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Distributed MPC and partition-based MHE for distributed output feedback

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Abstract: For the first time, a distributed output feedback control scheme is presented which combines distributed model predictive control with distributed moving horizon estimation. More specifically, we combine the iterative methods of sensitivity-driven distributed model predictive control (S-DMPC) with sensitivity-driven partition-based moving horizon estimation (S-PMHE). To that end, S-PMHE is extended such that it can handle inputs of S-DMPC. The resulting distributed output feedback scheme is then applied to an alkylation benchmark process from the literature. We find that its control performance is comparable to that of fully centralized MPC and MHE but our distributed output feedback scheme is faster.

1. INTRODUCTION

Distributed approaches to model predictive control (MPC) and moving horizon estimation (MHE) have become active research topics in the process control community. These distributed MPC (DMPC) and distributed or partition-based MHE (DMHE / PMHE) [1] algorithms describe several communicating MPC and MHE agents, each assigned to a particular subsystem of the overall process. Therefore, these distributed control and estimation schemes are naturally suited for spatially distributed systems, such as irrigation channels [Negenborn et al., 2009], and chemical processes consisting of a number of interacting unit operations [Liu et al., 2010]. Since each DMPC and PMHE problem considers only a subset of the variables of the centralized problem, a reduction in computation time is also expected. Computation time is further decreased by iterative approaches which try to solve the DMPC and PMHE problems in parallel. Finally, current research aims to make the design of distributed control solutions simpler and more robust. To that end, the plug-and-play behaviour of individual agents is currently under study [Riverso et al., 2012, Riverso et al., 2013].

Distributed control and estimation schemes are also the subject of extensive literature reviews. For instance, distributed MPC methods are reviewed by Scattolini [2009] and more recently by Christofides et al. [2013]. The latter paper also briefly treats distributed state estimation. In particular, Farina et al. [2010] contributed a number of prominent PMHE algorithms. Schneider et al. [2013] presented an alternative approach based on sensitivity-driven coordination, called S-PMHE.

Overall, it is striking that most papers on DMPC assume state feedback whereas most papers on PMHE work with autonomous systems or open-loop inputs. The practically more important case of distributed output feedback is rarely treated in the literature. Notable exceptions are papers that combine DMPC with centralized estimators, such as Zheng et al. [2009], Hu and El-Farra [2013], or combinations with distributed Luenberger observers [Venkat et al., 2005, Farina and Scattolini, 2011, Giselsson, 2013], or with distributed Kalman filters [Venkat et al., 2006, Mercangöz and Doyle III, 2007, Menighed et al., 2009, Roshany-Yamchi et al., 2013]. Interestingly, and to the best of our knowledge, DMPC was not yet combined with PMHE, even though both rely on distributed optimization techniques. In this work, we will close this gap and discuss optimal distributed output feedback control by combining sensitivity-driven DMPC (S-DMPC) [Scheu and Marquardt, 2011] and S-PMHE [Schneider et al., 2013].

To this end, we first recall both methods briefly. Since S-PMHE was not yet applied to non-autonomous systems, we will present an extended theory in order to apply the method to linear discrete-time systems with external inputs as well. Finally, we validate our distributed output feedback approach on a case study.

2. PROBLEM FORMULATION

We consider an optimal tracking control problem for the following overall system Σ:

$$\dot{\tilde{x}}(t) = f(\tilde{x}(t), \tilde{u}(t)), \quad \tilde{x}(0) = \tilde{x}_0, \tag{1a}$$
$$\tilde{y}(t_k) = h(\tilde{x}(t_k)) + v(t_k), \tag{1b}$$

where $\tilde{x}(t) \in \mathbb{R}^n$ is the state at time t with initial condition $\tilde{x}_0$, $\tilde{u}(t) \in \mathbb{R}^r$ are the inputs, and $\tilde{y}(t_k) \in \mathbb{R}^m$ are the measurements at sampling time t_k, corrupted by measurement noise $v(t_k) \in \mathbb{R}^m$. Let us denote the deviation of the state, input, and measured variables from the steady state (x_s, u_s, y_s) as $x = \tilde{x} - x_s$, $u = \tilde{u} - u_s$, and $y = \tilde{y} - y_s$.

