Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Distributed Optimization Methods for Wide-Area Damping Control of Power System Oscillations ⋆

Aranya Chakrabortty

Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27695, USA, E-mail: achakra2@ncsu.edu

Abstract:
This paper presents a distributed optimization algorithm to address the problem of wide-area damping control of large-scale electric power systems using Synchrophasors. Our approach consists of a three-step strategy. First, Synchrophasors from selected nodes in a power network are used to identify offline dynamic models of the dominant areas of the network. Thereafter, a linear controller is designed for this reduced-order model to shape the inter-machine oscillation dynamics. Finally, algorithms are developed to invert this design to realistic local controllers in each area by optimizing the controller parameters until their interarea response matches the closed-loop inter-machine response achieved in the second step. A model reference control design following this three-step strategy was recently proposed in [1] using a centralized controller. Our results in this paper extend that design by posing the problem purely from a perspective of distributed optimization.

Keywords: Electric Power Systems, Wide-area Control, Synchrophasors, Distributed Control

1. INTRODUCTION

Following the Northeast blackout of 2003, Wide-Area Measurement System (WAMS) technology using Phasor Measurement Units (PMUs) has largely matured for the North American grid [2]. However, as the number of PMUs scales up into the thousands in the next few years under the US Department of Energy's smart grid demonstration initiative, Independent System Operators (ISO) and utility companies are struggling to understand how the resulting gigantic volumes of real-time data can be efficiently harvested, processed, and utilized to solve wide-area monitoring and control problems for any realistic power system interconnection. It is rather intuitive that the current state-of-the-art centralized communication and information processing architecture of WAMS will no longer be sustainable under such a data explosion, and a completely distributed cyber-physical architecture will need to be developed [3].

Motivated by this challenge, in this paper we address the problem of distributed wide-area damping control using Synchrophasor feedback. We assume the system to be composed of multiple areas with a given set of PMU locations. Our strategy is to first derive an offline reference model for the closed-loop system using model reduction [1], categorize the available PMUs into *area-level* disjoint sets, and finally run a distributed optimization problem for tuning the controller parameters of each area-level power system stabilizer (or group of stabilizers) (PSS) to match the cumulative output of the actual model with that of the reference model. The main idea behind our design is a so-called, novel *control inversion* framework which allows PMU-based linear power system stabilizers (PSS) designs, developed for reduced-order power systems, to be inverted to PSS controllers in higher-order systems via suitable optimization methods. A model reference control design following this strategy was presented for two-area power systems in our recent work [1], but the implementation was still centralized. This paper extends that design by posing the control problem purely from a distributed optimization perspective. The approach consists of three precise steps, namely:

1. *Model Reduction/Dynamic Equivalencing* - where PMU data are used offline to identify equivalent models of the oscillation clusters of the entire power system based on the differences in their coupling strengths. Detailed derivations of these measurement-based equivalencing methods have been presented in [1], and, therefore, will not be our focus in this paper. Our objective is to design the PSS control for damping the oscillations between these areas in a distributed fashion, for which we will simply assume that the area models are available to us by prior identification methods.

⋆ This work was supported in part by NSF Grants ECCS 1054394 and 1062811

2. *Aggregate Control* - where output-feedback based linear PSS are designed to achieve a desired closed-loop transient response between every pair of clusters in the reduced-order system, and

3. *Control Inversion*- where the aggregate control design is distributed and tuned back to actual realistic controllers at the generator terminals until the inter-area responses of the full-order power system matches the respective inter-machine responses of the reduced-order system.

The most generic way to formulate the control inversion problem in the final step is to use functional optimization. This means that assuming standard second-order swing dynamics with first-order excitation for the j^{th} hypothetically aggregated machine in the reduced-order power system model, one may first design output-feedback excitation controllers:

$$u_j = f(y_1(t),\, y_2(t), ..., y_m(t),\, k_1,\, k_2, ..., k_m) \qquad (1)$$

where $y_i(t)$ is a chosen set of variables (eg. voltage magnitude/phase angle, frequency, etc.) measured over time $t \geq 0$ by a PMU installed at the i^{th} bus in the reduced network, and $f(\cdot)$ is a smooth, nonlinear damping function producing a desired inter-machine transient response. Next, u_j needs to be distributed to each local machine belonging to the j^{th} area. A plausible approach for this would be to construct nonlinear functions $\rho(\cdot)$ mapping each of the feedback gains $(k_1, k_2, ..., k_m)$ to each such machine. Stacking these functions $\rho(\cdot)$ and the gains k_j into vectors $\mathcal{R}$ and $\mathcal{K}$, respectively, the problem that we must, therefore, solve is:

$$\min_{\mathcal{R}(\mathcal{K})} \sum_{i=1}^{n^*} \int_o^T ||x_{ij}(t, \mathcal{R}(\mathcal{K})) - \bar{x}_{ij}(t, \mathcal{K})||_2 \quad \text{st. } \mathcal{K} \in \mathcal{K}^*, \qquad (2)$$

for all $j \in \mathcal{N}_i$, over time $t \in [0, T]$, where: n^* is the total number of areas, $\mathcal{N}_i$ is the index set for the neighboring areas of area i, x_{ij} is the interarea state response (phase or frequency) between i^{th} and j^{th} areas in the full-order system, $\bar{x}_{ij}$ is the *designed* inter-machine state response (phase or frequency, respectively) between i^{th} and j^{th} machines in the reduced-order system, and $\mathcal{K}^*$ denotes a constraint set for the feedback gains specifying their allowable upper and lower bounds.

The remainder of the paper is organized as follows. Section II presents a PMU-based model reduction method; Section III formulates the wide-area damping problem for a n-area power system as a distributed parametric minimization problem. Section IV illustrates the results through an example. Section V concludes the paper.

2. SWING MODELS AND MODEL REDUCTION

To formulate the distributed wide-area control problem we first present a brief discussion on the model reduction methods that can be used for steps 1 and 2 described in Section I.

2.1 Swing Oscillation Models

Consider a network of n synchronous generators connected to each other through m tie-lines (edges) with $m \leq n(n-1)/2$, forming a connected graph with cardinality (n, m), such that no more than one edge exists between any two nodes. Let the internal voltage phasor of the i^{th} machine be denoted as

$$\tilde{E}_i = E_i \angle \delta_i, \quad i = 1, 2, \ldots, n \qquad (3)$$

where, following synchronous machine theory [5], E_i is constant, δ_i is the angular position of the generator rotor, and $E_i \angle \delta_i$ denotes the polar representation $E_i\, e^{j\delta_i}$ ($j = \sqrt{-1}$). The transmission line connecting the p^{th} and the q^{th} machines is assumed to have an impedance $\tilde{z}_{pq} = r_{pq} + jx_{pq}$, where '$r$' denotes the resistive part and 'x' denotes the reactive part. Here $p \in \{1, 2, ..., n\}$ and $q \in \mathcal{N}_p$ where $\mathcal{N}_p$ is the set of nodes to which the p^{th} node is connected. It follows that the total number of tuples formed by pairing p and q is m. We will denote the edge connecting the p^{th} and the q^{th} node by e_{pq}. If two nodes do not share a connection then the impedance corresponding to that non-existing edge is infinite (i.e., open circuit), or equivalently, $\tilde{y}_{pq} = 1/\tilde{z}_{pq} = 0 \ \ \forall q \notin \mathcal{N}_p$ where $\tilde{y}_{pq}$ is the admittance of e_{pq}. The mechanical inertia of the i^{th} machine is denoted as H_i. We assume that the network structure is known, i.e., the set $\mathcal{N}_i$ for all $i = 1, 2, \ldots, n$.

Defining the small-signal state variables as

$$\Delta\delta = \text{col}(\Delta\delta_1, \Delta\delta_2, \ldots, \Delta\delta_n) \qquad (4)$$

$$\Delta\omega = \text{col}(\Delta\omega_1, \Delta\omega_2, \ldots, \Delta\omega_n). \qquad (5)$$

and assuming that the control input u enters the system through the j^{th} node, $j \in \{1, 2, \ldots, n\}$, the linearized swing model of the system can be written as

$$\left[\frac{\Delta\dot{\delta}}{\Delta\dot{\omega}}\right] = \underbrace{\left[\begin{array}{c|c} 0 & I \\ \hline \mathcal{M}^{-1}\mathcal{L} & 0 \end{array}\right]}_{A} \left[\frac{\Delta\delta}{\Delta\omega}\right] + \underbrace{\left[\frac{0}{\mathcal{E}_j}\right]}_{B} u \qquad (6)$$

