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Adaptive Cubature Strong Tracking Information Filter Using Variational Bayesian Method ⋆

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Abstract: For most practical nonlinear state estimation problems, the conventional nonlinear filters do not usually work well for some cases, such as inaccurate system model, sudden change of state-interested and unknown variance of measurement noise. In this paper, an adaptive cubature strong tracking information filter using variational Bayesian (VB) method is proposed to cope with these complex cases. Firstly, the strong tracking filtering (STF) technology is used to integrally improve performance of cubature information filter (CIF) aiming at the first two cases and an iterative scheme is presented to effectively evaluate a strong tracking fading factor. Secondly, the VB method is introduced to iteratively evaluate the unknown variance of measurement noise. Finally, the novel adaptive cubature information filter is obtained by perfectly combining the STF technology with the VB method, where the variance estimation provided by the VB method guarantees normal running of the strong tracking functionality.

Keywords: Nonlinear system, unknown variance of measurement noise, cubature information filter, strong tracking filtering, variational Bayesian.

1. INTRODUCTION

Nonlinear filtering or state estimation is a popular topic in various fields such as signal processing, target tracking, data fusion and control all the time (Chandra and Gu *et al*, 2011; Ge and Wen *et al*, 2013; Xu and Zhang *et al*, 2012; Li and Wang, 2012; Musicki and Song *et al*, 2012; Xu and Ding *et al* , 2013; Li and Jia, 2012). Due to complexity of nonlinear systems and finiteness of ways to deal with nonlinear filtering, the study on nonlinear filtering are suffering many difficulties and challenges in theory and applications and currently it gets a slow rate of progress. For most of practical target tracking systems composed of many distributed sensors, the tracking models are usually nonlinear. To realize excellent fusion tracking performance, it is basic and important to design high-powered nonlinear filters which can cope with many complex application problems. In recent decades, the nonlinear filtering under the minimum mean square error (MMSE) sense has been paid many attention from many researchers and engineers (Dallil *et al*, 2013; Ristic and Arulampalam, 2010; Arasaratnam and Haykin, 2009; Sun and Tang, 2013; Chandra and Gu *et al*, 2013; Ge and Xu *et al*, 2014; Zhou and Li *et al*, 2010). But, it is still a difficult problem how to obtain high performance estimation when parameters of estimation systems are not accurate and unknown and sudden change of state appears (Chen, 2008).

The Kalman filter (KF) proposed originally by R.E Kalman only deals with state estimation of linear dynamic systems and it is optimal in the sense of linear minimum mean square error when system models are accurate (LMMSE)(Kalman,1960; Ge and Wen *et al*, 2012; Han *et al*, 2010). Consequently, for the nonlinear state estimation, the extended Kalman filter (EKF)(Jwo and Wang, 2007; Lee, 2008; Xu and Zhang *et al*, 2012), the unscented Kalman filter (UKF)(Julier and Uhlmann, 2004), and the cubature Kalman filter (CKF)(Arasaratnam and Haykin, 2009) were sequentially presented to deal with state estimation of nonlinear systems. From the current research results, the CKF has the best performance among the three nonlinear filter mentioned above. Later in Chandra

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and Gu *et al* (2013), the associated information filters CIF and SRCIF were proposed. For the case with two kind of correlated noises, the design of cubature Kalman estimators and fusion algorithms has been developed (Ge and Xu *et al*, 2013; Ge and Wen *et al*, 2014).

These nonlinear filters generally require known and accurate systematic parameters and work poorly with the sudden change of state (Zhou, 1999; Li and Ge, 2010; Ge and Li *et al*, 2011). For the two cases, the common result is that the used system in filtering process mismatches the practical one. Accordingly, adaptive filtering and robust filtering technologies have been presented to improve estimation performance and some effective nonlinear filtering methods have been established (Zhou, 1999; Wang, 2010; Luo *et al*, 2012; Blom and Bar-Shalom, 1988; Ge and Li *et al*, 2011; Deisenroth and Turner *et al*, 2012; Hajiyev and Soken, 2014). Among these methods, the strong tracking filtering (STF) is relatively popular. Its basic idea is to increase prediction error covariance by using a fading factor, which is related to the current measurement innovation and can adaptively adjust the gain matrix in the STF. Thereby, the so-called strong tracking performance is taken (Jwo and Wang, 2007). For the target maneuvering or the sudden change of state with inaccurate variance of measurement noise sequence, the STF can be also used to improve the estimation performance. Unfortunately, the inaccurate variance cannot be estimated and corrected in real-time. In the current work, variational Bayesian (VB) method has been presented to dynamically estimate the inaccurate or unknown variance, but the ability to deal with the target maneuvering or the sudden change of state is weak for the adaptive filter with only the VB method (Sarkka, 2009; Ge and Wen *et al*, 2014).