Suppose now that Σ consists of N interacting subsystems Σ_i, i.e. $i \in \mathcal{N} = \{1, ..., N\}$. Then, a linearized form of Σ in terms of its subsystems Σ_i is given as

$$\dot{x}_i(t) = \sum_{j \in \mathcal{N}} \left(A_{ij} x_j(t) + B_{ij} u_j(t) \right), \quad x_i(0) = x_{i,0}, \tag{2}$$

[1] Farina et al. [2010] distinguish between distributed and partition-based MHE.

$\forall i \in \mathcal{N}$. $x(t) = \langle x_1(t), \ldots, x_N(t)\rangle$ [2], with $x_i(t) \in \mathbb{R}^{n_i}$, is the state vector, and $u(t) = \langle u_1(t), \ldots, u_N(t)\rangle$ is the aggregated input vector, where $u_i(t) \in \mathbb{R}^{r_i}$ is the local input vector of subsystem Σ_i. $A = [A_{ij}]_{i,j\in\mathcal{N}}$, with $A_{ij} \in \mathbb{R}^{n_i\times n_j}$, refers to the system matrix, and $B = [B_{ij}]_{i,j\in\mathcal{N}}$, with $B_{ij} \in \mathbb{R}^{n_i\times r_j}$, denotes the input matrix.

Due to physical limits of actuators and operational restrictions, the contoller has to cope with the additional input and state constraints [3]

$$u_i^{\min} \le u_i(t) \le u_i^{\max}, \tag{3a}$$

$$x_i^{\min} \le x_i(t) \le x_i^{\max}, \quad \forall i \in \mathcal{N}, \tag{3b}$$

which are naturally considered by MPC. The DMPC will be formulated for the continuous-time model (2) to allow a (possibly) flexible parameterization of the input variables.

Since we do not assume state feedback in this work, we require an estimate of the current state. Such an estimate is conveniently computed on arrival of each new measurement sample. To be consistent with the sampling nature of the measurements, we prefer a discrete-time LTI model for state estimation. Similar to (2), it is given in terms of the subsystems Σ_i as follows:

$$x_i(k+1) = \sum_{j\in\mathcal{N}} \left(A_{ij}^d x_j(k) + B_{ij}^d u_j(k)\right), \quad x_i(0) = x_{i,0}, \tag{4a}$$

$$y_i^\diamond(k) = \sum_{j\in\mathcal{N}} C_{ij}^d x_j(k) + v_i(k), \tag{4b}$$

$\forall i \in \mathcal{N}$, where the superscript d indicates the discrete-time setting, k is shorthand for t_k, and the sampling time is given as $\Delta t = t_k - t_{k-1}$. Also, $C^d = [C_{ij}^d]_{i,j\in\mathcal{N}}$ denotes the output matrix, with $C_{ij}^d \in \mathbb{R}^{m_i\times n_j}$. Depending on the discretization of the optimal, continuous-time inputs, the system matrices in (4) can be derived systematically from these in (2). For instance, a zero-order hold can be used in case of piecewise constant inputs.

3. DISTRIBUTED MPC FORMULATION

The distributed MPC algorithm used in this paper is S-DMPC as presented by Scheu and Marquardt [2011]. This algorithm was found to be a competitive DMPC scheme in a recent benchmark [Alvarado et al., 2011]. We present the distributed MPC formulation here, while the sensitivity-driven solution approach will be presented later in Section 5.

The control objectives discussed in Section 2 can be summarized in the optimal control problem for the overall system Σ at time t_k

$$\min_u \Phi \triangleq \sum_{i\in\mathcal{N}} \Phi_i(x_i, u_i) \tag{5a}$$

$$\text{s.t. } \Phi_i = \frac{1}{2}\int_{t_k}^{t_k+t_p} x_i^T Q_i x_i + u_i^T R_i u_i \, dt, \tag{5b}$$

$$\dot{x} = Ax + Bu, \quad x(t_k) = x_k, \tag{5c}$$

$$0 \le D\langle x, u\rangle + e, \tag{5d}$$

on a finite (receding) horizon $[t_k, t_k + t_p]$, with a separable (or additive), quadratic objective function (cf. Eq. (5a)) and symmetric positive definite weighting matrices Q_i and R_i. The input and state constraints (3) are gathered in Eq. (5d) with $D = \langle D_1, \ldots, D_N\rangle$ and $e = \langle e_1, \ldots, e_N\rangle$. Note, that the constraints need not to be separable allowing the implementation of appropriate terminal regions.

