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# $H_\infty$ fault detection for a class of T-S fuzzy model-based nonlinear networked control systems ⋆

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**Abstract:** In this paper, a robust $H_\infty$ fault detection filter is designed for a class of discrete-time nonlinear networked control systems via T-S fuzzy model with multiple bounded state delay and random packet dropout induced by the limited bandwidth of communication networks. Our aim in this paper is to analyze and design a robust $H_\infty$ full-order fault detection filter, such that the filtering error dynamics is asymptotically mean-square stable with a prescribed $H_\infty$ performance level. Sufficient conditions for the existence of the desired filter are presented. Finally, an example is given to illustrate the effectiveness and applicability of the proposed new design method.

*Keywords:* Fault detection, networked control systems, fuzzy systems.

## 1. INTRODUCTION

Fault detection (FD) has attracted researchers' attention over the past decades because of the increasing demand for high safety and high reliability in many industrial processes. Fruitful results can be found in several books Chen et al. (1999), Ding X. (2008) and excellent papers Jiang et al. (2006), Zhong et al. (2010). Recently, with the rapid developments in network technologies, the interests on FD of networked control systems (NCSs) are increasing.

Networked control systems (NCSs) are closed-loop feedback control systems, where sensor-controller and controller-actuator signal link is through a shared network. Compared with conventional systems, NCSs have great advantages, such as low cost, reduced weight and increased system agility, which leads to the applications of NCSs for many fields ranging from DC motors, advanced aircraft to manufacturing process. However, the introduction of communication networks also brings communication constraints to the control systems, e.g., network-induced delays and packet dropouts, which all might be potential causes to poor performance, instability, and even faults. Therefore, many excellent results have been carried out for NCSs with time delays and data drops. For examples, Dong et al. (2010) develops a robust $H_\infty$ filter for NCSs with multiple stochastic communication delays and packet dropouts under discrete-time model. Gao et al. (2008) presents a new approach to solve the problem of stabilization for networked control systems. Yue et al. (2009) concerns the design of stabilization controllers for linear systems with stochastic input delays. The abovementioned works focus on the design of controller, observer and filter. However, it should be noted that faults in actuators, sensors and other components existing in NCSs are similar to other control systems. Thus, it is important to study FD schemes for these systems to avoid the failures. There exist some results to design FD for NCSs. Zhang et al. (2011) designs a robust FD filter for NCSs with delay distribution characterization under discrete-time model. Wang et al. (2009) proposes a fault detection approach for NCSs with communication constrains. Mao et al. (2007) develops the scheme of robust FD for networked control systems with large transfer delays . Unfortunately, the most research works on FD for NCSs are reported for linear systems, and only limited results are concerned on nonlinear systems.

In recent years, many fuzzy control approaches and fault diagnosis schemes have been developed for nonlinear systems because of its approximate performance (e.g., see Su et al. (2012), Zhou et al. (2013), Jiang et al. (2010)). Zhang et al. (2007) designs the fuzzy filter for signal estimation of nonlinear discrete-time systems with multiple time delays and unknown bounded disturbances. Tong et al.

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(2013) concerns the problem of adaptive fuzzy tracking control for a class of multi-input and multi-output strict-feedback nonlinear systems with both unknown nonsymmetric dead-zone inputs and immeasurable states. Since nonlinear systems can be approximated by T-S fuzzy models, an applicable design scheme of the fault detectors for nonlinear systems can be transformed into the fault detection problem for T-S fuzzy systems. On the other hand, preserving some nonlinearities in T-S fuzzy systems can make the systems close to the reality. Thus, in this paper, the nonlinear NCSs via T-S fuzzy model is taken into account.

In this paper, the fault detection problem for a discrete-time nonlinear NCSs with time-varying state delays and random data dropouts is considered. Based on Lyapunov functions, sufficient conditions for FD filter design are established. The designed FD filter can detect the fault with the limited information successfully.

The paper is organized as follows. Section 2 describes the system model and presents some assumptions and definitions. The $H_\infty$ FD filter design method based on Lyapunov functions is derived in section 3. A simulation example is included in section 4, followed by some concluding remarks in section 5.

## 2. PROBLEM STATEMENT

In this paper, we consider the following class of discrete-time fuzzy systems with sector-bounded nonlinearity:

$\triangle$Plant Rule $i$: IF $\theta_1(k)$ is $F_{i1}$ and $\cdots$ and $\theta_p(k)$ is $F_{ip}$, THEN

$$\begin{cases} x_{k+1} = A_{1i}g(x_k) + A_{2i}x_k + A_{di}x_{k-\tau_k} + B_iw_k + E_if_k \\ z_k = C_{1i}g(x_k) + C_{2i}x_k + C_{di}x_{k-\tau_k} + D_{1i}w_k \\ y_k = C_ix_k + D_iw_k \\ x_k = \varphi_k, k = -\tau_u, -\tau_u+1, \ldots, 0 \\ i = 1, \ldots, r \end{cases} \tag{1}$$

where $F_{ij}$ is the fuzzy set, $r$ denotes the number of IF-THEN rules, $\theta_k = [\theta_1(k), \theta_2(k), \ldots, \theta_p(k)]$ is the premise variable vector. $x_k \in R^n$ is the state vector; $z_k \in R^{n_1}$ is the signal to be estimated; $y_k \in R^{n_2}$ is the measured output vector; $w_k \in R^{n_w}$ is the unknown disturbance belonging to $l_2 \in [0, \infty)$ and $f_k$ is the fault vector. $A_{1i}$, $A_{2i}$, $A_{di}$, $B_i$, $C_i$, $C_{1i}$, $C_{2i}$, $C_{di}$, $D_i$, $D_{1i}$, and $E_i$ are all constant matrices with appropriate dimensions. $\tau_k$ is the time-varying state delay with lower and upper bounds $\tau_l \le \tau \le \tau_u$, where $\tau_l$ and $\tau_u$ are constant positive scalars. $\varphi_k$ is a given real initial sequence on $[-\tau_u, 0]$. $g(x_k)$ is the nonlinear function satisfying the following assumption.

*Assumption 1:* The nonlinear function $g(\cdot)$ in system (1) satisfies

$$[g(x_k) - F_1x_k]^T[g(x_k) - F_2x_k] \le 0, \quad \forall x_k \in R^n \tag{2}$$

where $F_1 \in R^{n\times n}$ and $F_2 \in R^{n\times n}$ are known real constant matrices and $F = F_1 - F_2$ is a symmetric positive definite matrix.

According to (2), for any scalar $\varepsilon > 0$, it has

$$\varepsilon \begin{bmatrix} \tilde{x}_k \\ g(x_k) \end{bmatrix}^T \begin{bmatrix} Z^T\tilde{F}_1Z & Z^T\tilde{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ g(x_k) \end{bmatrix} \le 0, \tag{3}$$

where $\tilde{F}_1 = (F_1^TF_2 + F_2^TF_1)/2$, $\tilde{F}_2 = -(F_1^T + F_2^T)/2$, and $Z = [I \quad 0]$.

Consider the measurement signal with random data dropouts as the following form,

$$y_k = \lambda_kC_ix_k + D_iw_k \tag{4}$$

where the stochastic variable $\lambda_k$ distributes in the known interval $[\alpha, \beta]$, $(0 \le \alpha \le \beta \le 1)$, with its mathematical expectation $E\{\lambda_k\} = \rho$ and variance $Cov\{\lambda_k\} = \sigma^2$. $\alpha$, $\beta$, $\rho$ and $\sigma$ are known real scalars.

*Remark 1:* Here, system (4) is used to describe the random data dropouts induced by the limited bandwidth as He et al. (2009), which is usually used to describe the characters of net. When $\alpha = \beta = 1$, the data is transferred successfully. While $\alpha = \beta = 0$, the data is loss at all.

The overall fuzzy model can be inferred as follows:

$$\begin{cases} x_{k+1} = \sum_{i=1}^{r}\mu_i(\theta_k)[A_{1i}g(x_k) + A_{2i}x_k + A_{di}x_{k-\tau_k} \\ \quad +B_iw_k + E_if_k] \\ z_k = \sum_{i=1}^{r}\mu_i(\theta_k)[C_{1i}g(x_k) + C_{2i}x_k + C_{di}x_{k-\tau_k} \\ \quad +D_{1i}w_k] \\ y_k = \sum_{i=1}^{r}\mu_i(\theta_k)[\lambda_kC_ix_k + D_iw_k] \end{cases} \tag{5}$$

where the fuzzy basis functions are given by

$$\mu_i(\theta_k) = \frac{\eta_i(\theta_k)}{\sum_{i=1}^{r}\eta_i(\theta_k)}, \quad \eta_i(\theta_k) = \Pi_{j=1}^{p}F_{ij}(\theta_j(k))$$

$F_{ij}(\theta_j(k))$ represents the grade of membership value of $\theta_j(k)$ in $F_{ij}$. Therefore, $\mu_i(\theta_k)$ has the following basic property:

$$\eta_i(\theta_k) \ge 0, \quad i = 1, 2, \ldots, r, \quad \textstyle\sum_{i=1}^{r}\eta_i(\theta_k) \ge 0, \quad \forall k$$
$$\mu_i(\theta_k) \ge 0, \quad i = 1, 2, \ldots, r, \quad \textstyle\sum_{i=1}^{r}\mu_i(\theta_k) = 1, \quad \forall k.$$