As a matter of fact, for most of practical target tracking systems, the inaccurate or unknown variance of the measurement noise and the sudden change of the state appear synchronously. In order to obtain high estimation performance, it is necessary to require that nonlinear filters have two abilities which are to estimate the inaccurate measurement noise variance in real time and to have the strong tracking ability on the sudden change of the target state. So, we propose a nonlinear filter based on an information filtering form of the cubature Kalman filter (Chandra and Gu *et al*, 2011; Ge and Xu *et al*, 2014) by combining the STF with the VB method for a kind of nonlinear system with an unknown variance of measurement noise in this paper. The proposed filter not only has the strong tracking functionality on the sudden change of state but can estimate the unknown variance of measurement noise online.

The rest of the paper is organized as follows. We provide the problem formulation in Section 2. A cubature strong tracking information filter (CSTIF) is proposed in Section 3. Section 4 presents an adaptive CSTIF using the VB method, which is called VB-ACSTIF. In Section 5, a brief analysis is given. Simulation examples are demonstrated in Section 6. Finally, we conclude the paper.

2. PROBLEM FORMULATION

Consider a kind of nonlinear dynamic system expressed by

$$\begin{cases} \mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k,k-1} \\ \mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k \end{cases} \tag{1}$$

where $k \geq 1$ is the time index, and $\mathbf{z}_k \in \Re^{p\times 1}$ is a measurement vector of $\mathbf{x}_k \in \Re^{n\times 1}$ which is the system state. $f_{k-1}(\mathbf{x}_{k-1})$ and $h_k(\mathbf{x}_k)$ are both differentiable functions. $\mathbf{w}_{k,k-1} \in \Re^{n\times 1}$ and $\mathbf{v}_k \in \Re^{p\times 1}$ are zero mean Gaussian white noises and

$$\begin{cases} E\{\mathbf{w}_{k,k-1}\mathbf{w}_{k,k-1}^T\} = \mathbf{Q}_{k,k-1}, E\{\mathbf{w}_{k,k-1}\mathbf{v}_k^T\} = \mathbf{0} \\ E\{\mathbf{v}_k\mathbf{v}_k^T\} = \mathbf{R}_k = diag(\sigma_{1,k}^2, \sigma_{2,k}^2, \cdots, \sigma_{p,k}^2) \end{cases} \tag{2}$$

where $\mathbf{R}_k$ is unknown. The initial state $\mathbf{x}_0$ with mean $\widehat{\mathbf{x}}_{0|0}$ and variance $\mathbf{P}_{0|0}$ is unrelated to $\mathbf{w}_{k,k-1}$ and $\mathbf{v}_k$.

Aiming at various complex situations in practical systems, a desirable universal nonlinear filter should have four features: 1) The basic nonlinear filter should be high-performance. 2) It should have an expression that can be computed efficiently and can be easily extended to multisensor case (Lee, 2008; Ge and Wen *et al*, 2013). 3) The strong tracking performance and favorable robustness can be taken for the cases with inaccurate system model and the sudden change of state, and the computation of Jacobian matrices should be avoided in solving for the fading factor. 4) For the case with the unknown variance of measurement noise, it can evaluate this variance online. In this paper, we design a nonlinear filter by combining the STF with VB method based on the CIF. For this novel nonlinear filter, the VB method guarantees the normal running of the STF functionality by providing an estimate of the unknown variance $\mathbf{R}_k$ at every time.

