Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Adaptive Leader-following Consensus of Multiple Uncertain Rigid Spacecraft Systems ⋆

He Cai * **Jie Huang** *

* *Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong. (e-mail: {hcai,jhuang}@mae.cuhk.edu.hk).*

Abstract: In this paper, we consider the leader-following consensus problem for multiple uncertain rigid spacecraft systems with the attitude being represented by unit quaternion. Existing results on this problem rely on the assumption that all parameters of the rigid spacecraft system are known exactly. By employing a nonlinear distributed observer for the leader system, we first convert the leader-following consensus problem into a global adaptive stabilization problem of a well defined error system. Then, under the standard assumption that the state of the leader can reach every follower through a path, we further show that this stabilization problem is solvable by a distributed adaptive control law.

Keywords: Multi-agent system, attitude consensus, nonlinear distributed observer, adaptive control.

1. INTRODUCTION

One of the key issues in formation flying of a group of spacecraft systems is to asymptotically aline the attitude and angular velocity of all spacecraft systems to the desired attitude and angular velocity generated by a reference system called leader system. Such a problem is also called leader-following consensus of multiple spacecraft systems.

Depending on whether or not the state of the leader is accessible to all followers, there are roughly two control schemes for dealing with the leader-following consensus problem: decentralized control and distributed control. The former one assumes that the state of the leader is available to all the followers (Abdessameud and Tayebi (2009); VanDyke and Hall (2006)), while the latter one only requires the state of the leader can pass to each of the followers through a path (Bai et al. (2008); Cai and Huang (2014); Ren (2007)). When some followers cannot access the state of the leader, the first scheme is not applicable. One can only make use of local information determined by a communication graph to handle the problem, which has to resort to a distributed control law. Recently, some attempts have been made to deal with the second scenario Bai et al. (2008), Cai and Huang (2014), Ren (2007). The results in Bai et al. (2008) and Ren (2007) have both achieved leader-following consensus for angular velocity. However, in Bai et al. (2008), the consensus for attitude has been realized in a leaderless way, i.e., the attitudes of all followers will converge to a common trajectory determined by the initial condition. In Ren (2007), the leader-following attitude consensus is only applicable to some special type of communication topologies such as a tree. More recently, the authors of this paper solved the leader-following consensus problem for multiple rigid spacecraft systems in Cai and Huang (2014) under the same assumption on the communication graph as in Bai et al. (2008). The result in Cai and Huang (2014) has two features. First, a marginally stable linear system is introduced to generate the desired angular velocity. This scheme enables the control law to handle a class of reference trajectories including step signal with arbitrary magnitude, sinusoidal signal with arbitrary amplitude and initial phase and the combination of the step signal and the sinusoidal signal. Second, the control law achieves both attitude and angular velocity tracking. It is noted that a key technique developed in Cai and Huang (2014) is a nonlinear distributed observer for the leader system which will also play an important role in this paper.

Like all previous papers on leader-following consensus problem of multiple spacecraft systems, the result in Cai and Huang (2014) assumed that all parameters in the spacecraft system are known precisely. This is a quite unrealistic assumption as the mass distribution of the spacecraft system is practically uncertain and may change with time due to fuel consumption or spacecraft reconfiguration (Ahmed et al. (1998); Luo et al. (2005)). To make the result of Cai and Huang (2014) more practically useful, in this paper, we will further consider the leader-following attitude consensus problem without the exact knowledge of the inertial matrix of the spacecraft system. To this end, by employing the same nonlinear distributed observer for the leader system as the one in Cai and Huang (2014), we first convert the leader-following consensus problem into a global adaptive stabilization problem of a well defined error system. Then, under the

⋆ This work has been supported in part by the Research Grants Council of the Hong Kong Special Administration Region under grant No. 412810, and in part by National Natural Science Foundation of China under grant No. 61174049.

standard assumption that the state of the leader can reach every follower through a path, we further show that this stabilization problem is solvable by a distributed adaptive control law. Finally, the effectiveness of our control scheme is evaluated by simulation.

For the rest of this paper, we use the following notation. $\otimes$ denotes the Kronecker product of matrices. 1_N denotes an N dimensional column vector whose components are all 1. $||x||$ denotes the Euclidean norm of vector x and $||A||$ denotes the Euclidean norm of matrix A. For $x_i \in R^{n_i}$, $i = 1, ..., m$, $\mathrm{col}(x_1, \cdots, x_m) = [x_1^T, \cdots, x_m^T]^T$. For $x = \mathrm{col}(x_1, x_2, x_3) \in R^3$, define

$$x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

It can be verified that $x^T x^\times = 0$.

2. PROBLEM FORMULATION

We consider a group of N rigid spacecraft systems with the following motion equations:

$$\dot{\hat{q}}_i = \frac{1}{2}\hat{q}_i^\times \omega_i + \frac{1}{2}\bar{q}_i\omega_i, \quad \dot{\bar{q}}_i = -\frac{1}{2}\hat{q}_i^T \omega_i \tag{1a}$$

$$J_i\dot{\omega}_i = -\omega_i^\times J_i\omega_i + u_i, \ i = 1, \dots, N \tag{1b}$$

where $q_i = \mathrm{col}(\hat{q}_i, \bar{q}_i)$ with $\hat{q}_i \in R^3$, $\bar{q}_i \in R$ is the unit quaternion expression of the attitude of the body frame $\mathcal{B}_i$ of the i^{th} spacecraft relative to the inertial frame $\mathcal{I}$. $\omega_i \in R^3$ is the angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$; $J_i \in R^{3\times 3}$ is the positive definite inertia matrix; $u_i \in R^3$ is the control torque. ω_i, J_i and u_i are all expressed in $\mathcal{B}_i$.

