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Distributed Sensor Fault Detection and Isolation over Network

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Abstract: Consensus and diffusion based observers have been developed to address distributed state estimation over a sensor network. In this paper, the way sensor faults affect the state estimation of such distributed state observers is investigated. Next, a methodology to design a distributed system for sensor fault detection and isolation (FDI) is proposed. The aim is to achieve a specified fault detection (FD) performance while limiting the complexity of the algorithm at each node of the sensor network. To this end, proper selection of the measurements used for FDI at each node is performed thanks to a fault distinguishability measure based on the Kullback-Leibler divergence.

1. INTRODUCTION

State estimation over distributed architectures has been the object of an important research effort in the recent years as surveyed in Farina et al. [2010]. The applications of such sensor networks are numerous in environmental monitoring, domotics and structural health monitoring notably. Yet to be able to build a reliable state estimate, one has to make sure that the measurements issued by the different sensors are fault free. Therefore, our aim is to build a distributed fault detection and isolation (FDI) system that could be combined with a distributed state observer in order to provide reliable state estimation. The methodology should ensure that the FDI system performance can be characterized in terms of false detection and isolation probability and correct detection and isolation probability and that the computing load at each node is kept as small as possible.

In the present work, the focus lies on the situation where each of the sensor node estimates the whole state vector of the monitored system. Among the approaches considered in this framework one can distinguish distributed Kalman filter based on consensus on measurements Olfati-Saber [2005], on consensus on measurements and estimates Olfati-Saber [2007], on consensus on estimates with optimality properties Olfati-Saber [2009], as well as diffusion Kalman filters Cattivelli and Sayed [2010]. All these approaches involve exchanging measurements and possibly local state estimates between the different nodes. This exchange of information also results in a propagation of sensor faults over the whole sensor network, which can be seen as a drawback of these approaches. This problem motivates the use of specific FDI systems for sensor faults in this context. The output of the FDI system is expected to induce a reconfiguration of the distributed state observer in order to eliminate the faulty measurement or to correct it.

Distributed fault diagnosis over sensor networks has been the object of several research works. The methods can be classified according to the prior information each node has about the system model. Two main classes of FDI problems are presently under investigation: distributed FDI problems and partition-based FDI problems. In the first category, each node has the full knowledge of the whole system model while in the second category each node only has information on a subsystem and the relation with its neighbors. The problems stated in Franco et al. [2006] and Reppa et al. [2012] correspond to the distributed FDI problems while Davoodi et al. [2012] and Reppa et al. [2013] solved partition-based FDI problems. In Franco et al. [2006], a distributed fault diagnosis scheme is proposed for detecting and isolating faults which induce a change of dynamics. The dynamics of each node is the same and a bank of Kalman filters is embedded into each node to estimate the state. Then consensus filters are used to synchronize the estimates at each node for the purpose of fault diagnosis. The scheme can detect and isolate the process faults under the assumption that the possible models of faulty nodes are known. But sensor fault diagnosis is not considered in this paper. In Reppa et al. [2012], a sensor fault diagnosis algorithm is proposed in which each node monitors the whole non-linear system. Nonlinear observers are built with adaptive threshold for fault detection and isolation but a hierarchical center is still needed for decision, so this method is not completely distributed. A set of non-homogeneous nodes is considered in Davoodi et al. [2012] in which each node has its own linear dynamical system. A model of each node is built according to all the measurement information from its neighbors and itself. Next an optimal FDI filter is designed with this model by solving a set of linear matrix inequalities. However the hypotheses behind the problem statement strongly limit the range of applications. Indeed, it is in particular assumed that there is no coupling between the dynamics associated to each node. In Reppa et al. [2013], a multiple sensor fault detection and isolation scheme is developed for interconnected nonlinear subsystems. Each subsystem has a sensing system and a fault diagnosis observer which does not exchange information with its

neighbors. Therefore, this strategy can only solve the FDI problem of the local subsystem, and it can not use the sensing information from its neighbors for reconfiguration.

Here we depart from the previous works in the following way:

- A parity space approach combined with a statistical change detection and isolation algorithm monitors the sensors of each node.
- The FDI system uses as little information as possible from the neighboring nodes in order to limit computation burden while ensuring the required performance.
- The measurements used at each node are selected on the basis of a recently proposed approach based on Kullback-Leibler divergence in Eriksson et al. [2013].