3. CUBATURE STRONG TRACKING INFORMATION FILTER

3.1 Cubature Information Filter

In this subsection, an information form of the CKF, the cubature information filter (CIF), is briefly reviewed. Generally, nonlinear state estimators under Kalman filtering frame solve a set of equations that contain conditional expectations on $\widehat{\mathbf{x}}_{k|k-1}$, $\mathbf{P}_{k|k-1}$, $\widehat{\mathbf{z}}_{k|k-1}$, $\widehat{\mathbf{x}}_{k|k}$, and $\mathbf{P}_{k|k}$. The main difference is how to compute these conditional expectations in details. The CKF solves for these variables based on the following cubature rule

$$\int_{\Re^n} f_{k-1}(\mathbf{x})N(\mathbf{x};\mu,P)d\mathbf{x} \approx \frac{1}{2n}\sum_{i=1}^{2n} f_{k-1}(\mu + \sqrt{P}\xi_i), \tag{3}$$

where $\sqrt{P}$ is the square root of the covariance P, and $\xi_i = \sqrt{n}[1]_i$ where $[1]_i$ is the i^{th} column of point set (Arasaratnam and Haykin, 2009; Ge and Xu *et al*, 2014).

The CIF is obtained by embedding CKF in an extended information filter (EIF) framework (Chandra and Gu *et al*, 2013). It not only has good computational performance but is most useful in multisensor fusion (Ge and Xu *et al*, 2014). The formulas of the CIF are reviewed as follows:

Time Update

1) Evaluate information matrix and information state vector

$$\mathbf{Y}_{k|k-1} = \mathbf{P}_{k|k-1}^{-1}, \quad \widehat{\mathbf{y}}_{k|k-1} = \mathbf{Y}_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}, \tag{4}$$

where if let $\mathbf{Z}^{k-1}$ be a set of $\mathbf{z}_l(l=1,2,\cdots,k-1)$, we have

$$\begin{aligned}\mathbf{P}_{k|k-1} &= E\{[\mathbf{x}_k-\widehat{\mathbf{x}}_{k|k-1}][\mathbf{x}_k-\widehat{\mathbf{x}}_{k|k-1}]^T|\mathbf{Z}^{k-1}\}\\ &= \frac{1}{2n}\sum_{i=1}^{2n}\mathbf{X}^*_{i,k|k-1}\mathbf{X}^{*T}_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{x}}^T_{k|k-1}\\ &\quad+\mathbf{Q}_{k,k-1},\end{aligned} \quad (5)$$

$$\widehat{\mathbf{x}}_{k|k-1}=E\{\mathbf{x}_k|\mathbf{Z}^{k-1}\}=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{X}^*_{i,k|k-1}, \quad (6)$$

$$\mathbf{X}^*_{i,k|k-1}=f_{k-1}(\widehat{\mathbf{x}}_{k-1|k-1}+\sqrt{\mathbf{P}_{k-1|k-1}}\xi_i), \quad (7)$$

Measurement Update

2) Compute the cubature points and the propagated cubature points $(i=1,2,...,2n)$

$$\mathbf{X}_{i,k|k-1}=\sqrt{\mathbf{P}_{k|k-1}}\xi_i+\widehat{\mathbf{x}}_{k|k-1},\quad \mathbf{Z}_{i,k|k-1}=h_k(\mathbf{X}_{i,k|k-1}), (8)$$

3) Evaluate the predicted measurement

$$\widehat{\mathbf{z}}_{k|k-1}=E\{\mathbf{z}_k|\mathbf{Z}^{k-1}\}=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{Z}_{i,k|k-1}, \quad (9)$$

4) Estimate the cross-covariance

$$\mathbf{P}_{\mathbf{XZ},k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{X}_{i,k|k-1}\mathbf{Z}^T_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{z}}^T_{k|k-1}, \quad (10)$$

5) Evaluate information state contribution $\mathbf{I}_k$ and its associated information matrix $\mathbf{i}_k$

$$\mathbf{I}_k=\mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{H}_k,\quad \mathbf{i}_k=\mathbf{H}_k^T\mathbf{R}_k^{-1}(\upsilon_k+\mathbf{H}_k\widehat{\mathbf{x}}_{k|k-1}), \quad (11)$$

where $\mathbf{H}_k$ and υ_k are pseudo measurement matrix and innovation vector respectively, and

$$\mathbf{H}_k=\mathbf{P}^T_{\mathbf{XZ},k|k-1}\mathbf{Y}^T_{k|k-1},\quad \upsilon_k=\mathbf{z}_k-\widehat{\mathbf{z}}_{k|k-1}, \quad (12)$$

