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Tuning Nonlinear Controllers with the Virtual Reference Approach

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Abstract:
Virtual Reference Feedback Tuning (VRFT) is a well established method for data-driven tuning of linear controllers. In VRFT the design is performed in "one-shot", that is, with only one batch of input-output data, without the need of iterative data collection procedures. Its core concept consists in treating input-output data collected from the plant to be controlled as if they had been obtained from a *virtual experiment*, in which a particular reference signal - the *virtual reference* - would have been applied to the closed-loop system. A key feature of the VRFT method is that it greatly simplifies the design procedure with respect to standard model reference design: for linear and linearly parametrized controllers, it results in convexification of the design, so that its solution can be found by a single least squares method. In this paper this paradigm is applied to the tuning of nonlinear controllers. We propose specific design procedures for two classes of nonlinear plants: rational plants and plants of the Wiener type. Statistical properties of the controllers thus designed in noisy environments are also illustrated.

Keywords: auto-tuning, identification, process control, nonlinear systems

1. INTRODUCTION

In the past two decades, a number of data-driven control design methods have been proposed - Hjalmarsson et al. (1994, 1998); Campi et al. (2002); Karimi et al. (2004); Kammer et al. (2000), among others - where a parametrized controller structure is chosen a priori, and the controller tuning is based directly on input and output data collected from the plant, without the use of a model of this plant. A common theoretical framework for these data-driven - also called data-based - methods, which are based on the model reference control design formulation, is given in Bazanella et al. (2012). Some of these methods, like Iterative Feedback Tuning are iterative in nature: the optimal controller is obtained as a sequence of controllers that operate on the actual plant, and experimental data are collected on the corresponding sequence of closed-loop plants Hjalmarsson et al. (1994, 1998). Other methods are "one-shot", that is, non-iterative: they directly estimate the controller's parameters on the basis of one batch of input-output data: Virtual Reference Feedback Tuning (VRFT) - Campi et al. (2002), Model Reference Tuning by Prediction Error Identification - Campestrini et al. (2012) and a non-iterative version of Correlation-based Tuning - Karimi et al. (2007) - are representative of this class.

In VRFT the design is performed after computing, from the data collected from the system, a *virtual reference*, which is the signal that should be applied as the input to the reference model in order to produce the signals that have been collected. With this virtual reference, an alternative objective function representing the desired performance is formed which, contrary to the original model reference objective function, does not depend explicitly on the plant. When the controller is linearly parametrized, this alternative objective function is quadratic, allowing its minimization through a simple least squares procedure, which is probably the major asset of the VRFT method.

Over the past decade the virtual reference (VR) paradigm has matured and has continuously driven further developments in data-driven control design. VRFT has been successfully applied to relevant industrial applications, such as in Formentin et al. (2013). The methodology has been extended to nonminimum phase plants in Campestrini et al. (2011) and to linear-parameter varying plants in Formentin and Savaresi (2011). The VRFT approach in a nonlinear setup has been introduced in Campi and Savaresi (2006), where a general theory has been presented for a certain class of controller parameterizations.

In this paper we advance the application of the VR paradigm in a nonlinear setup by proposing design procedures for two major classes of controller parameterizations: rational controllers and Hammerstein-type controllers. We treat controller parameterizations that do not fit in the class studied in Campi and Savaresi (2006). On the other hand, the convenience of its implementation being a major feature of the VR paradigm, we propose specific solutions for the implementation issues for the two classes of nonlinear controllers considered. We also illustrate, by means

[1] This work was partially supported by CNPq.

of case studies, the statistical properties of the proposed solutions when applied in a noisy environment.

The paper is organized as follows. In Section 2 we present some notations and definitions. The virtual reference paradigm is presented in Section 3. Detailed solutions following this paradigm are presented for two classes of processes, each of which are presented in one of the following two sections. Finally, some conclusions are given in Section 6.