The paper is organized as follows. In section 2, the propagation of sensor faults in distributed observer schemes is illustrated in the case of the diffusion Kalman filter. In section 3, a centralized FDI scheme based on the parity space approach and log-likelihood ratio tests is reviewed and its performance is characterized. Next, the proposed approach for distributed FDI system design is described and illustrated in simulation respectively in section 4 and section 5.

2. FAULT PROPAGATION IN A DIFFUSION KALMAN FILTER

Let us consider the following discrete-time dynamic system

$$x(k+1) = Ax(k) + B_u u(k) + B_v v(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$and $v(k) \in \mathbb{R}^{n_v}$ is a zero mean white noise sequence with variance Q. The initial state $x(0)$ is assumed to be a zero mean random vector with variance $\Pi_0 > 0$. It is supposed to be uncorrelated with $v(k)$. The system is monitored by a sensor network of M nodes spatially distributed over some region. This network is characterized by its adjacency matrix W, a $M \times M$ matrix with Boolean entries. The $(i,j)^{th}$ entry is equal to 1 if nodes i and j are connected, namely if they can communicate directly with each other. Otherwise, the entry is equal to 0. The neighborhood of a given node, say i, is denoted by $\mathcal{N}_i$. It is defined as the set of nodes that are directly connected to node i. Each node is characterized by its measurement equation:

$$y_i(k) = C_i x(k) + D_{u,i} u(k) + f_i(k) + \varepsilon_i(k) \tag{2}$$

where $y_i(k) \in \mathbb{R}^{p_i}$ is the measurement vector at node i, $f_i(k) \in \mathbb{R}^{p_i}$ is the fault vector and $\varepsilon_i(k)$ is a zero mean white noise sequence with variance R_i. It is assumed that $\varepsilon_i(k)$ and $\varepsilon_j(l)$ are mutually uncorrelated for any i, j, k, l with $i \neq j$ and/or $k \neq l$, and the measurement noise is uncorrelated with the process noise, as well as with the initial state $x(0)$.

In order to present the algorithm of the diffusion Kalman filter for system (1) equipped with the sensor network characterized by (2) together with matrix W, let us introduce the $M \times M$ diffusion matrix $\mathbf{Z}$ with the following properties $\mathbf{1}^{\mathrm{T}}\mathbf{Z} = \mathbf{1}^{\mathrm{T}}$ with $z_{i,j} = 0$ if $i \notin \mathcal{N}_j$ and $0 \leqslant z_{i,j} \leqslant 1$ otherwise. Here $\mathbf{1}$ is the M dimensional vector with entries equal to 1 and $z_{i,j}$ denotes the $(i,j)^{th}$ entry of $\mathbf{Z}$. The latter represents weights used in the algorithm of the diffusion Kalman filter to combine estimates from a given neighborhood. This algorithm can now be described as in Cattivelli and Sayed [2010]:

Initialization of the on-line algorithm at node i:

$$\hat{x}_i(0 \mid -1) = 0, P_i(0 \mid -1) = \Pi_0$$

At every time instant k, compute at every node i:

Step 1: Incremental measurement update

$$\psi_i(k) \leftarrow \hat{x}_i(k \mid k-1)$$
$$P_i(k) \leftarrow P_i(k \mid k-1)$$

Then for every neighboring node $l \in \mathcal{N}_i$, repeat

$$Re \leftarrow R_i + C_i P_i(k) C_i^{\mathrm{T}}$$
$$\psi_i(k) \leftarrow \psi_i(k) + P_i(k) C_i^{\mathrm{T}} Re^{-1} \left(y_l(k) - C_l \psi_i(k) - D_l u(k)\right)$$
$$P_i(k) \leftarrow P_i(k) - P_i(k) C_l^{\mathrm{T}} Re^{-1} C_l P_i(k)$$

Step 2: Diffusion update

$$\hat{x}_i(k \mid k) \leftarrow \sum_{l \in \mathcal{N}_i} z_{i,l} \psi_l(k)$$
$$P_i(k \mid k) \leftarrow P_i(k)$$

$$\hat{x}_i(k+1 \mid k) = A\hat{x}_i(k \mid k) + B_u u(k)$$
$$P_i(k+1 \mid k) = AP_i(k \mid k)A^{\mathrm{T}} + B_v Q B_v^{\mathrm{T}}$$

The convergence of this algorithm to an unbiased state estimate is ensured provided $\left(A, B_v Q^{\frac{1}{2}}\right)$ is stabilizable and $\left(C_i^{loc}, A\right)$ is detectable for all $i = 1, \ldots, M$. Here, $C_i^{loc} = \left[C_{i_1}^{\mathrm{T}}, \ldots, C_{i_{n_i}}^{\mathrm{T}} \right]^{\mathrm{T}}$ where nodes $i_1, \ldots, i_{n_i}$ belong to $\mathcal{N}_i$.