6) Evaluate the estimated information vector and the information matrix

$$\mathbf{Y}_{k|k}=\mathbf{Y}_{k|k-1}+\mathbf{I}_k,\quad \hat{\mathbf{y}}_{k|k}=\widehat{\mathbf{y}}_{k|k-1}+\mathbf{i}_k, \quad (13)$$

7) Evaluate final estimate $\widehat{\mathbf{x}}_{k|k}$ and its covariance $\mathbf{P}_{k|k}$

$$\widehat{\mathbf{x}}_{k|k}=\mathbf{Y}_{k|k}\widehat{\mathbf{y}}_{k|k},\quad \mathbf{P}_{k|k}=\mathbf{Y}^{-1}_{k|k}, \quad (14)$$

3.2 Cubature Strong Tracking Information Filter

We propose a cubature strong tracking information filter by combing the strong tracking filter with the CIF in this subsection. Although the CIF has better computational efficiency than the CKF, it still requires accurate system parameters in order to achieve good estimation result as with the CKF. To address the problem, we introduce the strong tracking filtering technology to improve the tracking result of the CIF with the inaccurate system model. Then, for the CIF, a modified state prediction error covariance with the fading factor λ_k is given as follows

$$\begin{aligned}\mathbf{P}_{k|k-1}&=\lambda_k(\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{X}^*_{i,k|k-1}\mathbf{X}^{*T}_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{x}}^T_{k|k-1})\\ &\quad+\mathbf{Q}_{k,k-1},\end{aligned} \quad (15)$$

where the fading factor λ_k can be solved by minimizing $\mathbf{P}_{k|k}$ with constraint $E\{\upsilon_{k+d}\upsilon_k^T\}=\mathbf{0}$ $(k=1,2,\cdots;d\geq 1)$.

A popular suboptimal solution of λ_k is computed by dynamically using measurement innovation (Zhou, 1999; Ge and Li *et al*, 2011). Unfortunately, the fading factor cannot be directly computed by using the fundamental suboptimal formulas for the CIF. This is because the computation of $\mathbf{H}_k$ and λ_k depend on each other at this moment when we try to avoid the computation of the Jacobian matrix. For this problem, an iterative method is presented to compute λ_k in real time. The iterative formulas of $\lambda_k^j(1\leq j\leq N_1)$ are as follows:

$$\lambda_k^j=\begin{cases}1, j=1\\ \lambda_{j,k}, 2\leq j\leq N_1\end{cases},\quad \lambda_{j,k}=\begin{cases}c_{j,k}, c_{j,k}>1\\ 1,\ c_{j,k}\leq 1\end{cases}, \quad (16)$$

where $c_{j,k}=Tr(\mathbf{N}_{j,k})/Tr(\mathbf{M}_{j,k})$, and

$$\begin{cases}\mathbf{N}_{j,k}=\mathbf{V}^{j-1}_{0,k}-\kappa_0\mathbf{R}_k-\mathbf{H}_k^{j-1}\mathbf{Q}_{k,k-1}(\mathbf{H}_k^{j-1})^T\\ \mathbf{M}_{j,k}=\mathbf{H}_k^{j-1}\Phi_{k,k-1}\mathbf{P}_{k-1|k-1}\Phi^T_{k,k-1}(\mathbf{H}_k^{j-1})^T,\end{cases}, \quad (17)$$

$$\mathbf{V}^{j-1}_{0,k}=\begin{cases}\upsilon_1^{j-1}(\upsilon_1^{j-1})^T,\ k=1\\ \dfrac{\rho_1\mathbf{V}_{0,k-1}+\upsilon_k^{j-1}(\upsilon_k^{j-1})^T}{1+\rho_0},\ k>1,\end{cases} \quad (18)$$

