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Unit Commitment with Wind Generation and Reversible-Hydro System in Islands

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Abstract: The large share of renewable generation in electric power systems has promoted a big change in the way of scheduling the generation of power plants. Renewable energy is subject to a high variability and the forecast of future renewable production becomes a difficult task. Wind power uncertainty has influenced the operation of electrical power systems. In order to maintain the reliability and the power balance of a power system, conventional units have to be operated more flexibly. In isolated places, like islands, the influence of wind uncertainty on the operation is more pronounced. This paper tries to model the unit commitment for an isolated area with different generation technologies, such as thermal, wind, and hydro power. The model aims to reduce the operational costs of the thermal units while satisfying technical constraints and finding the optimal amount of reserves.

NOTATION

Sets

J	Set of indexes of the thermal generating units.
T	Set of indexes of the time periods.
S	Set of indexes of the scenarios.

Parameters

A_j	Coefficient of the piecewise linear production cost function of thermal unit j.
$c_{jt}^{r,up}$	Programmed up-reserve cost of thermal unit j in period t [$/MWh].
$c_{jt}^{r,dw}$	Programmed down-reserve cost of thermal unit j in period t [$/MWh].
$\hat{c}_{jt}^{r,up}$	Cost of using thermal unit j for up-reserve in period t [$/MWh].
$\hat{c}_{jt}^{r,dw}$	Cost of using thermal unit j for down-reserve in period t [$/MWh].
$\hat{c}_{nt}^{r,up}$	Cost of using the turbine for up-reserve in period t [$/MWh].
$\hat{c}_{pt}^{r,dw}$	Cost of using the pump for down-reserve in period t [$/MWh].
$\hat{c}_{t}^{lr}$	Cost of loss of spinning reserve in period t [$/MWh].
d_t	Load [MW].
DT_j	Minimum down time of thermal unit j [h].
η_n	Efficiency of the reversible-hydro plant as turbine.
η_p	Efficiency of the reversible-hydro plant as pump.
F_j	Number of periods during which thermal unit j must be initially offline due to its minimum down time constraint.
F^{cost}	Fuel cost [€/MWh].
hx_{lj}	Upper limit of block l of the piecewise linear production cost function of thermal unit j.
L_j	Number of periods during which thermal unit j must be initially online due to its minimum up-time.
M_{lj}	Slope of block l of the piecewise linear production cost function of thermal unit j.
NL_j	Number of segments of the piecewise linear production cost function of thermal unit j.
ONT_j^0	Time that thermal plant j has been online before planning [h].
$OFFT_j^0$	Time that thermal plant j has been offline before planning [h].
RD_j	Ramp-down limit of thermal unit j [MW/h].
RU_j	Ramp-up limit of thermal unit j [MW/h].
SUC_j	Startup cost of thermal unit j [$].
SDC_j	Shutdown cost of thermal unit j [$].
SD_j	Shutdown ramp limit of thermal unit j [MW/h].
SU_j	Startup ramp limit of thermal unit j [MW/h].
UT_j	Minimum up time of thermal unit j [h].
$\underline{v}^u$	Minimum capacity of the upper reservoir [MWh].
$\overline{v}^u$	Maximum capacity of the upper reservoir [MWh].
$\underline{v}^l$	Minimum capacity of the lower reservoir [MWh].
$\overline{v}^l$	Maximum capacity of the lower reservoir [MWh].
v^{u0}	Initial and final level of the upper reservoir [MWh].
v^{l0}	Initial and final level of the lower reservoir [MWh].
$\overline{X}_j$	Capacity of thermal unit j [MW].
$\underline{X}_j$	Minimum power output of thermal unit j [MW].
$\overline{X}_n$	Maximum turbine power limit for the reversible-hydro plant [MW].
$\underline{X}_n$	Minimum turbine power limit for the reversible-hydro plant [MW].
$\overline{X}_p$	Maximum pumping power limit for the reversible-hydro plant [MW].
$\underline{X}_p$	Minimum pumping power limit for the reversible-hydro plant [MW].

$w_t(s)$ Wind power generation in period t and scenario s [MW].