Following the notation introduced as follows (Tayebi (2008)), for two quaternion $q_i = \mathrm{col}(\hat{q}_i, \bar{q}_i)$ and $q_j = \mathrm{col}(\hat{q}_j, \bar{q}_j)$ with $\hat{q}_i, \hat{q}_j \in R^3$, $\bar{q}_i, \bar{q}_j \in R$, the product of q_i and q_j is given by

$$q_i \odot q_j = \begin{pmatrix} \bar{q}_i\hat{q}_j + \bar{q}_j\hat{q}_i + \hat{q}_i^\times \hat{q}_j \\ \bar{q}_i\bar{q}_j - \hat{q}_i^T \hat{q}_j \end{pmatrix}$$

and the conjugate of q_i is given by $q_i^* = \mathrm{col}(-\hat{q}_i, \bar{q}_i)$. If q_i is a unit quaternion, then its inverse is given by $q_i^{-1} = q_i^*$.

As in Chen and Huang (2009), we assume that the desired attitude q_0 of system (1) is generated by the following system

$$\dot{\hat{q}}_0 = \frac{1}{2}\hat{q}_0^\times \omega_0 + \frac{1}{2}\bar{q}_0\omega_0, \quad \dot{\bar{q}}_0 = -\frac{1}{2}\hat{q}_0^T \omega_0 \tag{2}$$

where $q_0 = \mathrm{col}(\hat{q}_0, \bar{q}_0)$ with $\hat{q}_0 \in R^3$, $\bar{q}_0 \in R$ represents the attitude of the leader frame $\mathcal{B}_0$ relative to the inertial frame $\mathcal{I}$; $\omega_0 \in R^3$ is the angular velocity of $\mathcal{B}_0$ relative to $\mathcal{I}$, expressed in $\mathcal{B}_0$.

Like in Cai and Huang (2014), we view the system composed of (1) and (2) as a multi-agent system of $(N+1)$ agents with (2) as the leader and the N subsystems of (1) as N followers. Given (1) and (2), we can define a graph [1] $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ with $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. Here the node 0 is associated with the leader system (2) and the node i, $i = 1, \dots, N$, is associated with the i^{th} subsystem of the follower system (1). For $i = 0, 1, \dots, N$, $j = 1, \dots, N$, $(i, j) \in \bar{\mathcal{E}}$ if and only if u_j can use the full state of agent i for control. Let $\bar{\mathcal{N}}_i$ denote the neighbor set of the node i of $\bar{\mathcal{G}}$. We can further define a subgraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of $\bar{\mathcal{G}}$ where $\mathcal{V} = \{1, \dots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is obtained from $\bar{\mathcal{E}}$ by removing all the edges between node 0 and the nodes in $\mathcal{V}$.

[1] See Appendix for a summary of graph.

In terms of $\bar{\mathcal{G}}$, we can describe a distributed control law as follows, for $i = 1, \dots, N$,

$$u_i = k_i(q_i, \omega_i, \varphi_i, \psi_i) \tag{3a}$$

$$\dot{\psi}_i = f_i(q_i, \omega_i, \varphi_i) \tag{3b}$$

$$\dot{\varphi}_i = g_i(\varphi_i, \varphi_j - \varphi_i, j \in \bar{\mathcal{N}}_i) \tag{3c}$$

where k_i, f_i and g_i are smooth functions, and $\varphi_0 = \mathrm{col}(q_0, \omega_0)$.

We introduce the attitude and angular velocity errors between systems (1) and (2) as follows:

$$\epsilon_i = q_0^{-1} \odot q_i, \tag{4a}$$

$$\hat{\omega}_i = \omega_i - C_i\omega_0 \tag{4b}$$

where $\epsilon_i = \mathrm{col}(\hat{\epsilon}_i, \bar{\epsilon}_i)$, $\hat{\epsilon}_i \in R^3$, $\bar{\epsilon}_i \in R$ and $C_i = (\bar{\epsilon}_i^2 - \hat{\epsilon}_i^T\hat{\epsilon}_i)I_3 + 2\hat{\epsilon}_i\hat{\epsilon}_i^T - 2\bar{\epsilon}_i\hat{\epsilon}_i^\times$ is called the direction cosine matrix, which represents the relative attitude between $\mathcal{B}_i$ and $\mathcal{B}_0$. Then, we have

$$\dot{\hat{\epsilon}}_i = \frac{1}{2}\hat{\epsilon}_i^\times \hat{\omega}_i + \frac{1}{2}\bar{\epsilon}_i\hat{\omega}_i, \quad \dot{\bar{\epsilon}}_i = -\frac{1}{2}\hat{\epsilon}_i^T \hat{\omega}_i \tag{5a}$$

$$J_i\dot{\hat{\omega}}_i = -\omega_i^\times J_i\omega_i + J_i(\hat{\omega}_i^\times C_i\omega_0 - C_i\dot{\omega}_0) + u_i. \tag{5b}$$

Remark 1. Note that for $i = 0, 1, \dots, N$, $||q_i(0)|| = 1$ implies $||q_i(t)|| = 1$ for all $t \geq 0$. Also, by Proposition 1 of Yuan (1988), $\mathcal{B}_i$ and $\mathcal{B}_0$ coincide if and only if $\hat{\epsilon}_i = 0$.