For the purpose of residual generation, the following filter can be constructed:

$$\begin{cases} \hat{x}_{k+1} = \sum_{j=1}^{r}\mu_j(\theta_k)[G_j\hat{x}_k + K_jy_k] \\ \hat{z}_k = \sum_{j=1}^{r}\mu_j(\theta_k)L_j\hat{x}_k \end{cases} \tag{6}$$

where $\hat{x}_k$ is the observer state vector, $\hat{z}_k$ is an estimation for $z_k$. $G_j$, $K_j$ and $L_j$ are the parameters to be determined. By defining $\tilde{x}_k = [x_k^T \quad \hat{x}_k^T]^T$, $\tilde{z}_k = z_k - \hat{z}_k$ and $d_k = [f_k^T \quad w_k^T]^T$, the filter-error dynamic system can be obtained:

$$\begin{cases} \tilde{x}_{k+1} = \left[\,\tilde{A}_0 + (\lambda_k - \rho)\bar{A}_0\,\right]\tilde{x}_k + \tilde{A}_dZ\tilde{x}_{k-\tau_k} \\ \quad +\tilde{B}_0g(x_k) + \bar{B}_0d_k \\ \tilde{z}_k = \tilde{C}_0\tilde{x}_k + \bar{C}_dZ\tilde{x}_{k-\tau_k} + \tilde{D}_0g(x_k) + \bar{D}_0d_k \end{cases} \tag{7}$$

where

$$\tilde{A}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\begin{bmatrix} A_{2i} & 0 \\ \rho K_jC_i & G_j \end{bmatrix} := \begin{bmatrix} A_2 & 0 \\ \rho KC & G \end{bmatrix}$$

$$\bar{A}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\begin{bmatrix} 0 & 0 \\ K_jC_i & 0 \end{bmatrix} := \begin{bmatrix} 0 & 0 \\ KC & 0 \end{bmatrix}$$

$$\tilde{A}_d = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\begin{bmatrix} A_{di} \\ 0 \end{bmatrix} := \begin{bmatrix} A_d \\ 0 \end{bmatrix}$$

$$\tilde{B}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\begin{bmatrix} A_{1i} \\ 0 \end{bmatrix} := \begin{bmatrix} A_1 \\ 0 \end{bmatrix}$$

$$\bar{B}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\begin{bmatrix} E_i & B_i \\ 0 & K_jD_i \end{bmatrix} := \begin{bmatrix} E & B \\ 0 & KD \end{bmatrix}$$

$$\tilde{C}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\,[\, C_{2i} \ -L_j \,] := [\, C_2 \ -L \,]$$

$$\bar{C}_d = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)C_{di} := C_d$$

$$\bar{D}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)\,[\, 0 \ D_{1i} \,] := [\, 0 \ D_1 \,]$$

$$\tilde{D}_0 = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta_k)\mu_j(\theta_k)C_{1i} := C_1.$$

Considering the existence of the stochastic variable $\lambda_k$, we recall the definition of stochastic stability in the mean-square sense for the filter-error system.

*Definition 1 Mao X. (1997):* The FD filter-error dynamic system (7) is said to be asymptotically mean-square stable if, with $d_k = 0$, for any initial conditions,

$$\lim_{k\to\infty} E\{\|\tilde{x}_k\|^2\} = 0 \tag{8}$$

Due to the existence of the random variable $\lambda_k$, in the rest of the paper, our aim is to design a robust full-order FD filter (6) for system (5). For all disturbance $d_k$ and the random variable $\lambda_k$, determine the FD filter-gain matrix $G_j$, $K_j$ and $L_j$, such that FD filter-error system satisfies both the following requirements:

1) The FD filter-error system (7) is asymptotically mean-square stable.

2) Under the zero-initial conditions, the FD filer-error $\tilde{z}_k$ satisfies

$$\sum_{k=0}^{\infty} E\{\|\tilde{z}_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{\|d_k\|^2\} \tag{9}$$

for all nonzero $d_k$, where $\gamma > 0$ is a prescribed scalar.

To detect the system faults, the residual evaluation function is selected as

$$J(k) = \left\{ \sum_{s=k_0}^{k_0+L_e} r_s^T r_s \right\}^{1/2}, \quad J_{th} = \sup_{w_k\in l_2, F_k=0} J(k) \tag{10}$$

where $k_0$ is the initial evolution time instant; $L_e$ is the evolution time steps and $J_{th}$ is the threshold. Based on this, the occurrence of faults can be detected by

$$\begin{cases} J(k) < J_{th} \Rightarrow no \quad fault \quad occurs \\ J(k) \geq J_{th} \Rightarrow alarm \quad for \quad fault \end{cases} \tag{11}$$

From (11), it can be seen that the threshold $J_{th}$ is proportional to the performance index $\gamma$. For the no fault case, $\gamma$ determines the influence of the disturbance to FD filter. The smaller $\gamma$ is, the better of the robustness of FD filter to the disturbance will be.

## 3. FD FILTER DESGIN

In this section, we aim to develop an innovative approach to guarantee the FD filter-error system (7) satisfying the requirements 1) and 2). The following theorem provides a sufficient condition for the existence of a full-order FD filter for system (5).

