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Analytical Method of the Performance of the Rotational INS based on the Spatial Accumulation of the Inertial Instrument Biases

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Abstract: Rotational inertial navigation system (INS) make the biases accumulate averagely to the directions of the navigation coordinate frame by rotating the inertial measurement unit, and eliminate the influence of the biases using the integration of the navigation computation, so that the precision of the INS can be improved effectively. According to the error suppression principle of the rotational INS, this work analyzes the accumulation of the biases in the navigation frame space per unit time by approximately discretizing the IMU rotation path, in order to compare the performance of the rotation modulation schemes with different angular motion statuses. By analyzing the constant angular rate rotation scheme, the linearly varied angular rate rotation schemes and the sine angular rate rotation scheme, the conclusion can be drawn that with the extension of the varying angular rate process, the spatial accumulation of the biases increase, which decreases the error suppression effect of rotation modulation. The conclusion was verified by the simulation of navigation errors. Therefore, the design principle of the rotation modulation schemes is reached that the angular rate should be kept constant in the single-axis rotation modulation and the dual-axis alternate rotation modulation

Keywords: rotational inertial navigation system; spatial accumulation of the biases; rotation modulation method; error compensation

1. INTRODUCTION

In recent years, with the development of solid-state gyros like optic gyros, rotation modulation technology, (Sun, 2009) becomes the focus of inertial navigation research field. A rotational inertial navigation system (INS) has similar frames and spindles like a gimbal INS, which only work as rotation actuators and angle transducers. The calculation procedure of a rotational INS is the same as a strapdown INS. By rotating the inertial measurement unit (IMU), the rotational INS makes the biases of the inertial instruments (gyros and accelerometers) accumulate averagely in each direction of navigation coordinate frame, which modulates the biases into the production of sine and cosine functions, and suppresses the navigation errors using the integration of the navigation calculation (Ben, 2011). The research on effective rotation schemes is one of the main contents of rotation modulation technology. Effective rotation schemes include multi-position rotation (Long, 2010), dual-axis alternate rotation (Liu, 2009) and dual-axis continuous rotation (Wang, 2007). The relative factors of rotation schemes include the structure of the system, the type of the inertial instruments, the application environment of the carrier, and so on.

The angular rate of the rotation modulation is an important component of the rotation scheme. This work this work analyzes the accumulation of the biases in the navigation frame space per unit time based on the model of instrument biases under the rotation modulation, in order to compare the performance of the rotation modulation schemes with different angular motion statuses, and reaches the conclusion that the rotation scheme with constant angular rate has sound performance.

2. MATHEMATICAL MODEL OF THE MODULATED BIASES

The model of modulated biases in navigation coordinate frame, which is east-north-up (ENU) geographical coordinate frame, is (Hu, 2010)

$$\begin{aligned} \begin{bmatrix} \Delta_E & \Delta_N & \Delta_U \end{bmatrix}^T &= C_p^n \begin{bmatrix} \Delta_x & \Delta_y & \Delta_z \end{bmatrix}^T \\ \begin{bmatrix} \varepsilon_E & \varepsilon_N & \varepsilon_U \end{bmatrix}^T &= C_p^n \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z \end{bmatrix}^T \end{aligned} \quad (1)$$

where subscript p stands for the IMU coordinate frame, whose axes x, y, z respectively point to the right, forward and up directions of the IMU, and when the angles of rotation actuators' spindles are all zero, the spindles coinciding with the x, y, z axes of the coordinate p are respectively named x_p, y_p, z_p; Subscript n stands for the navigation coordinate; Δ_k and ε_k are the biases of accelerometers and gyros (k=x, y, z); C_p^n is the attitude matrix of IMU, which is the product of the carrier's attitude matrix C_b^n and the rotation matrix C_p^b, which is the direction cosine matrix between the p frame and the carrier coordinate frame b.

When the rotational INS is single-axis system with the zp spindle, the rotation matrix is (Sun, 2009)

$$C_p^b = \begin{bmatrix} \cos\alpha_z & \sin\alpha_z & 0 \\ -\sin\alpha_z & \cos\alpha_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and when the rotational INS is dual-axis whose outer spindle is x_p and inner spindle is z_p the rotation matrix is (Wang, 2007)

$$\begin{aligned} C_p^b &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_x & \sin\alpha_x \\ 0 & -\sin\alpha_x & \cos\alpha_x \end{bmatrix} \begin{bmatrix} \cos\alpha_z & \sin\alpha_z & 0 \\ -\sin\alpha_z & \cos\alpha_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos\alpha_z & \sin\alpha_z & 0 \\ -\sin\alpha_z\cos\alpha_x & \cos\alpha_z\cos\alpha_x & \sin\alpha_x \\ \sin\alpha_z\sin\alpha_x & -\cos\alpha_z\sin\alpha_x & \cos\alpha_x \end{bmatrix} \end{aligned} \tag{3}$$

where α_x and α_z are the angles of spindles x_p and z_p.

