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Synthesis of Decentralized Variable Gain Robust Controllers for a Class of Large-Scale Serially Connected Systems with Nonlinear Perturbations via Piecewise Lyapunov Functions

Hidetoshi OYA * **Tomohiro KUBO** ** **Kojiro HAGINO** ***

* *The Institute of Technology and Science, The University of Tokushima, 2-1 Minamijosanjima, Tokushima, 770-8506 JAPAN (e-mail: hide-o@ee.tokushima-u.ac.jp).*
** *The Institute of Technology and Science, The University of Tokushima, 2-1 Minamijosanjima, Tokushima, 770-8506 JAPAN (e-mail: kubo@ee.tokushima-u.ac.jp).*
*** *The Department of Systems Engineering, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo, 182-8585 JAPAN (e-mail: hagino@se.uec.ac.jp)*

Abstract: In this paper, we present a decentralized variable gain robust controller for a class of large-scale serially connected systems with nonlinear perturbations via piecewise Lyapunov functions. The nonlinear perturbation means the interconnections between the subsystems and uncertainties, and the proposed decentralized variable gain robust controller is designed so as to reduce the effect of uncertainties and interconnections. In this paper, on the basis of the concept of piecewise Lyapunov functions, we show sufficient conditions for the existence of the proposed decentralized variable gain robust controller. Finally, numerical examples are presented.

Keywords: decentralized control, variable gain robust controllers, large-scale serially connected systems, nonlinear perturbations, piecewise Lyapunov functions

1. INTRODUCTION

It is well-known that robust control problems for uncertain dynamical systems have received much attention for a long time (e.g. Barmish [1983] and references therein). Additionally, with the rapid development of modern industry, computer engineering, communication technique and so on, many practical systems become complex, and can be considered as large-scale interconnected systems composed of lower dimensional subsystems. Thus a large number of the existing results in decentralized robust control of uncertain large-scale interconnected systems have also been widely studied (e.g Davison [1978], Gong [1995]), because when a large-scale interconnected system is concerned, the centralized pattern often fails to hold due to either lack of the overall information or lack of the centralized computing capability. From this viewpoint, we have proposed a decentralized robust controller with compensation inputs for large-scale serially connected systems (Oya and Hagino [2002]). From the practical point of view, large-scale serially connected systems are also important class of large-scale interconnected systems, since many actual control plant such as production line, cold rolling processes and so on, are composed of many similar subprocesses placed after one another, in such a way that a subprocess is influenced only by its neighbour subprocesses.

By the way, most of robust controllers of the existing results have fixed structure and they are based on worst-case design. Therefore, they become cautious when the perturbation region of the uncertainties has been estimated larger than the proper region. In contrast with these, design problems of robust control systems with adjustable parameters, i.e. variable gain robust controllers have also been studied. Yamamoto and Yamauchi [1999] have proposed a design method of a robust controller with the ability to adjust control performances adaptively. In the works of Maki and Hagino [1999] and Oya and Hagino [2004], variable gain robust controllers which achieve not only asymptotical stability but also good transient performance in time response have been shown and these variable gain robust controllers are tuned by on-line information about parameter uncertainties. In addition, for both centralized robust control and decentralized one, the concept of quadratic stabilization via fixed quadratic Lyapunov functions plays an important role in dealing with the controller design.

On the other hand, during the last two decades, piecewise Lyapunov functions and parameter-dependent Lyapunov functions for robust stability analysis and/or robust controller synthesis have been investigated (e.g. Gahinet et al. [1996], Veselý [2002], Boyd et al. [1994], Xie et al. [1997]), because the quadratic stability based on fixed Lyapunov functions can be lead sometimes to very conser-

$$\begin{aligned}\frac{d}{dt}x_1(t) &= A_{11}x_1(t) + B_1u_1(t) + h_1(x_1,t)\\ \frac{d}{dt}x_i(t) &= A_{ii}x_i(t) + A_{ij}x_j(t) + B_iu_i(t) + h_i(x_i,x_j,t) \quad (i=2,\cdots,\mathcal{N},\ j=i-1)\end{aligned} \tag{1}$$

$$h_1^T(x_1,t)h_1(x_1,t) \le \alpha_1^2 x_1^T(t)\Psi_1 x_1(t) \ \text{ and } \ h_i^T(x_i,x_j,t)h_i(x_i,x_j,t) \le \alpha_i^2 x_i^T(t)\Psi_i x_i(t) + \alpha_{ij}^2 x_j^T(t)\Psi_{ij}x_j(t) \tag{2}$$

vative results. For polytopic uncertain systems, parameter-dependent Lyapunov functions are introduced and sufficient conditions for affine quadratic stability are presented (Gahinet et al. [1996], Veselý [2002]). In the work of Boyd et al. [1994], Lyapunov functions constructed as the maximum or the minimum of quadratic terms have been adopted for analysis of robust stability for time-varying uncertain linear systems, and by using $\mathcal{S}$-procedure necessary and sufficient conditions for the existence of these piecewise Lyapunov functions have been obtained in terms of LMIs. Furthermore, numerically efficient conditions for robust stability of convex combinations of two matrices have been studied (Xie et al. [1997]). Besides, Oya et al. [2011] have also suggested a design method of variable gain controllers based on piecewise Lyapunov functions for a class of uncertain linear systems with state delays. However, so far the design problem of decentralized variable gain robust controllers for large-scale serially connected systems with uncertain nonlinear perturbations has not been discussed.

From the above, this paper deals with a design problem of decentralized variable gain robust controllers for a class of large-scale serially connected systems with nonlinear perturbations via piecewise Lyapunov functions. In this paper, on the basis of the works of Oya and Hagino [2002] and Oya et al. [2011], the proposed decentralized variable gain robust controller consists of a fixed gain controller and a variable gain one. The fixed gain controller is determined by using the nominal subsystem and the variable gain controller is designed by using piecewise Lyapunov functions in order to reduce the effect of uncertainties and interactions. In this paper, we show that sufficient conditions for the existence of the proposed decenralized robust controller are given in terms of LMIs.