*Theorem 1.* Consider system (5) with a full-order FD filter of the form (6). Given a prescribed $H_\infty$ attenuation level $\gamma > 0$. If there exist matrixes $P = P^T > 0$, $Q = Q^T > 0$ and a constant scalar $\varepsilon > 0$ such that the following inequality holds:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix} < 0 \tag{12}$$

where

$$\begin{aligned}
&\Omega_{11} = \tilde{A}_0^T P\tilde{A}_0 + \sigma^2\bar{A}_0^T P\bar{A}_0 - P - \varepsilon Z^T\tilde{F}_1 Z \\
&+\tilde{C}_0^T\tilde{C}_0 + \nu Z^T QZ, \quad \Omega_{12} = \tilde{A}_0^T P\tilde{A}_d + \tilde{C}_0^T C_d, \\
&\Omega_{13} = \tilde{A}_0^T P\tilde{B}_0 - \varepsilon Z^T\tilde{F}_2 + \tilde{C}_0^T\tilde{D}_0, \\
&\Omega_{14} = \tilde{A}_0^T P\bar{B}_0 + \tilde{C}_0^T\bar{D}_0, \quad \Omega_{22} = \tilde{A}_d^T P\tilde{A}_d - Q + C_d^T C_d, \\
&\Omega_{23} = \tilde{A}_d^T P\tilde{B}_0 + C_d^T\tilde{D}_0, \quad \Omega_{24} = \tilde{A}_d^T P\bar{B}_0 + C_d^T\bar{D}_0 \\
&\Omega_{33} = \tilde{B}_0^T P\tilde{B}_0 - \varepsilon I + \tilde{D}_0^T\tilde{D}_0, \quad \Omega_{34} = \tilde{B}_0^T P\bar{B}_0 + \tilde{D}_0^T\bar{D}_0, \\
&\Omega_{44} = \bar{B}_0^T P\bar{B}_0 - \gamma^2 I + \bar{D}_0^T\bar{D}_0, \quad \nu = \tau_u - \tau_l + 1,
\end{aligned}$$

then the FD filter-error system (7) is asymptotically mean-square stable with the prescribed $H_\infty$ attenuation level bound $\gamma$ given in (9).

**Proof.** Consider the following Lyapunov functions:

$$\begin{aligned}
&V(\tilde{x}_k) = V_1(\tilde{x}_k) + V_2(\tilde{x}_k) + V_3(\tilde{x}_k) \\
&V_1(\tilde{x}_k) = \tilde{x}_k^T P\tilde{x}_k, \quad V_2(\tilde{x}_k) = \sum_{i=k-\tau_k}^{k-1} \tilde{x}_i^T Z^T QZ\tilde{x}_i \\
&V_3(\tilde{x}_k) = \sum_{j=k-\tau_u+1}^{k-\tau_l} \sum_{i=j}^{k-1} \tilde{x}_i^T Z^T QZ\tilde{x}_i
\end{aligned} \tag{13}$$

where $P = P^T > 0$ and $Q = Q^T > 0$. From the characters of the random variable $\lambda_k$, it has:

$$E\{\lambda_k - \rho\} = 0, \quad E\{[\lambda_k - \rho]^2\} = \sigma^2 \tag{14}$$

For $d_k = 0$, the difference of each Lyapunov functions can be calculated from (7):

$$E\{\Delta V_1\} = E\{V_1(\tilde{x}_{k+1})\} - V_1(\tilde{x}_k)$$

$$
\begin{aligned}
&= \begin{bmatrix} \tilde{x}_k \\ Z\tilde{x}_{k-\tau_k} \\ g(x_k) \end{bmatrix}^T \begin{bmatrix} M_1 & M_2 & M_3 \\ * & M_4 & M_5 \\ * & * & M_6 \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ Z\tilde{x}_{k-\tau_k} \\ g(x_k) \end{bmatrix} \\
E\{\Delta V_2\} &= E\{V_2(\tilde{x}_{k+1})\} - V_2(\tilde{x}_k) \\
&\leq E\Bigg\{\tilde{x}_k^T Z^T Q Z \tilde{x}_k - \tilde{x}_{k-\tau_k}^T Z^T Q Z \tilde{x}_{k-\tau_k} \\
&\quad + \sum_{i=k+1-\tau_u}^{k-\tau_l} \tilde{x}_i^T Z^T Q Z \tilde{x}_i \Bigg\} \\
E\{\Delta V_3\} &= E\{V_3(\tilde{x}_{k+1})\} - V_3(\tilde{x}_k) \\
&= E\Bigg\{(\tau_u - \tau_l)\tilde{x}_k^T Z^T Q Z \tilde{x}_k \\
&\quad - \sum_{i=k+1-\tau_u}^{k-\tau_l} \tilde{x}_i^T Z^T Q Z \tilde{x}_i \Bigg\}
\end{aligned}
$$