2. PRELIMINARIES

Let $u(t)$ and $y(t)$ stand respectively for the input and output of a system at time t. The plant to be controlled is a time-invariant single-input single-output "real system" described by a fixed yet unknown difference equation:

$$\mathcal{S}: \quad y(t) - P(\psi_P(t), \epsilon(t)) = 0 \tag{1}$$

where $\psi_P(t) = [y(t-1), \ldots, y(t-n_P), u(t-1), \ldots, u(t-m_P)]$, and $\epsilon(t) = [e(t), e(t-1), \ldots, e(t-l_P)]$, $e(t)$ being a white noise sequence with variance σ_e^2, and the nonlinear function $P(\cdot,\cdot) : \Re^{n_P+m_P} \times \Re^{l+1} \to \Re$ is Lipschitz.

The controller is similarly described by a difference equation

$$\mathcal{C}: \quad u(t) - C(\psi_C(t), \rho) = 0 \tag{2}$$

where $\psi_C(t) = [y(t), y(t-1), \ldots, y(t-n_C), u(t-1), \ldots, u(t-m_C), r(t), r(t-1), \ldots, r(t-l_C)]$, $r(t)$ is the reference signal, and $\rho \in \Re^d$ is the parameter vector to be tuned. The nonlinear function $C(\cdot,\cdot) : \Re^{n_C+m_C+l_C+2} \times \Re^d \to \Re$ is Lipschitz and defines the controller structure, which is fixed a priori. Notice that $\mathcal{S}$ designates a single system, the real unknown plant, whereas $\mathcal{C}$ designates a whole class of systems, one for each parameter value ρ. The control design consists in tuning the parameter vector ρ, that is, in choosing one controller among all those in the class $\mathcal{C}$.

Model Reference control design consists in specifying a desired closed-loop behavior and then solving the following optimization problem

$$\min_{\rho} J^{MR}(\rho) \tag{3}$$

$$J^{MR}(\rho) \triangleq \frac{1}{N}\sum_{t=1}^{N}\left[(y(t,\rho) - y_d(t))^2\right] \tag{4}$$

where $y_d(t)$ is the desired output. The desired output $y_d(t)$ is usually specified by means of a desired transfer function $M(q)$, called the reference model, such that

$$y_d(t) = M(q)r(t)$$

where q is the forward shift operator defined by $qx(t) = x(t+1)$. Assume that there exists a feedback control law $u(t) = C_d(\psi_d(t))$ such that the input-output map $r(t) \to y(t)$ [2] of the closed-loop system is exactly the one specified by the model reference $M(q)$, where $\psi_d(t) = [y(t), y(t-1), \ldots, y(t-n_d), u(t-1), \ldots, u(t-m_d), r(t), r(t-1), \ldots, r(t-l_d)]$ and $C_d(\cdot) : \Re^{n_d+m_d+l_d+2} \to \Re$. This control law is called the *ideal controller*. Whether or not the ideal controller can be exactly matched by one of the controllers in the class considered for the design is a crucial issue in model reference control design - Bazanella et al. (2012). This is an assumption that is tacitly made in most of what follows, so let us formalize it.

[2] That is, the relationship between $r(t)$ and $y(t)$ assuming that $e(t) \equiv 0$.

Assumption 2.1. Matching condition - $\mathcal{C}_d \in \mathcal{C}$

$$\exists \rho_0 : \quad C(\psi_C(t), \rho_0) \equiv C_d(\psi_d(t))$$

◇

The matching condition 2.1 is equivalent with the assumption that the real system belongs to the model set, which is ubiquitous in the context of system identification plus model-based control design - Bazanella et al. (2012); Ljung (1999). The following example, extracted from Campi and Savaresi (2006), illustrates these definitions.

Example 2.1. Consider the plant:

$$y(t) = y(t-1) + u(t-1)^3 + e(t) \tag{5}$$

and the controller class

$$u(t) = \rho[r(t) - y(t)]^{1/3} \tag{6}$$

and also the reference model:

$$y_d(t) = r(t-1) \tag{7}$$

that is, $M(q) = q^{-1}$.

If the control law is $u(t) = [r(t) - y(t)]^{\frac{1}{3}}$ and $e(t) \equiv 0$, then the closed-loop becomes

$$\begin{aligned} y(t) &= y(t-1) + \{[r(t-1) - y(t-1)]^{\frac{1}{3}}\}^3 \\ &= y(t-1) + [r(t-1) - y(t-1)] \\ &= r(t-1) \end{aligned}$$

as desired. Hence this control law represents the ideal controller and it belongs to the controller class, with $\rho_0 = 1$ - the matching condition is satisfied.