We now state our problem as follows.

Problem 1. Given systems (1), (2) and the graph $\bar{\mathcal{G}}$, design a control law of the form (3) such that, for $i = 1, \dots, N$,

$$\lim_{t\to\infty} \hat{\epsilon}_i(t) = 0 \text{ and } \lim_{t\to\infty} \hat{\omega}_i(t) = 0$$

for all $\omega_i(0)$ and all $q_i(0)$ satisfying $||q_i(0)|| = 1$.

Remark 2. When $N = 1$, the above problem reduces to the problem studied in Ahmed et al. (1998). If every follower can receive the state from the leader, the approach in Ahmed et al. (1998) can be directly extended to handle Problem 1 and this kind of control scheme is called decentralized control. What makes our current problem interesting is that, without the assumption that every follower can receive the state from the leader, we can still solve the leader-following attitude consensus problem by a distributed control law of the form (3).

We need the following assumptions.

Assumption 1. $\bar{\mathcal{G}}$ contains a spanning tree with the node 0 as the root.

Assumption 2. The desired angular velocity ω_0 is generated by the following system

$$\dot{\omega}_0 = S\omega_0 \tag{6}$$

which is marginally stable with constant matrix $S \in R^{3\times 3}$.

Remark 3. Assumption 1 is the standard assumption in consensus problem that imposes a mild constraint on the information exchange among agents. Assumption 2 is made so that the desired angular velocity ω_0 can be estimated by every follower. System (6) can generate step functions of arbitrary magnitudes and sinusoidal functions of arbitrary amplitudes and initial phases.

3. MAIN RESULT

Let $\bar{\mathcal{A}} = [a_{ij}]$ be any weighted adjacency matrix of $\bar{\mathcal{G}}$. For $i = 1, \ldots, N$, define a nonlinear distributed compensator as follows:

$$\dot{\hat{\eta}}_i = \frac{1}{2}\hat{\eta}_i^\times \xi_i + \frac{1}{2}\bar{\eta}_i\xi_i + \mu_1 \sum_{j=0}^{N} a_{ij}(\hat{\eta}_j - \hat{\eta}_i) \quad (7a)$$

$$\dot{\bar{\eta}}_i = -\frac{1}{2}\hat{\eta}_i^T \xi_i + \mu_1 \sum_{j=0}^{N} a_{ij}(\bar{\eta}_j - \bar{\eta}_i) \quad (7b)$$

$$\dot{\xi}_i = S\xi_i + \mu_2 \sum_{j=0}^{N} a_{ij}(\xi_j - \xi_i) \quad (7c)$$

where $\eta_i = \text{col}(\hat{\eta}_i, \bar{\eta}_i)$, $\hat{\eta}_i \in R^3$, $\bar{\eta}_i \in R$, $\xi_i \in R^3$, and $\eta_0 = q_0$, $\xi_0 = \omega_0$. μ_1, μ_2 are some positive numbers.

It was shown in Cai and Huang (2014) that, under Assumption 1 and the assumption that the communication among the followers is bidirectional, i.e., the matrix $\bar{\mathcal{A}}$ is symmetric, (7) is a distributed observer of the leader system. Here we will further remove the assumption that the matrix $\bar{\mathcal{A}}$ is symmetric as shown by the following lemma.

Lemma 1. Consider (2) and (7). Under Assumptions 1 and 2, for all $\eta_i(0)$, $\xi_i(0)$ and any μ_1, $\mu_2 > 0$, for $i = 1, \ldots, N$, $\eta_i(t)$ and $\xi_i(t)$ exist and are bounded for all $t \geq 0$ and satisfy

$$\lim_{t\to\infty}(\eta_i(t) - q_0(t)) = 0, \ \lim_{t\to\infty}(\xi_i(t) - \omega_0(t)) = 0. \quad (8)$$

Also, for $i, j = 0, 1, \ldots, N$, $\int_0^\infty ||\xi_i(\tau) - \xi_j(\tau)||d\tau$ and $\int_0^\infty ||\eta_i(\tau) - \eta_j(\tau)||d\tau$ are bounded.

Proof: Consider the system composed of (6) and (7c). Let $\mathcal{L}$ be the Laplacian of $\mathcal{G}$. Let $\xi = \text{col}(\xi_1, \ldots, \xi_N)$ and $\omega = \xi - 1_N \otimes \omega_0$. Then we have

$$\dot{\omega} = ((I_N \otimes S) - (\mu_2 H \otimes I_3))\omega \quad (9)$$

where $H = \mathcal{L} + \text{diag}\{a_{10}, \ldots, a_{N0}\}$. Note that, H is not symmetric since $\bar{\mathcal{A}}$ is not. Nevertheless, by Lemma 1 of Su and Huang (2012), under Assumption 1, all the eigenvalues of H have positive real parts and therefore H is an M matrix. By Remark 4 of Su and Huang (2012), for any $\mu_2 > 0$, $\lim_{t\to\infty}(\xi_i(t) - \omega_0(t)) = 0$ exponentially, for $i = 1, \ldots, N$. Then, $\int_0^\infty ||\xi_i(\tau) - \omega_0(\tau)||d\tau$ exists and is bounded. It follows from the inequality $||\xi_i(\tau) - \xi_j(\tau)|| \leq ||\xi_i(\tau) - \omega_0(\tau)|| + ||\xi_j(\tau) - \omega_0(\tau)||$ that $\int_0^\infty ||\xi_i(\tau) - \xi_j(\tau)||d\tau$ also exists and is bounded for $i, j = 0, 1, \ldots, N$. Under Assumption 2, ω_0 is bounded, so is ξ_i.