Binary variables

u_{jt} Thermal unit j on/off status.
u_{nt} Turbine on/off status.
u_{pt} Pump on/off status.

Variables

c_{jt}^{p} Production cost of thermal unit j in period t [\$/MWh].
c_{jt}^{u} Startup cost of thermal unit j in period t [\$].
c_{jt}^{d} Shutdown cost of thermal unit j in period t [\$].
c_{jt}^{f} Fixed cost of thermal unit j in period t [\$].
c_{nt}^{u} Startup cost of the turbine of the reversible-hydro plant in period t [\$].
c_{nt}^{d} Shutdown cost of the turbine of reversible-hydro plant in period t [\$].
c_{pt}^{u} Startup cost of the pump of the reversible-hydro plant in period t [\$].
c_{pt}^{d} Shutdown cost of the pump of the reversible-hydro plant in period t [\$].
c_{pt}^{p} Production cost of the pump of the reversible-hydro plant in period t [\$/MWh].
δ_{ljt} Power produced in block l of the piecewise linear production cost of thermal unit j in period t [MW].
$lr_t(s)$ Loss of spinning reserve in period t and scenario s [MW].
r_{jt}^{up} Programmed up-reserve of thermal unit j in period t [MW].
r_{jt}^{dw} Programmed down-reserve of thermal unit j in period t [MW].
$\overline{r}_{jt}^{up}$ Maximum available output power of thermal unit j for up-reserve in period t [MW].
$\overline{r}_{jt}^{dw}$ Maximum available output power of thermal unit j for down-reserve in period t [MW].
$\hat{r}_{jt}^{up}(s)$ Up-reserve of thermal unit j in period t and scenario s [MW].
$\hat{r}_{jt}^{dw}(s)$ Down-reserve of thermal unit j in period t and scenario s [MW].
$\overline{r}_{nt}^{up}$ Maximum available output power of turbine of the reversible-hydro plant for up-reserve in period t [MW].
$\overline{r}_{pt}^{dw}$ Maximum available input power of pump of the reversible-hydro plant for down-reserve in period t [MW].
$\hat{r}_{nt}^{up}(s)$ Up-reserve of turbine of reversible-hydro plant in period t and scenario s [MW].
$\hat{r}_{pt}^{dw}(s)$ Down-reserve of pump of the reversible-hydro plant in period t and scenario s [MW].
x_{jt} Power output of thermal unit j in period t [MW].
x_{nt} Turbine power output of the reversible-hydro plant in period t [MW].
x_{pt} Pumping power output of the reversible-hydro plant in period t [MW].
$\underline{x}_{jt}$ Minimum available power of thermal unit j in period t [MW].
$\overline{x}_{jt}$ Maximum available power of thermal unit j in period t [MW].
v_t^u Energy stored in the upper reservoir in period t [MWh].
v_t^l Energy stored in the lower reservoir in period t [MWh].

1. INTRODUCTION

The Unit Commitment (UC) problem aims at determining the schedule combination of the available generating units to satisfy the forecasted demand with the minimum total production cost. The UC problem has been traditionally solved in centralized power systems to determine when to start-up or shut-down thermal generation units and how to dispatch generators to meet demand and spinning reserve requirements while satisfying generation constraints. These constraints reduce the freedom to choose to start-up and shut-down generating units.

Renewable generation has produced many changes in the way electric systems are operated. In particular, wind and solar systems are highly variable and their forecast is challenging.

In the case of islands, uncertainty in the operation is more pronounced due to the low-response capacity of the other non-renewable generation units. Imbalances in the system should only be compensated with the programmed reserves, unlike continental areas, where, if a high imbalance occurs, it can be solved with power from other areas.

The scheduling model is framed as a stochastic optimization model (Birge and Louveaux, 1997) that includes stochasticity of wind generation and demand.

In the scheduling problem defined in this work, the objective function is the cost of running the thermal units. Like several previous papers (Takriti, et al., 1996), (García-González, et al., 2008) and (Meibom, et al., 2011), the proposed model takes into account stochastic demand and wind power in order to analyze how reserves are influenced by both.