By means of a proper discretization of the inputs u_i, e.g. a piecewise constant realization, the ODEs can be solved and the optimal control problem (5) can be transcribed into the following QP [Scheu and Marquardt, 2011]:

$$\min_{\boldsymbol{z}} \sum_{i\in\mathcal{N}} \Phi_i \tag{6a}$$

$$\text{s.t. } \Phi_i(\boldsymbol{z}) = \frac{1}{2}\boldsymbol{z}^T \mathcal{T}_i \boldsymbol{z} + \mathcal{S}_i \boldsymbol{z} + const, \tag{6b}$$

$$0 \ge g_i(\boldsymbol{z}) = \mathcal{D}_{ii}\boldsymbol{z}_i + \sum_{j\in\mathcal{N}\setminus i} \mathcal{D}_{ij}\boldsymbol{z}_j + d_i, \tag{6c}$$

$\forall i \in \mathcal{N}$, where $\boldsymbol{z}_i$ indicates the parameters for the discretization of the input variables u_i.

4. PARTITION-BASED MHE WITH INPUTS

Originally, S-DMPC required state feedback for closed-loop operation. However, if the process state is not readily available at every sampling instant, it must be estimated from available measurements. Since centralized estimators are inconsistent with the goals of distributed control, we will rather employ a partition-based MHE scheme.

The following derivation of S-PMHE is close to the one presented by Schneider et al. [2013]. However, the novel aspect here is that the resulting algorithm will be able to consider past inputs from the controller to the plant along the estimation horizon. This extension finally allows us to run S-PMHE in closed loop with S-DMPC.

To that end, we first formulate the centralized MHE (CMHE) optimization problem for system Σ. For the current time instant, indexed k', CMHE computes an estimate of the process state $x(k)$ from K measurement samples $y^\diamond(k)$, $k \in \{k^0, \ldots, k'-1\}$, $k^0 = k' - K$, on a moving estimation horizon:

$$\min_{\Delta x(k^0), x, w, v} \frac{1}{2}\left(\left\|\Delta x(k^0)\right\|_{\tilde{P}}^2 + \sum_{k=k^0}^{k'-1} \|w(k)\|_{\tilde{Q}}^2 + \|v(k)\|_{\tilde{R}}^2\right) \tag{7a}$$

$$\text{s.t. } x(k^0) = \bar{x}(k^0) + \Delta x(k^0), \tag{7b}$$

$$x(k+1) = A^d x(k) + B^d u(k) + w(k), \tag{7c}$$

$$y^\diamond(k) = C^d x(k) + v(k). \tag{7d}$$

Note that in the beginning, i.e. when $k' \le K$, the problem is solved on a growing horizon, processing only those measurements up to time $t_{k'-1}$, i.e. $k^0 = 0$ in (7). Above, we have further introduced the a-priori estimate of the state at the beginning of the horizon, $\bar{x}(k^0)$. We also defined vectors of state estimates $x = \langle x(k^0), \ldots, x(k')\rangle$ and of estimated measurement noise $v = \langle v(k^0), \ldots, v(k'-1)\rangle$. In addition to the variables in the nominal process model (1), process noise $w = \langle w(k^0), \ldots, w(k'-1)\rangle$ is estimated here to account for model uncertainties, such as, e.g., linearization errors. Finally, $\tilde{P} \in \mathbb{R}^{n\times n}$, $\tilde{Q} \in \mathbb{R}^{n\times n}$ and $\tilde{R} \in \mathbb{R}^{m\times m}$ are weighting matrices, assumed to be symmetric and positive semi-definite.

[2] $\langle a, \ldots, b\rangle$ is used as a shorthand for $[a^T, \ldots, b^T]^T$.

[3] Output constraints could be considered as well if output equations were added to the controller model.