3. SPATIAL ACCUMULATION OF BIASES OF THE ROTATION SCHEMES

Considering the rotation schemes such as single-axis rotation (Sun, 2009) and dual-axis alternate rotation (Long, 2010), (Liu, 2009) whose rotation axes are orthogonal to the sensing axes of the inertial instruments are frequently-used, this paper will discuss the effect of the rotation modulation to the biases according to the variation of the bias inside the two-dimension plane orthogonal to the rotation axis. To simplify the discussion, suppose the two-dimension plane is the $x_n oy_n$ plane of the navigation frame n. Because the rotation modulation effect to accelerometer biases is the same as that to gyro biases, only the rotation effect to the gyro biases is analyzed.

Suppose that the modulus of the bias is ε (°/h), and its initial position coincides with the ox_n axis of the $x_n oy_n$ plane. When the bias rotates an angle of α across the axis orthogonal to the plane, its components along ox_n axis and oy_n axis are

$$\begin{aligned} \varepsilon_x &= \varepsilon\cos\alpha \\ \varepsilon_y &= \varepsilon\sin\alpha \end{aligned} \tag{4}$$

Because the errors of the INS are generated from the integration of the biases, the spatial accumulations of the bias in a single revolve can be viewed as the product of the bias component and the time that it takes to pass an angular position range. Then the spatial accumulations of the bias along ox_n axis and oy_n axis can be expressed as

$$\begin{aligned} \mathrm{E}_x &= \int_{\alpha_1}^{\alpha_2} \varepsilon\cos\alpha \frac{d\alpha}{\omega} \\ \mathrm{E}_y &= \int_{\alpha_1}^{\alpha_2} \varepsilon\sin\alpha \frac{d\alpha}{\omega} \end{aligned} \tag{5}$$

where E_x and E_y are the spatial accumulations of the bias along ox_n axis and oy_n axis when the gyro's sensing axis rotates from angle α_x to α_z. Devide the angle range [0, 2π] of the spindle's rotation in to N sections by the step size $\Delta\alpha$. Suppose that Δα is small enough so that in each section the rotation angular rate can be viewed as constant, and the spatial accumulations of the bias in No. k angle section $[(k-1)\Delta\alpha, k\Delta\alpha]$ along ox_n axis and oy_n axis are

$$\begin{aligned} \mathrm{E}_{x,k} &= \int_{(k-1)\Delta\alpha}^{k\Delta\alpha} \varepsilon\cos\alpha \frac{1}{\omega_k} d\alpha = \frac{\sin(k\Delta\alpha) - \sin[(k-1)\Delta\alpha]}{\omega_k}\varepsilon \\ \mathrm{E}_{y,k} &= \int_{(k-1)\Delta\alpha}^{k\Delta\alpha} \varepsilon\sin\alpha \frac{1}{\omega_k} d\alpha = \frac{-\cos(k\Delta\alpha) + \cos[(k-1)\Delta\alpha]}{\omega_k}\varepsilon. \end{aligned} \tag{6}$$

Therefore, the total spatial accumulation of the bias along ox_n axis within the whole revolve is (The angle range is [0, 2π])

$$\begin{aligned} \sum_{k=1}^{N} \mathrm{E}_{x,k} = \varepsilon \Bigg\{ & \frac{\sin(N\Delta\alpha) - \sin[(N-1)\Delta\alpha]}{\omega_N} \\ & + \frac{\sin[(N-1)\Delta\alpha] - \sin[(N-2)\Delta\alpha]}{\omega_{N-1}} + \cdots \\ & + \frac{\sin[(k+1)\Delta\alpha] - \sin(k\Delta\alpha)}{\omega_{k+1}} \\ & + \frac{\sin(k\Delta\alpha) - \sin[(k-1)\Delta\alpha]}{\omega_k} + \cdots \\ & + \frac{\sin(2\Delta\alpha) - \sin(\Delta\alpha)}{\omega_2} + \frac{\sin(\Delta\alpha) - \sin 0}{\omega_1} \Bigg\} \\ = \varepsilon & \left\{ \frac{\sin(2\pi)}{\omega_N} - \frac{\sin 0}{\omega_1} \right\} \\ & + \varepsilon \sum_{k=1}^{N-1} \sin(k\Delta\alpha) \left(\frac{-1}{\omega_{k+1}} + \frac{1}{\omega_k} \right) \\ = \varepsilon & \sum_{k=1}^{N-1} \sin(k\Delta\alpha) \left(\frac{-1}{\omega_{k+1}} + \frac{1}{\omega_k} \right) \end{aligned} \tag{7}$$