2. MODEL FORMULATION

The objective function of the proposed mixed-integer linear programming (MILP) stochastic UC is defined by two terms. The first one corresponds to the scheduling costs of power and reserves of generating plants (deterministic costs), Φ, and the second one represents the expected costs, $\Gamma(s)$, related to the use of reserves with the purpose of maintaining power balance in each period. The corrective actions depend on the scenarios, s. Note that there is no cost associated to wind generation since the only cost taken into account in the objective function is the fuel cost needed to start-up, shut-down or run the different components. Fixed costs are disregarded in the model. Although wind turbines do not need fuel when generating electricity, they have production costs, but, since their values are much lower than the production cost of thermal units, this is not considered. The same occurs with the turbine.

2.1 Objective Function

The objective function, Υ, is composed of two terms that are described as follows.

$$\Upsilon = \{\Phi + \mathbb{E}[\Gamma(s)]\} \quad (1)$$

The scheduled power and programmed reserves costs are defined in (2), whereas the cost of using reserves depending on the scenario is defined in (3). Equation (2) includes the start-up and shut-down costs of thermal units and the pump and turbine of the reversible-hydro plant, as well as the production cost of conventional and pumping units. For simplicity, hourly periods are considered.

$$\Phi = \sum_t \{\sum_j (c_{jt}^p + c_{jt}^u + c_{jt}^d) + (c_{nt}^u + c_{nt}^d + c_{pt}^u + c_{pt}^d + c_{pt}^p) + \sum_j (c_{jt}^{r,up} r_{jt}^{up} + c_{jt}^{r,dw} r_{jt}^{dw})\} \quad (2)$$

$$\Gamma(s) = \sum_t \{\hat{c}_{jt}^{r,up} \hat{r}_{jt}^{up}(s) + \hat{c}_{jt}^{r,dw} \hat{r}_{jt}^{dw}(s) + \hat{c}_{pt}^{r,dw} \hat{r}_{pt}^{dw}(s) + \hat{c}_{nt}^{r,up} \hat{r}_{nt}^{up}(s) + lr_t(s)\hat{c}_t^{lr}\} \quad (3)$$

Thus, the problem becomes:

$$\min \{\Phi + \mathbb{E}[\Gamma(s)]\} \quad (4)$$

subject to:

2.2 Power Balance Constraint

Constraint (5) represents the power balance in all periods and scenarios. Since wind power depends on scenarios, reserves are also calculated for each scenario. However, scheduling of the thermal units and the reversible-hydro plant outputs are unique for each period. When a high imbalance occurs and there is not enough power to supply all demand requirements, variable $lr_t(s)$ quantifies the value of the energy not supplied. The left-hand side of (5) represents the sum of the energy produced by the thermal plants, wind units, turbine, and reserves. The right-hand side is the total consumption, including demand, pump and pump reserves.

$$\sum_j x_{jt} + w_t(s) + x_{nt} + \sum_j \left(\hat{r}_{jt}^{up}(s) - \hat{r}_{jt}^{dw}(s)\right) + \hat{r}_{nt}^{up}(s) + lr_t(s) = d_t + x_{pt} + \hat{r}_{pt}^{dw}(s), \quad \forall t, s \quad (5)$$

2.3 Thermal Constraints

The model used for the deterministic UC of thermal units is taken from (Carrión and Arroyo, 2006).

a) Generation limits and ramping constraints

Constraint (6) bounds the minimum available power output of unit j in period t, $\underline{x}_{jt}$, by the minimum power output and the generation, which is also lower than the maximum available power output of unit j in period t, $\overline{x}_{jt}$, and the maximum power output (7).

$$\underline{X}_j u_{jt} \le \underline{x}_{jt} \le x_{jt}, \quad \forall j, t \quad (6)$$

$$x_{jt} \le \overline{x}_{jt} \le \overline{X}_j u_{jt}, \quad \forall j, t \quad (7)$$

Ramp rates are defined as follows: variable x_{jt} defines the power output of thermal unit j in period t constrained by the ramp-up rates (8), the shut-down rates (9) and $\underline{x}_{jt}$ in (10).