This paper is organized as follows. In Sec. 2, notation and useful lemmas which are used in this paper are shown, and in Sec. 3, we introduce the class of large-scale serially connected systems under consideration. Sec. 4 contains the main results. The design method of the proposed decentralized variable gain robust controller is presented. Finally, a simple numerical example is included to illustrate the results developed in this paper.

2. PRELIMINARIES

In this section, we show notations and useful and well-known lemmas (Boyd et al. [1994], Gantmacher [1960]) which are used in this paper.

For a matrix $\mathcal{A}$, the transpose of the matrix $\mathcal{A}$ and it's inverse are denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$, respectively and rank $\{\mathcal{A}\}$ represents the rank of the matrix $\mathcal{A}$. Also, $H_e\{\mathcal{A}\}$ and I_n represent $\mathcal{A}+\mathcal{A}^T$ and n-dimensional identity matrix, respectively. The notation diag $(\mathcal{A}_1,\cdots,\mathcal{A}_N)$ denotes a block diagonal matrix composed of matrices $\mathcal{A}_i$ for $i=1,\cdots,N$. For real symmetric matrices $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} > \mathcal{B}$ (resp. $\mathcal{A} \ge \mathcal{B}$) means that $\mathcal{A}-\mathcal{B}$ is positive (resp. nonnegative) definite matrix. For a vector $\alpha \in \mathbb{R}^n$, $||\alpha||$ denotes standard Euclidian norm and for a matrix $\mathcal{A}$, $||\mathcal{A}||$ represents its induced norm. The intersection of sets Υ_k $(k=1,\cdots,\mathcal{M})$ is denoted by $\bigcap_{k=1}^{\mathcal{M}}\Upsilon_k$, and the symbols "$\stackrel{\triangle}{=}$" and "$\star$" mean equality by definition and symmetric blocks in matrix inequalities, respectively. Besides, for a symmetric matrix $\mathcal{P}$, $\lambda_{\max}\{\mathcal{P}\}$ (resp. $\lambda_{\min}\{\mathcal{P}\}$) represents the maximal eigenvalue (resp. minimal eigenvalue).

Lemma 1. (Schur complement) For a given constant real symmetric matrix Ξ, the following items are equivalent.

(i) $\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22}\end{pmatrix} > 0$

(ii) $\Xi_{11} > 0$ and $\Xi_{22} - \Xi_{12}^T\Xi_{11}^{-1}\Xi_{12} > 0$

(iii) $\Xi_{22} > 0$ and $\Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^T > 0$

Lemma 2. ($\mathcal{S}$-procedure) Let $\mathcal{F}(x)$ and $\mathcal{G}(x)$ be two arbitrary quadratic forms over $\mathbb{R}^n$. Then $\mathcal{F}(x) < 0$ for $\forall x \in \mathbb{R}^n$ satisfying $\mathcal{G}(x) \le 0$ if and only if there exists a nonnegative scalar τ such that

$$\mathcal{F}(x) - \tau\mathcal{G}(x) \le 0 \quad \forall x \in \mathbb{R}^n$$

Lemma 3. (Barbalat's lemma) Let $\phi : \mathbb{R} \to \mathbb{R}$ be a uniformly continuous function on $[0,\infty)$. Suppose that $\lim_{t\to\infty}\int_0^t \phi(\tau)d\tau$ exists and is finite. Then

$$\phi(t) \to 0 \ \text{ as } \ t \to \infty.$$

3. PROBLEM FORMULATION

Consider the uncertain large-scale serially connected system composed of subsystems described by (1). In (1), $x_l(t) \in \mathbb{R}^{n_l}$ and $u_l(t) \in \mathbb{R}^{m_l}$ $(l=1,\cdots,\mathcal{N})$ are the vectors of the state and the control input for the l-th subsystem, respectively. The matrices A_{ll}, A_{ij} and B_l represent the nominal values of the system and the matrix B_l has full column rank, i.e. rank$\{B_l\} = m_l$. Furthermore the functions $h_1 : \mathbb{R}^1 \times \mathbb{R}^{n_1} \to \mathbb{R}^{n_1}$ and $h_i : \mathbb{R}^1 \times \mathbb{R}^{n_i+n_j} \to \mathbb{R}^{n_i}$ are non-linear perturbation functions and satisfy the quadratic constraint conditions of (2) where $\alpha_1 \in \mathbb{R}^1$, $\alpha_i \in \mathbb{R}^1$ and $\alpha_{ij} \in \mathbb{R}^1$ and $\Psi_1 \in \mathbb{R}^{n_1\times n_1}$, $\Psi_i \in \mathbb{R}^{n_i\times n_i}$ and $\Psi_{ij} \in \mathbb{R}^{n_j\times n_j}$ are known positive scalars and known constant positive definite matrices, respectively. Besides, we introduce the integer $\mathcal{M}_l \in \mathbb{Z}^+$ $(l=1,2,\cdots,\mathcal{N})$ defined as

$$\mathcal{M}_l \stackrel{\triangle}{=} \arg\min_{\mathcal{Z}\in\mathbb{Z}^+}\{\mathcal{Z} \mid (\mathcal{Z}m_l - n_l) \ge 0\} \tag{3}$$

and we assume that there exist symmetric positive definite matrices $\mathcal{S}_{k_l}^{(l)} \in \mathbb{R}^{n_l\times n_l}$ which satisfy the following relation (Oya et al. [2011]).