In the same way, the total spatial accumulation of the bias along oy_n axis within the whole revolve is

$$\begin{aligned} \sum_{k=1}^{N} \mathrm{E}_{y,k} &= \varepsilon \sum_{k=1}^{N-1} \cos(k\Delta\alpha) \left(\frac{1}{\omega_{k+1}} - \frac{1}{\omega_k} \right) \\ &+ \varepsilon \left[\frac{-\cos(N\Delta\alpha)}{\omega_N} + \frac{\cos 0}{\omega_1} \right]. \end{aligned} \tag{8}$$

For the periodically rotating IMU, considering the realizability of the system, the difference between ω_1 and ω_N should be small enough, so $1/\omega_1 - 1/\omega_N$ should be 0 or an very small value that can be ignored, and the spatial accumulation is mainly decided by the cumulation items. Name the coefficients of ε of the cumulation items in (7) and (8) as the spatial averaged bias factors (SABF), and their expressions are

$$\begin{aligned} \varsigma_x &= \sum_{k=1}^{N-1} \sin(k\Delta\alpha) \left(\frac{-1}{\omega_{k+1}} + \frac{1}{\omega_k} \right) \\ \varsigma_y &= \sum_{k=1}^{N-1} \cos(k\Delta\alpha) \left(\frac{1}{\omega_{k+1}} - \frac{1}{\omega_k} \right) \end{aligned} \tag{9}$$

Because the bias ε is a constant value, the error suppression performance of the rotation scheme is decided by the value of the SABFs, and the absolute values of ζx and ζy can be used as the evaluation index of the performance of the rotation scheme.

4. ROTATION SCHEME ANALYSIS BASED ON SABF METHOD

It can be known from the mathematical model of the SABFs that when the angular rate remains constant in the whole revolve, $\varsigma_x = \varsigma_y = 0$, and the bias of the instruments orthogonal to the spindle can be completely compensated by rotation modulation; when the angular rate varies, because the weighting coefficients, $\sin(k\Delta\alpha)$ and $\cos(k\Delta\alpha)$, are constant, the performance of rotation modulation is decided by the sum of the angular rate variation in the whole revolve. Therefore, the rotation angular rate should be kept constant in the design of orthogonal-spindle rotation schemes.

The errors of inertial instruments contains not only biases, but also scale errors (Wu, 2012), installation errors (Gao, 2012) and random noises (Sun, 2012). As mention above, the biases can be compensated by rotating the IMU periodically, while scale errors and installation errors coupled with the angular rate of the IMU will also generate equivalent biases. In a rotational INS, because the IMU is in continuous angular motion status, the coupled errors are usually more than ordinary INS. To avoid causing extra navigation errors, the rotational INS usually changes rotating direction every integer revolves. The oscillation generated during the IMU rotating will also cause errors to the attitude computation (Lai, 2012), so some rotational INSs use the rotation schemes which avoid the angular rate changes by a large margin to make the rotation more stable. To research the effect of the rotation angular rate variation to the performance of rotation schemes, this work analyzes constant angular rate rotation scheme, the linearly varied angular rate rotation schemes and sine angular rate rotation scheme using the SABF method.

The compared rotation schemes are as follows:

1) Constant angular rate rotation. The IMU rotates around the z_p spindle continuously, and changes direction every revolve. The constant angular rate ω is 6 °/s.

2)~4) Linearly varied angular rate rotation schemes with constant rate process. The IMU rotates around the zp spindle continuously, and changes direction every revolve. The mean angular rate ω is also 6 °/s. In every revolve, the IMU firstly rotates in uniformly accelerated angular motion, then rotates with constant angular rate, and then rotates in uniformly retarded angular motion. The starting and finishing angular rates in every revolve are both 0 °/s. The angular rate variation procedures of schemes 2), 3) and 4) last sepreately 1/3, 1/2, 2/3 of the period of one revolve.

5) Linearly varied angular rate rotation schemes without constant rate process. The IMU rotates around the zp spindle continuously, and changes direction every revolve. The mean angular rate ω is also 6 °/s. In every revolve, the IMU firstly rotates in uniformly accelerated angular motion, and then rotates in uniformly retarded angular motion. The starting and finishing angular rates in every revolve are both 0 °/s.

