]^T$ and $\hat{q}_i^* = q_i^* - w_{i1}q_1^* = [0, l_d\hat{\theta}_i]^T$, $i = 1,\cdots,N$. Due to $\dot{\hat{q}}_i = \dot{q}_i - w_{i1}\dot{q}_1 = \dot{q}_i$, the control protocol (7) can be rewritten as

$$\begin{aligned} \dot{\hat{q}}_i &= -\sum_{j\in N_i}(\hat{q}_i^* - \hat{q}_j^*) - \sum_{j\in N_i}(\hat{q}_i - \hat{q}_j) - H_i\sum_{j\in N_i}(\dot{\hat{p}}_i - \dot{\hat{p}}_j) \\ &\quad - H_i\sum_{j\in N_i}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|). \end{aligned} \tag{8}$$

Let $\tilde{p} = [\hat{p}_{11}^T,\cdots,\hat{p}_{1N}^T,\cdots,\hat{p}_{N1}^T,\cdots,\hat{p}_{NN}^T]^T$, $\hat{q} = [\hat{q}_2^T,\cdots,\hat{q}_N^T]^T$, and $\hat{\theta} = [\hat{\theta}_2,\cdots,\hat{\theta}_N]^T$. The set of followers who have one leader neighbor at time t is expressed by $N_l(t) = \{i | w_{i1} = 1, i \in \mathbb{F}\}$. Consider the following energy function as the common Lyapunov function

$$E(\tilde{p},\hat{q},\hat{\theta}) = \frac{1}{2}V + \frac{1}{2}\sum_{i\in\mathbb{F}}\hat{q}_i^T\hat{q}_i + \frac{1}{2}\Theta \tag{9}$$

where $V = \sum_{i\in\mathbb{F}}\sum_{j\in N_i}V(\|\hat{p}_{ij}\|) + \sum_{j\in N_l}V(\|\hat{p}_{1j}\|)$ and $\Theta = l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\hat{\theta}_i(\hat{\theta}_i - \hat{\theta}_j)$. With $\hat{\theta}_1 = 0$, Θ can be rewritten by $\Theta = l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i/\{1\}}\hat{\theta}_i(\hat{\theta}_i - \hat{\theta}_j) + l_d^2\sum_{i\in N_l}\hat{\theta}_i^2$. Furthermore, we have

$$\begin{aligned} \dot{\Theta} &= 2l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i/\{1\}}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) + 2l_d^2\sum_{i\in N_l}\dot{\hat{\theta}}_i\hat{\theta}_i \\ &= 2l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j). \end{aligned} \tag{10}$$

Then the derivative of $E(\tilde{p},\hat{q},\hat{\theta})$ w.r.t. the time $t \in [t_r, t_{r+1})$ is

$$\begin{aligned} \frac{dE}{dt} &= \frac{1}{2}\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{p}}_{ij}^T\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|) + \frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{1j}^T\nabla_{\hat{p}_{1j}}V(\|\hat{p}_{1j}\|) \\ &\quad + \sum_{i\in\mathbb{F}}\hat{q}_i^T\dot{\hat{q}}_i + l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) \\ &= \sum_{i\in\mathbb{F}}\dot{\hat{p}}_i^T\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|) + \frac{1}{2}\sum_{i\in N_l}\dot{\hat{p}}_{i1}^T\nabla_{\hat{p}_{i1}}V(\|\hat{p}_{i1}\|) \\ &\quad + \frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{1j}^T\nabla_{\hat{p}_{1j}}V(\|\hat{p}_{1j}\|) + \sum_{i\in\mathbb{F}}\hat{q}_i^T\dot{\hat{q}}_i \\ &\quad + l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j). \end{aligned} \tag{11}$$

Due to $\dot{\hat{p}}_{ij}^T = -\dot{\hat{p}}_{ji}^T$ and the symmetric nature of $V(\|\hat{p}_{ij}\|)$, one gets

$$\begin{aligned} \frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{1j}^T\nabla_{\hat{p}_{1j}}V(\|\hat{p}_{1j}\|) &= \frac{1}{2}\sum_{j\in N_l}\dot{\hat{p}}_{j1}^T\nabla_{\hat{p}_{j1}}V(\|\hat{p}_{j1}\|) \\ &= \frac{1}{2}\sum_{i\in N_l}\dot{\hat{p}}_{i1}^T\nabla_{\hat{p}_{i1}}V(\|\hat{p}_{i1}\|). \end{aligned} \tag{12}$$