$$\bigcap_{k_l=1}^{\mathcal{M}_l} \Omega_{\mathcal{S}_{k_l}^{(l)}} = \{0\} \tag{4}$$

$$\begin{aligned} &\frac{d}{dt}x_1(t) = A_{K_1}x_1(t) + h_1(x_1,t) + B_1\psi_1(x,t) \\ &\frac{d}{dt}x_i(t) = A_{K_i}x_i(t) + A_{ij}x_j(t) + h_i(x_i,x_j,t) + B_i\psi_i(x,t) \quad (i = 2, \cdots, \mathcal{N},\ j = i-1) \end{aligned} \tag{8}$$

$$\begin{aligned} &H_e\left\{\left(\mathcal{P}_1^{(1)} + \mathcal{P}_2^{(1)}\right)A_{K_1}\right\} + \gamma_1^{(1)}\mathcal{P}_1^{(1)}B_1B_1^T\mathcal{P}_1^{(1)} - \gamma_1^{(1)}\mathcal{P}_2^{(1)}B_1B_1^T\mathcal{P}_2^{(1)} + \mathcal{Q}_1^{(1)} + \mathcal{W}_1 < 0 \\ &H_e\left\{\left(\mathcal{P}_1^{(1)} + \mathcal{P}_2^{(1)}\right)A_{K_1}\right\} + \gamma_2^{(1)}\mathcal{P}_2^{(1)}B_1B_1^T\mathcal{P}_2^{(1)} - \gamma_2^{(1)}\mathcal{P}_1^{(1)}B_1B_1^T\mathcal{P}_1^{(1)} + \mathcal{Q}_2^{(1)} + \mathcal{W}_1 < 0 \end{aligned} \tag{9}$$

$$\begin{aligned} &\left(\begin{array}{c|c} H_e\left\{\left(\mathcal{P}_1^{(2)} + \mathcal{P}_2^{(2)}\right)A_{K_2}\right\} + \gamma_1^{(2)}\mathcal{P}_1^{(2)}B_2B_2^T\mathcal{P}_1^{(2)} - \gamma_1^{(2)}\mathcal{P}_2^{(2)}B_2B_2^T\mathcal{P}_2^{(2)} + \mathcal{Q}_1^{(2)} + \mathcal{W}_2 & \mathcal{S}_1^{(2)}A_{21} \\ \hline \star & -\mathcal{W}_1 + \vartheta_2\alpha_{21}^2\Psi_{21} \end{array}\right) < 0 \\ &\left(\begin{array}{c|c} H_e\left\{\left(\mathcal{P}_1^{(2)} + \mathcal{P}_2^{(2)}\right)A_{K_2}\right\} + \gamma_2^{(2)}\mathcal{P}_2^{(2)}B_2B_2^T\mathcal{P}_2^{(2)} - \gamma_2^{(2)}\mathcal{P}_1^{(2)}B_2B_2^T\mathcal{P}_1^{(2)} + \mathcal{Q}_2^{(2)} + \mathcal{W}_2 & \mathcal{S}_2^{(2)}A_{21} \\ \hline \star & -\mathcal{W}_1 + \vartheta_2\alpha_{21}^2\Psi_{21} \end{array}\right) < 0 \end{aligned} \tag{10}$$

$$\begin{aligned} &\psi_1(x_1,t) \triangleq -\frac{1}{2}\frac{\left(\xi_1(x_1,t) + 2x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}A_{K_1}x_1(t)\right)^2}{\left(\sigma_1(t) + \xi_1(x_1,t) + 2x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}A_{K_1}x_1(t)\right)\left\|B_1^T\mathcal{S}_{k_1}^{(1)}x_1(t)\right\|^2}B_1^T\mathcal{S}_{k_1}^{(1)}x_1(t) \\ &\psi_2(x_2,t) \triangleq -\frac{1}{2}\frac{\left(\xi_2(x_2,t) + 2x_2^T(t)\mathcal{P}_{k_2}^{(2)}B_2B_2^T\mathcal{P}_{k_2}^{(2)}A_{K_2}x_2(t)\right)^2}{\left(\sigma_2(t) + \xi_2(x_2,t) + 2x_2^T(t)\mathcal{P}_{k_2}^{(2)}B_2B_2^T\mathcal{P}_{k_2}^{(2)}A_{K_2}x_2(t)\right)\left\|B_2^T\mathcal{S}_{k_2}^{(2)}x_2(t)\right\|^2}B_2^T\mathcal{S}_{k_2}^{(2)}x_2(t) \end{aligned} \tag{11}$$

$$\mathcal{V}_1(x_1,t) \triangleq x_1^T(t)\mathcal{S}_{k_1}^{(l)}x_1(t) \quad \text{for} \quad k_1 = \arg\max_{k_1}\left\{x_1^T(t)\mathcal{P}_{k_1}^{(l)}B_1B_1^T\mathcal{P}_{k_1}^{(l)}x_1(t)\right\} \tag{15}$$

where $\Omega_{\mathcal{S}_{k_l}^{(l)}}$ represents a subspace defined as

$$\Omega_{\mathcal{S}_{k_l}^{(l)}} \triangleq \left\{x_l \in \mathbb{R}^{n_l} \mid B_l^T\mathcal{S}_{k_l}^{(l)}x_l = 0\right\}. \tag{5}$$

The nominal subsystem, ignoring the unknown parameters and the interactions in (1), is given by

$$\frac{d}{dt}\overline{x}_l(t) = A_{ll}\overline{x}_l(t) + B_l\overline{u}_l(t). \tag{6}$$

In this paper, first of all, we adopt the standard LQ control theory for the nominal subsystem of (6), i.e. the control input is given by $\overline{u}_l(t) = -\mathcal{R}_l^{-1}B_l^T\mathcal{X}_l\overline{x}(t)\ (= K_l\overline{x}(t))$ *

Now on the basis of the works of Oya and Hagino [2002] and Oya et al. [2011], by using the fixed gain matrix $K_l \in \mathbb{R}^{m_l\times n_l}$ for the l-th nominal system of (6) we consider the following control input.

$$u_l(t) \triangleq K_lx_l(t) + \psi_l(x_l,t) \tag{7}$$

where $\psi_l(x_l,t) \in \mathbb{R}^{m_l}$ is a decentralized compensation input for the l-th subsystem. From (1), (6) and (7), we have the closed-loop system for the l-th subsystem of (8). In (8), A_{K_l} is the matrix given by $A_{K_l} = A_l + B_lK_l$.