where I is the n-dimensional identity matrix, $\mathcal{E}_j$ is the j^{th} unit vector with all elements zero except the j^{th} element which is 1, $\mathcal{M} = \text{diag}(M_1, M_2, \ldots, M_n)$, $M_i = 2H_i$ is the inertia of the i^{th} generator, and $\mathcal{L}$ is the $n \times n$ Laplacian matrix with elements:

$$\mathcal{L}_{ii} = -\sum_{k \in \mathcal{N}_i} \frac{E_i E_k}{p_{ik}} \sin(\delta_{i0} - \delta_{k0} + \alpha_{ik}), \qquad (7)$$

$$\mathcal{L}_{ik} = \frac{E_i E_k}{p_{ik}} \sin(\delta_{i0} - \delta_{k0} + \alpha_{ik}), \quad k \in \mathcal{N}_i, \qquad (8)$$

$$\mathcal{L}_{ik} = 0, \quad \text{otherwise} \qquad (9)$$

for $i = 1, 2, \ldots, n$. It follows that if $M_i = M_j, \forall(i, j)$, then $A = A^T$. However, in general each machine will have distinct inertia as a result of which the symmetry property does not hold. We, therefore, refer to $\mathcal{M}^{-1}\mathcal{L}$ as the unsymmteric Laplacian matrix for the linearized swing model.

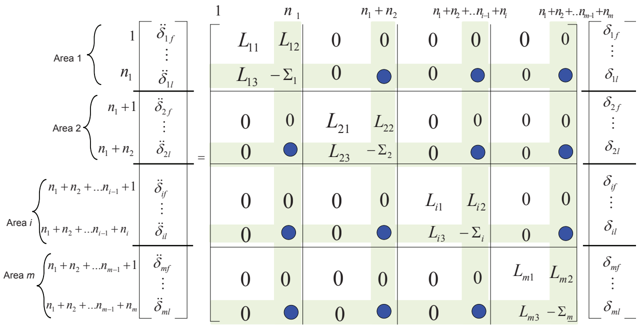


Fig. 1. Linearized Laplacian matrix denoting time-scale separation

It is obvious from (6) that the coupling strengths p_{ik} of the links are contained in this matrix, and will decide the separation of areas depending on the differences in the strengths. For example, if the areas are separated by exactly one boundary node in each cluster, then the Laplacian matrix can be written in the form shown in Figure 2.1, assuming that the phase angles are stacked according to each area with the last state of each stack being that of the boundary node for that particular area. The blue dots in the state matrix, under such a partition, will indicate the *inter-area* coupling strengths between any pair of areas, and can be used to reduce the full-order network into a dynamic equivalent system of n-equivalent machines using the parameter identification methods outlined in [1].

2.2 PMU Data Analysis & Model Identification

We next review the typical data analysis, filtering, signal separation, and model identification methods that can be used for estimating the aggregated dynamic swing models presented above for any given operating condition for the grid using PMU measurements [4].

Filtering and Signal Separation Before the voltage, phase angle, and frequency data can be used for identifying a dynamic model of interest, the measurements must be properly filtered and massaged with the objective of extracting the correct et of frequency components that are relevant to that model. Typically such measurements have both high frequency measurement noise and undesired low frequency oscillations arising from governer effects. If the goal is to construct an equivalent model capturing the interarea oscillations that typically range between 0.1 to 1 Hz, then neither of these components are necessary for the estimation. A simple band pass filter, therefore, may be used to filter the raw data and retain its components only within the desired frequency range. However, the outputs of the filter may still retain several modes that do not qualify as slow modes. A second round of filtering is, therefore, done by using subspace identification algorithms such as Eigenvalue Realization Algorithm (ERA). The computational scheme for applying ERA on multiple PMU data is shown in Figure 2. The basic ERA algorithm for separating the slow modal components of any given PMU

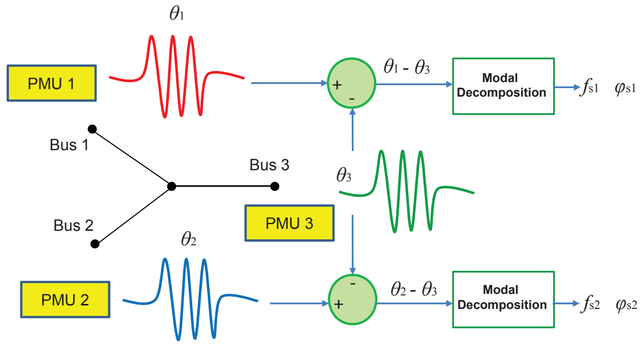


Fig. 2. Modal decomposition of multiple PMU data streams

signal $y(t)$ defined over N samples, can be described as follows. Consider a discrete-time LTI system