◇

Under assumption 2.1 it is clear that ρ_0 is the global minimum of the cost function $J^{MR}(\rho)$ and thus the solution of the optimization (3). But finding this solution starting only from input-output data collected from the plant (without its model) can be quite involved due to the lack of convexity of the cost function $J^{MR}(\rho)$, thus requiring iterative procedures and several experiments on the plant. This issue can be tackled by a Virtual Reference approach, presented in the next Section.

3. THE VIRTUAL REFERENCE PARADIGM

The solution of (3) often becomes quite troublesome for two reasons: the objective function depends of the unknown plant and it is nonconvex, even for linear systems - Hjalmarsson et al. (1998); Bazanella et al. (2012, 2008). One approach to eliminate, or at least mitigate, these problems is the Virtual Reference (VR) paradigm. Based on signals $y(t)$ and $u(t)$ measured from the real system, define the *virtual reference* $\bar{r}(t)$ as $M(q)\bar{r}(t) = y(t)$. This is the signal that, when applied to the reference model, would produce the measured data; but this is not how the data have actually been generated, hence the signal's designation. Figure 1 shows the virtual system and the related signals.

In the VR approach one proceeds as if the experiment depicted in Figure 1 had actually taken place. If this were

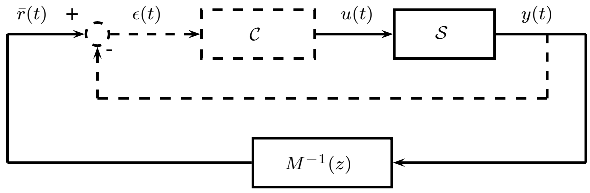

Fig. 1. Block representation of the virtual closed-loop system, with the virtual reference $\bar{r}(t)$, which would have generated the signals that have been measured. The dashed lines represent the virtual part of this fictitious system.

the case, the ideal controller would have been in the loop and then we could identify it from its input output data by solving the following optimization.

$$\min_{\rho} J^{VR}(\rho) \tag{8}$$

$$J^{VR}(\rho) \triangleq \frac{1}{N}\sum_{t=1}^{N}\left[\left(u(t) - C(\bar{\psi}_C(t), \rho)\right)^2\right]. \tag{9}$$

where $C(\bar{\psi}_C(t), \rho)$ is the controller function (2) with $r(t)$ replaced by the virtual reference $\bar{r}(t)$. The two problems (8) and (3) have the same global minimum provided that Assumption 2.1 is satisfied.

Solving the optimization problem (8) instead of (3) is the cornerstone of the VR control design paradigm, the advantage being that (8) is usually much simpler to solve than (3). First, (8) is purely data-dependent, that is, it does not depend explicitly on the unknown plant. Second, the shape of the cost function (8) often is much more amenable to optimization than (3). In the particular situation in which the controller is linearly parameterized (an ubiquitous situation), (8) is quadratic in the decision variable ρ and thus can be solved by Least Squares. The direct solution of (3), on the other hand, requires optimization methods that not only are complicated because of the dependence on the plant but also tend to get trapped in local minima due to lack of convexity - see Bazanella et al. (2008).

The nonlinear VRFT has been presented in Campi and Savaresi (2006), but there only the first feature (the data-dependent nature) of the method has been explored. In order to obtain a complete and powerful design method, effective means for solving the optimization are needed and these will be different for different classes of processes. Moreover, in this paper we consider controllers in the form (2), which can depend on the reference and on the input independently. This is somewhat more general than the controller structure considered in Campi and Savaresi (2006), which requires that the control action depends only on the tracking error. We will see that for the control of rational systems, for instance, this more general controller structure comes in handy.

4. RATIONAL SYSTEMS

A rational model is one in which the function $P(\cdot,\cdot)$ consists in the ratio of two polynomials, each of which a function of past input and output data: $[y(t-1), \ldots, y(t-n_C), u(t), u(t-1), \ldots, u(t-m_C)]$. When the real process is a rational system, so will be the ideal controller, as illustrated in the case study to be presented shortly. It is then only reasonable to use a rational controller structure when controlling a rational process.