From the above discussion, our control objective in this paper is to design the decentralized variable gain robust control of (7) such that the resultant closed-loop system is robustly stable. That is to design the decentralized compensation input $\psi_l(x_l,t) \in \mathbb{R}^{m_l}$ such that asymptotical stability of the overall system composed of the $\mathcal{N}$ subsystems of (8) is guaranteed.

* Note that $\mathcal{X}_l \in \mathbb{R}^{n_l\times n_l}$ is the unique solution of the algebraic Riccati equation $H_e\left\{A_{ll}^T\mathcal{X}_l\right\} - \mathcal{X}_lB_l\mathcal{R}_l^{-1}B_l^T\mathcal{X}_l + \mathcal{Q}_l = 0$ where the matrices $\mathcal{Q}_l \in \mathbb{R}^{n_l\times n_l}$ and $\mathcal{R}_l \in \mathbb{R}^{m_l\times m_l}$ are design parameters and $\mathcal{Q}_l$ is selected such that the pair $(A_{ii},\mathcal{C}_l)$ is detectable where $\mathcal{C}_l$ is any matrix satisfying $\mathcal{Q}_l = \mathcal{C}_l\mathcal{C}_l^T$.

4. MAIN RESULTS

In this section, we show a design method of the decentralized variable gain robust controller via piecewise Lyapunov functions such that the overall system is asymptotically stable. In the sequel, we consider the case of $\mathcal{M}_l = 2$ and $\mathcal{N} = 2$ for simplicity, because the results for the case of $\mathcal{M}_l > 2$ and $\mathcal{N} > 2$ can easily be obtained by the following result and the existing results (Oya and Hagino [2002] and Oya et al. [2011]).

The following theorem gives sufficient conditions for the existence of the proposed decentralized controller.

Theorem 1. Consider the closed-loop system with nonlinear perturbations of (8) and the control input of (7).

If there exist symmetric positive definite matrices $\mathcal{S}_{k_l}^{(l)} \in \mathbb{R}^{n_l\times n_l}$ $\left(\mathcal{S}_{k_l}^{(l)} \triangleq \mathcal{P}_1^{(l)} + \mathcal{P}_2^{(l)} + \mathcal{P}_{k_l}^{(l)}B_lB_l^T\mathcal{P}_{k_l}^{(l)}\right)$ and $\mathcal{P}_{k_l}^{(l)} \in \mathbb{R}^{n_l\times n_l}$ $(k_l = 1, 2,\ l = 1, 2)$ and positive constants $\gamma_{k_l}^{(l)}$ and ϑ_i which satisfy matrix inequalities of (9) and (10) and the relation of (3), then by using the matrices $\mathcal{P}_{k_l}^{(l)} \in \mathbb{R}^{n_l\times n_l}$, the decentralized compensation input $\psi_l(x_l,t) \in \mathbb{R}^{m_l}$ is determined as (11) ** where $\xi_l(x_i,t) \in \mathbb{R}^1$ is the function given by

$$\begin{aligned} &\xi_1(x_1,t) \triangleq x_1^T(t)\left(\frac{1}{\vartheta_1}\mathcal{S}_{k_1}^{(1)}\mathcal{S}_{k_1}^{(1)} + \alpha_1^2\vartheta_1\Psi_1\right)x_1(t) \\ &\xi_2(x_2,t) \triangleq x_2^T(t)\left(\frac{1}{\vartheta_2}\mathcal{S}_{k_2}^{(2)}\mathcal{S}_{k_2}^{(2)} + \alpha_2^2\vartheta_2\Psi_2\right)x_2(t) \end{aligned} \tag{12}$$

and $\sigma_l(t) \in \mathbb{R}^1$ is any positive uniform continuous and bounded function which satisfies

$$\int_{t_0}^{t}\sigma_l(\tau)d\tau \leq \sigma_l^* < \infty \tag{13}$$

** We find that the decentralized compensation input $\psi_l(x_l,t) \in \mathbb{R}^{m_l}$ can be calculated except for $x_l(t) \equiv 0$ (Oya et al. [2011]).

$$\frac{d}{dt}\mathcal{V}_1(x_1,t) = x_1^T(t)\left[H_e\left\{\mathcal{S}_{k_1}^{(1)}A_{K_1}\right\}\right]x_1(t) + H_e\left\{x_1^T(t)\mathcal{S}_{k_1}^{(1)}h_1(x_1,t) + x_1^T(t)\mathcal{S}_{k_1}^{(1)}B_1\psi_1(x_1,t)\right\} \tag{16}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_1(x_1,t) \le{}& x_1^T(t)\left[H_e\left\{\left(\mathcal{P}_1^{(1)}+\mathcal{P}_2^{(1)}\right)A_{K_1}\right\}\right]x_1(t) + 2x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}A_{K_1}x_1(t)\\ &+x_1^T(t)\left(\frac{1}{\vartheta_1}\mathcal{S}_{k_1}^{(1)}\mathcal{S}_{k_1}^{(1)} + \alpha_1^2\vartheta_1\Psi_1\right)x_1(t) + H_e\left\{x_1^T(t)\mathcal{S}_{k_1}^{(1)}B_1\psi_1(x_1,t)\right\}\\ &\text{for}\quad k_1 = \arg\max_{k_1}\left\{x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}x_1(t)\right\}\end{aligned} \tag{18}$$

$$\frac{d}{dt}\mathcal{V}_1(x_1,t) \le x_1^T(t)\left[H_e\left\{\left(\mathcal{P}_1^{(1)}+\mathcal{P}_2^{(1)}\right)A_{K_1}\right\}\right]x_1(t) + \sigma_1(t) \quad \text{for}\quad k_1 = \arg\max_{k_1}\left\{x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}x_1(t)\right\} \tag{20}$$