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k), \tag{10}$$

where $x(k) \in \mathbb{R}^n$ (n is known quantity), $k = 0, .., N$. The impulse response of the system will be given as

$$y(k) = CA^{k-1}B. \tag{11}$$

Given measurement $y(k)$, we next construct two $M \times p$ Hankel matrices H_0 and H_1 as:

$$H_0 = \left[\ \mathbf{y}_0^0 \mid \mathbf{y}_1^0 \mid \cdots \mid \mathbf{y}_p^0\ \right], \tag{12a}$$

$$H_1 = \left[\ \mathbf{y}_0^1 \mid \mathbf{y}_1^1 \mid \cdots \mid \mathbf{y}_p^1\ \right], \tag{12b}$$

where,

$$\mathbf{y}_i^0 = \left[y(i),\ y(i+1), \ldots,\ y(i+M-1)\right]^T, \tag{13a}$$

$$\mathbf{y}_i^1 = \left[y(i+1),\ y(i+2), \ldots,\ y(i+M)\right]^T. \tag{13b}$$

It can be easily shown that $H_0 = \mathcal{OC}$ and $H_1 = \mathcal{O}A\mathcal{C}$, where $\mathcal{O}$ and $\mathcal{C}$ are observability and controllability matrices for (10), respectively. We next consider the truncated SVD of H_0 as:

$$\hat{H}_0 = \hat{R}\hat{\Sigma}\hat{S}^T. \tag{14}$$

Defining $E_p^T = [I_p\ 0_p\ \cdots\ 0_p]$ and $E_q^T = [I_q\ 0_q\ \cdots\ 0_q]$, where p and q are the number of inputs and outputs, respectively, estimates for triplet of $(\hat{A}, \hat{B}, \hat{C})$ can be easily calculated as follows:

$$\hat{A} = \hat{\Sigma}^{-1/2}\hat{R}^T H_1 \hat{S}\hat{\Sigma}^{-1/2}, \tag{15a}$$

$$\hat{B} = \hat{\Sigma}^{1/2}\hat{S}^T E_p, \quad \hat{C} = E_q^T \hat{R}\hat{\Sigma}^{1/2}. \tag{15b}$$

From $(\hat{A}, \hat{B}, \hat{C})$, one can write:

$$y(t) = \underbrace{\alpha_0}_{\text{DC value}} \underbrace{+\alpha_1 e^{\pm j\Omega_1 t} + \cdots + \alpha_{r-1} e^{\pm j\Omega_{r-1} t}}_{y^s(t),\ \text{slow oscillations}} + \underbrace{\alpha_r e^{\pm j\Omega_r t} + \cdots + \alpha_{n-1} e^{\pm j\Omega_{n-1} t}}_{y^f(t),\ \text{fast oscillations}}, \tag{16}$$

where, by assumption, there are $r-1$ slow modes (or, equivalently r areas). The constants α's and Ω's are known from estimation. By designating an upper bound on Ω_{r-1}, say 1 Hz, one can, therefore, easily extract $y^s(t)$ from $y(t)$ using (15).

2.3 Model Identification

Although the oscillation model derived in (6) motivates the time-scale separation between interarea modes, it also implicitly assumes that the coherent areas in the system are connected directly to their neighboring areas without the presence of any other bus in between. In an actual power system, however, this connection may not be direct and will possibly involve intermediate buses, also referred to as PQ buses or load buses depending on if a load is being tapped from that bus. We, therefore, pose the model identification problem using both generator buses (or PV buses) and PQ buses. Without any loss of generality, we assume that every area is connected to every other area through an equivalent line with an equivalent impedance. Due to the presence of buses where there may not be any generation or load, the state-space model for this system will be differential-algebraic, which can be converted to a completely differential model using Kron reduction [6]. The small-signal model is given as:

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0_{5\times5} & \omega_s I_{5\times5} \\ \mathrm{diag}(\frac{-L}{M_i}) & -\mathrm{diag}(\frac{2D_i}{M_i}) \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0_{5\times5} \\ \mathrm{diag}(\frac{1}{M_i}) \end{bmatrix} \Delta P_m \tag{17}$$

where the expressions for the matrices in the RHS can be found in [5], and are skipped here for brevity. Considering that PMU measurements of voltage, phase angle and frequency are available from the terminal buses, the output equation can be written as

$$\begin{bmatrix} \Delta V \\ \Delta\theta \\ \Delta f \end{bmatrix} = \begin{bmatrix} F & 0 \\ G & 0 \\ 0 & \omega_s G \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}. \tag{18}$$

The unknown parameters in (17) include the aggregate inertias of each equivalent machine, the internal impedance of each area connecting the machine internal node to its corresponding terminal bus, and the impedances of each transmission line. These parameters can be estimated from (18) using standard least squares techniques. Due to changing operating conditions of the grid, the model may be estimated periodically every hour by the operator, and the most updated model may further be used for designing damping controllers as discussed in the next section.

3. DISTRIBUTED DAMPING CONTROL

We next describe a distributed optimization approach for designing PSS controllers interacting across areas for damping the interarea oscillation modes λ_s. The first step of the design is based on the reduce-order model identified using PMU data, as discussed in the previous section, followed by a control inversion strategy, recently developed in [1]. These may be summarized as follows:

1. A linear control design is performed for this reduced-order model to guarantee a desired dynamic performance for all the inter-machine power flows (which are equivalent to the inter-area flows in the full-order model). These damped power flow signals are used as references for the wide-area design in the next step.

2. A distributed optimization problem is solved for tuning actual PSS parameters in the full-order system until the *interarea* response of this system replicates the closed-loop *inter-machine* reference obtained in the previous step.

Mathematically, the second step can be posed as follows. Let the state-variable model for the j^{th} chosen generator with a tunable PSS, for $j = 1, ...m$, be given as

$$\dot{\delta}_j = \omega_j \tag{19}$$

$$M_j\dot{\omega}_j = P_{mj} - D_j\omega_j - P_{ej} \tag{20}$$

$$\tau_j \dot{E}_j = -\frac{x_{dj}}{x'_{dj}}E_j + \frac{x_{dj} - x'_{dj}}{x'_{dj}}\cos(\delta_j - \theta_j) + E_{Fj} \tag{21}$$

where (21) represents the excitation system dynamics of the generator, and E_{Fj} is the excitation control feedback for the PSS. Let $\mathcal{M}$ be the set of bus indices where a PMU is installed, and $\mathcal{M}^j$ be the subset of $\mathcal{M}$ that are available for output-feedback to the j^{th} PSS. The measurements in the set $\mathcal{M}^j$ are denoted as $y_{\mathcal{M}^j}(t)$. Let the set of boundary buses separating the areas be $\mathcal{E}_b$, and the communication graph between the different controllers be $\mathcal{G}$. The distributed control problem then reduces to designing the function $\psi(\cdot)$ for

$$E_{Fj}(t) = \psi_j(y_{\mathcal{M}^k}(t), x_{k\in\mathcal{N}_j}(t - \tau_{jk}), t), \quad j = 1, .., m \tag{22}$$

where $\mathcal{N}_j$ is the neighbor set of j^{th} controller following from $\mathcal{G}$, and τ_{jk} is the communication delay in the channel connecting the j^{th} and k^{th} controllers, such that all closed-loop state responses are bounded over time, and the 'slow' oscillation component of the relative phase angle difference $x_{pq}(t) \triangleq (x_p(t) - x_q(t))$ between every pair of boundary nodes $(p, q) \in \mathcal{E}_b$ satisfy a desired response, which is precisely the corresponding inter-machine reference signal designed in Step 2. However, we must remember that in the actual system the signal $x_{pq}(t)$ will contain the contribution of both local and inter-area modes. Hence, for an accurate tracking we must filter this signal through a band-pass filter (BPF), whose pass-band is designed to cover the typical inter-area frequency spectrum (0.1-1 Hz). We denote this BPF as $G(s)$, and design it using standard Butterworth filters. The filter coefficients can be designed, for example, using convex optimization. Furthermore, we consider our PSS designs to be linear, which means that essentially we need to design an output feedback controller of the form