Upon occurrence of a fault at node i, namely the appearance of a non zero entry in $f_i(k)$, the state estimate at the nodes can become biased. Since both the considered system and the diffusion Kalman filter are linear, the effect of a fault can be characterized by an additive term in the state estimation error, $\widetilde{x}_i(k \mid k) = x(k) - \hat{x}_i(k \mid k)$. Namely,

$$\widetilde{x}_i(k \mid k) = \widetilde{x}_{i,0}(k \mid k) + \beta_i(k) \tag{3}$$

where $\widetilde{x}_{i,0}(k \mid k)$ is the zero mean fault free estimation error, and $\beta_i(k)$ is the bias introduced by the fault. An expression for $\beta_i(k)$ is provided in Kinnaert and Hao [2014]. Our aim is to develop a diagnostic system to be implemented at each node in order to detect and isolate sensor fault at this node. The corresponding faulty measurements should then be eliminated or propagation of the error should be avoided possibly by retuning the weighting matrix $\mathbf{Z}$.

3. CENTRALIZED FDI SYSTEM

In this section we briefly review the work presented in Eriksson et al. [2013] and propose a statistical change detection and isolation scheme which is compatible with the approach. The considered setting is based on a standard linear system instead of a descriptor system as in Eriksson et al. [2013] to keep coherency with section 2.

3.1 Problem Statement

Let us consider the following state space model:

$$\begin{cases} x(k+1) = Ax(k) + B_u u(k) + B_v v(k) \\ y(k) = Cx(k) + D_u u(k) + f(k) + \varepsilon(k) \end{cases} \quad (4)$$

The model can be reformulated by considering a fixed time horizon N and introducing the following vectors characterizing the system behavior over this time horizon:

$$z^N(k) = [y(k-N+1)^{\mathrm{T}},\ldots,y(k)^{\mathrm{T}},u(k-N+1)^{\mathrm{T}},\ldots,u(k)^{\mathrm{T}}]^{\mathrm{T}}$$
$$x^N(k) = [x(k-N+1)^{\mathrm{T}},\ldots,x(k)^{\mathrm{T}},x(k+1)^{\mathrm{T}}]^{\mathrm{T}}$$
$$f^N(k) = [f(k-N+1)^{\mathrm{T}},\ldots,f(k)^{\mathrm{T}}]^{\mathrm{T}}$$
$$e^N(k) = [v(k-N+1)^{\mathrm{T}},\ldots,v(k)^{\mathrm{T}},\varepsilon(k-N+1)^{\mathrm{T}},\ldots,\varepsilon(k)^{\mathrm{T}}]^{\mathrm{T}}$$

where $z^N(k) \in \mathbb{R}^{\mathrm{N(p+m)}}$,$x^N(k) \in \mathbb{R}^{\mathrm{(N+1)n}}$,$f^N(k) \in \mathbb{R}^{\mathrm{Np}}$ and $\left\{e^N(k)\right\}$ is a Gaussian zero mean random sequence with known variance (directly deduced from the variance of the state and measurement noise sequences). The variance of $e^N(k)$ will be denoted Λ_e.

Then a sliding window model can be written:

$$Lz^N(k) = Hx^N(k) + Ff^N(k) + Ee^N(k) \quad (5)$$

where

$$L = \begin{bmatrix} 0_{nN\times pN} \vdots - I_N \otimes B_u \\ I_N \otimes I_p \vdots - I_N \otimes D_u \end{bmatrix}$$
$$H = \begin{bmatrix} H_{up} \\ I_N \otimes C \vdots 0_{pN\times n} \end{bmatrix}$$
$$F = \begin{bmatrix} 0_{Nn\times Np} \\ I_N \otimes I_p \end{bmatrix}$$
$$E = \begin{bmatrix} I_N \otimes B_v \vdots 0_{Nn\times Np} \\ 0_{Np\times n_v} \quad \vdots I_N \otimes I_p \end{bmatrix}$$

with $H_{up} = \left[I_N \otimes A \vdots 0_{Nn\times n} \right] - \left[0_{Nn\times n} \vdots -I_N \otimes I_n \right]$. In these expressions $\otimes$ denotes matrix Kronecker product and $0_{l\times q}$ denotes an $l \times q$ matrix with null elements.