Note that this CMHE formulation is slightly different from the one presented in Schneider et al. [2013], where an a-posteriori state estimate is computed. Here, an a-priori state estimate is computed. In fact, (7) is identical to the formulation of Rao et al. [2001], except for the factor ½ which does not alter the resulting optimal state estimate. A particular advantage of this a-priori CMHE for closed-loop control is that the estimator can begin to solve the CMHE problem directly after the measurement at time $t_{k'-1}$ is available; possibly finishing the state estimation and subsequent estimation-based control input calculation before time $t_{k'}$. Thus, the control inputs can be directly applied at the next sampling time, without further delay.

In order to fully appreciate S-PMHE later, it is useful to derive an even more compact formulation of CMHE. Since this derivation is mostly a matter of rearranging terms in (7), it is omitted here for space reasons. However, all steps are very similar to those shown by Schneider et al. [2013]. The only major difference is the definition of the auxiliary variable $\bar{X}_i(k^0)$: To account for known external inputs, $\bar{X}_i(k^0)$ must be redefined here as

$$\bar{X}_i(k^0) = \left\langle \bar{x}_i(k^0), \sum_{j\in\mathcal{N}} B_{ij}u_j(k^0), \ldots, \sum_{j\in\mathcal{N}} B_{ij}u_j(k'-1) \right\rangle.$$

We finally obtain the compact CMHE formulation:

$$\min_{\boldsymbol{z}} \sum_{i\in\mathcal{N}} \Phi_i \tag{8a}$$

$$\text{s.t. } \Phi_i = \frac{1}{2}\sum_{j\in\mathcal{N}} \boldsymbol{z}_i^T \mathcal{T}_{ij}\boldsymbol{z}_j, \tag{8b}$$

$$c_i = \mathcal{A}_{ii}\boldsymbol{z}_i + \sum_{j\in\mathcal{N}\setminus i} \mathcal{A}_{ij}\boldsymbol{z}_j + X_i = 0, \tag{8c}$$

where $\forall i \in \mathcal{N}$. $\boldsymbol{z}_i$ are the subsystem partitions of the decision variables $\Delta x(k^0)$, x, w, and v in (7).

5. SENSITIVITY-DRIVEN SOLUTION APPROACH AND DISTRIBUTED OUTPUT FEEDBACK

In the previous two sections, the centralized MPC and MHE problems were reformulated in Eqs. (6) and (8) as quadratic programs. Each of these QPs contains objective functions and constraints that are coupled in terms of subsystem optimization variables. To solve this kind of optimization problem in a distributed manner, we present in this section an iterative, sensitivity-driven algorithm. This algorithm solves, in every iteration l, and for each subsystem $i \in \mathcal{N}$, the following optimization problem:

$$\boldsymbol{z}_i^{[l+1]} = \arg\min_{\boldsymbol{z}_i}\ \Phi_i^* \tag{9a}$$

$$\begin{aligned}\text{s.t. } \Phi_i^* = \Phi_i + \Bigg[& \sum_{j\in\mathcal{N}\setminus i} \left.\frac{\partial \Phi_j}{\partial \boldsymbol{z}_i^T}\right|_{\boldsymbol{z}^{[l]}} + \left(\lambda_j^{[l]}\right)^T \left.\frac{\partial c_j}{\partial \boldsymbol{z}_i^T}\right|_{\boldsymbol{z}^{[l]}} \\ & + \left(\mu_j^{[l]}\right)^T \left.\frac{\partial g_j}{\partial \boldsymbol{z}_i^T}\right|_{\boldsymbol{z}^{[l]}} \Bigg] (\boldsymbol{z}_i - \boldsymbol{z}_i^{[l]}),\end{aligned} \tag{9b}$$

$$0 = c_i, \tag{9c}$$

$$0 \geq g_i. \tag{9d}$$

$\lambda_i^{[l]}$ and $\mu_i^{[l]}$ are the Lagrange multipliers. As the number of iterations increases, the algorithm converges towards the optimal solutions $\boldsymbol{z}_i, i \in \mathcal{N}$. In this work, we terminate the algorithm after a fixed number of iterations L.