$$C_j(s) = \frac{\rho_{j0} + \rho_{j1}s + \rho_{j2}s^2 + ... + \rho_{jj_a}s^{j_a}}{\vartheta_{j0} + \vartheta_{j1}s + \vartheta_{j2}s^2 + ... + \vartheta_{jj_b}s^{j_b}} \tag{23}$$

where j_a and j_b are fixed integers (practically, both should be less than or equal to 3 since high-order controllers increase processing delay). Denote the controller parameter set as

$$\mathcal{R}_j = \{\rho_{j0}, ..., \rho_{jj_a}, \vartheta_{j0}, ..., \vartheta_{jj_b}\}. \tag{24}$$

The distributed control design problem then simply reduces to a distributed parametric optimization problem for finding the optimal $\mathcal{R}_j$, $j = 1, ..., m$, that guarantees:

$$x(t) \in l_2, \ \min ||G(s)[x_{pq}](t) - x^d_{pq}(t)||_2, \ \forall (p, q) \in \mathcal{E}_t \quad (25)$$

over $t \in [0,\, t_f]$, where, $x^d_{pq}(t)$ is the desired power flow response following from the pre-designed model in Step 2.

3.1 Parallel Variable Distribution

We wrap up this section by briefly describing a candidate optimization method that we use for solving (25) in the simulations of Section IV, namely Parallel Variable Distribution (PVD) [7]. In this method, processors not only bear the responsibility of optimizing and updating a local set of variables but also optimize a directed step for the other variables related to other processors. The parallelization step is followed by a synchronization step wherein an affine hull is searched from the optimal values of each parallel processor. A convergence proof for the unconstrained case is given in [7] considering Lipschitz continuous differentiability of the search space. The theoretical approach is briefly described as follows. From (25) it follows that the number of optimization variables is $m(j_a + j_b)$. Let these variables be denoted by the vector θ with elements θ_i, $i = 1, .., m(j_a + j_b)$. Let the 2-normed objective function averaged over the time interval $t \in [0,\, t_f]$ be denoted as $f(\theta)$. The vector θ is partitioned into m blocks $\theta_1, \theta_2, \cdots \theta_m$, where $\theta_l \in \Re^{n_l +}$ (the superscript $+$ denotes positive real numbers as the controller coefficients must guarantee stability), $\sum_{l=1}^{m} n_l = m(j_a + j_b)$, among m controllers, each belonging to a designated PSS. In a certain i^{th} step a processor l is responsible for updating block $\theta^i_l \in \Re^{n_l +}$ of the iterate θ^i by solving the next step. The **parallelization** step is given as:

$$(y^i_l, \lambda^i_{\bar{l}}) \in argmin_{(\theta^i_l, \lambda_{\bar{l}})} f(\theta_l, \theta^i_{\bar{l}} + D_{\bar{l}} \lambda_{\bar{l}}) \quad (26)$$

such that

$$[(\theta_l, \theta^i_{\bar{l}} + D_{\bar{l}} \lambda_{\bar{l}}) \in \Theta], \ [\theta^i_l : \ = (y^i_l, \theta^i_{\bar{l}} + D^i_{\bar{l}} \lambda^i_{\bar{l}})]. \quad (27)$$

Here, Θ denotes a bounded set, $\bar{l}$ denotes complement of l in $(1, 2, \cdots, m)$, $\lambda_{\bar{l}} \in \Re^{(m-1)+}$. The matrix $D^i_{\bar{l}}$ is a $n_{\bar{l}} \times (m-1)$ matrix formed by arbitrary directions $d^i \in \Re^{n+}$, breaking it into blocks of $d^i_s \in \Re^{n_s +}, s = 1, 2, \cdots, m$, consistent with the distribution of the variables. The direction d^i can be chosen as,

$$d_i = -\nabla f(\theta_i) / \left\| \nabla f(\theta_i) \right\|. \quad (28)$$

After the points θ^i_l for $l = 1, 2, \cdots, m$ have been computed by the m parallel controllers during an iteration, the best estimate of the iteration is obtained by a **synchronization** step as shown below:

$$\theta^{i+1} = \mu^i_0 \theta^i + \sum_{k=1}^{p} \mu^i_k \theta^{i_k}$$
$$(\mu^i_0, \mu^i_1, ...\mu^i_p) = \underset{\mu_0, \mu_1, ...\mu_p}{\arg\min} \ f(\mu^i_0 \theta^i + \sum_{k=1}^{p} \mu^i_k \theta^{i_k})$$
$$\mu_0 \theta^i + \sum_{k=1}^{p} \mu_k \theta^{i_k} \in \Theta, \ \ \mu_0 + \sum_{k=1}^{p} \mu_k = 1 \quad (29)$$