A fault on the i^{th} sensor is represented by a non-zero i^{th} component in $f(k)$. The term associated to the fault in (5) will be rewritten as follows in this situation.

$$Ff^N(k) = F_i f_i^N(k)$$

where $f_i^N(k) = [f_i(k-N+1),\ldots,f_i(k)]^{\mathrm{T}}$ with $f_i(k)$ denoting the i^{th} component of $f(k)$, and F_i is made of the relevant columns of F (columns $i, p+i, \ldots, (N-1)\,p+i$). In the sequel a vector $\theta_i \in \Theta_i \subset \mathbb{R}^{\mathrm{N}}$ will be used to represent an admissible fault time profile of $f_i^N(k)$.

Our aim is to detect and isolate sensor faults on the basis of this sliding window model.

3.2 Residual generation

To generate a residual signal, an input-output model is determined from (5) by multiplying this equations on the left by N_H, a matrix whose rows make an orthonormal basis of the left null space of H. This yields:

$$N_H Lz^N(k) = N_H Ff^N(k) + N_H Ee^N(k) \quad (6)$$

In order to ease the development to follow, a normalization of (6) is performed so that the covariance matrix of the noise term is the identity matrix. This can be achieved as follows.

Let Σ be defined as: $\Sigma = N_H E \Lambda_e E^{\mathrm{T}} N_H^{\mathrm{T}}$ and determine a matrix Γ, by Cholesky factorization for instance, such that $\Sigma = \Gamma\Gamma^{\mathrm{T}}$. Then it is straightforward to show that left multiplication of (6) by Γ^{-1} yields the desired normalization. The resulting residual vector can be written:

$$\begin{aligned} r(k) &= \Gamma^{-1} N_H L z^N(k) \\ &= \Gamma^{-1} N_H F f^N(k) + \Gamma^{-1} N_H E e^N(k) \end{aligned}$$

Letting $\overline{L} = \Gamma^{-1} N_H L$, $\overline{F} = \Gamma^{-1} N_H F$, $\overline{E} = \Gamma^{-1} N_H E$, these expressions can be rewritten:

$$r(k) = \overline{L} z^N(k) = \overline{F} f^N(k) + \overline{E} e^N(k)$$

The probability law of $r(k)$ can be described as follows in fault free mode (hypothesis $\mathcal{H}_0$) and in the i^{th} faulty mode (hypothesis $\mathcal{H}_{\mathrm{i}}$) :

$$\begin{aligned} \mathcal{H}_0 &: r(k) \sim \mathcal{N}(0, I_{n_r}) \\ \mathcal{H}_{\mathrm{i}} &: r(k) \sim \mathcal{N}(\overline{F}_i \theta_i, I_{n_r}) \end{aligned} \quad (7)$$

where $\overline{F}_i$is made of column $i, i+p, \ldots, (N-1)p+i$ of $\overline{F}$.

In Eriksson et al. [2013], the possibility to distinguish a fault i with given profile θ_i from a fault j with profile $\theta_j \in \Theta_j$ has been quantified thanks to the Kullback-Leibler divergence. This notion has also proved useful for the design of robust residual generators Romano and Kinnaert [2006a], Romano and Kinnaert [2006b]. The main results on distinguishability are reviewed next.

3.3 Distinguishability between two faults

The Kullback-Leibler divergence between the pdfs of the residual in faulty modes i and j is defined as:

$$K\left(p^i \parallel p^j\right) = \int_{-\infty}^{+\infty} \log p^i(r) \frac{p^i(r)}{p^j(r)} dr = E_{p^i}\left[\log \frac{p^i}{p^j}\right]$$

where $p^i(r)(p^j(r))$ denotes the pdf of the residual vector under fault i (fault j) and $E_{p^i}(.)$ is the expected value when the pdf of the residual corresponds to p^i.

More precisely, distinguishability $\mathcal{D}_{ij}(\theta_i)$ of a fault f^i with given fault time profile θ_i from fault mode f^j is defined as

$$\mathcal{D}_{ij}\left(\theta_i\right) = \min_{\theta_j \in \mathbb{R}^{\mathrm{N}}} K\left(p^i_{\theta_i} \parallel p^j_{\theta_j}\right)$$

where in our problem setting $p^i_{\theta_i}$ is the pdf associated to the Gaussian distribution $\mathcal{N}\left(\overline{F}_i\theta_i, I_{n_r}\right)$ with n_r denoting the dimension of the residual vector.