where $\Phi_{k,k-1}=\frac{f_{k-1}(\mathbf{X}_{k-1})}{\partial\mathbf{X}_{k-1}}|_{\mathbf{X}_{k-1}=\hat{\mathbf{X}}_{k-1|k-1}}$, $0<\rho_0\leq 1$ and $\kappa_0\geq 1$. υ_k^{j-1} and $\mathbf{H}_k^{j-1}$ are the innovation vector and the pseudo measurement matrix taken respectively in $(j-1)^{th}$ iteration. Finally, $\lambda_k=\lambda_k^{N_1}$. Then, the new cubature information filter is called Cubature Strong Tracking Information Filter (CSTIF). We summarize the procedure of the CSTIF as follows:

i) Evaluate $\hat{\mathbf{x}}_{k|k-1}$ in terms of Eqs.(7) and (6)

ii) Set $j=1$ and N_1, and iteration loop begins

∘ **Iteration Loop**

iii) If $j=1(\lambda_k^1=1)$, $\mathbf{H}_k^1$ and υ_k^1 can be taken by directly using Eqs.(4)-(10) and (12), and go to x).

iv) If $2\leq j\leq N_1$, then evaluate λ_k^j by using Eqs.(16)-(18)

v) Compute the state prediction error covariance $\mathbf{P}^j_{k|k-1}$

$$\begin{aligned}\mathbf{P}^j_{k|k-1}&=\lambda_k^j(\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{X}^*_{i,k|k-1}\mathbf{X}^{*T}_{i,k|k-1}-\widehat{\mathbf{x}}_{k|k-1}\widehat{\mathbf{x}}^T_{k|k-1})\\ &\quad+\mathbf{Q}_{k,k-1},\end{aligned} \quad (19)$$

vi) Evaluate

$$\mathbf{Y}^j_{k|k-1}=(\mathbf{P}^j_{k|k-1})^{-1},\quad \widehat{\mathbf{y}}^j_{k|k-1}=\mathbf{Y}^j_{k|k-1}\widehat{\mathbf{x}}_{k|k-1}, \quad (20)$$

vii) Evaluate the measurement prediction and innovation

$$\widehat{\mathbf{z}}^j_{k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{Z}^j_{i,k|k-1},\quad \upsilon_k^j=\mathbf{z}_k-\widehat{\mathbf{z}}^j_{k|k-1}, \quad (21)$$

where

$$\begin{cases}\mathbf{Z}^j_{i,k|k-1}=h_k(\mathbf{X}^j_{i,k|k-1})\\ \mathbf{X}^j_{i,k|k-1}=\sqrt{\mathbf{P}^j_{k|k-1}}\xi_i+\widehat{\mathbf{x}}_{k|k-1},\end{cases} \quad (22)$$

viii) Compute the cross-covariance

$$\begin{aligned}\mathbf{P}^j_{\mathbf{XZ},k|k-1}&=\frac{1}{2n}\sum_{i=1}^{2n}\mathbf{X}^j_{i,k|k-1}(\mathbf{Z}^j_{i,k|k-1})^T\\ &\quad-\widehat{\mathbf{x}}_{k|k-1}(\widehat{\mathbf{z}}^j_{k|k-1})^T,\end{aligned} \quad (23)$$

ix) Evaluate the pseudo measurement matrix

$$\mathbf{H}_k^j = (\mathbf{P}^j_{\mathbf{XZ},k|k-1})^T(\mathbf{Y}^j_{k|k-1})^T, \tag{24}$$

x) If $j \leq N_1$, let $j = j+1$ and go to iv); else go to xi).

∘ **Iteration Over**

xi) Let $\mathbf{H}_k = \mathbf{H}_k^{N_1+1}$, $\upsilon_k = \upsilon_k^{N_1+1}$, $\mathbf{V}_{0,k} = \mathbf{V}_{0,k}^{N_1}$, $\mathbf{Y}_{k|k-1} = \mathbf{Y}_{k|k-1}^{N_1+1}$ and $\widehat{\mathbf{y}}_{k|k-1} = \widehat{\mathbf{y}}_{k|k-1}^{N_1+1}$.

xii) Evaluate $\widehat{\mathbf{x}}_{k|k}$ and $\mathbf{P}_{k|k}$ according to Eqs.(11),(13),(14).