The algorithm is illustrated for $m = 6$ in the next section. The local optimization steps have been performed using the Quasi-Newton Method of gradient based minima search. Synchronization is obtained by a sequence of message passing among the processors to find the best among the local optimization results, thereby capturing the communication delays between the controllers. The results are quite satisfactory in terms of convergence rate, even if m is varied.

4. EXAMPLES

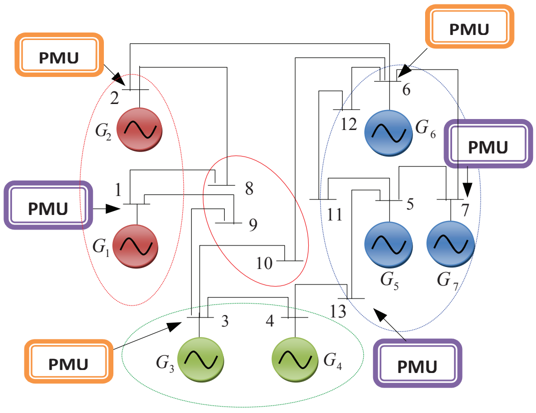
In this section we illustrate the results of Section III by considering a 3-area model of Pacific AC intertie. The structure of this system is shown in Figure 3(a), and is based on the WA-CA north south power oscillation characteristics for which detailed PMU data analysis has been done in [1]. The system is first reduced to an equivalent 3-machine system characterized by 3 aggregate inertias. All the machines are classical generator models with identical parameters except for the machine inertias in each area, as given in the Appendix. A relatively low value of H_3 makes the system act almost like a two-area system with a dominant slow mode of approximately 0.5 Hz. Hence, a second order Butterworth band-pass filter is designed for filtering the modes from the PMU measurements, as described in Section III, of the form:

$$G(s) = \frac{s(\omega_u - \omega_l)}{s^2 + (\omega_u - \omega_l)s + \omega_u \omega_l} \quad (30)$$

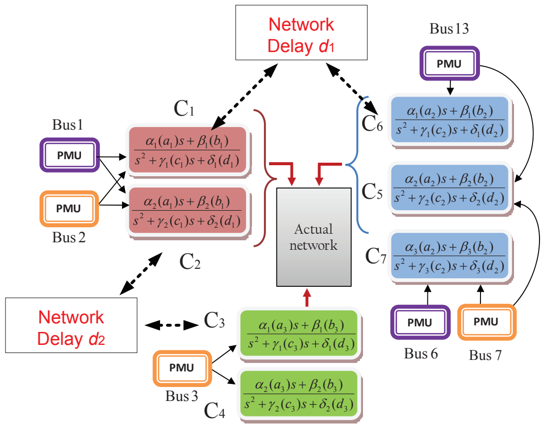
with $\omega_u = 0.9$ Hz and $\omega_l = 0.2$ Hz. A simple lead controller for Area 1 is designed to generate desired dynamic responses for the inter-machine power flows. For the actual system in Figure 3(b) $C_7(s)$ is kept fixed, and $C_1(s)$ through $C_6(s)$ are designed using 2 zeros and 3 poles each, i.e.

$$C_i(s) = \frac{a_{i1}s + a_{i2}}{s^2 + b_{i2}s + b_{i3}}, \ \ i = 1, .., 6. \quad (31)$$

The controller parameters are provided in the Appendix. An all-to-all communication between the 6 controllers is assumed. The communication between C_1-C_6 and $C_2 - C_3$ are considered to be most prone to communication delays. The closed loop response of the inter-area component of the phase angle difference between bus 3 and bus 9, i.e., Area 1 and 3, are shown in Figure 4 for various values of these two delays (in milli seconds), denoted as d_1 and d_2. It can be seen that the closed-loop matching deteriorates as the delay increases, and after a certain threshold the matching becomes unacceptable. By virtue of the global



(a) 3-area, 7-machine power system



(b) Wide-area control distribution

Fig. 3. Distributed parametric optimization

convergence property of PVD, it can be verified that the integrated error between the distributed and centralized solution over $t \in [0, 5]$ is less than a set threshold of $\gamma = 10^{-4}$ for $d_1 = d_2 = 0$. The simulations are repeated for 450 MW power transfer between Area 1 and 2. Figure 5 shows the change in the modal frequency compared to the previous case, where only 300 MW is transferred.