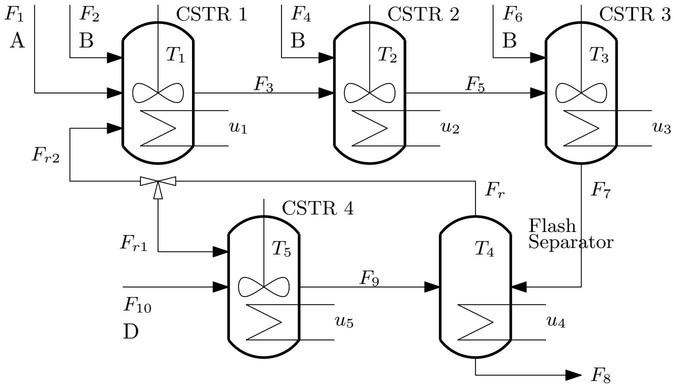

Fig. 1. Process flow diagram for alkylation of benzene.

This iterative, sensitivity-driven solution approach can be applied to both the MPC problem, Eq. (6), and the MHE problem, Eq. (8), with small modifications: In case of Eq. (6), there are no equality constraints (9c) and thus no Lagrange multipliers λ. The resulting distributed control algorithm is called S-DMPC. In case of Eq. (8), we have neither inequality constraints (9d) nor corresponding Lagrange multipliers μ. The resulting partition-based estimation algorithm is called S-PMHE.

To achieve distributed output feedback, the most recent measurements are first processed by S-PMHE to compute an estimate of the current state. This state estimate is subsequently used by S-DMPC as the initial state to compute optimal inputs. Upon arrival of a new measurement, this procedure is repeated.

Finally, note that the MPC and MHE problems cannot be solved simultaneously as one large QP. Doing so would result in unwanted interactions between the estimated states and optimized inputs. For example, instead of minimizing the output error, the "optimal" state estimate may be chosen close to the desired state set point in order to minimize the control objective. Erroneously assuming that the set point has been reached, inadequate control actions will be taken and the true plant state may diverge.

6. CASE STUDY: ALKYLATION OF BENZENE

6.1 Process description & control objective

We illustrate our distributed output feedback approach on a simulated chemical process for the alkylation of benzene [4], depicted in Fig. 1. The plant consists of four continuous stirred-tank reactors (CSTR) and one flash separator; it produces ethylbenzene (C), and in addition the by-product diethylbenzene (D), by reaction of benzene (A) and ethene (B). A and B are fed into the cascaded CSTR 1-3, while D is fed into CSTR 4. The model consists of material balances for each component, an energy balance for each unit of the plant, nonlinear reaction kinetics, and a nonlinear model of the phase equilibrium in the flash separator. We assume all volumetric feed flows to be constant, while the heat flows are assumed to be the manipulated variables $u_i(t)$.

[4] The corresponding process model was first presented by Liu et al. [2010]. However, this work uses the modified model developed by Scheu and Marquardt [2011].

The simulated test cycle consists of one setpoint change: Starting from an operating point defined by the temperatures $T_0 = [443.0, 437.1, 428.4, 433.1, 457.6]$ K, the goal is to stabilize the plant at its steady-state with $T_s = [472.32, 472.35, 472.39, 472.00, 472.49]$ K.

6.2 Controller & estimator configuration

The full nonlinear model described above is used exclusively to simulate the plant. For control and estimation, linear models (2) and (4) are derived. For optimization-based control of each unit i of the plant, we consider the objective function Φ_i (cf. Eq. (5b)) with weighting matrices $Q_i = \text{diag}[1, 1, 1, 1, 1]$ and $R_i = 10^{-8}$. The inputs $u_i(t)$ are discretized by a piecewise constant approximation with a horizon length of five samples. The sampling time is $\Delta t = 5$ sec. Finally, the inputs are constrained with $|u_1| < 0.75$ MJ/s, $|u_2| < 0.5$ MJ/s, $|u_3| < 0.5$ MJ/s, $|u_4| < 0.6$ MJ/s, $|u_5| < 0.6$ MJ/s.