Given a controller class of rational type, we propose to apply the virtual reference paradigm to design the controller's parameters. Then the controller design amounts to identification of the ideal controller with a rational model and an appropriate algorithm must be used. We propose the use of the prediction error algorithm in Billings and Zhu (1991), that is tailored for rational systems. This algorithm minimizes the prediction error through the iterative solution of a series of least squares problems and has been shown to be quite effective; it can be applied, with minor adaptations, to the minimization of $J^{VR}(\rho)$ in this case. This is still a one-shot solution, in the sense that only one batch of input-output data are needed.

4.1 Case study

Consider the control of the nonlinear system $\mathcal{S}$ described by:

$$y(t) = \frac{0.5u(t-1)y(t-1) + u(t-1)}{1 + 0.25y^2(t-2)} + e(t) \tag{10}$$

where the desired closed-loop response is specified by means of the following *linear* reference model:

$$M(q) = \frac{0.4}{q-0.6} \tag{11}$$

The input-output relationship in (11) can also be described by the following difference equation.

$$y(t) = 0.4r(t-1) + 0.6y(t-1) \tag{12}$$

Equating (10) with (12) and isolating the signal $u(t)$ we find the equation that describes the control law corresponding to the ideal controller, which is given by:

$$u(t) = \frac{0.4r(t) + 0.6y(t) + 0.1y^2(t-1)r(t) + 0.15y(t)y^2(t-1)}{1 + 0.5y(t)} \tag{13}$$

Assume that the matching condition is satisfied, that is, the structure of the ideal controller is known and only its numerical parameters are unknown. Then the controller structure is

$$u(t) = \frac{\rho_1 r(t) + \rho_2 y(t) + \rho_3 y^2(t-1)r(t) + \rho_4 y(t)y^2(t-1)}{1 + \rho_5 y(t)} \tag{14}$$

and the controller class (14) is seen to contain the ideal controller (13), thus satisfying the matching condition. Notice that the ideal controller and the controller class depend on the output and on the reference independently, and not on the tracking error only.

After exciting the system with a Pseudo Random Binary Signal (PRBS) with $N = 254$ and with a noise with variance $\sigma_e^2 = 0.005$, we performed the control design as described. In order to assess the statistical properties of the control design, 100 Monte Carlo experiments have been run. The average controller obtained was:

$$\rho_{\text{av}} = [0.4000\ 0.5999\ 0.1001\ 0.1501\ 0.5000]^T$$

with the following sample covariance matrix:

1.0×10^{-6}

$$\begin{bmatrix} 0.0498 & 0.0585 & -0.0353 & -0.0362 & -0.0018 \\ 0.0585 & 0.4714 & -0.0449 & -0.3690 & 0.0057 \\ -0.0353 & -0.0449 & 0.0581 & 0.0745 & -0.0040 \\ -0.0362 & -0.3690 & 0.0745 & 0.4647 & -0.0070 \\ -0.0018 & 0.0057 & -0.0040 & -0.0070 & 0.0911 \end{bmatrix}.$$

The variance of the estimates is illustrated in Figure 2, in which the results of all the 100 Monte Carlo experiments performed are depicted, in the form of the projections of the parameters obtained in each case onto the $\rho_1 \times \rho_2$ plane. The controller's performance can be assessed by the cost value obtained with the typical (average) controller, which was $J^{MR}(\rho_{\rm av}) = 5.8442 \times 10^{-3}$. It is important to interpret this number: it is the per sample difference between the output obtained with the controller thus designed and the desired output $y_d(t)$. Since the noise variance is of this same order (recall that $\sigma_e^2 = 0.005$), one could not expect (from any control design method) the cost value to attain values significantly lower than what has been obtained.

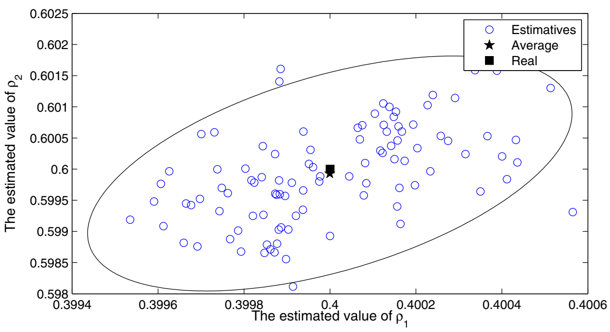

Fig. 2. ρ_1 and ρ_2 obtained in 100 Monte Carlo experiments and the estimated confidence ellipsoid with $\chi^2 = 95\%$.