6) Sine angular rate rotation scheme. The angular rate changes as sine function in every revolve with the mean angular rate ω of 6 °/s.

The rotation angular rates of rotation schemes 1) to 5) are shown in Fig. 1. The rotation angular rate and angle position of rotation scheme 6) are shown in Fig. 2.

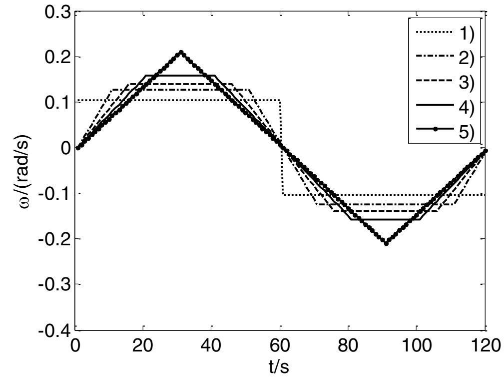

Fig. 1. Rotation angular rate of schemes 1) to 5).

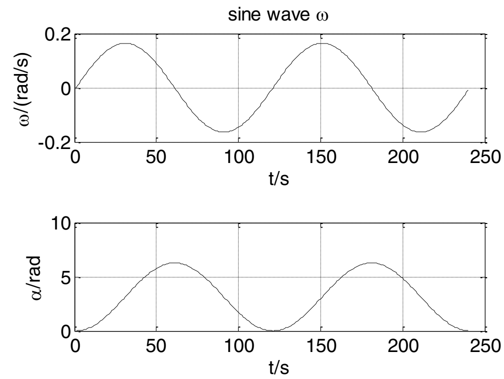

Fig. 2. Rotation rate and angle of scheme 6).

Calculate the absolute values of SABFs of rotation schemes 1) to 6) according to Equation (9), supposing Δα=0.001, and the result are shown in Fig. 3.

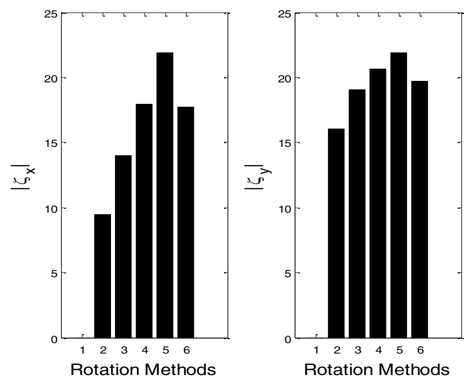

Fig. 3. Absolute values of SABFs of rotation schemes 1) to 6).

According to the analysis results, the absolute SABF of constant rate scheme is 0, which is the ideal method for rotational INS. Among linearly varied rate schemes 2) to 5), the absolute SABF increases with the duration of the rate variation process, which indicates that the performances of the schemes decrease progressively. The SABF of the sine angular rate rotation scheme is nearly equal to that of the 2/3 linearly varied angular rate rotation scheme.

5. NAVIGATION ERROR SIMULATION

To test the accuracy of the analysis results, use MATABLE to simulate the navigation errors of the rotational INSs using the rotation schemes 1) to 6) mentioned above according to the INS error equations (Hu, 2010), (Sun, 2011). The simulation parameters are set as follows: The initial latitude of the carrier is 30° northern latitude; its heading is 30° north by east; its ground velocity is 10 m/s. The biases of accelerometers and gyros are 10^{-4}g and 0.01°/h; all the scale errors are 10^{-5}; all the installation error angles are 10″.

Two groups of simulation experiments are carried out:

1. The rotational INS is the single-axis system whose spindle is z_p. The tested rotation schemes A1) to A6) are the same as 1) to 6) as mentioned in last section and the no rotation strapdown system is also added in as scheme A7).

2. The rotational INS is the dual-axis system whose structure is as described in Equation (3). The rotation schemes B1) to B7) are dual-axis alternate rotation, and in each cycle the rotation schemes of the currently rotating spindle are the same as those of A1) to A6).

Simulate the 24 h navigation errors of the above two groups of rotation schemes, and the 24-hour maximum absolute values of the resultant navigation error parameters (ground velocity errors δV_E and δV_N, latitude error δL, longitude error $\delta\lambda$, attitued errors ϕ_E, ϕ_N and $\phi_{U)}$ are shown in Table 1 and 2. In Group 1 and Group 2, the longitude errors of rotation schemes 1), 3), 5), 6) and 7) are shown in Fig. 4 and 5.