$$x_1^T(t)\left[H_e\left\{\left(\mathcal{P}_1^{(1)}+\mathcal{P}_2^{(1)}\right)A_{K_1}\right\}\right]x_1(t) < 0 \quad \text{for}\quad k_1 = \arg\max_{k_1}\left\{x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}x_1(t)\right\} \tag{21}$$

$$\begin{aligned}&\mathcal{V}_2(x_2,x_1,t) \stackrel{\triangle}{=} x_2^T(t)\mathcal{S}_{k_2}^{(2)}x_2(t) + \mathcal{V}_1(x_1,t)\\ &\qquad\text{for}\quad k_2 = \arg\max_{k_2}\left\{x_2^T(t)\mathcal{P}_{k_2}^{(l)}B_2B_2^T\mathcal{P}_{k_2}^{(l)}x_2(t)\right\} \quad\text{and}\quad k_1 = \arg\max_{k_1}\left\{x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}x_1(t)\right\}\end{aligned} \tag{24}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_2(x_2,x_1,t) \le{}& x_2^T(t)\left[H_e\left\{\left(\mathcal{P}_1^{(2)}+\mathcal{P}_2^{(2)}\right)A_{K_2}\right\}\right]x_2(t) + H_e\left\{x_2^T(t)\mathcal{S}_{k_2}^{(2)}A_{21}x_1(t)\right\} + \frac{d}{dt}\mathcal{V}_1(x_1,t)\\ &+\vartheta_2\alpha_{21}^2x_1^T(t)\Psi_{21}x_1(t) + \sigma_2(t)\\ &\text{for}\quad k_2 = \arg\max_{k_2}\left\{x_2^T(t)\mathcal{P}_{k_2}^{(l)}B_2B_2^T\mathcal{P}_{k_2}^{(l)}x_2(t)\right\} \quad\text{and}\quad k_1 = \arg\max_{k_1}\left\{x_1^T(t)\mathcal{P}_{k_1}^{(1)}B_1B_1^T\mathcal{P}_{k_1}^{(1)}x_1(t)\right\}\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_2(x_2,x_1,t) \le{}& \begin{pmatrix}x_2(t)\\ x_1(t)\end{pmatrix}^T\begin{pmatrix}H_e\left\{\left(\mathcal{P}_1^{(2)}+\mathcal{P}_2^{(2)}\right)A_{K_2}\right\} & \mathcal{S}_{k_2}^{(2)}A_{21}\\ \star & -\mathcal{W}_1+\vartheta_2\alpha_{21}^2\Psi_{21}\end{pmatrix}\begin{pmatrix}x_2(t)\\ x_1(t)\end{pmatrix} + \sigma_1(t) + \sigma_2(t)\\ &\text{for}\quad k_2 = \arg\max_{k_2}\left\{x_2^T(t)\mathcal{P}_{k_2}^{(2)}B_2B_2^T\mathcal{P}_{k_2}^{(2)}x_2(t)\right\}\end{aligned} \tag{26}$$

$$\frac{d}{dt}\mathcal{V}_2(x_2,x_1,t) \le -x_2^T(t)\left(\mathcal{Q}_{k_2}+\mathcal{W}_2\right)x_2(t) + \sigma_1(t) + \sigma_2(t) \tag{27}$$

where σ_l^* is any positive constant and t_0 denotes an initial time. In (9) and (10), $\mathcal{Q}_{k_l}^{(l)}$ and $\mathcal{W}_l$ are design parameters. Then the overall system composed of 2 subsystems is bounded and the following relation holds.

$$\lim_{t\to\infty} x(t;t_0,x(t_0)) = 0 \tag{14}$$

where $x(t) \stackrel{\triangle}{=} \left(\, x_1^T(t)\ x_2^T(t)\,\right)^T$. Namely, asymptotical stability of the resultant closed-loop system is ensured.

Proof: Firstly, let us define the piecewise quadratic functions $\mathcal{V}_1(x_1,t)$ of (15) *** at the top of the previous page. The time derivative of the piecewise quadratic functions $\mathcal{V}_1(x,t)$ of (15) along the trajectory of the l-th subsystem ($l = 1,\cdots,3$) can be written as (16). By using the well-known relation

$$2a^Tb \le \frac{1}{\mu}a^Ta + \mu b^Tb \tag{17}$$

for arbitrary vectors a and b with appropriate dimensions and a positive constant μ, we can obtain the relation of (18). Furthermore we see from the decentralized compensation input of (10) and the well-known inequality

$$0 \le \frac{\alpha\beta}{\alpha+\beta} \le \alpha \quad \forall\alpha,\beta > 0 \tag{19}$$

that some trivial manipulations give the inequality of (20) for the time derivative of the piecewise quadratic function $\mathcal{V}_1(x_l,t)$ of (15).

Let us consider the condition of (21). By applying **Lemma 2** ($\mathcal{S}$-procedure) to the inequalities of (21) one can see from the definition of the matrices $\mathcal{S}_k$ that the inequalities of (21) are satisfied if there exist $\mathcal{P}_{k_1}^{(1)} > 0$ and $\gamma_{k_1}^{(1)} \ge 0$ satisfying inequalities of (9). Therefore if the condition of (9) holds then we have the inequality

$$\frac{d}{dt}\mathcal{V}_1(x_1,t) \le -x_1^T(t)\left(\mathcal{Q}_{k_1}+\mathcal{W}_1\right)x_1(t) + \sigma_1(t). \tag{22}$$

Besides, the inequality of (22) can be reduced to

$$\frac{d}{dt}\mathcal{V}_1(x_1,t) \le -\zeta_1^*\left\|x_1(t)\right\|^2 + \sigma_1(t) \tag{23}$$

where $\zeta_1^* \stackrel{\triangle}{=} \min_{k_1}\{\mathcal{Q}_{k_1}+\mathcal{W}_1\}$. Thus, it is proved by same procedure as the proof of **Theorem 1** in Oya et al. [2011] that $x_1(t)$ is uniformly continuous, and by using **Lemma 3** (Barbalat's lemma) the relation $\|x(t)\| \to 0 \quad (t\to\infty)$ holds. Namely, asymptotical stability of the 1st subsystem is ensured.