5. WIENER PROCESSES

A Wiener system is one that can be described by (see Figure 3)

$$z(t) = G_0(q)u(t) + H_0(q)e(t) \quad (15)$$

$$y(t) = \phi(z(t)) \quad (16)$$

where $G_0(q)$ and $H_0(q)$ are rational transfer functions, both causal and BIBO-stable, and $\phi(\cdot) : \mathcal{D} \to \mathcal{D}$, with $\mathcal{D} \subset \Re$; the system (15)-(16) is a particular case of the general structure (1).

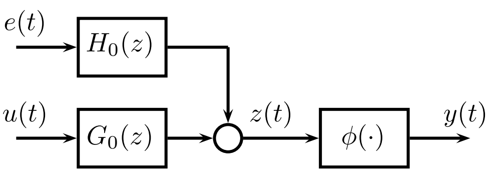

Fig. 3. A Wiener system as described in (15) and (16)

It is assumed that the output map $\phi(\cdot)$ is right invertible in the whole operating range $\mathcal{D}$, that is, there exists a map $\phi_R^{-1}(\cdot)$ such that $\phi(\phi_R^{-1}(x)) = x \quad \forall x \in \mathcal{D}$. It is not hard to see that for this case the ideal controller is of the following form:

$$u(t) = C'_d(q)v(t) \quad (17)$$

$$v(t) = r(t) - \psi(y(t)) \quad (18)$$

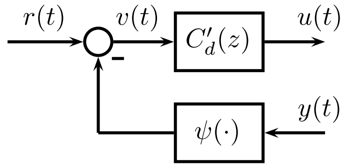

Fig. 4. A Hammerstein controller as described in (17) and (18)

Indeed, if $\psi(\cdot) = \phi_R^{-1}(\cdot)$ and

$$C'_d(q) = \frac{M(q)}{G_0(q)(1 - M(q))} \quad (19)$$

then the closed-loop system is linear with transfer function $M(q)$. This is a standard solution in control systems practice, as commercial controllers often include a customizable linearizing function $\psi(\cdot)$ in the feedback loop. It is clearly seen that in this standard case the ideal controller is not a function of the tracking error $\varepsilon(t) = r(t) - y(t)$, but instead a function of $r(t)$ and of $y(t)$ separately.

Assuming that the reference is a given constant, an alternative topology can be used for the controller that makes the controller dependent of the tracking error only. Define the controller class as

$$u(t) = C'(q, \rho)v(t) \quad (20)$$

$$v(t) = \psi(r(t) - y(t)) \quad (21)$$

where the function $\psi(\cdot)$ must be such that the control system behaves as if both nonlinearities were not present, that is:

$$\psi(r(t) - y(t)) = \alpha[r(t) - z(t)] \quad (22)$$

for some $\alpha \in \Re^+$. Figure 5 shows a system with a Wiener process with a Hammerstein controller as described before.

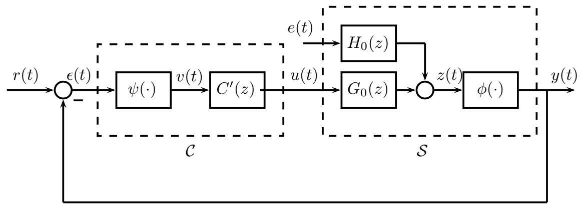

Fig. 5. Wiener process/Hammerstein controller block diagram.

In so doing, the ideal controller does depend only on the tracking error, but the nonlinearity $\psi(\cdot)$ that forms the ideal controller depends on the reference value, and will be a different function for each different constant value. Moreover, for non constant references, the inverse nonlinearity would not be autonomous. Both choices for the controller structure can be treated similarly within the VR framework; let us present the treatment for the later structure, (20)-(21).