Since H is an M matrix, by Lemma 2 in the Appendix, there is a positive definite diagonal matrix $D = \text{diag}\{d_1, \ldots, d_N\}$ such that $\bar{H} = DH + H^T D$ is positive definite. Let $\eta = \text{col}(\eta_1, \ldots, \eta_N)$, $x = \eta - 1_N \otimes q_0$ and

$$\bar{V} = x^T (D \otimes I_4)x. \quad (10)$$

Then,

$$\begin{aligned}
\dot{\bar{V}} &= \sum_{i=1}^{N} d_i(\hat{\eta}_i^T - \hat{q}_0^T, \bar{\eta}_i - \bar{q}_0) \\
&\begin{bmatrix} \hat{\eta}_i^\times \xi_i + \bar{\eta}_i\xi_i + \sum_{j=0}^{N} 2a_{ij}\mu_1(\hat{\eta}_j - \hat{\eta}_i) - \hat{q}_0^\times \omega_0 - \bar{q}_0\omega_0 \\ -\hat{\eta}_i^T \xi_i + \sum_{j=0}^{N} 2a_{ij}\mu_1(\bar{\eta}_j - \bar{\eta}_i) + \hat{q}_0^T \omega_0 \end{bmatrix} \\
&= \sum_{i=1}^{N}\sum_{j=0}^{N} 2d_i a_{ij}\mu_1(\eta_i^T - q_0^T)(\eta_j - \eta_i) \\
&+ \sum_{i=1}^{N} d_i \left(-\hat{q}_0^T \hat{\eta}_i^\times \xi_i + \bar{\eta}_i\hat{\eta}_i^T \xi_i - \bar{\eta}_i\hat{q}_0^T \xi_i - \hat{\eta}_i^T \hat{q}_0^\times \omega_0 - \bar{q}_0\hat{\eta}_i^T \omega_0 \right. \\
&\left. + \bar{q}_0\hat{q}_0^T \omega_0 - \bar{\eta}_i\hat{\eta}_i^T \xi_i + \bar{q}_0\hat{\eta}_i^T \xi_i + \bar{\eta}_i\hat{q}_0^T \omega_0 - \bar{q}_0\hat{q}_0^T \omega_0\right) \\
&= \sum_{i=1}^{N}\sum_{j=0}^{N} 2d_i a_{ij}\mu_1(\eta_i^T - q_0^T)(\eta_j - \eta_i) \\
&+ \sum_{i=1}^{N} d_i \left[\hat{q}_0^T (\xi_i^\times - \omega_0^\times)\hat{\eta}_i + (\bar{q}_0\hat{\eta}_i^T - \bar{\eta}_i\hat{q}_0^T)(\xi_i - \omega_0)\right] \\
&= -x^T (2\mu_1 DH \otimes I_4)x + \phi(t) \\
&= -x^T (\mu_1 \bar{H} \otimes I_4)x + \phi(t)
\end{aligned} \quad (11)$$

where

$$\phi(t) = \sum_{i=1}^{N} d_i \left[\hat{q}_0^T (\xi_i^\times - \omega_0^\times)\hat{\eta}_i + (\bar{q}_0\hat{\eta}_i^T - \bar{\eta}_i\hat{q}_0^T)(\xi_i - \omega_0)\right].$$

Since q_0 is bounded and $\lim_{t\to\infty}(\xi_i(t) - \omega_0(t)) = 0$ exponentially, there exist $m_1, m_2, m_3 > 0$ such that

$$\begin{aligned}
\phi(t) &\leq m_1 e^{-m_2 t}||\eta|| \\
&\leq m_1 e^{-m_2 t}(||x|| + ||1_N \otimes q_0||) \\
&\leq m_1||x|| + m_3.
\end{aligned} \quad (12)$$

Since H is positive definite, from (11) and (12), there exists $m_4 > 0$ such that

$$\dot{\bar{V}} \leq -m_4||x||^2 + m_1||x|| + m_3. \quad (13)$$

Therefore, if $||x|| > (m_1 + \sqrt{m_1^2 + 4m_4 m_3})/2m_4$, we have $\dot{\bar{V}} < 0$. Also noticing that $\bar{V}$ is positive definite in x and $\dot{\bar{V}}$ is continuous, by Theorem 2.5.7 of Lewis et al. (1999), we conclude that x is bounded. Therefore η and hence η_i are also bounded, which implies $\lim_{t\to\infty}\phi(t) = 0$ exponentially.

Let λ^* denote the smallest eigenvalue of H, $\lambda^* > 0$. Therefore, by (10) and (11),

$$\dot{\bar{V}} \leq -2\lambda^*\mu_1 x^T x + \phi(t) = -2\lambda^*\mu_1\bar{V} + \phi(t). \quad (14)$$

Define a linear stable differential equation as follows:

$$\dot{W} = -2\lambda^*\mu_1 W + \phi(t). \quad (15)$$

Then, clearly, the solution $W(t)$ of (15) will approach zero as t tends to infinity. By the comparison lemma as can be found in, say, Lemma 3.4 of Khalil (2002), we conclude $\bar{V}(t) \leq W(t)$ for $t \geq 0$. Since $\bar{V}(t) \geq 0$ for all $t \geq 0$, we have $\lim_{t\to\infty}\bar{V}(t) = 0$. Thus $\lim_{t\to\infty}(\eta_i(t) - q_0(t)) = 0$ for $i = 1, \ldots, N$. From (14)

$$\bar{V} \leq -\frac{1}{2\lambda^*\mu_1}(\dot{\bar{V}} - \phi(t)) \quad (16)$$

which implies

$$\int_0^\infty \bar{V}(\tau)d\tau \leq -\frac{1}{2\lambda^*\mu_1}(\bar{V}(\infty) - \bar{V}(0) - \int_0^\infty \phi(\tau)d\tau).$$