Next, we introduce the piecewise quadratic function $\mathcal{V}_2(x,t)$ for the 2nd subsystem given by (24). Then by using the similar manipulations for the 1st subsystem, we have the relation of (25) for the piecewise quadratic function $\mathcal{V}_2(x,t)$. Since the function $\mathcal{V}_1(x_1,t)$ for the 1st subsystem satisfies the inequality of (22), one can see that the relation of (25) can be transformed into the inequality of (26). Moreover, one can see from the inequalaities of (10) that by applying **Lemma 2** ($\mathcal{S}$-procedure) to the inequalities of (26), the relation of (27) holds. Thus, we see robust stability of the 2nd subsystem can also be shown by using **Lemma 3** (Barbalat's lemma).

*** Note that the function $\mathcal{V}_1(x_1,t)$ of (15) is continuous and its level set is closed.

$$
\begin{aligned}
&H_e\left\{\mathcal{P}_1 A_{K_1}\right\}+\gamma_1^{(1)}\mathcal{P}_1^{(1)}B_1B_1^T\mathcal{P}_1^{(1)}\leq 0 \text{ and } H_e\left\{\mathcal{P}_2^{(1)}A_{K_1}\right\}-\gamma_1\mathcal{P}_2^{(1)}B_1B_1^T\mathcal{P}_2^{(1)}+\mathcal{Q}_1^{(1)}+\mathcal{W}_1<0\\
&H_e\left\{\mathcal{P}_2 A_{K_1}\right\}+\gamma_2^{(1)}\mathcal{P}_2^{(1)}B_1B_1^T\mathcal{P}_2^{(1)}\leq 0 \text{ and } H_e\left\{\mathcal{P}_1^{(1)}A_{K_1}\right\}-\gamma_2\mathcal{P}_1^{(1)}B_1B_1^T\mathcal{P}_1^{(1)}+\mathcal{Q}_2^{(1)}+\mathcal{W}_1<0
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\left(\begin{array}{c:c}\varOmega_1\left(\mathcal{P}_1,\gamma_1\right) & \mathcal{P}_1^{(2)}A_{21}+\mathcal{P}_1^{(2)}BB^T\mathcal{P}_1^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1\end{array}\right)<0 \quad\text{and}\quad \left(\begin{array}{c:c}\varPi_1\left(\mathcal{P}_2,\gamma_1\right) & \mathcal{P}_1^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1+\vartheta_2\alpha_{21}^2\varPsi_{21}\end{array}\right)<0\\
&\left(\begin{array}{c:c}\varOmega_2\left(\mathcal{P}_2,\gamma_2\right) & \mathcal{P}_2^{(2)}A_{21}+\mathcal{P}_2^{(2)}BB^T\mathcal{P}_2^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1\end{array}\right)<0 \quad\text{and}\quad \left(\begin{array}{c:c}\varPi_2\left(\mathcal{P}_1,\gamma_2\right) & \mathcal{P}_1^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1+\vartheta_2\alpha_{21}^2\varPsi_{21}\end{array}\right)<0
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&\varOmega_{k_2}\left(\mathcal{P}_{k_2},\gamma_{k_2}\right)=H_e\left\{\mathcal{P}_{k_2}^{(2)}A_{K_2}\right\}+\gamma_{k_2}^{(2)}\mathcal{P}_1^{(2)}B_2B_2^T\mathcal{P}_1^{(2)}\\
&\varPi_{k_2}\left(\mathcal{P}_{\zeta_k},\gamma_{k_2}\right)=H_e\left\{\mathcal{P}_{\zeta_k}^{(2)}A_{K_2}\right\}-\gamma_{k_2}^{(2)}\mathcal{P}_{\zeta_k}^{(2)}B_2B_2^T\mathcal{P}_{\zeta_k}^{(2)}+\mathcal{Q}_{k_2}^{(2)}+\mathcal{W}_2
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&\left(\begin{array}{c:c}\varOmega\left(\mathcal{P}_1,\gamma_1\right)+\mathcal{P}_1^{(2)}BB^T\mathcal{P}_1^{(2)}BB^T\mathcal{P}_1^{(2)} & \mathcal{P}_1^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1+A_{21}^T\mathcal{P}_1^{(2)}A_{21}\end{array}\right)<0 \quad\text{and}\quad \left(\begin{array}{c:c}\varPi_1\left(\mathcal{P}_2,\gamma_1\right) & \mathcal{P}_2^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1+\vartheta_2\alpha_{21}^2\varPsi_{21}\end{array}\right)<0\\
&\left(\begin{array}{c:c}\varOmega\left(\mathcal{P}_2,\gamma_2\right)+\mathcal{P}_2^{(2)}BB^T\mathcal{P}_2^{(2)}BB^T\mathcal{P}_2^{(2)} & \mathcal{P}_2^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1+A_{21}^T\mathcal{P}_2^{(2)}A_{21}\end{array}\right)<0 \quad\text{and}\quad \left(\begin{array}{c:c}\varPi_2\left(\mathcal{P}_1,\gamma_2\right) & \mathcal{P}_1^{(2)}A_{21}\\ \hdashline \star & -\mathcal{Z}_1+\vartheta_2\alpha_{21}^2\varPsi_{21}\end{array}\right)<0
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&H_e\left\{A_{K_1}\mathcal{Y}_1^{(1)}\right\}+\gamma_1^{(1)}B_1B_1^T\leq 0 \text{ and } \left(\begin{array}{c:c}H_e\left\{A_{K_1}\mathcal{Y}_2^{(1)}\right\}-\gamma_1^{(1)}B_1B_1^T & \mathcal{Y}_2^{(1)}\\ \hdashline \star & -\left(\mathcal{Q}_1^{(1)}+\mathcal{W}_1\right)^{-1}\end{array}\right)<0\\
&H_e\left\{A_{K_1}\mathcal{Y}_2^{(1)}\right\}+\gamma_2^{(1)}B_1B_1^T\leq 0 \text{ and } \left(\begin{array}{c:c}H_e\left\{A_{K_1}\mathcal{Y}_1^{(1)}\right\}-\gamma_2^{(1)}B_1B_1^T & \mathcal{Y}_1^{(1)}\\ \hdashline \star & -\left(\mathcal{Q}_2^{(1)}+\mathcal{W}_1\right)^{-1}\end{array}\right)<0
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
&\left(\begin{array}{c:c:c:c}\varOmega_1^*\left(\mathcal{Y}_1^{(2)},\gamma_1^{(2)}\right) & A_{21} & B_2^TB_2 & 0\\ \hdashline \star & -\mathcal{Z}_1 & 0 & A_{21}^T\\ \hdashline \star & \star & -\mathcal{Y}_1^{(2)} & 0\\ \hdashline \star & \star & \star & -\mathcal{Y}_1^{(2)}\end{array}\right)<0 \text{ and } \left(\begin{array}{c:c:c}\varPi_1^*\left(\mathcal{Y}_2^{(2)},\gamma_1^{(2)}\right) & A_{21} & \mathcal{Y}_2^{(2)}\\ \hdashline \star & -\mathcal{Z}_1+\vartheta_2\alpha_{21}^2\varPsi_{21} & 0\\ \hdashline \star & \star & -\mathcal{T}_1^{(2)}\end{array}\right)<0\\
&\left(\begin{array}{c:c:c:c}\varOmega_2^*\left(\mathcal{Y}_2^{(2)},\gamma_2^{(2)}\right) & A_{21} & B_2^TB_2 & 0\\ \hdashline \star & -\mathcal{Z}_1 & 0 & A_{21}^T\\ \hdashline \star & \star & -\mathcal{Y}_2^{(2)} & 0\\ \hdashline \star & \star & \star & -\mathcal{Y}_2^{(2)}\end{array}\right)<0 \text{ and } \left(\begin{array}{c:c:c}\varPi_2^*\left(\mathcal{Y}_1^{(2)},\gamma_2^{(2)}\right) & A_{21} & \mathcal{Y}_1^{(2)}\\ \hdashline \star & -\mathcal{Z}_1+\vartheta_2\alpha_{21}^2\varPsi_{21} & 0\\ \hdashline \star & \star & -\mathcal{T}_2^{(2)}\end{array}\right)<0
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&\varOmega_{k_2}^*\left(\mathcal{Y}_{k_2}^{(2)},\gamma_{k_2}^{(2)}\right)=H_e\left\{A_{K_2}\mathcal{Y}_{k_2}^{(2)}\right\}+\gamma_{k_2}^{(2)}BB^T,\quad \varPi_{k_2}^*\left(\mathcal{Y}_{\zeta_k}^{(2)},\gamma_{k_2}^{(2)}\right)=H_e\left\{A_{K_2}\mathcal{Y}_{\zeta_k}^{(2)}\right\}-\gamma_{k_2}^{(2)}B_2B_2^T,\\
&\mathcal{T}_1=\left(\mathcal{Q}_1^{(2)}+\mathcal{W}_1\right)^{-1} \quad\text{and}\quad \mathcal{T}_2=\left(\mathcal{Q}_2^{(2)}+\mathcal{W}_1\right)^{-1}
\end{aligned}
\tag{34}
$$