With $\dot{\hat{p}}_{i1}^T = \dot{\hat{p}}_i^T - \dot{\hat{p}}_1^T = \dot{\hat{p}}_i^T$, we have

$$\begin{aligned} \frac{dE}{dt} &= \sum_{i\in\mathbb{F}}\dot{\hat{p}}_i^T\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|) + \sum_{i\in N_l}\dot{\hat{p}}_{i1}^T\nabla_{\hat{p}_{i1}}V(\|\hat{p}_{i1}\|) \\ &\quad + \sum_{i\in\mathbb{F}}\hat{q}_i^T\dot{\hat{q}}_i + l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) \\ &= \sum_{i\in\mathbb{F}}\dot{\hat{p}}_i^T\sum_{j\in N_i/\{1\}}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|) + \sum_{i\in N_l}\dot{\hat{p}}_i^T\nabla_{\hat{p}_{i1}}V(\|\hat{p}_{i1}\|) \\ &\quad + \sum_{i\in\mathbb{F}}\hat{q}_i^T\dot{\hat{q}}_i + l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) \\ &= \sum_{i\in\mathbb{F}}\dot{\hat{p}}_i^T\sum_{j\in N_i}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|) + \sum_{i\in\mathbb{F}}\hat{q}_i^T\dot{\hat{q}}_i + l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) \\ &= \sum_{i\in\mathbb{F}}\hat{q}_i^T H_i\sum_{j\in N_i}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|) + l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) \\ &\quad + \sum_{i\in\mathbb{F}}\hat{q}_i^T(-\sum_{j\in N_i}(\hat{q}_i^* - \hat{q}_j^*) - \sum_{j\in N_i}(\hat{q}_i - \hat{q}_j) \\ &\quad - H_i\sum_{j\in N_i}(\dot{\hat{p}}_i - \dot{\hat{p}}_j) - H_i\sum_{j\in N_i}\nabla_{\hat{p}_{ij}}V(\|\hat{p}_{ij}\|)) \\ &= l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) - \sum_{i\in\mathbb{F}}\hat{q}_i^T\sum_{j\in N_i}(\hat{q}_i^* - \hat{q}_j^*) \\ &\quad - \sum_{i\in\mathbb{F}}\hat{q}_i^T\sum_{j\in N_i}(\hat{q}_i - \hat{q}_j) - \sum_{i\in\mathbb{F}}\hat{q}_i^T H_i\sum_{j\in N_i}(\dot{\hat{p}}_i - \dot{\hat{p}}_j) \\ &= l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) - \sum_{i\in\mathbb{F}}\sum_{j\in N_i}\hat{q}_i^T(\hat{q}_i^* - \hat{q}_j^*) \\ &\quad - \sum_{i\in\mathbb{F}}\hat{q}_i^T\sum_{j\in N_i}(\hat{q}_i - \hat{q}_j) - \sum_{i\in\mathbb{F}}\dot{\hat{p}}_i^T\sum_{j\in N_i}(\dot{\hat{p}}_i - \dot{\hat{p}}_j). \end{aligned} \tag{13}$$

From $\hat{q}_i^T(\hat{q}_i^* - \hat{q}_j^*) = l_d^2\hat{\omega}_i(\hat{\theta}_i - \hat{\theta}_j)$ and $\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) = \hat{\omega}_i(\hat{\theta}_i - \hat{\theta}_j)$, it is easy to get that $l_d^2\sum_{i\in\mathbb{F}}\sum_{j\in N_i}\dot{\hat{\theta}}_i(\hat{\theta}_i - \hat{\theta}_j) = \sum_{i\in\mathbb{F}}\sum_{j\in N_i}\hat{q}_i^T(\hat{q}_i^* - \hat{q}_j^*)$. $\frac{dE}{dt}$ can be simplified by

$$\frac{dE}{dt} = -\sum_{i\in\mathbb{F}} \hat{q}_i^T \sum_{j\in N_i} (\hat{q}_i - \hat{q}_j) - \sum_{i\in\mathbb{F}} \dot{\hat{p}}_i^T \sum_{j\in N_i} (\dot{\hat{p}}_i - \dot{\hat{p}}_j). \tag{14}$$