where $M_1 = \tilde{A}_0^T P \tilde{A}_0 + \sigma^2 \bar{A}_0^T P \bar{A}_0 - P$, $M_2 = \tilde{A}_0^T P \tilde{A}_d$, $M_3 = \tilde{A}_0^T P \tilde{B}_0$, $M_4 = \tilde{A}_d^T P \tilde{A}_d$, $M_5 = \tilde{A}_d^T P \tilde{B}_0$ and $M_6 = \tilde{B}_0^T P \tilde{B}_0$. Further, we obtain that

$$
\begin{aligned}
E\{\Delta V\} &= E\{\Delta V_1\} + E\{\Delta V_2\} + E\{\Delta V_3\} \\
&\leq \zeta_k^T \begin{bmatrix} \tilde{M}_1 & M_2 & M_3 \\ * & \tilde{M}_4 & M_5 \\ * & * & M_6 \end{bmatrix} \zeta_k^T \qquad (15)
\end{aligned}
$$

where $\zeta_k^T = \left[\, \tilde{x}_k^T \ \tilde{x}_{k-\tau_k}^T Z^T \ g^T(x_k) \,\right]$, $\tilde{M}_1 = \tilde{A}_0^T P \tilde{A}_0 + \sigma^2 \bar{A}_0^T P \bar{A}_0 - P + \nu Z^T Q Z$, $\tilde{M}_4 = \tilde{A}_d^T P \tilde{A}_d - Q$.

Substituting (3) into (15), one can obtain

$$
\begin{aligned}
&E\{\Delta V(\tilde{x}_k)\} \leq E\{\Delta V(\tilde{x}_k)\} \\
&- E\left\{\varepsilon \begin{bmatrix} \tilde{x}_k \\ g(x_k) \end{bmatrix}^T \begin{bmatrix} Z^T \tilde{F}_1 Z & Z^T \tilde{F}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ g(x_k) \end{bmatrix}\right\} \\
&= \zeta_k^T \begin{bmatrix} \tilde{M}_1 - \varepsilon Z^T \tilde{F}_1 Z & M_2 & M_3 - \varepsilon Z^T \tilde{F}_2 \\ * & \tilde{M}_4 & M_5 \\ * & * & M_6 - \varepsilon I \end{bmatrix} \zeta_k \\
&:= \zeta_k^T \Xi \zeta_k \qquad (16)
\end{aligned}
$$

Using Theorem 1, one can verify that

$$
\Xi < -\begin{bmatrix} \tilde{C}_0^T \tilde{C}_0 & \tilde{C}_0^T C_d & \tilde{C}_0^T \tilde{D}_0 \\ * & C_d^T C_d & C_d^T \tilde{D}_0^T \\ * & * & \tilde{D}_0^T \tilde{D}_0 \end{bmatrix} < 0 \qquad (17)
$$

Thus, for all $d_k = 0$, we have $E\{\Delta V(\tilde{x}_k)\} < 0$, that is system (7) is asymptotically stable in the mean-square sense.

Next, for any nonzero $d_k$, it follows from (7), (12) and (16) that

$$
\begin{aligned}
&E\{\Delta V(\tilde{x}_k)\} + E\{\tilde{z}_k^T \tilde{z}_k\} - \gamma^2 E\{d_k^T d_k\} \\
&\leq E\left\{\xi_k^T \Omega \xi_k\right\} < 0 \qquad (18)
\end{aligned}
$$

where $\xi_k := \left[\, \tilde{x}_k^T \ \tilde{x}_{k-\tau_k}^T Z^T \ g^T(x_k) \ d_k^T \,\right]^T$. Now, summing up this relationship from 0 to $\infty$ with respect to $k$ yields

$$
\sum_{k=0}^{\infty} E\{\tilde{z}_k^T \tilde{z}_k\} < \sum_{k=0}^{\infty} \gamma^2 E\{d_k^T d_k\} - E\{V_\infty\} + E\{V_0\} \qquad (19)
$$

Since the system (7) is asymptotically mean-square stable, it is obvious that (9) holds under the zero initial condition. This concludes the proof. □

Theorem 1 provides a sufficient condition for the design of the FD filter, which contains the coupled matrix variables. To overcome the difficulties caused by the coupling in (12), a decoupling techniques as Gao et al. (2005) is used, by which, (12) can be transformed into a new form.

*Corollary 1.* Consider system (5) with a full-order FD filter of the form (6). Given a prescribed scalar $\gamma > 0$, if there exist matrixes $\bar{P} = \bar{P}^T > 0$, $Q = Q^T > 0$, $F > 0$, $\bar{K}$, $\bar{G}$, $\bar{L}$ and a constant scalar $\varepsilon > 0$ such that the following LMI holds:

$$
\begin{bmatrix}
\Pi_{11} & 0 & \Pi_{13} & 0 & \Pi_{15} & \Pi_{16} & \Pi_{17} \\
* & -Q & 0 & 0 & C_d^T & \Pi_{26} & 0 \\
* & * & -\varepsilon I & 0 & \tilde{D}_0^T & \Pi_{36} & 0 \\
* & * & * & -\gamma^2 I & \tilde{D}_0^T & \Pi_{46} & 0 \\
* & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 \\
* & * & * & * & * & * & \Pi_{77}
\end{bmatrix} < 0 \qquad (20)
$$