5. CONCLUSIONS

In this paper we derived distributed algorithms for wide-area damping control using Synchrophasors collected from different spatial nodes in a large power system. We proposed a distributed message passing strategy for cooperative damping control between area-level PSS controllers. These architecture behind these designs can be cast on top of NASPInet, and used for any generic wide-area monitoring, estimation and control purposes. Future work on this topic will include extension of the estimation and control methods to more unstructured power systems.

REFERENCES

[1] A. Chakrabortty, "Wide-Area Control of Large Power Systems Using Dynamic Clustering and TCSC-Based Redesigns," *IEEE Transactions on Smart Grid*, vol. 3(3), Sep. 2012.

[2] A. G. Phadke, J. S. Thorp, and M. G. Adamiak, "New Measurement Techniques for Tracking Voltage Phasors, Local System Frequency, and Rate of Change of Frequency," *IEEE Transactions on Power Apparatus and Systems*, vol. 102, pp. 1025-1038, May 1983.

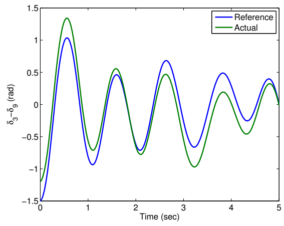
[3] J. E. Dagle, "Data Management Issues Associated with the August 14, 2003 Blackout Investigation," *IEEE PES General Meeting*, June, 2004.

[4] A. Chakrabortty, J. H. Chow and A. Salazar, "A Measurement-based Framework for Dynamic Equivalencing of Power Systems using Wide-Area Phasor Measurements," *IEEE Transactions on Smart Grid*, vol. 1(2), pp. 68-81, 2011.

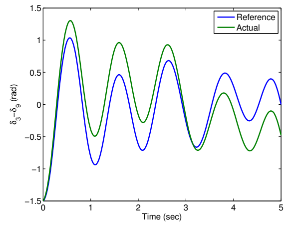
[5] A. R. Bergen and V. Vittal, *Power System Analysis*, second edition, Prentice Hall, 1999.

[6] B. Pal and B. Chaudhuri, *Robust Control in Power Systems*, Springer, 2000.

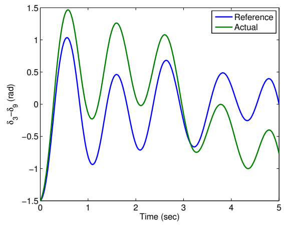
[7] M. C Ferris and O. L. Mangasarian, "Parallel Variable Distribution," *SIAM Journal on Optimization*, vol. 4(4), pp. 815-832, 1994.



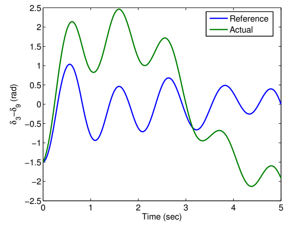
(a) $d_1 = 0$, $d_2 = 0$



(b) $d_1 = 0$, $d_2 = 5$ ms

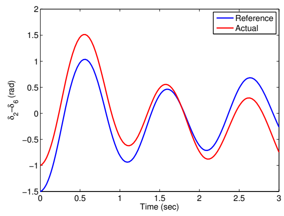


(c) $d_1 = 10$, $d_2 = 10$ ms

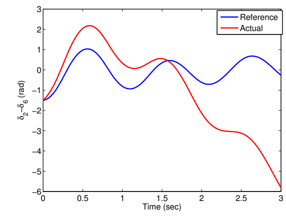


(d) $d_1 = 40$ ms, $d_2 = 50$ ms

Fig. 4. Phase angle response between Area 1 and 3 via distributed control, 300 MW case



(a) $d_1 = 0$, $d_2 = 0$



(b) $d_1 = 40$, $d_2 = 50$ ms

Fig. 5. Phase angle response between Area 1 and 2 via distributed control, 450 MW case