Owing to $\hat{q}_1 = 0$, one gets

$$\begin{aligned} & -\sum_{i\in\mathbb{F}} \hat{q}_i^T \sum_{j\in N_i} (\hat{q}_i - \hat{q}_j) \\ &= -\sum_{i\in\mathbb{F}} \hat{q}_i^T \sum_{j\in N_i/\{1\}} (\hat{q}_i - \hat{q}_j) - \sum_{i\in N_l} \hat{q}_i^T \hat{q}_i \\ &= -\hat{q}^T (L_{N-1}\otimes I_2)\hat{q} - \sum_{i\in N_l} \hat{q}_i^T \hat{q}_i \le 0 \end{aligned} \tag{15}$$

where I_2 denotes the 2×2 identity matrix, and L_{N-1} is symmetric and positive semi-definite. Similarly, due to $\dot{\hat{p}}_1 = \mathbf{0}$, we have

$$-\sum_{i\in\mathbb{F}} \dot{\hat{p}}_i^T \sum_{j\in N_i} (\dot{\hat{p}}_i - \dot{\hat{p}}_j) = -\dot{\hat{p}}^T (L_{N-1}\otimes I_2)\dot{\hat{p}} - \sum_{i\in N_l} \dot{\hat{p}}_i^T \dot{\hat{p}}_i \le 0 \tag{16}$$

where $\hat{p} = [\hat{p}_2^T, \cdots, \hat{p}_N^T]^T$. Hence,

$$\begin{aligned} \frac{dE}{dt} &= -\hat{q}^T (L_{N-1}\otimes I_2)\hat{q} - \sum_{i\in N_l} \hat{q}_i^T \hat{q}_i - \dot{\hat{p}}^T (L_{N-1}\otimes I_2)\dot{\hat{p}} \\ &\quad - \sum_{i\in N_l} \dot{\hat{p}}_i^T \dot{\hat{p}}_i \\ &\le 0. \end{aligned} \tag{17}$$

The initial potential energy of the system is finite, and the initial speeds and initial heading angles of all agents are also finite. Thus, the initial energy $E(\tilde{p}(0), \hat{q}(0), \hat{\theta}(0))$ of the system is finite, and the supremum of E is obviously its initial value $E(\tilde{p}(0), \hat{q}(0), \hat{\theta}(0))$. Then, the potential energy V of the system is finite, and the potential energy $V(\|\hat{p}_{ij}(t)\|)$ between agent i and j is finite. Furthermore, the thrusting acceleration and the rotational acceleration of each agent are also finite.

3.1 Velocity Alignment

Assume that there are $m_r \in \mathbb{N}$ new links being added to the interaction network at switching time t_r, $r = 1, 2, \cdots$. We have supposed that the initial communication topology should satisfy the following two conditions: (1) the interaction network among the followers is an undirected connected graph; (2) there exists at least one follower having a leader neighbor. Let G_1 denote the initial communication topology of the system, and G_c denote the set of all graphs meeting the proposed two conditions on the vertices. The proposed control algorithm can guarantee that the sequence of switching topologies G_{r+1} within $[t_r, t_{r+1})$ consists of such graphs satisfying $G_{r+1} \in G_c$. The number of the vertices is finite, thus G_c is a finite set. Assume that there are at most $M \in \mathbb{N}$ new links that can be added to the initial communication topology G_1. Clearly, $0 < m_r \le M$ and $r \le M$. Therefore, the number of switching times of the system is finite, and the interaction network $G(t)$ eventually becomes fixed. Suppose that the last switching time is t_f, the following discussions are restricted on the time interval $[t_f, \infty)$.

Note that the distance between neighbors is not longer than D. Hence, the set

$$B = \{\tilde{p} \in D_p, \hat{q}, \hat{\theta} | E(\tilde{p}, \hat{q}, \hat{\theta}) \le E(\tilde{p}(0), \hat{q}(0), \hat{\theta}(0))\}, \tag{18}$$

is positively invariant, where $D_p = \{\tilde{p} | \|\hat{p}_{ij}\| \in (0, 2D), \forall (i,j) \in \varepsilon\}$. Since $G(t)$ is connected for all $t \ge 0$, one gets $\|\hat{p}_{ij}\| < 2(N-1)D$ for all i, j. Since $E(\tilde{p}(t), \hat{q}(t), \hat{\theta}(t)) \le E(\tilde{p}(0), \hat{q}(0), \hat{\theta}(0))$, we have $\hat{q}^T\hat{q} \le 2E(\tilde{p}(0), \hat{q}(0), \hat{\theta}(0))$, that is, $\|\hat{q}\| \le \sqrt{2E(\tilde{p}(0), \hat{q}(0), \hat{\theta}(0))}$. In addition, $\hat{\theta}_i \in (-2\pi, 2\pi), i = 1, \cdots, N$. Thus, the set B is closed and bounded, hence compact. Note that the system (1) with control input (6) and (7) is an autonomous system on the concerned time interval $[t_f, \infty)$.