where

$$
\Pi_{11} = \begin{bmatrix} \nu Q - \bar{P} - \varepsilon \tilde{F}_1 & -F \\ -F & -F \end{bmatrix}, \quad \Pi_{13} = \begin{bmatrix} -\varepsilon \tilde{F}_2 \\ 0 \end{bmatrix},
$$

$$
\Pi_{15} = \begin{bmatrix} C_2^T \\ -\bar{L}^T \end{bmatrix}, \quad \Pi_{17} = \sigma \begin{bmatrix} C^T \bar{K}^T & C^T \bar{K}^T \\ 0 & 0 \end{bmatrix},
$$

$$
\Pi_{16} = \begin{bmatrix} A_2^T \bar{P} + \rho C^T \bar{K}^T & A_2^T F + \rho C^T \bar{K}^T \\ \bar{G}^T & \bar{G}^T \end{bmatrix},
$$

$$
\Pi_{26} = \left[\, A_d^T \bar{P} \ \ A_d^T F \,\right], \quad \Pi_{36} = \left[\, A_1^T \bar{P} \ \ A_1^T F \,\right],
$$

$$
\Pi_{46} = \begin{bmatrix} E^T \bar{P} & E^T F \\ B^T \bar{P} + D^T \bar{K}^T & B^T \bar{P} + D^T \bar{K}^T \end{bmatrix},
$$

$$
\Pi_{66} = \Pi_{77} = \begin{bmatrix} -\bar{P} & -F \\ -F & -F \end{bmatrix}.
$$

**Proof.** Suppose condition (12) holds for matrices $P^T = P > 0$, $Q = Q^T > 0$, $\tilde{A}_0$, $\bar{A}_0$, $\tilde{A}_d$, $\tilde{B}_0$, $\bar{B}_0$, $\tilde{C}_0$, $\tilde{D}_0$, $\bar{D}_0$ and $Z = [\, I \ 0 \,]$. Using Schur complement Boyd et al. (1997), one can show that (12) is equivalent to

$$
\begin{bmatrix}
\bar{\Pi}_{11} & 0 & -\varepsilon Z^T \tilde{F}_2 & 0 & \tilde{C}_0^T & \tilde{A}_0^T P & \sigma \bar{A}_0^T P \\
* & -Q & 0 & 0 & C_d^T & \tilde{A}_d^T P & 0 \\
* & * & -\varepsilon I & 0 & \tilde{D}_0 & \tilde{B}_0^T P & 0 \\
* & * & * & -\gamma^2 I & \tilde{D}_0^T & \tilde{B}_0^T P & 0 \\
* & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & -P & 0 \\
* & * & * & * & * & * & -P
\end{bmatrix} < 0 \qquad (21)
$$

where $\bar{\Pi}_{11} = \nu Z^T Q Z - P - \varepsilon Z^T \tilde{F}_1 Z$. Let the matrix $P$ be partitioned as $P = \begin{bmatrix} \bar{P} & \bar{S} \\ \bar{S} & \bar{W} \end{bmatrix}$, where $\bar{P} > 0$, $\bar{W} > 0$ and $\bar{S}$ is invertible. Define the invertible matrix $J = \begin{bmatrix} I & 0 \\ 0 & \bar{S}\bar{W}^{-1} \end{bmatrix}$.

Performing congruence transformations to (21) by $diag\{J^T, I, I, I, I, J^T, J^T\}$ yields (20) with changes of variables as:

$F = \bar{S}\bar{W}^{-1}\bar{S}$, $\bar{K} = \bar{S}K$, $\bar{G} = \bar{S}G\bar{W}^{-1}\bar{S}$, $\bar{L} = L\bar{W}^{-1}\bar{S}$.

This ends the proof. □

With the aid of the above corollary, we are now in a position to present the filter gain design algorithm.

*Theorem 2.* Given system (5) with a full-order $H_\infty$ FD filter in the form of filter (6) exists, if there exist matrices $\bar{P} = \bar{P}^T > 0$, $Q^T = Q > 0$, $F > 0$, $\bar{K}_j$, $\bar{G}_j$, $\bar{L}_j$ and a constant scalar $\varepsilon > 0$ such that the following LMIs holds for a given scalar $\gamma > 0$:

$$\Phi_{ij} + \Phi_{ji} < 0, \quad i \le j \tag{22}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Gamma_{11} & 0 & \Gamma_{13} & 0 & \Gamma_{15} & \Gamma_{16} & \Gamma_{17} \\ * & -Q & 0 & 0 & C_{di}^T & \Gamma_{26} & 0 \\ * & * & -\varepsilon I & 0 & C_{1i}^T & \Gamma_{36} & 0 \\ * & * & * & -\gamma^2 I & \Gamma_{45} & \Gamma_{46} & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & \Gamma_{66} & 0 \\ * & * & * & * & * & * & \Gamma_{77} \end{bmatrix}$$