In particular, distinguishability from the fault free case, characterized by the pdf p_0 is defined as $\mathcal{D}_{i,0}\left(\theta_i\right) = K\left(p^i_{\theta_i} \parallel p_0\right)$, and $\mathcal{D}_{ij}\left(\theta_i\right) \leq \mathcal{D}_{i,0}\left(\theta_i\right)$.

The following result provides a straightforward way to compute the distinguishability for the sliding window model (6) (see Eriksson et al. [2013] and appendix):

$$\mathcal{D}_{ij}(\theta_i) = \frac{1}{2} \parallel N_{\left[\overline{H}\ \widetilde{F}_j\right]}\ \widetilde{F}_i\theta_i \parallel^2 \tag{8}$$

where $\Gamma^{-1}N_H = N_{\overline{H}}\overline{\Gamma}$ with $N_{\overline{H}}$ an orthonormal matrix and $\widetilde{F}_j = \overline{\Gamma}F_j$, $\widetilde{F}_i = \overline{\Gamma}F_i$, $N_{\overline{H}}\overline{H} = 0$ where $\overline{H}$ is an orthonormal basis for the null space of $N_{\overline{H}}$. $N_{\left[\overline{H}\ \widetilde{F}_j\right]}$ is an orthonormal basis for the left null space of $\left[\overline{H}\ \widetilde{F}_j\right]$.

This measure of fault diagnosibility will be used in the next section to determine the best structure for the distributed fault diagnosis system. θ_i is typically a design parameter that can be chosen as a constant vector whose entries correspond to the fault with the smallest magnitude to be detected.

Let us now turn to the design of the statistical change detection and isolation algorithm to be used for residual processing.

3.4 Decision system

For fault detection, Neyman-Pearson lemma indicates that choosing between hypothesis $\mathcal{H}_0$ and $\mathcal{H}_\mathrm{i}$ in (7) can be performed through the test Basseville and Nikiforov [1993]:

$$\Lambda_{i,0}(\theta_i) = \log\frac{p^i_{\theta_i}(r)}{p_0(r)} = \theta_i^{\mathrm{T}}\overline{F}_i^{\mathrm{T}}\left(r - \frac{1}{2}\overline{F}_i\theta_i\right) \gtrless h_d$$

where h_d is the detection threshold. When $\Lambda_{i,0}(\theta_i) > h_d$, the decision $\mathcal{H}_\mathrm{i}$ is issued, otherwise $\mathcal{H}_0$ is issued.

With regards to fault isolation, a GLR approach is considered, in accordance with the distinguishability measure introduced before, namely:

$$\begin{aligned}\Lambda_{i,j}(\theta_i) &= \log\frac{p^i_{\theta_i}(r)}{\max\limits_{\theta_j} p^j_{\theta_j}(r)}\\ &= -\frac{1}{2}\left(r - \overline{F}_i\theta_i\right)^{\mathrm{T}}\left(r - \overline{F}_i\theta_i\right)\\ &\quad + \frac{1}{2}r^{\mathrm{T}}\left(I - \overline{F}_j\left(\overline{F}_j^{\mathrm{T}}\overline{F}_j\right)^{-1}\overline{F}_j\right)r\end{aligned}$$

The test will then detect and isolate fault i when

$$\Lambda_{i,0}(\theta_i) > h_d \ and \min_{j\neq i}\Lambda_{i,j}(\theta_i) > h_{isol} \tag{9}$$

where h_{isol} is an isolation threshold.

This test is known to be optimal in the case of simple hypotheses Nikiforov [1998]. The properties of the change detection and isolation algorithm (9) in terms of probability of false alarm, false isolation, correct detection and isolation can be computed by Monte Carlo simulations on the basis of the known distribution of the residual. The Receiver Operating Characterized (ROC) curve associated to (9) can thus be estimated, allowing to set h_d and h_{isol} at best for a given horizon N of the residual. Notice that increasing the horizon will normally increase the power of the test for a given false detection and isolation probability. Yet this also increases the delay for detection if a sliding window is considered for on-line implementation. We now turn to the development of a method to design a distributed diagnostic system on the basis of the above described tools.