It is desired that in closed loop the system behaves linearly and as specified by $M(q)$. To achieve that, the controller should cancel all nonlinear effects of $\phi(z(t))$. In other words it is desired that in closed loop the signal $v(t)$ obeys (22). However the system depicted in Figure 5 behaves as:

$$v(t) = \psi(r(t) - y(t))$$

and thus we can obtain the relationship between $\psi(\cdot)$ and $\phi(\cdot)$ as:

$$\psi(r(t) - \phi(z(t))) = \alpha[r(t) - z(t)]. \tag{23}$$

It is clear that when there is no reference signal, $r(t) \equiv 0$, the relationship between the functions $\psi(\cdot)$ and $\phi(\cdot)$ is simplified to:

$$\psi(\phi(z(t))) = -\alpha z(t)$$

so that in this case (but only in this case) $\psi(\cdot)$ must be the right inverse of $\phi(\cdot)$, apart from a real constant.

5.1 A linear parameterization

In order to obtain a linear parameterization, and thus a convex optimization, let us consider that the controller and the nonlinear function are both linearly parameterized, that is

$$C'(q, \theta) = \frac{1}{D(q)}\theta^T N(q) \tag{24}$$

$$\psi(\varepsilon(t)) = \eta^T E(t) \tag{25}$$

with $D(q)$ a known monic polynomial,

$$N(q) \triangleq [1 \quad q^{-1}q^{-2} \ldots q^{-p}]^T$$

$$E(t) \triangleq [\varepsilon(t) \quad \varepsilon^2(t) \ldots \varepsilon^k(t)]^T$$

and $\varepsilon(t)$ is the tracking error; $\theta \in \Re^{p+1}$ and $\eta \in \Re^k$ are the controller's parameters to be adjusted. The parameterization (25) for the nonlinear map can be seen as the truncated Taylor series of the ideal nonlinear function - the one which results in the exact matching of the closed-loop desired response. Hence the truncation at its k^{th} term implies that the matching assumption is, in general, not satisfied.

Defining $u_f(t) = D(q)u(t)$ we can write

$$u_f(t) = \sum_{i=0}^{p} \theta_i \psi(\varepsilon(t-i)) = \sum_{i=0}^{p} \theta_i [\sum_{j=1}^{k} \eta_j \varepsilon^j(t-i)] \tag{26}$$

where θ_i are the elements of the vector θ and likewise for η_j. Finally, define the new parameters $\rho_1 \triangleq \theta_0\eta_1,\ \rho_2 \triangleq \theta_0\eta_2,\ \ldots\ \rho_{(p+1)k} \triangleq \theta_p\eta_k$ and the corresponding parameter vector $\rho = \left[\rho_1 \ldots \rho_{(p+1)k}\right]^T$, so that (26) can be rewritten as

$$u_f(t) = \sum_{i=0}^{p}\sum_{j=1}^{k} \rho_{i+j}\varepsilon^j(t-i) = \rho^T \varpi(t) \tag{27}$$

where the regressor $\varpi(t)$ has also been defined. Now the vector ρ is the parameter to be identified and the parameterization is linear. The original parameters θ and η can be recovered from ρ after these have been determined, but this is unnecessary for the implementation of the control law. Notice that the number of parameters is now $(p+1) \times k$ instead of the original number $p+1+k$.

The numerical example below aims to demonstrate this method proposed for tuning Hammerstein controllers.

5.2 Case study

Consider a Wiener type nonlinear plant $\mathcal{S}$ as shown in Figure 5 whose linear block and static nonlinearity are given by:

$$G_0(q) = \frac{0.5}{q - 0.9} \tag{28}$$

$$\phi(z(t)) = y(t) = 1.5z(t) + 0.2z^3(t) \tag{29}$$

The noise is white ($H_0(q) = 1$) and the noise variance is $\sigma_e^2 = 0.05$.