The MHE horizon comprises $K = 5$ measurement samples. Due to this sampling, MHE uses the discrete-time model (4). The corresponding system matrices are obtained from the continuous-time model (2) through a zero-order hold. By this procedure, the one-to-one assignment between inputs and their corresponding subsystems is obliterated. Thus, in the discrete-time model each input affects more than one subsystem.

In the following, three different measurement configurations are considered: In the first configuration C1, all states are assumed to be measured, i.e. all matrices $C_i^d \in \mathbb{R}^{5\times5}$ are identity matrices. In practice, this configuration requires temperature and composition measurements inside each CSTR and in the separation unit. The condition number of the corresponding linear discrete-time observability matrix is 66.7, promising accurate state estimation.

In the second configuration C2, only temperature and total density are measured on every unit. This is resembled by C_i^d matrices containing the molar masses of each component in $g\ mol^{-1}$:

$$C_i^d = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 78.11 & 28.05 & 106.17 & 134.22 & 0 \end{bmatrix}.$$

In contrast to C1, this configuration would not require expensive composition measurement devices, such as spectrometers, but rather simple density probes. However, the loss of information is reflected by a much larger condition number of the observability matrix of 1.2×10^6.

In the third configuration C3, the total density is only measured at the flash separator. In addition, the temperatures inside all units are measured. This measurement configuration is least expensive but also yields the least information, indicated by a very large condition number of the discrete-time observability matrix of 6.5×10^8.

To keep computations simple, $\tilde{P}$ in (7) was kept constant on all horizons. In particular, we first computed the steady state Riccati covariance matrix P_s from A^d and C^d, as well as from $\tilde{Q}^{-1}$ and $\tilde{R}^{-1}$. The diagonal elements of $\tilde{Q}$ and $\tilde{R}$ were tuned to reflect zero-mean Gaussian process and measurement noise with expected standard deviations of 1% of the steady state values x_s and y_s: $\tilde{Q}_{ii} = 1\times10^4/x_{s,i}^2$, $i \in \{1, \ldots, n\}$, and $\tilde{R}_{ii} = 1\times10^4/y_{s,i}^2$, $i \in \{1, \ldots, m\}$. This heuristic was inspired by Schneider and Georgakis [2013] and aims to compensate for the different orders of magnitude of the state and output variables involved. $\tilde{P}$ was then formed from the diagonal blocks of P_s^{-1} to reduce coupling between the S-PMHE subsystem computations.

6.3 Closed-loop simulation results

We apply the proposed distributed output feedback scheme to the alkylation process. After simulating the process for $t_f = 3000$ seconds, we evaluate its performance based on three key indicators. First, we evaluate the MPC objective function Φ in (5a) using the true states and inputs to the process. A lower value of Φ indicates a successful control performance and a fast transition to the desired set point. Second, we analyse the estimation error e. In order to have a scalar indicator, we take the norm of the estimation error at every time sample:

$$e = \left\| \left\langle \|x(0) - \hat{x}(0)\|_2, \ldots, \|x(k_f) - \hat{x}(k_f)\|_2 \right\rangle \right\|_2.$$

Note that each combination of control and estimation scheme leads to a different closed-loop trajectory of the plant. Thus, estimation quality and control performance are not independent: Certain combinations of measurement configurations, measurement noise, initial state estimate and control actions may lead to state trajectories that give a small estimation error even when the estimator itself is poor. The third performance indicator is the total computation time $\bar{t}$ for one simulation. Even though some computations in S-DMPC and S-PMHE could be computed in parallel, the current implementation is sequential: In every iteration, the independent optimization problems of all subsystems are computed sequentially. All computations were performed in Matlab 2011b (64 bit) on a single fixed core of a 3.2 GHz Intel i5 desktop computer.

As a reference, we simulate a scenario similar to the one reported by Scheu and Marquardt [2011], i.e. the case with centralized MPC (CMPC) as a controller and state feedback. The resulting objective function value is $\Phi = 2.347\times10^9$. The corresponding input and state trajectories can be found elsewhere [Scheu and Marquardt, 2011].