4. DISTRIBUTED SENSOR FAULT DETECTION AND ISOLATION

Our aim is to ensure that each node is able to detect any malfunction of its own sensors with a given level of performance. The latter is specified for the i^{th} node, by a lower bound $\underline{\beta_i}$ on the probability of correct detection, and an upper bound $\overline{\alpha}_i$ on the probability of false detection. In order to achieve this goal, when the instrumentation allows for it, the minimum required data from neighboring nodes will be used, so that the complexity of the FDI system at each node is the lowest possible.

Remember that the following model corresponds to the measurements available at node i:

$$\begin{cases} x(k+1) = Ax(k) + B_u u(k) + B_v v(k)\\ y_i(k) = C_i x(k) + D_{u,i}u(k) + f(k) + \varepsilon_i(k)\end{cases} \tag{10}$$

Let $\mathcal{Y}_i$ denotes the set of measurements available in the neighborhood of node i, namely, $\mathcal{Y}_i = \{y_{jl}\ s.t.\ y_{jl} \in \mathcal{N}_i\}$. Here, y_{jl} stands for the j^{th} measurement at node l.

Then our aim is to determine a vector $\overline{y}_i(k)$ with the lowest dimension whose components belong to $\mathcal{Y}_i$, and such that any sensor fault at node i can be detected with the required performance $\overline{\alpha}_i$ and $\underline{\beta_i}$ on the basis of the following model:

$$\begin{aligned}&x(k+1) = Ax(k) + B_u u(k) + B_v v(k)\\ &\begin{bmatrix} y_i(k)\\ \overline{y}_i(k)\end{bmatrix} = \begin{bmatrix} C_i\\ \overline{C}_i\end{bmatrix}_{x(k)} + \begin{bmatrix} D_{u,i}\\ \overline{D}_{u,i}\end{bmatrix}_{u(k)} + \begin{bmatrix} f_i(k)\\ \overline{f}_i(k)\end{bmatrix} + \begin{bmatrix} \varepsilon_i(k)\\ \overline{\varepsilon}_i(k)\end{bmatrix}\end{aligned} \tag{11}$$

This problem statement is motivated by the fact that increasing the number of measurements can only increase the distinguishability of a fault *w.r.t.* the no fault situation Kinnaert and Hao [2014].

To determine the components of $\overline{y}_i$, an iterative approach is used. First a FDI system is built on the basis of model (10) for each node, using the approach of section 3.2 and 3.4, and its performance is evaluated. This amounts to checking whether the probability of correct fault detection, β_i^q, for sensor q at node i fulfills $\beta_i^q > \underline{\beta_i}$ for $q = 1, \cdots, p_i$, and the probability of false alarm, α_i^q, fulfills $\alpha_i^q < \overline{\alpha}_i$. If these inequalities are not verified, one looks for the additional measurement that improves the most the lowest fault detectability.

To explain the selection procedure, let us introduce the notation $\mathcal{D}^{i,s}_{q,0}(\theta_q)$ for the distinguishability between fault q ($q = 1, \cdots, p_i$) and the no fault case at node i, based on model (11), when $\overline{y}_i(k)$ is made of s components. Then the additional measured variable $y_{jl} \in \mathcal{N}_i$ which makes the first component of $\overline{y}_i(k)$ is determined by solving

$$\max_{y_{jl}\in\mathcal{N}_i}\min_{q=1,\cdots,p_i}\mathcal{D}^{i,1}_{q,0}(\theta_q) \tag{12}$$

More generally if the performance is not fulfilled at the s^{th} iteration, namely when $\overline{y}_i$ is made of s components, the additional measured variable y_{jl} is determined by solving:

$$\max_{y_{jl}\in\mathcal{P}_i^{s+1}} \min_{q=1,\cdots,p_i} \mathcal{D}_{q,0}^{i,s+1}(\theta_q) \tag{13}$$

where $\mathcal{P}_i^{s+1} = \mathcal{N}_i \setminus \{\overline{y}_{i,1}, \cdots, \overline{y}_{i,s}\}$ is the set of measured variables that are still available for inclusion in $\overline{y}_i$. Here $\overline{y}_{i,l}$ denotes the l^{th} component of $\overline{y}_i$. The algorithm stops for node i either when $\beta_i^q > \underline{\beta_i}$ and $\alpha_i^q < \overline{\alpha}_i$ $q = 1, \cdots, p_i$ or when $\mathcal{P}_i^{s+1} = \emptyset$.