Since $\lim_{t\to\infty}\phi(t)=0$ exponentially, $\int_0^\infty \phi(\tau)d\tau$ is bounded. Since $\lim_{t\to\infty}\bar{V}(t)=0$, $\int_0^\infty \bar{V}(\tau)d\tau$ is bounded. Thus, $\int_0^\infty ||\eta_i(\tau)-q_0(\tau)||d\tau$ is bounded. It follows from the inequality $||\eta_i(\tau)-\eta_j(\tau)||\leq ||\eta_i(\tau)-q_0(\tau)||+||\eta_j(\tau)-q_0(\tau)||$ that $\int_0^\infty ||\eta_i(\tau)-\eta_j(\tau)||d\tau$ is bounded for $i,j=0,1,\ldots,N$. □

Based on the distributed observer of (7), we further define the errors as follows:

$$e_i = \eta_i^* \odot q_i \tag{17a}$$

$$\bar{\omega}_i = \omega_i - \hat{C}_i\xi_i + k_{i1}\hat{e}_i \tag{17b}$$

where $k_{i1}>0$, $e_i=\mathrm{col}(\hat{e}_i,\bar{e}_i)$ with $\hat{e}_i\in R^3$, $\bar{e}_i\in R$ and $\hat{C}_i=(\bar{e}_i^2-\hat{e}_i^T\hat{e}_i)I_3+2\hat{e}_i\hat{e}_i^T-2\bar{e}_i\hat{e}_i^\times$, which leads to the following error system:

$$\dot{\hat{e}}_i = \frac{1}{2}(\hat{e}_i^\times+\bar{e}_iI_3)(\bar{\omega}_i-k_{i1}\hat{e}_i)+\alpha_i(t) \tag{18a}$$

$$\dot{\bar{e}}_i = -\frac{1}{2}\hat{e}_i^T(\bar{\omega}_i-k_{i1}\hat{e}_i)+\beta_i(t) \tag{18b}$$

$$\begin{aligned}J_i\dot{\bar{\omega}}_i =& -\omega_i^\times J_i\omega_i + J_i((\bar{\omega}_i-k_{i1}\hat{e}_i)^\times\hat{C}_i\xi_i-\hat{C}_iS\xi_i\\ &+\frac{1}{2}k_{i1}(\hat{e}_i^\times+\bar{e}_iI_3)(\bar{\omega}_i-k_{i1}\hat{e}_i))+\gamma_i(t)+u_i\end{aligned} \tag{18c}$$

where

$$\begin{aligned}\alpha_i(t) =& \sum_{j=0}^N a_{ij}\mu_1\left((\bar{\eta}_j-\bar{\eta}_i)\hat{q}_i-(\hat{\eta}_j-\hat{\eta}_i)^\times\hat{q}_i-\bar{q}_i(\hat{\eta}_j-\hat{\eta}_i)\right)\\ &-\frac{1}{2}(e_i^Te_i-1)(\hat{e}_i^\times-\bar{e}_iI_3)\xi_i\end{aligned} \tag{19a}$$

$$\begin{aligned}\beta_i(t) =& \sum_{j=0}^N a_{ij}\mu_1\left(\hat{q}_i^T(\hat{\eta}_j-\hat{\eta}_i)+\bar{q}_i(\bar{\eta}_j-\bar{\eta}_i)\right)\\ &-\frac{1}{2}(e_i^Te_i-1)\hat{e}_i^T\xi_i\end{aligned} \tag{19b}$$

$$\gamma_i(t) = -J_i\left(\varsigma_i(t)\xi_i+\mu_2\hat{C}_i\sum_{j=0}^N a_{ij}(\xi_j-\xi_i)-k_{i1}\alpha_i(t)\right) \tag{19c}$$

with $\varsigma_i(t)=2\bar{e}_i\beta_i(t)I_3-2\hat{e}_i^T\alpha_i(t)I_3+2\alpha_i(t)\hat{e}_i^T+2\hat{e}_i\alpha_i(t)^T-2\beta_i(t)\hat{e}_i^\times-2\bar{e}_i\alpha_i(t)^\times$.

Remark 4. Let $\bar{V}_i=e_i^Te_i$. It was shown in Cai and Huang (2014) that $\lim_{t\to\infty}\bar{V}_i(t)=1$ and e_i is bounded over $t\geq 0$. Moreover, $\alpha_i(t)$, $\beta_i(t)$ and $\gamma_i(t)$ will tend to zero as t tends to infinity and $\int_0^\infty||\alpha_i(\tau)||d\tau$, $\int_0^\infty||\beta_i(\tau)||d\tau$ and $\int_0^\infty||\gamma_i(\tau)||d\tau$ are all bounded over $t\geq 0$.

Remark 5. It was shown by Lemma 4.1 of Cai and Huang (2014) that for $i=1,\ldots,N$, for any piecewise continuous time function $\bar{\omega}_i(t)$ defined for $t\geq 0$ satisfying $\lim_{t\to\infty}\bar{\omega}_i(t)=0$, the solution of the subsystem composed of (18a) and (18b) is bounded for all $t\geq 0$ and satisfies $\lim_{t\to\infty}\hat{e}_i(t)=0$.