We now compare the performance of a fully distributed output feedback scheme using S-DMPC and S-PMHE with one iteration each to a fully centralized scheme using CMPC and CMHE. We distinguish two cases: the *nominal* case without measurement noise and a *noisy* case with zero-mean Gaussian measurement noise with covariance $\tilde{R}^{-1}$, i.e. the same covariance as used for tuning the moving horizon estimators. For the three different measurement configurations discussed earlier, the results are shown in Table 1. Not surprisingly, we find that both the overall control performance as well as the estimation errors are better in the centralized setting than in the distributed case. Even though the estimation error of the distributed scheme is at most 37 times as large as the best centralized solution, the largest difference in control performance across all configurations is only less than 10%. This indicates a certain robustness of the controllers with respect to possibly poor state estimates. The quality of the state estimates highly depends on the measurement configuration. In line with the degree of observability computed earlier, the best state estimates are obtained when all states are measured. However, and in contrast to the observability analysis,

	Centralized output feedback			Distributed output feedback (1 it.)		
	C1	C2	C3	C1	C2	C3
Φ_{nom}	2.347×10^9	2.373×10^9	2.371×10^9	2.356×10^9	2.576×10^9	2.578×10^9
$\Phi_\sim$	2.344×10^9	2.387×10^9	2.396×10^9	2.363×10^9	2.541×10^9	2.565×10^9
e_{nom}	639.6	4147.8	3953.5	2525.8	22988.8	22941.5
$e_\sim$	3250.3	9208.6	5735.3	4170.3	22796.0	23331.9

Table 1. Comparison of measurement configurations C1 - C3. The subscript $\sim$ indicates the presence of measurement noise.

configuration C2 does not always outperform C3 in terms of estimation performance. This can only be explained by favourable interactions of the estimation results with the control input computations. Finally, we see that the effect of measurement noise on the control performance is small. Since the estimators know the true covariance, they compensate the measurement noise very well.

We now focus our analysis on the scenario with measurement noise of known covariance and measurement configuration C3. This time, the estimators are provided an initial state estimate whose values are 10% larger than the initial state of the plant. Some input and state trajectories resulting from the combination of CMPC with CMHE and from S-DMPC with S-PMHE, each of which uses a single iteration, are shown in Fig. 2. More combinations are given in Table 2. We find that good control performance can be achieved with distributed solutions. In particular, by comparing the two cases with CMHE we see that S-DMPC is already comparable to CMPC at only one iteration. More attention must be paid to choosing the right number of iterations for S-PMHE. Since in this scenario, the initial state estimate is off from the true initial state, good state estimation performance is important. This becomes clear from Table 2 as well: Increasing the iterations of S-PMHE both improves estimation and control performance. Overall, it appears as if the combination of S-DMPC with one iteration with S-PMHE with two iterations comes closest to the performance of a fully centralized solution.

The first two rows of Table 2 deserve an additional remark. There, CMPC with CMHE slightly outperforms CMPC with state feedback. This may seem irritating but can be explained by the fact that the overall control performance is computed for the full simulation while the on-line objective function of MPC is only a short time approximation of an optimal control problem on a longer horizon. Thus, it may turn out at the end of the simulation that suboptimal control inputs caused by poor state estimates were in fact a good choice.

Finally, the computation times are quite remarkable. Even though they can be reproduced only with an accuracy of $\pm 5\%$, the fully centralized set-up is the most time-consuming. More specifically, and judging from the two CMHE simulations, CMHE seems to be the bottleneck. Overall, we find that using distributed output feedback is faster than centralized output feedback, even if the distributed algorithms are computed sequentially. Neglecting communication overhead and assuming that each subproblem could be solved in parallel on different CPUs, a further speed-up is expected. For instance, the distributed set-up with two iterations would solve five times as fast as the centralized set-up. Considering the well-known deterioration of control performance resulting from computation delay, the proposed distributed output feedback scheme seems to be a promising alternative.

Control (it.)	Estimation (it.)	Φ	e	$\bar{t}$
CMPC	State feedback	2.347×10^9	0.0	49
CMPC	CMHE	2.339×10^9	8078.2	153
CMPC	S-PMHE (1)	2.625×10^9	23495.0	88
S-DMPC (1)	State feedback	2.346×10^9	0.0	57
S-DMPC (1)	CMHE	2.339×10^9	8362.9	162
S-DMPC (1)	S-PMHE (1)	2.605×10^9	23133.7	94
S-DMPC (1)	S-PMHE (2)	2.364×10^9	7667.5	130
S-DMPC (2)	S-PMHE (2)	2.398×10^9	9063.4	143

Table 2. Comparison of different output feedback schemes for configuration C3 with measurement noise and +10% error in the initial state estimate. Iteration number in brackets.