5. SIMULATION

A sensor network made of 3 mutually connected nodes is considered. The adjacency matrix of this network is thus $W = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. This network provides noisy measurements of the position of a projectile. The states variables of the system are the 3 components of the velocity and the position of the projectile. At node 1 and node 3, the two components associated with the displacement in the horizontal directions are measured while at node 2, one horizontal and the vertical displacement are measured. The system can be described as:

$$\begin{cases} x(k+1) = Ax(k) + B_u u(k) + v(k) \\ \qquad y_i(k) = C_i x(k) + \varepsilon_i(k) \qquad\quad k = 1, ..., N \end{cases} \tag{14}$$

where $A = \begin{bmatrix} I_3 & 0 \\ 0.1 * I_3 & I_3 \end{bmatrix}$,$B_u = 0.1 * I_6$,$C_1 = C_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. The state noise and measurement noise are white noise sequences with respective covariance $Q = 0.1 * I_6$ and $R = 0.1 * I_2$. The simulations are performed for an initial state $x_0 = [10; 2; 8; 0.1; 0.1; 0.1]$ and a constant input $u = [0; 0; 0; -10; 0; 0]$ which corresponds to the effect of gravity on the projectile. The window size used for residual generation is set to $N = 6$, and the fault profile is set to $\theta = [0; 0; 1; 1; 1; 1]$ for all sensors.

The results presented next correspond to the application of the procedure described in section 4 for the selection of the sensors of the FDI system at node 1. The specifications impose $\overline{\alpha}_1 = 0.05$ and $\underline{\beta_1} = 0.9$. Figures 1 and 2 show, in green, the ROC curves corresponding to a FDI system that is based on the sensors at node 1 only. The specifications are obviously not fulfilled in this case. The distinguishability matrix obtained for a FDI system made of the 2 sensors at node 1 is given in table 1.

The diagonal elements of the distinguishability matrix indicate the detectability of the corresponding sensor, namely $\mathcal{D}_{i,0}^{1,0}(\theta_i), for\, i = 1, 2$ and the other elements characterize the isolability, namely $\mathcal{D}_{i,j}^{1,0}(\theta_i), for\, (i,j) = (1,2)\, and\, (2,1)$. Solving the optimization problem stated in (12) leads to the selection of sensor 2 at node 3 as an additional measurement for building the FDI system at node 1. The resulting distinguishability matrix is provided in table 2, where $\mathcal{D}_{1,0}^{1,1}(\theta)$ and $\mathcal{D}_{2,0}^{1,1}(\theta)$ correspond to the $(1,1)$ and $(2,2)$ elements. The ROC curves obtained for the FDI system based on three sensors correspond to the blue curves in Fig 1 and 2. The specifications are now fulfilled with regard to sensor 2 but not for sensor 1. A significant increase is observed in $\mathcal{D}_{2,0}^{1,1}(\theta)$.

Solving the optimization problem (13), for $s = 1$ and for $\mathcal{P}_1^2$ made of sensor 1 at node 3 and the two sensors of node 2, yields the distinguishability matrix of table 3 where again elements $(1,1)$ and $(2,2)$ correspond to distinguishability w.r.t. the fault free mode. Now, $\mathcal{D}_{1,0}^{1,2}(\theta)$ and $\mathcal{D}_{2,0}^{1,2}(\theta)$ both reach high values and the magenta ROC curves depicted in Fig. 1 and 2 indicate that the required performance is fulfilled.

It should be noticed that the proposed methodology does not account for possible requirements of fault isolation performance. However, the distinguishability matrix provides information on this issue through its off-diagonal elements.

	f_1	f_2
f_1	1.2615	1.2615
f_2	1.2615	1.2615

Table 1: Computed distinguishability at node 1 before adding any measurement

	f_1	f_2	f_3
f_1	1.2615	1.2615	1.2615
f_2	10.4654	10.4654	1.2615

Table 2: Computed distinguishability at node 1 after adding one extra measurement

	f_1	f_2	f_3	f_4
f_1	10.4654	10.4654	10.4654	1.2615
f_2	10.4654	10.4654	1.2615	10.4654

Table 3: Computed distinguishability at node 1 after adding two extra measurement

6. CONCLUSION

A systematic way to design a local FDI system at each node of a sensor network has been presented. It allows one to determine the minimum amount of data to be exchanged between the different nodes in order to achieve specified fault detection performance. It resorts to a distinguishability measure between faulty and fault free modes based on the Kullback-Leibler divergence.