$$\overline{x}_{jt} \le x_{j,t-1} + RU_j u_{j,t-1} + SU_j(u_{j,t} - u_{j,t-1}) + \overline{X}_j(1 - u_{jt}), \ \forall j, t \quad (8)$$

$$x_{jt} \ge x_{j,t-1} - RD_j u_{jt} - SD_j(u_{j,t-1} - u_{jt}) - \overline{X}_j(1 - u_{j,t-1}), \quad \forall j, t \quad (9)$$

$$\underline{x}_{jt} \ge x_{jt} - RD_j u_{jt}, \quad \forall j, t \quad (10)$$

b) Minimum up- and down-time constraints

They are defined as:

$$\sum_{t=1}^{L_j} (1 - u_{jt}) = 0, \quad \forall j \quad (11)$$

$$\sum_{k=t}^{t+UT_j-1} u_{jk} \ge UT_j \cdot (u_{jt} - u_{j,t-1}), \quad \forall j \in J, \forall t = L_j + 1 \cdots T - UT_j + 1 \quad (12)$$

$$\sum_{k=t}^{T} \{u_{jk} - (u_{jt} - u_{j,t-1})\} \ge 0, \quad \forall j \in J, \ \forall t = T - UT_j + 2 \cdots T \quad (13)$$

where L_j is the number of initial periods during which unit j must be online. L_j is mathematically expressed as $L_j = \min\{T, (UT_j - ONT_j^0) u_{jt}\}$, where ONT_j^0 indicates the number of periods unit j has been online before being scheduled. Constraint (11) is related to the initial status of the units, as defined by L_j. Constraint (12) is used for the subsequent periods to satisfy the minimum up-time constraint during all possible set of consecutive periods of size UT_j. Constraint (13) models the final $UT_j - 1$ periods in which, if unit j is started up, it remains online until the end of the time horizon. Analogously, minimum down-time constrains are formulated as follows:

$$\sum_{t=1}^{F_j} u_{jt} = 0, \quad \forall j \quad (14)$$

$$\sum_{k=t}^{t+DT_j-1} (1 - u_{jk}) \ge DT_j \cdot (u_{j,t-1} - u_{jt}), \quad \forall j \in J, \ \forall t = F_j + 1 \cdots T - DT_j + 1 \quad (15)$$

$$\sum_{k=t}^{T} \{1 - u_{jk} - (u_{j,t-1} - u_{jt})\} \ge 0, \quad \forall j \in J, \ \forall t = T - DT_j + 2 \cdots T \quad (16)$$

where F_j is the number of initial periods during which unit j must be offline, where $OFFT_j^0$ is the number of periods unit j has been offline before being scheduled. F_j is mathematically expressed as $F_j = \min\{T, (DT_j - OFFT_j^0)(1 - u_{jt})\}$.

2.4 Thermal Reserves

The maximum available power for the up-spinning reserve of each thermal unit in each period, (17), can be computed as the difference between $\overline{x}_{jt}$ and x_{jt}. Similarly, the maximum available power for the down-spinning reserve is shown in (18) as the difference between x_{jt} and $\underline{x}_{jt}$. The maximum up- and down-reserves cannot be larger than the programmed reserves, (19) and (20). The limits of the reserve contribution of each unit in each period are described in (21) for the up-reserve and in (22) for the down-reserve, respectively.

$$\overline{r}_{jt}^{up} = \overline{x}_{jt} - x_{jt}, \quad \forall j, t \quad (17)$$

$$\overline{r}_{jt}^{dw} = x_{jt} - \underline{x}_{jt}, \quad \forall j, t \quad (18)$$

$$0 \le r_{jt}^{up} \le \overline{r}_{jt}^{up}, \quad \forall j, t \quad (19)$$

$$0 \leq r_{jt}^{dw} \leq \overline{r}_{jt}^{dw}, \quad \forall j,t \tag{20}$$

$$0 \leq \hat{r}_{jt}^{up}(s) \leq r_{jt}^{up}, \quad \forall j,t,s \tag{21}$$

$$0 \leq \hat{r}_{jt}^{dw}(s) \leq r_{jt}^{dw}, \quad \forall j,t,s \tag{22}$$

2.5 *Production, Start-up, and Shut-down Costs*

The production cost is typically expressed as a quadratic function of the power output, while the startup cost is usually modeled as a nonlinear function of the offline time prior the startup (Wood and Wollenberg, 1996). To calculate the production cost in this model, a linearization of the quadratic function is used. The quadratic cost function can be accurately approximated by a set of piecewise blocks. The analytic representation of this linear approximation is:

$$c_{jt}^{p} = A_j u_{jt} \sum_{l=1}^{NL_j} M_{lj} \delta_{ljt}, \quad \forall j,t \tag{23}$$

$$x_{jt} = \sum_{l=1}^{NL_j} \delta_{ljt} + \underline{X}_j u_{jt}, \quad \forall j,t \tag{24}$$

$$0 \leq \delta_{1jt} \leq hx_{1j} - \underline{X}_j u_{jt}, \ \forall j,t \tag{25}$$

$$0 \leq \delta_{ljt} \leq hx_{lj} - hx_{l-1,j}, \forall j \in J, \forall t \in T, \forall l = 2 \cdots NL_j - 1 \tag{26}$$

$$0 \leq \delta_{NL_j,jt} \leq \overline{X}_j - hx_{NL_j-1,j}, \quad \forall j,t \tag{27}$$

where

$$A_j = a_j + b_j \underline{X}_j + c_j \underline{X}_j^{\,2} \tag{28}$$

If enough segments are used, the piecewise linear function is almost equal to the original function. In (23), the slope formula is applied to get the production cost of plant j in each period, depending on the value of δ_{ljt}, which represents the power output of unit j in interval l, and M_{lj} is the slope of each interval.

The startup cost is considered a unique cost in the first period in which unit j is generating power. Also, a shutdown cost is considered when unit j changes from being online to being offline. To simplify the model, a constant startup cost, c_{jt}^{u}, is incurred if unit j is brought online due to the use of fuel. Similarly, this happens when unit j is brought offline, c_{jt}^{d}.

$$0 \leq c_{jt}^{u} \leq SUC_j\left(u_{jt} - u_{j,t-1}\right), \quad \forall j,t \tag{29}$$

$$0 \leq c_{jt}^{d} \geq SDC_j\left(u_{j,t-1} - u_{jt}\right), \quad \forall j,t \tag{30}$$

2.6 *Reversible-Hydro Plant*

The reversible-hydro plant considered is composed of an upper reservoir and a lower reservoir and it is supposed to have a turbine and a pump. The pump-turbine works as a turbine when water is released from the upper reservoir to the lower one, injecting its production into the network. Likewise, when pumping is taking place, the energy consumed is stored in the upper reservoir. Natural inflows in the reservoirs are not considered as the pumped storage unit is supposed to be isolated from the hydro chain. The model is developed from (García-González, et al., 2008).

a) Reservoir Levels

For each reservoir, the water balance equation (expressed in terms of energy) must be satisfied: (33) for the upper reservoir, v_t^u, and (34) for the lower reservoir, v_t^l. Also, both reservoirs must satisfy their capacity limits, (31) and (32). Constraints (35) and (36) limit the reservoirs to a minimum capacity at the end of the time horizon equal to the initial values, v^{u0} and v^{l0}.

$$\underline{v}^{u} \leq v_t^{u} \leq \overline{v}^{u} \tag{31}$$

$$\underline{v}^{l} \leq v_t^{l} \leq \overline{v}^{l} \tag{32}$$

$$v_t^{u} = v_{t-1}^{u} + \eta_{pu} \cdot x_{pt} - x_{nt}/\eta_n, \quad \forall t \tag{33}$$

$$v_t^{l} = v_{t-1}^{l} + x_{nt}/\eta_n - \eta_p \cdot x_{pt}, \quad \forall t \tag{34}$$

$$v_{t=0}^{u} = v_{t=T}^{u} = v^{u0}, \tag{35}$$

$$v_{t=0}^{l} = v_{t=T}^{l} = v^{l0}, \tag{36}$$

b) Pump and Turbine Limits

The pumping and turbine capacities are limited by the pump-turbine characteristics. In the case of the pump, the pumping limitation is given by the minimum and maximum limits per hour (37). The turbine is limited by the minimum and maximum power generation per hour (38). Finally, (39) guarantees that the pump-turbine plant does not work simultaneously as a pump and a turbine.