From the above, asymptotical stability of the resultant closed-loop system of (8) is guaranteed. Therefore the proof of **Theorem 1** is completed. ■

Theorem1 gives a sufficient condition for the existence of the proposed decentralized variable gain robust controller. Next, we consider the inequality conditions of (9) and (10) in **Theorem 1**.

The inequality conditions of (9) and (10) require positive definite matrices $\mathcal{P}_{k_l}^{(1)}\in\mathbb{R}^{n_l\times n_l}$ and positive scalars γ_{k_l} and ϑ_l for stability. Therefore, we consider the conditions of (28) and (29) instead of that of (9) and (10). In (29), $\mathcal{Z}_1=\frac{1}{2}\mathcal{W}_1$ and $\varOmega_{k_2}\left(\mathcal{P}_{k_2},\gamma_{k_2}\right)$ and $\varPi_{k_2}\left(\mathcal{P}_{\mathcal{S}},\gamma_{k_2}\right)$ are the matrices given by (30). Note that in (28), ζ_k is the integer defined as $\zeta_k=1$ if $k_2=2$ or $\zeta_k=2$ if $k_2=1$. By using the well-known relation of (17) again, we see that the inequality of (29) can be transformed into (31). Thus by introducing the matrices $\mathcal{Y}_{k_l}^{(1)}\triangleq(\mathcal{P}_{k_l}^{(1)})^{-1}$ and using **Lemma 4** (Schur complement), one can obtain (32) and (33) as sufficient conditions for inequalities of (28) and (29). In (33), the matrices $\varOmega_1^*(\mathcal{Y}_{k_2}^{(2)},\gamma_{k_2}^{(2)})$, $\varPi_{k_2}^*(\mathcal{Y}_{\mathcal{S}}^{(2)},\gamma_{k_2}^{(2)})$ and $\mathcal{T}_{k_2}^{(2)}$ are given by (34). Namely, we can obtain the proposed decentralized variable gain robust controller by solving LMIs of (32) and (33). As a result, we have the following theorem.

Theorem 2. Consider the closed-loop system with nonlinear perturbations of (8) and the control input (7).

If the LMIs of (32) and (33) are feasible, by using the solution of LMIs of (32) and (33) the decentralized compensation input $\psi_l(x_l,t)\in\mathbb{R}^{m_l}$ is determined as (11). Then the overall system composed of $\mathcal{N}$ subsystems is bounded and the relation of (14) holds.