In order to put equation (18c) in the standard form where the unknown parameters appear linearly, we adopt the following notation introduced in Ahmed et al. (1998). For any vector $x=\mathrm{col}(x_1,x_2,x_3)\in R^3$, define a linear operator L acting on x by

$$L(x)=\begin{bmatrix}x_1 & 0 & 0 & 0 & x_3 & x_2\\ 0 & x_2 & 0 & x_3 & 0 & x_1\\ 0 & 0 & x_3 & x_2 & x_1 & 0\end{bmatrix}.$$

Let J_i be denoted by

$$J_i=\begin{bmatrix}J_{i11} & J_{i12} & J_{i13}\\ J_{i12} & J_{i22} & J_{i23}\\ J_{i13} & J_{i23} & J_{i33}\end{bmatrix}.$$

Define $\Theta_i=\mathrm{col}(J_{i11},J_{i22},J_{i33},J_{i23},J_{i13},J_{i12})$. Then

$$J_ix=L(x)\Theta_i. \tag{20}$$

Thus, equation (18c) can be rewritten as

$$J_i\dot{\bar{\omega}}_i=\chi_i(t)\Theta_i+\gamma_i(t)+u_i \tag{21}$$

where

$$\begin{aligned}\chi_i(t)=&-\omega_i^\times L(\omega_i)+L((\bar{\omega}_i-k_{i1}\hat{e}_i)^\times\hat{C}_i\xi_i-\hat{C}_iS\xi_i\\ &+\frac{1}{2}k_{i1}(\hat{e}_i^\times+\bar{e}_iI_3)(\bar{\omega}_i-k_{i1}\hat{e}_i)).\end{aligned} \tag{22}$$

For $i=1,\ldots,N$, let

$$\dot{\hat{\Theta}}_i=\Lambda_i^{-1}\chi_i(t)^T\bar{\omega}_i \tag{23a}$$

$$u_i=-\chi_i(t)\hat{\Theta}_i-k_{i2}\bar{\omega}_i \tag{23b}$$

where $k_{i2}>0$, $\Lambda_i\in R^{6\times 6}$ is the positive definite gain matrix.

Theorem 1. Given systems (1), (2) and the graph $\bar{\mathcal{G}}$, under Assumptions 1 and 2, problem 1 is solvable by the control law composed of (7) and (23).

Proof: Let $\tilde{\Theta}_i=\Theta_i-\hat{\Theta}_i$. Then substituting (23b) into (21) gives

$$J_i\dot{\bar{\omega}}_i=\chi_i(t)\tilde{\Theta}_i-k_{i2}\bar{\omega}_i+\gamma_i(t). \tag{24}$$

Let

$$V=\frac{1}{2}\sum_{i=1}^N\left(\bar{\omega}_i^TJ_i\bar{\omega}_i+\tilde{\Theta}_i^T\Lambda_i\tilde{\Theta}_i\right). \tag{25}$$

Let $0<\varepsilon_i<2k_{i2}$ and $\tilde{k}_{i2}=k_{i2}-\varepsilon_i/2$. Then

$$\begin{aligned}\dot{V}=&\sum_{i=1}^N\bar{\omega}_i^T(\chi_i(t)\tilde{\Theta}_i-k_{i2}\bar{\omega}_i+\gamma_i(t))-\tilde{\Theta}_i^T\chi_i(t)^T\bar{\omega}_i\\ =&\sum_{i=1}^N -k_{i2}\bar{\omega}_i^T\bar{\omega}_i+\bar{\omega}_i^T\gamma_i(t)\\ \leq&\sum_{i=1}^N -k_{i2}\bar{\omega}_i^T\bar{\omega}_i+\frac{\varepsilon_i}{2}\bar{\omega}_i^T\bar{\omega}_i+\frac{1}{2\varepsilon_i}||\gamma_i(t)||^2\\ =&-\Omega(t)+\zeta(t)\end{aligned} \tag{26}$$

where

$$\Omega(t)=\sum_{i=1}^N\tilde{k}_{i2}\bar{\omega}_i^T\bar{\omega}_i,\ \zeta(t)=\sum_{i=1}^N\frac{1}{2\varepsilon_i}||\gamma_i(t)||^2. \tag{27}$$

By Remark 4, we have $\int_0^\infty\zeta(\tau)d\tau$ is bounded. Since $\Omega(t)\geq 0$ for all $t\geq 0$, we have

$$\dot{V}\leq\zeta(t). \tag{28}$$

Therefore

$$\int_0^\infty\dot{V}(\tau)d\tau\leq\int_0^\infty\zeta(\tau)d\tau \tag{29}$$

Since $\int_0^\infty\zeta(\tau)d\tau$ is bounded, V is bounded, which implies $\bar{\omega}_i$ and $\tilde{\Theta}_i$ are bounded. Since e_i and ξ_i are bounded, by (17b) and (22), ω_i and hence χ_i is bounded. Therefore, $\dot{\bar{\omega}}_i$ is bounded by (24). By (27), $\dot{\Omega}(t)$ is bounded, which implies $\Omega(t)$ is uniformly continuous.