Clearly, the fading factor can be effectively evaluated by the iterative computation of the pseudo-measurement matrix. At the same time, by introducing the time-variant fading factor based on the newest innovation, the CIF can have the adaptive strong tracking functionality and good robustness to the sudden change of state and inaccuracy of system model. Here, the variance $\mathbf{R}_k$ should be known in order to guarantee the normal running of the CSTIF. Accordingly, for the case with the unknown variance $\mathbf{R}_k$, the CSTIF does not work. Then, it is necessary to estimate $\mathbf{R}_k$ online before performing the CSTIF.

4. ADAPTIVE CSTIF USING VARIATIONAL BAYESIAN

4.1 Adaptive CIF Using Variational Bayesian

An adaptive cubature information filter by using the variational Bayesian method (VB-ACIF) is proposed to simultaneously estimate the system state and the unknown variance of the measurement noise based on the CIF in this subsection. The VB method uses many known distributions to approximate joint posterior distribution $p(\mathbf{x}_k, \mathbf{R}_k|\mathbf{Z}^k)$ of state and measurement noise variance, namely (Sarkka, 2009)

$$p(\mathbf{x}_k, \mathbf{R}_k|\mathbf{Z}^k) \approx \mathbf{N}(\mathbf{x}_k|\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}) \times \prod_{l=1}^{p} Inv-Gamma(\sigma^2_{l,k}|\alpha_{l,k|k-1}, \beta_{l,k|k-1}), \tag{25}$$

Then, the procedure of the VB-ACIF with an iterative estimation of the unknown $\mathbf{R}_k$ is summarized as follows:

I) Evaluate global parameters prediction

$$\alpha_{k|k-1} = \varrho\ .\alpha_{k-1}, \quad \beta_{k|k-1} = \varrho\ .\beta_{k-1}, \tag{26}$$

where $'.'$ indicates the point operation in Matlab software, and

$$\alpha_k = [\alpha_{1,k} \ \cdots \ \alpha_{p,k}]^T, \ \beta_k = [\beta_{1,k} \ \cdots \ \beta_{p,k}]^T, \tag{27}$$

$$\varrho_k = [\varrho_{1,k} \ \cdots \ \varrho_{p,k}]^T, \tag{28}$$

II) Parameters update: $\alpha_k = 1/2 + \alpha_{k|k-1}$ and $\beta_k^0 = \beta_{k|k-1}$.

III) Set $m = 0$ and N_2, and the iterative process begins.

IV) Iteratively evaluate the variance $\mathbf{R}_k$

$$\hat{\mathbf{R}}_k^m = diag(\beta_k^m./\alpha_k) = diag((\hat{\sigma}^m_{1,k})^2, \ \cdots, \ (\hat{\sigma}^m_{p,k})^2), \tag{29}$$

V) Evaluate state estimate $\widehat{\mathbf{x}}_{k|k}^{m+1}$ and the associated covariance $\mathbf{P}_{k|k}^{m+1}$ in terms of Eqs.(4)-(14) by using $\widehat{\mathbf{R}}_k^m$.

VI) If $m < N_2$, then

$$\beta_k^{m+1} = \beta_{k|k-1} + (\mathbf{z}_k - \mathbf{H}_k\widehat{\mathbf{x}}_{k|k}^{m+1})^{.2}/2 + diag(\mathbf{H}_k\mathbf{P}_{k|k}^{m+1}\mathbf{H}_k^T)/2, \tag{30}$$

Afterwards, let $m = m+1$ and go to III); else $\widehat{\mathbf{R}}_k = \widehat{\mathbf{R}}_k^{N_2}$.

4.2 VB-ACSTIF

In this subsection, a powered adaptive nonlinear filter is proposed by embedding the iterative esimation of $\mathbf{R}_k$ in the CSTIF given in Section 3 and is called VB-ACSTIF. From Eq.(30), every interation of estimating $\mathbf{R}_k$ depends on the state estimate and its associated covariance. For the CSTIF, the computation of the state estimate and the covariance requires the knowledge of λ_k, which iterative evaluation depends on $\mathbf{R}_k$. Thereby, the combination of the VB method and the STF also suffers the interdependent problem on the computation of some variables. To address this problem, we only send the iteration estimate result $\hat{\mathbf{R}}_k^0$ to the estimation of λ_k so that the strong tracking functionality in the VB-ACSTIF can normally work.