$$\underline{X}_p u_{pt} \leq x_{pt} \leq \overline{X}_p u_{pt}, \quad \forall t \tag{37}$$

$$\underline{X}_n u_{nt} \leq x_{nt} \leq \overline{X}_n u_{nt}, \quad \forall t \tag{38}$$

$$u_{pt} + u_{nt} \leq 1, \quad \forall t \tag{39}$$

c) Reserves

Similar to the thermal units, the reversible-hydro plant either provides up- or down-reserves, depending on the working state of the turbine or the pump in a certain period. The maximum available power output that can be used as reserve, $\overline{r}_{nt}^{up}$, is lower than the upper reservoir level in the previous period (40). In the case of the pump, the maximum available input power that can be used as a down reserve, $\overline{r}_{pt}^{dw}$, is lower than the level of the lower reservoir in the previous period (41). Taking this into account, $\overline{r}_{nt}^{up}$, can be defined as the maximum power generation per hour, $\overline{X}_n$, minus the power generation of the turbine in period t (42). In the same way, the maximum pump power consumption per hour, $\overline{r}_{pt}^{dw}$, is equal to the maximum pumping limit per hour minus the pumping power in period t (43). It is noted that, unlike thermal units, both the turbine and the pump can start-up or shut-down almost instantly, so it is not needed to be online in period t to generate or consume power as reserves. In this case, programmed reserves do not exist. However, the turbine is neither allowed to start-up if the pump is online nor the pump is allowed to start-up if the turbine is online. The limits of the reserve contribution of each unit in period t are described in (44) for the turbine, and in (45) for the pump, respectively.

$$\overline{r}_{nt}^{up} \leq v_{t-1}^{u}, \quad \forall t \tag{40}$$

$$\overline{r}_{pt}^{dw} \leq v_{t-1}^{l}, \quad \forall t \tag{41}$$

$$\overline{r}_{nt}^{up} = \overline{X}_n\left(1 - u_{pt}\right) - x_{nt}, \quad \forall t \tag{42}$$

$$\overline{r}_{pt}^{dw} = \overline{X}_p\left(1 - u_{nt}\right) - x_{pt}, \quad \forall t \tag{43}$$

$$0 \leq \hat{r}_{nt}^{up}(s) \leq \overline{r}_{nt}^{up}, \quad \forall t \tag{44}$$

$$0 \leq \hat{r}_{pt}^{dw}(s) \leq \overline{r}_{pt}^{dw}, \quad \forall t \tag{45}$$

d) Start-up and Shut-down Costs of the Turbine and Pump

The start-up cost for each unit is considered to be $c_t^u = 2.5\overline{X}$, as proposed in (Nilsson and Sjelvgren, 1997) and the shut-down cost has been fixed to be 10% of the start-up cost.

$$0 \le c_{nt}^u \le 2.5 \cdot \overline{X}_n(u_{nt} - u_{n,t-1}), \quad \forall t \tag{46}$$

$$0 \le c_{pt}^u \le 2.5 \cdot \overline{X}_p(u_{pt} - u_{p,t-1}), \quad \forall t \tag{47}$$

$$0 \le c_{nt}^d \le 0.1 \cdot 2.5 \cdot \overline{X}_n(u_{n,t-1} - u_{nt}), \quad \forall t \tag{48}$$

$$0 \le c_{pt}^d \le 0.1 \cdot 2.5 \cdot \overline{X}_p(u_{p,t-1} - u_{pt}), \quad \forall t \tag{49}$$

e) Production Cost of the Pump

The production cost of the pump is equal to the power of the pump multiplied by the fuel cost.

$$c_{pt}^p = F^{cost} \cdot x_{pt}, \quad \forall t \tag{50}$$

3. CASE STUDY

3.1 Input Data

The electric system of San Miguel Island, Azores, has a generation mix composed of diesel plants, a reversible-hydro power plant and a wind farm, with an annual peak load of 70 MW. There are 6 diesel plants installed in the island, comprising a total of 94 MW, $\overline{v}^u = \overline{v}^l$ =10MWh of reservoirs and 9 MW of wind power installed capacity. The simulation is run using 100 scenarios forming a scenario tree considering 25 wind power forecasts and 4 demand forecasts for the month of September, 2013. These forecasts have been provided by Smartwatt (SMARWATT, n.d.). The technical and economic characteristics are described in the following tables. In table 1, the maximum and minimum power of thermal units and the parameters of their cost production functions are defined, whereas the costs of reserves and loss of spinning reserves are shown in table 2.