$$\Gamma_{11} = \begin{bmatrix} \nu Q - \bar{P} - \varepsilon \tilde{F}_1 & -F \\ -F & -F \end{bmatrix}, \quad \Gamma_{13} = \begin{bmatrix} -\varepsilon \tilde{F}_2 \\ 0 \end{bmatrix},$$

$$\Gamma_{15} = \begin{bmatrix} C_{2i}^T \\ -\bar{L}_j^T \end{bmatrix}, \quad \Gamma_{17} = \sigma \begin{bmatrix} C_i^T \bar{K}_j^T & C_i^T \bar{K}_j^T \\ 0 & 0 \end{bmatrix},$$

$$\Gamma_{16} = \begin{bmatrix} A_{2i}^T \bar{P} + \rho C_i^T \bar{K}_j^T & A_{2i}^T F + \rho C_i^T \bar{K}_j^T \\ \bar{G}_j^T & \bar{G}_j^T \end{bmatrix},$$

$$\Gamma_{26} = \begin{bmatrix} A_{di}^T \bar{P} & A_{di}^T F \end{bmatrix}, \quad \Gamma_{36} = \begin{bmatrix} A_{1i}^T \bar{P} & A_{1i}^T F \end{bmatrix},$$

$$\Gamma_{46} = \begin{bmatrix} E_i^T \bar{P} & E_i^T F \\ B_i^T \bar{P} + D_i^T \bar{K}_j^T & B_i^T \bar{P} + D_i^T \bar{K}_j^T \end{bmatrix},$$

$$\Gamma_{45} = \begin{bmatrix} 0 \\ D_{1i}^T \end{bmatrix}, \quad \Gamma_{66} = \Gamma_{77} = \begin{bmatrix} -\bar{P} & -F \\ -F & -F \end{bmatrix}$$

In this case, the FD filter parameters in filter (6) are given by

$$K_j = F^{-1}\bar{K}_j, \quad G_j = F^{-1}\bar{G}_j, \quad L_j = \bar{L}_j. \tag{23}$$

**Proof.** Set $\triangle(k) = \sum_{i=1}^{r} \mu_i \triangle_i$, where $\triangle$ denotes matrices $A_1$, $A_2$, $A_d$, $B$, $C$, $C_1$, $C_2$, $C_d$, $D$, $D_1$, $\bar{K}$, $\bar{G}$ or $\bar{L}$. For simplicity, denote the matrix on the left side of inequality (20) by $\Phi(k)$. Then, from (22), one can obtain that

$$\begin{aligned} \Phi(k) &= \sum_{i=1}^{r} \mu_i \sum_{j=1}^{r} \mu_j \Phi_{ij} \\ &= \sum_{i=1}^{r} \mu_i^2 \Phi_{ii} + \sum_{i<j}^{r} \mu_i \mu_j (\Phi_{ij} + \Phi_{ji}) < 0 \end{aligned} \tag{24}$$

From Corollary 1, the $H_\infty$ FD filter design problem has been solved, and the parameter matrix functions are given by $K = \bar{S}^{-1}\bar{K}$, $G = \bar{S}^{-1}\bar{G}\bar{S}^{-1}\bar{W}$, $L = \bar{L}\bar{S}^{-1}\bar{W}$, where $\bar{W} > 0$, $\bar{S}$ is invertible and $F = \bar{S}\bar{W}^{-1}\bar{S}$. Under the transformation $\bar{S}^{-1}\bar{W}\tilde{x}_k$, the FD filter matrix functions can be of the following form:

$$\begin{aligned} K &= \bar{S}^{-1}\bar{W}(\bar{S}^{-1}\bar{K}) = F^{-1}\bar{K}, \\ G &= \bar{S}^{-1}\bar{W}(\bar{S}^{-1}\bar{G}\bar{S}^{-1}\bar{W})\bar{W}^{-1}\bar{S} = F^{-1}\bar{G}, \\ L &= (\bar{L}\bar{S}^{-1}\bar{W})\bar{W}^{-1}\bar{S} = \bar{L}. \end{aligned} \tag{25}$$

Hence, the parameter matrices in filter (6) are given by (23). The proof is completed. □

*Remark 2:* Since the mean value and the variance of the random variable $\lambda_k$ are known, the parameters of FD filter can be solved through Theorem 1 and Theorem 2. For the sensors, we just consider the same rate of the data drops. It is well-known that each channel has its own drop rate, i.e., the $\lambda_k$ for each sensor is different. The obtained results can be extend to the case of the different drop rate.

*Remark 3:* The $H_\infty$ fault detection filter design for nonlinear networked control systems can be readily found by solving the following problem: Minimize $\gamma^2$ subject to (22) and (23), which can be solved by the LMI tool box in MATLAB.