7. CONCLUSIONS

A novel distributed output feedback scheme was presented which uses S-PMHE for estimation and S-DMPC for control. Both methods employ the same iterative, sensitivity-driven solution approach to compute optimal state estimates and control inputs in a distributed manner. On a simulated alkylation process for the production of benzene, the distributed scheme performed comparative to fully centralized output feedback while being much faster. Theoretical proofs for these promising closed-loop properties remain challenges for future work.

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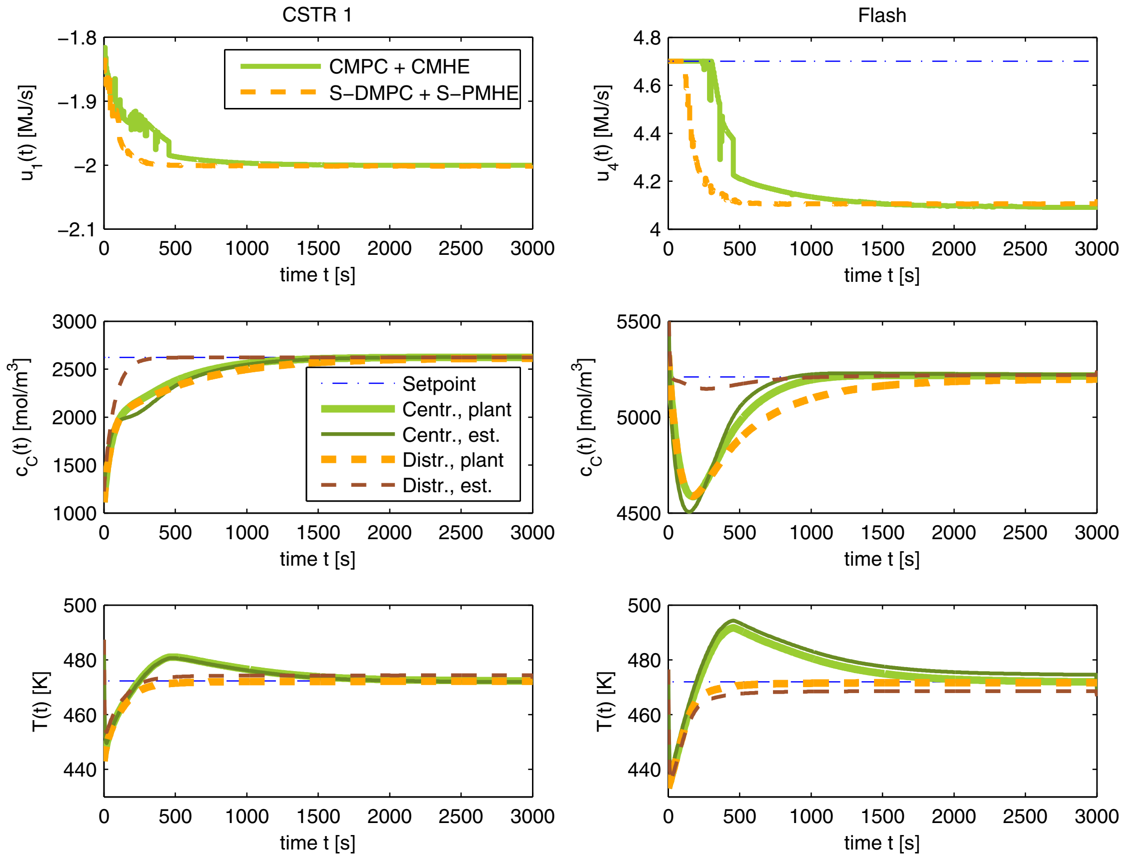

Fig. 2. Selected closed-loop trajectories for C3 with measurement noise. Initial state estimate is 10% larger than the true plant state. Distributed schemes are stopped after the first iteration.

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