Table 1. Technical and economic characteristics of the thermal units

	P1, P6	P2	P3	P4, P5
Max. Power (MW)	15	15	25	12
Min. Power (MW)	7	6	14	4
a ($/h)	85.74	85.74	108.98	44.39
b($/MWh)	22.46	22.46	22.53	13.20
c($/MW^{2}h)	0.603	0.603	0.214	0.514

Due to the small capacity of the thermal plants, the start-up cost is defined as one tenth of the cost of producing at the maximum power, and the shut-down cost is defined as one tenth of the cost of producing at the minimum power.

Table 2. Cost of reserves power

Up programmed	$(c_{jt}^{r,up})$	($/MWh)	20
Dw programmed	$(c_{jt}^{r,dw})$	($/MWh)	20
Up-reserve	$(\hat{c}_{jt}^{r,up})$	($/MWh)	80
Dw-reserve	$(\hat{c}_{jt}^{r,dw})$	($/MWh)	100
Loss spin. reserve	$(\hat{c}_t^{lr})$	($/MWh)	500

3.2 Simulation

This model has been tested for 24 hours. The MILP optimization model (1)-(50) has been simulated with CPLEX solver in GAMS 24.0 (Brooke, et al., 2003) and MATLAB R2012a (MATLAB, n.d.).

This work is mainly focused on the analysis of reserves of the electric system when a high penetration of non dispatchable renewable energy (wind) exists in an isolated area.

Fig. 1 shows the influence of reserves on the electric system depending on the scenarios. The multiple lines correspond to the different scenarios for both up- and down-reserves.

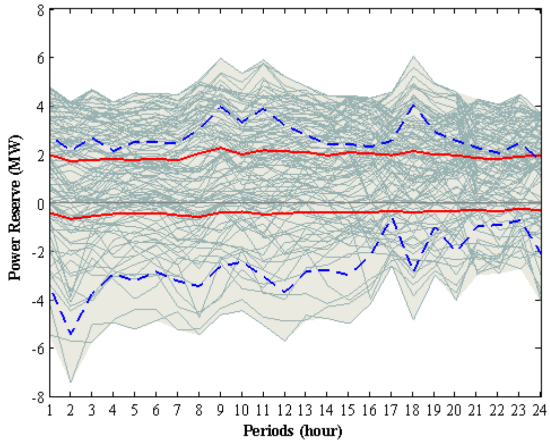

Fig. 1. Power reserves.

Note that in a particular scenario and period, only one of the reserves (up-reserve or down-reserve) can be different from zero. These lines plot the final used reserves for each scenario in order to solve the imbalances between the programmed generation (including wind power) and the load: $\hat{r}_{jt}^{up}(s) + \hat{r}_{nt}^{up}(s)$ for the up-reserve, and $\hat{r}_{jt}^{dw}(s) + \hat{r}_{pt}^{dw}(s)$ for the down-reserve. The thick continuous lines are the mean values of the up- (positive) and down-reserves (negative) of all scenarios and the discontinuous lines represent the values of the programmed thermal reserves in each period.

In Fig. 2, the programmed reserves are evaluated for several pump and turbine hourly power limits. It is observed that the more pumping and turbine, the less the thermal programmed reserves are needed, decreasing the total cost of the system.

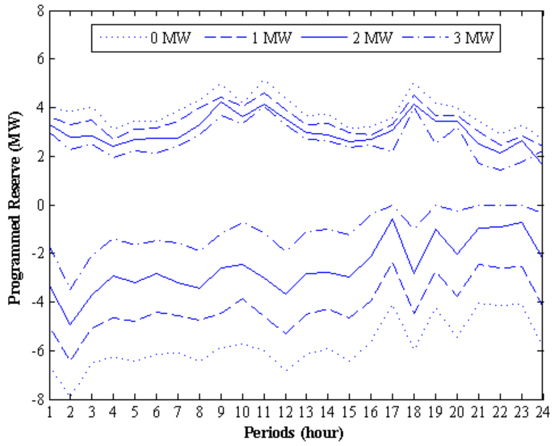

Fig. 2. Comparison of programmed reserves.