Then, according to the LaSalle-Krasovskii invariance principle Khalil (2002), the trajectories of the followers will converge to the invariant set $S = \{\tilde{p}, \hat{q}, \hat{\theta} | dE/dt = 0\}$. Clearly, $dE/dt = 0$ if and only if $\hat{q}_i = \hat{q}_j$ and $\dot{\hat{p}}_i = \dot{\hat{p}}_j$ for all $i \in \mathbb{F}$ and $j \in N_i$. Then, we have $\hat{q}_i = \hat{q}_j = \hat{q}_1$ for $i \in N_l$, $j \in N_i$. With $\hat{q}_1 = 0$, one gets $q_i = q_1$ and $q_j - w_{j1}q_1 = 0$. If $j \notin N_l$, one gets $q_j = 0$, which means that agent j stops moving. The state of agent j cannot be maintained, because its neighbor i always follows the moving leader and the distance between agent i and agent j will change to cause the control protocol (7) to work. Thus, there is only $j \in N_l$. As mentioned above, followers asymptotically approach a configuration that every follower has one leader neighbor, that is, $N_l = \mathbb{F}$. Discussion about $\dot{\hat{p}}_i = \dot{\hat{p}}_j$ is similar. In a word, $dE/dt = 0$ means $q_i = q_1$ and $\dot{p}_i = \dot{p}_1$ for $i = 2, \cdots, N$.

According to (1), $\dot{p}_i = \dot{p}_1$ is equivalent to

$$\begin{cases} \upsilon_i \cos\theta_i - \omega_i l_d \sin\theta_i = \upsilon_1 \cos\theta_1 - \omega_1 l_d \sin\theta_1 \\ \upsilon_i \sin\theta_i + \omega_i l_d \cos\theta_i = \upsilon_1 \sin\theta_1 + \omega_1 l_d \cos\theta_1 \end{cases} \tag{19}$$

In terms of $q_i = q_1$ and $l_d > 0$, (19) can be explicitly expressed by

$$\begin{cases} (\upsilon_1^2 + l_d^2\omega_1^2)\sin\theta_i = (\upsilon_1^2 + l_d^2\omega_1^2)\sin\theta_1 \\ (\upsilon_1^2 + l_d^2\omega_1^2)\cos\theta_i = (\upsilon_1^2 + l_d^2\omega_1^2)\cos\theta_1. \end{cases} \tag{20}$$

With $\omega_1 \ne 0$, thus we have $\theta_i = \theta_1 \in [0, 2\pi)$. As previously mentioned, the thrusting speed, the rotational speed, and the heading angle of each follower asymptotically become the same with those of the leader. Conclusion 1 of Theorem 1 is proved.

3.2 Potential Minimization

Due to $\dot{q}_1 = \mathbf{0}$ and $q_i = q_1$, one gets $\dot{q}_i = \mathbf{0}$, $i \in \mathbb{F}$. Here, $\mathbf{0}$ denotes the zero vector. With $\hat{\theta}_i = \hat{\theta}_j$, $\hat{q}_i = \hat{q}_j$, and $\dot{\hat{p}}_i = \dot{\hat{p}}_j$, $\dot{q}_i = \mathbf{0}$ can be simplified as

$$\begin{cases} -\sum_{j\in N_i} \nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|)^T t_i = 0 \\ -\sum_{j\in N_i} \nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|)^T n_i = 0 \end{cases} \tag{21}$$