This plant is to be controlled such that its closed-loop behavior is given by the following reference model:

$$M(q) = \frac{0.4}{q - 0.6} \tag{30}$$

The controller class is in form of (20) and (21) with $C'(q, \theta)$ as a PI controller:

$$C'(q, \theta) = \theta_1 \frac{1}{q-1} + \theta_2 \frac{q}{q-1} \tag{31}$$

and the nonlinear output map of the controller is a fourth order polynomial:

$$\psi(\varepsilon(t)) = \eta_1\varepsilon(t) + \eta_2\varepsilon^2(t) + \eta_3\varepsilon^3(t) + \eta_4\varepsilon^4(t) \tag{32}$$

The linear part of the ideal controller can be determined from (19), resulting in:

$$C'_d(q) = \frac{0.8q - 0.72}{q - 1} \tag{33}$$

which can be exactly described within the class of PI controllers (31). The nonlinear part of the ideal controller is the inverse of the nonlinear function $\phi(\cdot)$ given in (29), which will be approximated by the truncated Taylor series as in (32).

Making the necessary mathematical substitutions as described in the previous subsection, the following control law is obtained:

$$\begin{aligned} u_f(t) =& \rho_1\varepsilon(t) + \rho_2\varepsilon^2(t) + \rho_3\varepsilon^3(t) + \rho_4\varepsilon^4(t) + \\ & \rho_5\varepsilon(t-1) + \rho_6\varepsilon^2(t-1) + \rho_7\varepsilon^3(t-1) + \\ & \rho_8\varepsilon^4(t-1) \end{aligned} \tag{34}$$

where $u_f(t)$ denotes that $u(t)$ has been filtered by the denominator of $C'_d(q)$ in (33). This control law is linearly parameterized and the tuning of its parameters can be done by minimization of $J^{VR}(\rho)$, which can be performed by least squares. In order to determine the efficacy of the method and illustrate its behavior in a noisy environment, this design has been performed in one hundred Monte Carlo simulations, each time determining the controller parameters $\rho_1 \to \rho_8$. The average parameter obtained is:

$$\rho_{\rm av} = \begin{bmatrix} 0.44717 \\ 2.0216 \times 10^{-3} \\ -6.5181 \times 10^{-4} \\ -1.608 \times 10^{-5} \\ -0.40435 \\ -1.6163 \times 10^{-3} \\ 6.1265 \times 10^{-4} \\ 1.469 \times 10^{-5} \end{bmatrix}$$

with a sample variance whose norm is of the order of 10^{-6}. The resulting cost, which measures the average per sample difference between the desired output $y_d(t)$ and the actually obtained output, was $J^{MR}(\rho_{\rm av}) = 3.0078 \times 10^{-3}$. In Figure 6 it can be seen that the closed-loop system's response with the average controller to a step reference is very close to the desired step response specified by the model reference $M(q)$.

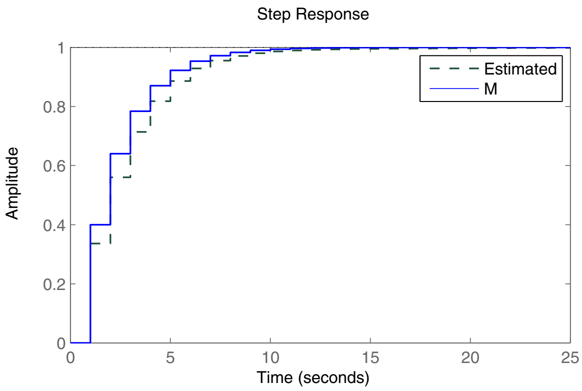

Fig. 6. Output of both systems: desired (continuous line) and designed (dashed line)

6. CONCLUSIONS

In this paper we have explored the design of nonlinear controllers using the virtual reference paradigm, inspired by VRFT. A major appeal of the VRFT design methodology is that it simplifies tremendously the controller tuning procedure, even convexifying, for a large class of problems, the model reference design. Our main concern was on extending this idea to the tuning of nonlinear controllers in such a way as to retain, at least partially, this mostly desirable feature. We have proposed specific solutions, in which this has been shown possible, for two broad classes of nonlinear plants: Wiener plants and rational plants.

For these two classes of plants, we have formally stated the design problem and detailed both the theoretical and practical formulation of the design problem. We have also provided case studies to illustrate the design issues and the effectiveness of our design methodologies, indicating the potential of the virtual reference paradigm for nonlinear controller tuning. In the results obtained for the two case studies presented it is seen that the statistical properties of the designed controller are quite similar to (in fact, hardly distinguishable from) the ones predicted by the theory in the case of linear processes and controllers.

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