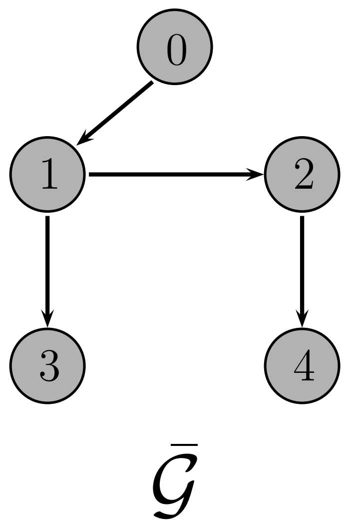

Fig. 1. The network topology $\bar{\mathcal{G}}$

We now claim $\lim_{t\to\infty}\Omega(t)=0$. Otherwise, since $\Omega(t)\geq 0$ for all $t\geq 0$, there exists $\varepsilon>0$ and a sequence $\{t_k\}$, $k=1,2,\ldots$ satisfying $t_k\to\infty$ as $k\to\infty$ such that $\Omega(t_k)>\varepsilon$. Since $\Omega(t)$ is uniformly continuous, there exists $\delta>0$, such that $|\Omega(t_k)-\Omega(t)|<\varepsilon/2$ whenever $|t_k-t|<\delta$. Without loss of generality, we can always choose $\{t_k\}$ such that $t_1>\delta$ and $t_{k+1}-t_k>2\delta$. Therefore, $\Omega(t)>\varepsilon/2$ whenever $|t_k-t|<\delta$.

Therefore,

$$\int_0^\infty \Omega(\tau)d\tau \geq \sum_{k=1}^{\infty}\int_{t_k-\delta}^{t_k+\delta}\Omega(\tau)d\tau=+\infty. \tag{30}$$

Since $\int_0^\infty \zeta(\tau)d\tau$ is bounded, $\int_0^\infty \dot{V}(\tau)d\tau=V(\infty)-V(0)=-\infty$, which contradicts with the fact that $V(t)\geq 0$ for all $t\geq 0$. Thus we have proved our claim, which implies $\lim_{t\to\infty}\bar{\omega}_i(t)=0$ and by Remark 5, $\lim_{t\to\infty}\hat{e}_i(t)=0$. Since $\lim_{t\to\infty}\bar{V}_i(t)=1$, we have $\lim_{t\to\infty}|\bar{e}_i(t)|=1$ and therefore $\lim_{t\to\infty}\hat{C}_i(t)=I_3$. By Lemma 1, $\lim_{t\to\infty}(\eta_i(t)-q_0(t))=0$ and therefore $\lim_{t\to\infty}\hat{e}_i(t)=0$ implies that

$$\lim_{t\to\infty}\hat{\epsilon}_i(t)=0$$

and hence $\lim_{t\to\infty}C_i(t)=I_3$.

By (17b), we have $\lim_{t\to\infty}(\omega_i(t)-\hat{C}_i\xi_i(t))=0$. By Lemma 1, $\lim_{t\to\infty}(\xi_i(t)-\omega_0(t))=0$. Note that both $C_i(t)$ and $\hat{C}_i(t)$ will tend to I_3 as t tends to infinity. By the following identity,

$$\begin{aligned}\hat{\omega}_i(t)&=\omega_i(t)-C_i\omega_0(t)\\&=(\omega_i(t)-\hat{C}_i\xi_i(t))+(\hat{C}_i\xi_i(t)-C_i\omega_0(t))\end{aligned} \tag{31}$$

we have

$$\lim_{t\to\infty}\hat{\omega}_i(t)=0.$$

□

4. SIMULATION

Consider four follower systems whose motion equations are described by (1), with the following parameters

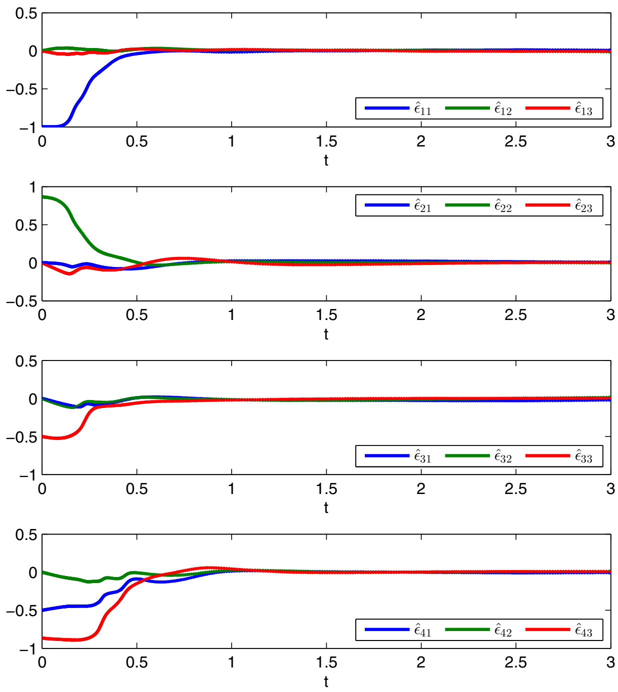

Fig. 2. Tracking performance of attitude.