5. ILLUSTRATIVE EXAMPLES

Consider the large-scale interconnected systems composed of three two-dimensional subsystems, i.e. $\mathcal{N}=3$. The nominal values of system parameters are given as (35) at the top of the next page. Besides, α_l, α_{ij}, $\varPsi_l$ and $\varPsi_{ij}$ are selected as $\alpha_l=\alpha_{ij}=1.0$ and $\varPsi_l=\varPsi_{ij}=\mathrm{diag}\,(5.0, 4.0)$, respectively.

$$
\begin{aligned}
&A_{11} = \begin{pmatrix} -1.0 & 1.0 \\ 0.0 & 1.0 \end{pmatrix}, \; A_{21} = \begin{pmatrix} 5.0 \times 10^{-1} & 1.0 \\ 0.0 & -5.0 \times 10^{-1} \end{pmatrix}, \; A_{22} = \begin{pmatrix} 5.0 \times 10^{-1} & -1.0 \\ 0.0 & 2.0 \end{pmatrix}, \\
&A_{33} = \begin{pmatrix} 2.0 & 1.0 \\ 1.0 & -1.0 \end{pmatrix}, \; A_{32} = \begin{pmatrix} 5.0 \times 10^{-1} & 0.0 \\ 2.5 \times 10^{-1} & -1.0 \end{pmatrix}, \; B_1 = \begin{pmatrix} 0.0 \\ 4.0 \end{pmatrix}, \; B_2 = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}, \; B_3 = \begin{pmatrix} 3.0 \\ 0.0 \end{pmatrix}
\end{aligned} \tag{35}
$$

$$
K_1 = \begin{pmatrix} -8.04556 \times 10^{-1} & -3.32649 \end{pmatrix}, \; K_2 = \begin{pmatrix} 1.50606 & -7.07401 \end{pmatrix}, \; K_3 = \begin{pmatrix} -3.22466 & -1.64832 \end{pmatrix} \tag{36}
$$

$$
\begin{aligned}
&\mathcal{Y}_1^{(1)} = \begin{pmatrix} 1.7290744 \times 10^{-1} & -1.2342716 \times 10^{-1} \\ \star & 2.45060 \end{pmatrix}, \; \mathcal{Y}_2^{(1)} = \begin{pmatrix} 1.74903 \times 10^{-1} & -1.30699 \times 10^{-1} \\ \star & 2.41107 \end{pmatrix}, \\
&\mathcal{Y}_1^{(2)} = \begin{pmatrix} 2.27376 & 1.54445 \\ \star & 1.61528 \end{pmatrix}, \; \mathcal{Y}_2^{(2)} = \begin{pmatrix} 2.68200 & 1.97840 \\ \star & 2.09495 \end{pmatrix}, \\
&\mathcal{Y}_1^{(3)} = \begin{pmatrix} 7.8344613 & -5.29204 \times 10^{-1} \\ \star & 7.21506 \end{pmatrix}, \; \mathcal{Y}_2^{(3)} = \begin{pmatrix} 1.168031 \times 10^{1} & -9.99192 \times 10^{-1} \\ \star & 1.19941 \times 10^{1} \end{pmatrix}, \\
&\gamma_1^{(1)} = 1.55376, \; \gamma_2^{(1)} = 1.15821, \; \gamma_1^{(2)} = 8.57891, \; \gamma_2^{(2)} = 9.14068, \; \gamma_1^{(3)} = 5.10568, \; \gamma_2^{(3)} = 6.13331 \\
&\vartheta_2 = 7.51474 \times 10^{-3}, \; \vartheta_3 = 9.09547 \times 10^{-1}
\end{aligned} \tag{37}
$$

Now, we select the weighting matrices $\mathcal{Q}_l \in \mathbb{R}^{2\times 2}$ and $\mathcal{R}_l \in \mathbb{R}^{1\times 1}$ $(i = 1, \cdots, 3)$ for the nominal subsystems such as $\mathcal{Q}_1 = \mathrm{diag}\,(6.0, 9.0)$, $\mathcal{Q}_2 = \mathrm{diag}\,(3.0, 9.0)$, $\mathcal{Q}_3 = \mathrm{diag}\,(5.0, 9.0)$, $\mathcal{R}_1 = 1.0$, $\mathcal{R}_2 = 1.0 \times 10^1$ and $\mathcal{R}_3 = 1.0$. Then, we have the fixed gain matrices $K_l \in \mathbb{R}^{1\times 2}$ of (36).

Next selecting the matrices $\mathcal{W}_1$, $\mathcal{W}_2$, $\mathcal{Q}_1^{(l)}$ and $\mathcal{Q}_2^{(l)}$ in (32) and (33) as $\mathcal{W}_1 = 5.0 \times I_2$, $\mathcal{W}_2 = 1.5 \times I_2$, $\mathcal{Q}_1^{(l)} = \mathrm{diag}\left(1.0 \times 10^{-1}, 2.0\right)$ and $\mathcal{Q}_2^{(l)} = \mathrm{diag}\left(3.0, 2.0 \times 10^{-1}\right)$ respectively, and solving the LMIs of (32) and (33), we have the solution of (37). From the solution of (37), one can easily see that the relation of (4) is satisfied. Thus the proposed decentralized variable gain robust controller can be obtained.

6. CONCLUSIONS

In this paper on the basis of the concept of the piecewise Lyapunov functions, we have proposed a decentralized variable gain robust controller for a class of uncertain large-scale serially connected systems with nonlinear perturbations. Besides, by numerical simulations, the effectiveness of the proposed decentralized variable gain robust controller has been presented.

For a class of large-scale serially connected systems with nonlinear perturbations, by using the concept of piecewise Lyapunov functions, we have shown that the proposed decentralized variable gain robust controller can be obtained by using the solutions of LMIs. One can see that the result presented in this paper is an extention of the existing results (e.g. Oya and Hagino [2009], Oya et al. [2011]).

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