APPENDIX

Computation of the distinguishablity

We will resort to the following lemma below:

Lemma (see Eriksson et al. [2013]): For a matrix $A \in \mathbb{R}^{\mathrm{n}\times\mathrm{m}}$and a vector $b \in \mathbb{R}^{\mathrm{n}}$ with $n > m$

$$\min_{x\in R^m} \parallel Ax - b \parallel^2 = \parallel N_A b \parallel^2$$

where the rows of N_A consist of an orthonormal basis for the left null space of A.

For two random vectors with Gaussian distribution p^i and $p^j$with unit variance and mean μ_i and μ_j respectively, the Kullback-Leibler divergence is computed as:

$$K\left(p^i \parallel p^j\right) = \frac{1}{2}\|\mu_i - \mu_j\|^2$$

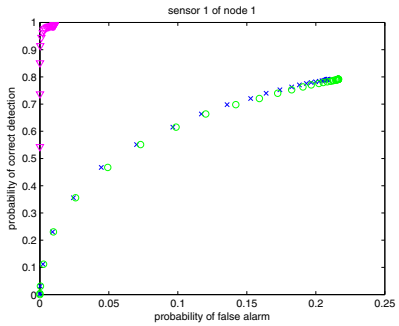

Fig. 1. ROC curves for sensor 1 at node 1. The green curve with $'\bigcirc'$ represents the ROC before adding any measurement; the blue curve with $'\times'$ stands for the ROC after adding one measurement and the magenta curve with $'\nabla'$ denotes the ROC after adding one more measurement.

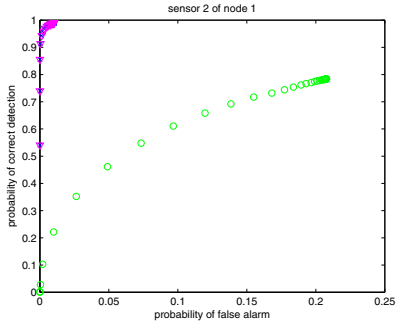

Fig. 2. ROC curves for sensor 2 at node 1. See caption of Fig.1 for the meaning of the symbols and colors.

Applying this expression to compute the distinguishability between fault nodes i and j on the basis of $r(k)$ yields:

$$\begin{aligned}\mathcal{D}_{ij}(\theta_i) &= \frac{1}{2}\min_{\theta_j} \parallel \overline{F}_i\theta_i - \overline{F}_j\theta_j \parallel^2 \\ &= \frac{1}{2}\min_{\theta_j} \parallel \Gamma^{-1}N_H F_i\theta_i - \Gamma^{-1}N_H F_j\theta_j \parallel^2 \quad (15)\end{aligned}$$

To be able to apply the above lemma $\Gamma^{-1}N_H$ has to be factorized so that $\Gamma^{-1}N_H = N_{\overline{H}}\overline{\Gamma}$ where $N_{\overline{H}}$ is made of orthonormal row vectors, and can be seen as a basis for the left null space of a matrix $\overline{H}$. To achieve this goal, let us consider the singular value decomposition of $\Gamma^{-1}N_H$.

$$\Gamma^{-1}N_H = U\,[\,\Sigma\ 0\,]\,V^{\mathrm{T}}$$

where U and V are orthogonal matrices and Σ is a diagonal matrix made of the singular values of $\Gamma^{-1}N_H$. This can be rewritten

$$\Gamma^{-1}N_H = U\,[\,I\ 0\,]\begin{bmatrix}\Sigma & 0\\ 0 & I\end{bmatrix}V^{\mathrm{T}}$$

Defining $N_{\overline{H}} = U\,[\,I\ 0\,]$ and $\overline{\Gamma} = \begin{bmatrix}\Sigma & 0\\ 0 & I\end{bmatrix}V^{\mathrm{T}}$ yields the desired matrices. Following the notation introduced in the main text, (15) can be rewritten:

$$\mathcal{D}_{ij}(\theta_i) = \frac{1}{2}\min_{\theta_j} \parallel N_{\overline{H}}\widetilde{F}_i\theta_i - N_{\overline{H}}\widetilde{F}_j\theta_j \parallel^2$$

A similar argument as in Eriksson et al. [2013] then yields (8).

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