The programmed reserves are scheduled before knowing the real wind power and the real demand, and these reserves have to compensate for all the imbalances. Thus, when the power of the reversible-hydro plant is close to zero, all the reserves needed in each scenario are provided by thermal units.

However, if the power of the reversible-hydro plant increases, it is possible to use the turbine and the pump to provide reserves, which produces a lower production cost, as can be seen in Fig. 3.

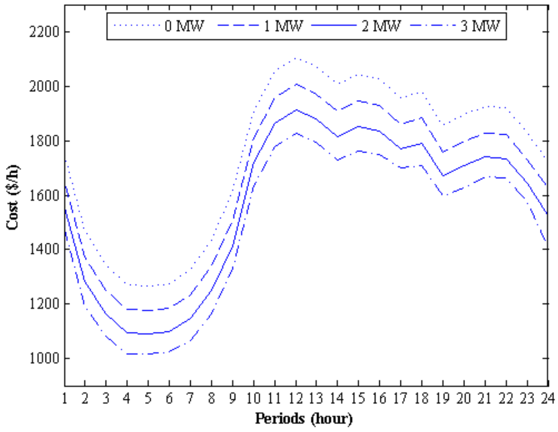

Fig. 3. Comparison of generation total costs for several capacities of the reversible-hydro plant.

Note that there is a variable, $lr_t(s)$, used to quantify the amount of energy that cannot be supplied. The value of this variable depends on the cost associated with it, $\hat{c}_t^{lr}$. This cost is different for each model or energy system. Some systems can sustain an energy loss by reducing the demand or disconnecting certain loads, but others cannot.

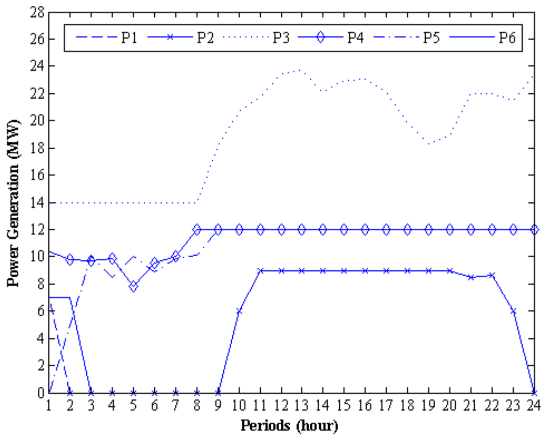

Fig. 4. Scheduled power generation of the thermal units.

In the first case, if the cost of providing reserves for all scenarios is high, it is possible to reduce $\hat{c}_t^{lr}$ and, if one of the most unfavorable scenarios occurs, the generation will be lower than the demand, and the imbalance will be solved by disconnecting the load. Nevertheless, if the system cannot sustain energy losses, the value of $\hat{c}_t^{lr}$ will be much higher, the reserves will be greater, and the demand will be satisfied.

Fig. 4 shows the scheduled power production of each thermal plant. In the simulation, the turbine and the pump of the reversible-hydro plant do not take part in the normal scheduling of the UC. Since wind power represents a high percentage of the load and the power of the reversible-hydro plant is smaller than the imbalance in many scenarios, this generation technology is only used to provide reserves. Note that the start-up and shut-down of the turbine and the pump are instantaneous and they can make the system more flexible and respond quickly to imbalances.

4. CONCLUSIONS

In this paper we solve a stochastic MILP Unit Commitment problem using conventional and renewable generation, where wind power and demand are stochastic. The model is mainly focused on the optimal deployment of reserves in an island.

The amount of programmed reserves of thermal units decreases as the reversible-hydro plant capacity increases. As a consequence of the reduction of the programmed reserves, the total cost of the electric system is lower.

Depending on the demand response to energy curtailments, it is possible to reduce the total cost by not allowing that all demand requirements in some scenarios are met.

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