For $t_i = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}$ and $n_i = \begin{bmatrix} -\sin\theta_i \\ \cos\theta_i \end{bmatrix}$ are unit vectors orthogonal to each other, as well as $l_d \ne 0$, (21) is equivalent to

$$\sum_{j\in N_i} \nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|) = 0, \quad i \in \mathbb{F} \tag{22}$$

where $N_i = \mathbb{L} \bigcup \mathbb{F}/\{i\}$. In further, we get

$$\sum_{i\in\mathbb{F}} \sum_{j\in N_i} \nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|) = 0 \tag{23}$$

which means that $\sum_{i\in\mathbb{F}} \sum_{j\in N_i/\{1\}} \nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|) + \sum_{i\in N_l} \nabla_{\hat{p}_{i1}} V(\|\hat{p}_{i1}\|) = 0$. Owing to $\sum_{i\in\mathbb{F}} \sum_{j\in N_i/\{1\}} \nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|) = 0$, one obtains that $\sum_{i\in N_l} \nabla_{\hat{p}_{i1}} V(\|\hat{p}_{i1}\|) = 0$, that is,

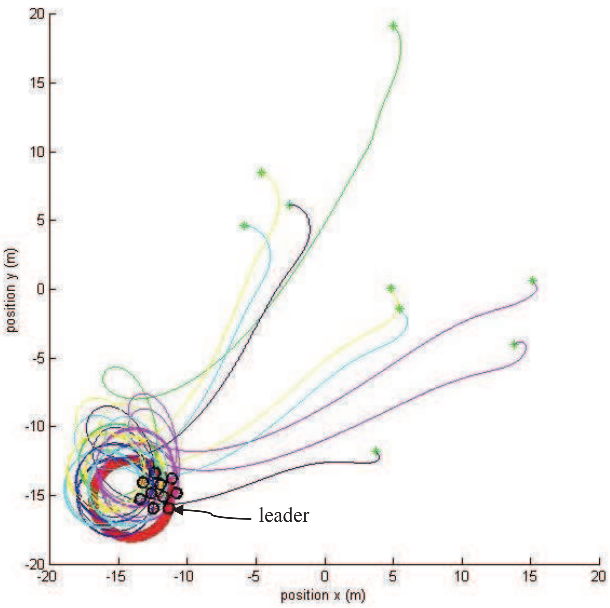
$$\sum_{j\in N_l} \nabla_{\hat{p}_{1j}} V(\|\hat{p}_{1j}\|) = 0 \tag{24}$$

where $N_l = \mathbb{F}$. (22) and (24) mean that conclusion 2 of Theorem 1 is proved. ■

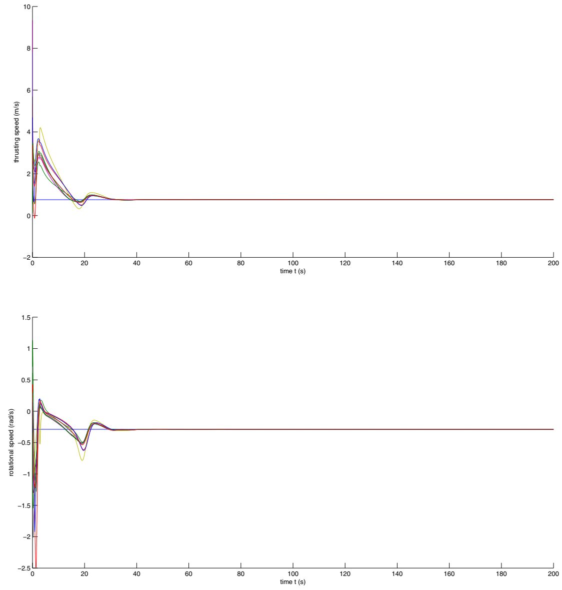
4. SIMULATION VALIDATION

According to definition 1, we design the following potential function

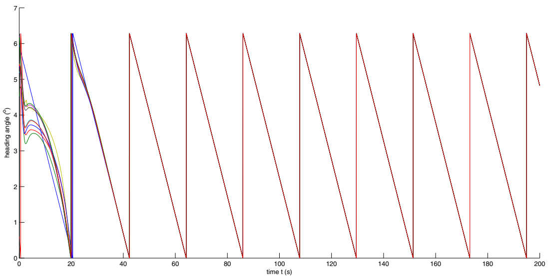
$$V(\|\hat{p}_{ij}\|) = \frac{b}{\|\hat{p}_{ij}\|^2} - a\ln(4D^2 - \|\hat{p}_{ij}\|^2) + c \tag{25}$$



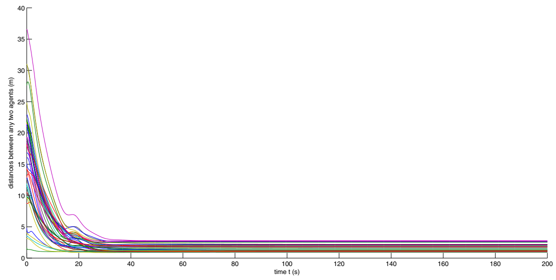
(a) Trajectories of ten agents.