## 4. ILLUSTRATIVE EXAMPLE

In this section, we give an illustrative example to demonstrate the effectiveness of presented method. Consider a T-S fuzzy system with three plant rules ($r = 3$) whose membership functions are shown in Figure 1. The param-

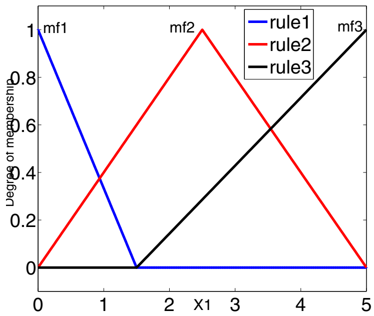


Fig. 1. Membership functions of the T-S fuzzy system

eters of the system are given as follows:

$$A_{11} = \begin{bmatrix} 1 & 0.31 \\ 0 & 0.33 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0.5 \\ 0.2 & 0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.8 \\ 1 \end{bmatrix}, \quad C_{11} = [\,0.2 \;\; 0\,]$$

$$C_{21} = [\,0.2 \;\; 0.5\,], \quad D_{11} = 0.15, \quad C_1 = [\,0.8948 \;\; 0.8\,]$$

$$A_{12} = \begin{bmatrix} 1 & 0.29 \\ 0 & 0.30 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_{12} = [\,0.1 \;\; 0\,]$$

$$C_{22} = [\,0.3 \;\; 0.4\,], \quad D_{12} = 0.15, \quad C_2 = [\,0.827 \;\; 0.8\,]$$

$$A_{13} = \begin{bmatrix} 0.9 & 0.35 \\ 0 & 0.28 \end{bmatrix}, \quad A_{23} = \begin{bmatrix} 0 & 0.45 \\ 0.154 & 0.24 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$A_{d3} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_{13} = [\,0.15 \;\; 0\,]$$

$$C_{23} = [\,0.24 \;\; 0.48\,], \quad D_{13} = 0.15, \quad C_3 = [\,0.8366 \;\; 0.8\,]$$

$$D_1 = D_2 = D_3 = 1, \quad C_{d1} = C_{d2} = C_{d3} = 0$$

$$g(x_k) = 0.5sin(x_k)$$

Suppose that the probabilistic variable $\lambda_k$ meets uniform distribution in interval [0.5  1], from which the expectation and variance can be easily calculated as $\rho = 0.75$, $\sigma =$

0.144. The unknown disturbance $w_k$ is assumed to be white noise with power of 0.5. The fault signal $f_k$ is simulated as 1 that occurs from 100 to 200 steps. Select $\gamma = 0.98$. Using Theorem 2, the $H_\infty$ filtering parameters can be calculated through the LMI toolbox:

$$
\begin{aligned}
&K_1 = \begin{bmatrix} -0.1144 \\ -0.0790 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -0.0405 & -0.0305 \\ -0.0257 & -0.0203 \end{bmatrix}, \\
&L_1 = [\,-0.2650 \;\; -0.1636\,] \\
&K_2 = \begin{bmatrix} -0.0915 \\ -0.1218 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -0.0219 & -0.0195 \\ -0.0305 & -0.0289 \end{bmatrix}, \\
&L_2 = [\,-0.2902 \;\; -0.2261\,] \\
&K_3 = \begin{bmatrix} -0.0922 \\ -0.1035 \end{bmatrix}, \quad G_3 = \begin{bmatrix} -0.0270 & -0.0263 \\ -0.0313 & -0.0309 \end{bmatrix}, \\
&L_3 = [\,-0.2875 \;\; -0.3102\,].
\end{aligned}
$$

Simulation results with the initial conditions $x_k = [0 \;\; 0]$, $\hat{x}_k = [0 \;\; 0]$ are shown in Figure 2 and Figure 3. From Figure 3, it can be seen that the residual evolution increases quickly when the fault $f_k \neq 0$. Choosing appropriate $J_{th}$, the fault $f_k$ can be detected as soon as its occurrence.

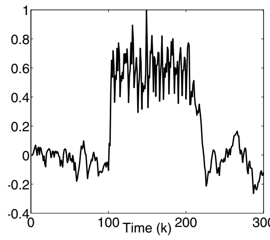


Fig. 2. Residual estimation based on common Lyapunv function

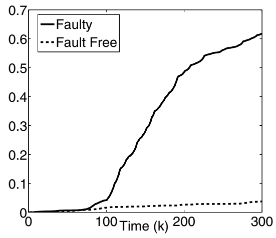


Fig. 3. Residual evolution based on common Lyapunov function

## 5. CONCLUSION

The problem of FD for a class of nonlinear NCSs with probabilistic data dropout and time-varying delay has been considered in this paper. Based on Lyapunov function approach, sufficient conditions for the FD filter design are established in terms of a set of LMIs, which guarantees the mean-square stability of the FD filtering-error systems, as well as prescribed $H_\infty$ performance requirement. By solving a LMIs, we obtain the FD filter parameters. In the future, the work about the fault diagnosis and fault-tolerant control for more general nonlinear models will be developed.

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