The procedure of the VB-ACSTIF is as follows:

1) Evaluate $\widehat{\mathbf{R}}_k^m$ in terms of steps I)-IV) in subsection 4.1.

2) If $m = 0$, let $\mathbf{R}_k = \widehat{\mathbf{R}}_k^0$ and go to 3); else go to 5).

3) Estimate $\widehat{\mathbf{x}}_{k|k-1}$ by using step i) in subsection 3.2.

4) Iteratively evaluate $\mathbf{H}_k, \upsilon_k, \mathbf{V}_{0,k}, \mathbf{Y}_{k|k-1}$ and $\widehat{\mathbf{y}}_{k|k-1}$ in terms of steps ii)-ix) in subsection 3.2.

5) Evaluate $\widehat{\mathbf{x}}_{k|k}^{m+1}$ and $\mathbf{P}_{k|k}^{m+1}$ by using $\widehat{\mathbf{R}}_k^m$, $\mathbf{H}_k$ and υ_k.

6) If $m < N_2$, compute β_k^{m+1} according to Eq.(30), let $m = m+1$ and go to 1); else go to 7).

7) Let $\widehat{\mathbf{x}}_{k|k} = \widehat{\mathbf{x}}_{k|k}^{N_2}$, $\mathbf{P}_{k|k} = \mathbf{P}_{k|k}^{N_2}$, $\widehat{\mathbf{R}}_k = \widehat{\mathbf{R}}_k^{N_2}$ and $\beta_k = \beta_k^{N_2}$. □

Actually, the VB-ACSTIF is formed by combining the CSTIF and the VB and this combination is naturally a reciprocally embedded process of the CSTIF algorithm and the VB method. In other words, the strong tracking functionality and the variational Bayesian method influence each other in the final VB-ACSTIF.

5. SIMULATION EXAMPLE

In this section, a simulation example with Bearings-only tracking is demonstrated. For this tracking system, two sensors are placed at the same level and their distance $D = 1000m$. It assumes that the interested target moves with a constant velocity and the target state $\mathbf{x}_k = [x_k \ \nu_{x,k} \ y_k \ \nu_{y,k}]^T$, namely it follows the CV model. Then, the state model is linear and

$$\mathbf{x}_k = \Phi_{k,k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k,k-1} \tag{31}$$

where the system transfer matrix

$$\Phi_{k,k-1} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

and the covariance matrix of the process noise

$$\mathbf{Q}_{k,k-1} = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} \times 0.5(m^3/s^2) \quad (33)$$

Two right angle measurements $\theta_{1,k}$, $\theta_{2,k}$ to the same target can be respectively taken by the two sensors. By intersecting both angles, we can get the nonlinear measurement equation as follows

$$\mathbf{z}_k = \begin{bmatrix} \theta_{1,k} \\ \theta_{2,k} \end{bmatrix} = \begin{bmatrix} \arccos\left[\dfrac{x_k}{\sqrt{x_k^2 + y_k^2}}\right] \\ \arccos\left[\dfrac{x_k - D}{\sqrt{(x_k - D)^2 + y_k^2}}\right] \end{bmatrix} + \mathbf{v}_k \quad (34)$$

and $\hat{\mathbf{x}}_{0|0} = [0\ 5\ 0\ 5]^T$, $\mathbf{P}_{0|0} = diag(8\ 1\ 8\ 1)$, $N_1 = 3$, $\rho_0 = 0.95$, and $\kappa_0 = 4$. For the VB method in Example 2, the corresponding parameters are $N_2 = 2$, $\varrho = [1 - e^{-8};\ 1 - e^{-8}]$, $\alpha_0 = [1;\ 1]$, and $\beta_0 = [0.05;\ 0.1]$. From time 50 to 60, sudden changes of x_k and y_k are set with 50 and 20 magnitudes respectively. All results are means of Monte-Carlo simulations.