(b) The thrusting speeds and the rotational speeds of ten agents with time t.



(c) The heading angles of ten agents with time t.



(d) The distances between any two agents with time t.

Fig. 2. Numerical simulation results of the leader-follower cohesive flocking task.

where a and b are positive constants, and c is constant. Therein, the first part is the repulsive potential term, the second part is the attractive potential term, and the third constant item c is just used to guarantee the potential function positive at all times. Then, the gradient of the potential function is obtained by

$$\nabla_{\hat{p}_{ij}} V(\|\hat{p}_{ij}\|) = 2\hat{p}_{ij}(-\frac{b}{\|\hat{p}_{ij}\|^4} + \frac{a}{4D^2 - \|\hat{p}_{ij}\|^2}) \tag{26}$$

Specially, it should be noted that parameters a and b at least satisfy the constraint condition $\frac{\sqrt{b^2+16abD^2}-b}{2a} < D^2$.

Agents with generic initial conditions are employed, on condition that the initial interaction topology among the followers is an undirected connected graph and there exists at least one follower having a leader neighbor at the initial time. According to Theorem 1, the connectivity of the communication network can be preserved at all times.

We assign ten agents to achieve the leader-follower cohesive flocking task. The simulation is conducted on Matlab. The sample step is $0.01s$, and the simulation time is $200s$. Let $S_i(t) = (x_i(t), y_i(t), \theta_i(t), \upsilon_i(t), \omega_i(t))$ denote the state vector of the robotic fish at time t, and the units of the components are respectively m, m, rad, m/s, rad/s. The initial conditions for the multi-agent system are given by

$$\begin{aligned}
&S_1(0) = (-12.2, -13.09, -0.34, 0.76, -0.29),\\
&S_2(0) = (3.7, -11.78, -1.76, 0.54, 0.7),\\
&S_3(0) = (13.8, -4, 0.6, 5.3, 0.08),\\
&S_4(0) = (4.82, 0.1, -1.1, 7.8, -0.38),\\
&S_5(0) = (-5.8, 4.6, -0.07, 9.34, 0.013),\\
&S_6(0) = (4.97, 19.2, -0.84, 1.3, -0.22),\\
&S_7(0) = (-2.6, 6.1, 0.34, 5.7, -0.275),\\
&S_8(0) = (15.1, 0.6, -0.92, 4.7, -0.34),\\
&S_9(0) = (-4.56, 8.45, -0.12, 0.12, 0.6),\\
&S_{10}(0) = (5.45, -1.42, -0.92, 3.37, 0.31).
\end{aligned} \tag{27}$$

In addition, the design parameters of the potential function (25) are $a = 625$, $b = 1$, $c = 5120$, and $D = 30$. The simulation results are shown in Fig. 2. In Fig. 2 (a), the green star point is the initial position of each agent, while the rough black circle denotes the final position of each agent at time $t = 200s$. The rough red circle line denotes the trajectory of the leader. It is clear to see that ten agents asymptotically converge to a stable flocking formation. Fig. 2 (b) and (c) tell us that the thrusting speed, the rotational speed, and the heading angles of the followers asymptotically become the same as those of the leader. Fig. 2 (d) gives the distances between any two agents. The distances are all asymptotically stable and there is no zero-value distance. Thus, no collision happens during the evolution process.

5. CONCLUSION

In this paper, we study the cohesive flocking problem of multiple robotic fish governed by extended second-order unicycles under the guidance of only one leader. By using the nearest neighbor interaction rules, a leader-follower flocking algorithm is proposed with the combination of consensus and attraction/repulsion functions. Provided that the initial interaction network among the followers is an undirected connected graph, and there exists at least one follower having a leader neighbor at the initial time, the followers asymptotically converge to a stable cohesive flocking formation based on a moving leader. The stability of the closed-loop system is analyzed by graph theory and LaSalle-Krasovskii invariance principle. Furthermore, simulations are conducted to validate the effectiveness of the proposed algorithm.

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