Table 1. 24-hour maximum absolute navigation errors of single-axis rotation schemes

Rotation schemes	δV_E/(m/s)	δV_N/(m/s)	δL/(′)	$\delta\lambda$/(′)	ϕ_E/(′)	ϕ_N /(′)	ϕ_U /(′)
A1)	0.7632	0.7318	3.7252	8.0859	0.4762	0.4583	2.5374
A2)	0.7817	0.8136	3.7288	8.6848	0.4999	0.5578	2.6644
A3)	0.8617	0.8599	3.7617	8.9980	0.5304	0.6211	3.0391
A4)	0.9306	0.9093	3.8487	9.2742	0.5635	0.6752	3.3699
A5)	1.0015	0.9720	3.9469	9.5568	0.5953	0.7294	3.7497
A6)	0.9274	0.9058	3.8440	9.2612	0.5615	0.6725	3.3524
A7)	1.6450	1.7715	5.5346	12.2418	0.6659	1.1153	8.0165

Table 2. 24-hour maximum absolute navigation errors of single-axis rotation schemes

Rotation schemes	δV_E/(m/s)	δV_N/(m/s)	δL/(′)	$\delta\lambda$/(′)	ϕ_E/(′)	ϕ_N /(′)	ϕ_U /(′)
B1)	0.9761	1.0349	3.1264	1.6231	0.6357	0.7870	3.2011
B2)	1.0604	1.1093	3.4573	3.0872	0.7914	0.8496	3.9441
B3)	1.1088	1.1485	3.6473	3.8798	0.7946	0.8906	4.3069
B4)	1.1571	1.1849	3.8100	4.5665	0.7974	0.9271	4.6438
B5)	1.2034	1.2240	3.9709	5.2875	0.7981	0.9624	4.9770
B6)	1.1535	1.1823	3.8009	4.5299	0.7979	0.9246	4.6256
B7)	1.6177	1.6939	5.4914	12.1722	0.8161	1.0747	8.0032

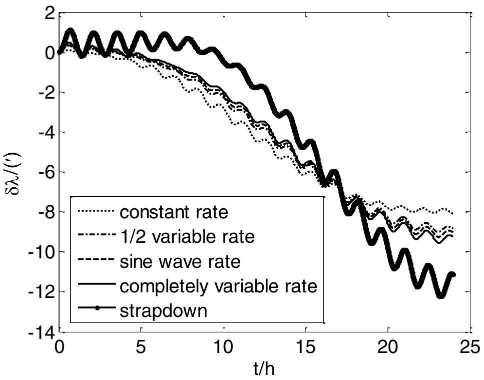

Fig. 4. Longitude errors of single-axis rotation schemes.

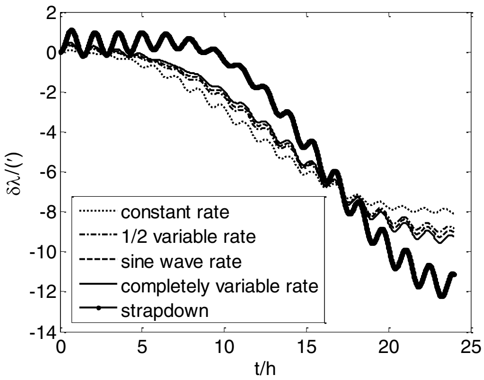

Fig. 5. Longitude errors of dual-axis alternate rotation schemes.

According to the simulation results, the simulated errors accord with the SABF based analysis. Then the conclusion is reached that in single-axis rotation schemes and dual-axis alternative schemes the rotation angular rate should be kept constant, which can reduce the angular rate difference along the rotation path. Because constant angular rate is difficult to be achieved in practical engineering application, the angular rate variation should be shortened to the greatest extent.

6. CONCLUSIONS

This work researches the spatial accumulation of the modulated bias in the plane which is orthogonal to the spindle under the status of rotation angular rate varies. The SABF based method is proposed to analyze the performance of the rotation modulation schemes. By analyzing the constant angular rate scheme, the linearly varied angular rate schemes and the sine angular rate scheme, the conclusion can be drawn that with the extension of the varying angular rate process, the spatial accumulation of the biases increases which cause decline of the error suppression performance. The simulation results of single-axis and dual-axis rotational INS consists with the analytical conclusion. Then the conclusion is reached that in orthogonal spindle rotation schemes the rotation angular rate should be kept constant, and the angular rate variation should be reduced to the greatest extent.

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