Example 1

This example is used to validate the proposed VB-ACSTIF with unknown variance of measurement noise sequence. A given value $\mathbf{R}_k = diag(\frac{1}{200}\ \frac{1}{200}) \times \pi/180$ is only used to generate simulation data. The results see Fig.1 to Fig.3. From Fig.1 to Fig.2, we know that the VB-ACSTIF is better than the VB-ACIF without the strong tracking ability on the estimation accuracy. It is apparent because the VB-ACSTIF has the strong tracking function when the sudden changes appear but the VB-ACIF does not. In addition, Fig.3 shows that the VB-ACSTIF can estimate effectively the unknown variance of measurement noise because it has the variational Bayesian module which can be used to evaluate the variance of measurement noise in real time. Furthermore, the variance estimation of the unknown measurement noise converges. However, the estimation accuracy of this variance depends closely on the initial value.

6. CONCLUSION

In this paper, the design of nonlinear state filters is developed in order to effectively cope with complex tracking cases and further improve estimation performance. The final nonlinear filter can be applied to dynamically estimate the system state by closely combining the STF with the VB method based on the cubature information filter when the sudden change of system state and the unknown variance of measurement noise appear simultaneously. Certainly, this combination is not simply an integration and is actually a closely embedded structure. There are some open topics, which include to study the associated multisensor fusion, to extend these filters to a more complex case with two kinds of correlated noises, and to discuss comparisons of variance estimations of the unknown measurement noise with different basic nonlinear filters and performance influence from different initial values on the variance estimation of the unknown measurement noise and so forth.

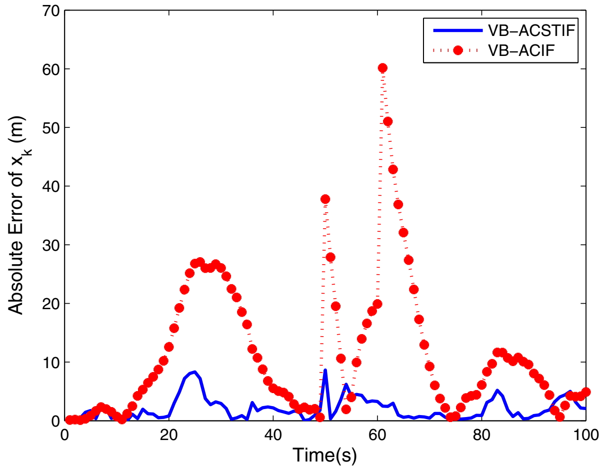

Fig. 1. Absolute estimation error of x-displacement

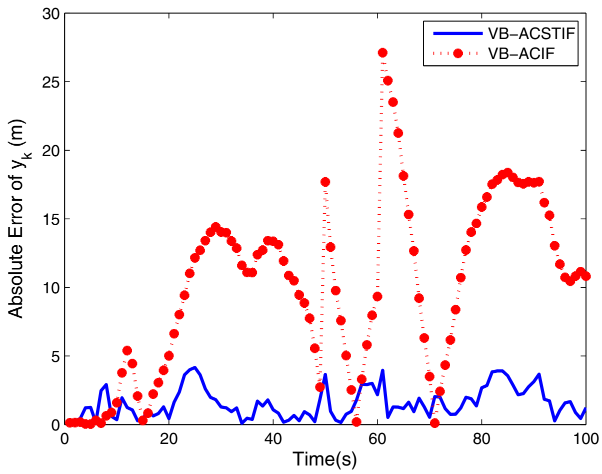

Fig. 2. Absolute estimation error of y-displacement

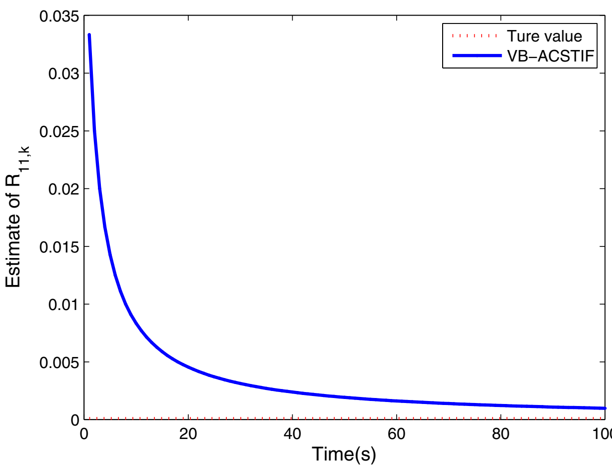

Fig. 3. Estimation result of $\mathbf{R}_{11,k}$

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