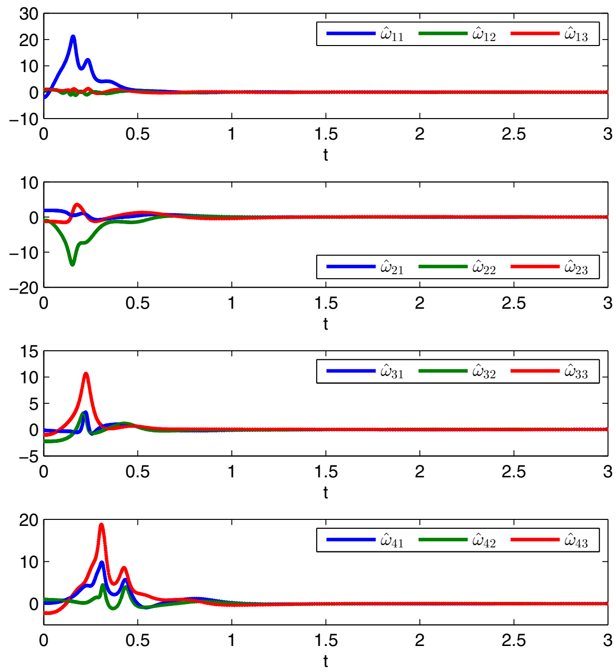

Fig. 3. Tracking performance of angular velocity.

$$\begin{aligned}\Theta_1&=\mathrm{col}(1.2,3.5,4.7,0,0,0)\\\Theta_2&=\mathrm{col}(5.4,1.3,7.2,0,0,0)\\\Theta_3&=\mathrm{col}(1.9,2.1,3.5,0,0,0)\\\Theta_4&=\mathrm{col}(4.6,5.1,3.1,0,0,0).\end{aligned}$$

Let the leader system be described by (2) with

$$S = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

Clearly, such a leader system satisfies Assumption 2 and can generate sinusoidal signals with arbitrary amplitudes and initial phases. Assume the graph $\bar{\mathcal{G}}$ is shown in Figure 1 which satisfies Assumption 1. Thus we can design a control law of the form of (7) and (23) with the following design parameters: $\mu_1 = 20$, $\mu_2 = 20$, $k_{i1} = 20$, $k_{i2} = 20$, $\Lambda_i = I_6$. We let $a_{ij} = 1$ whenever $(j,i) \in \bar{\mathcal{E}}$.

The performance of the control law is simulated with the following initial condition $q_0(0) = [1,0,0,0]^T$; $\omega_0(0) = [0,1,1]^T$; $q_1(0) = [0,0,0,1]^T$; $q_2(0) = [0.5,0,\sqrt{3}/2,0]^T$; $q_3(0) = [\sqrt{3}/2,0.5,0,0]^T$, $q_4(0) = [0,\sqrt{3}/2,0,0.5]^T$; $\omega_i(0) = \xi_i(0) = 0$, $\eta_i(0) = 0$, $\hat{\Theta}_i(0) = 0$.

Figures 2 and 3 show the tracking performance of the attitude and angular velocity of each coordinate of q_i and ω_i, respectively, for $i = 1,2,3,4$. It can be observed that the tracking performance is satisfactory and further validates the effectiveness of our control law.

5. CONCLUSION

In this paper, we have presented a result on the leader-following consensus problem for multiple uncertain rigid spacecraft systems. In comparison with the existing results, our control law can handle the case of unknown parameters of the spacecraft system. We first convert the leader-following consensus problem into a global adaptive stabilization problem of a well defined error system based on a nonlinear distributed observer for the leader system. Then, by proposing a distributed adaptive control law, this stabilization problem is solved under the standard assumption that each of the followers can receive the state of the leader through a path.

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Appendix A. GRAPH

We introduce some graph notation which can be found in Godsil and Royal (2001). A digraph $\mathcal{G} = (\mathcal{V},\mathcal{E})$ consists of a finite set of nodes $\mathcal{V} = \{1,\ldots,N\}$ and an edge set $\mathcal{E} = \{(i,j), i,j \in \mathcal{V}, i \neq j\}$. An edge from node i to node j is denoted by (i,j), and node i is called the neighbor of node j. If the digraph $\mathcal{G}$ contains a sequence of edges of the form $(i_1,i_2),(i_2,i_3),\ldots,(i_k,i_{k+1})$, then the set $\{(i_1,i_2),(i_2,i_3),\ldots,(i_k,i_{k+1})\}$ is called a path of $\mathcal{G}$ from i_1 to i_{k+1}, and node i_{k+1} is said to be reachable from node i_1. A graph is said to contain a spanning tree if there exists a node i such that any other node is reachable from node i. The node i is called the root of the spanning tree. The edge (i,j) is called undirected if $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$. The graph is called undirected if every edge in $\mathcal{E}$ is undirected. A graph $\mathcal{G}_s = (\mathcal{V}_s,\mathcal{E}_s)$ is called a subgraph of $\mathcal{G} = (\mathcal{V},\mathcal{E})$ if $\mathcal{V}_s \subseteq \mathcal{V}$ and $\mathcal{E}_s \subseteq \mathcal{E} \cap (\mathcal{V}_s \times \mathcal{V}_s)$. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ of $\mathcal{G}$ is defined as $a_{ii} = 0$; for $i \neq j$, $a_{ij} > 0 \Leftrightarrow (j,i) \in \mathcal{E}$ and $a_{ij} = a_{ji}$ if (i,j) is an undirected edge of $\mathcal{E}$. The Laplacian of $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}] \in R^{N\times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$.

Appendix B. LEMMA 2

Lemma 2. (Theorem 2.5.3 of Horn and Johnson (1991)), A matrix $A \in R^{N\times N}$ is called an M matrix if $a_{ij} \leq 0$ for $i \neq j$, $i,j = 1,\ldots,N$ and all the eigenvalues of A have positive real parts. Then, A is an M matrix if and only if there is a positive definite diagonal matrix $D = \text{diag}\{d_1,\ldots,d_N\}$ such that $DA + A^T D$ is positive definite.