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Passivity-based Integral Sliding Mode Active Suspension Control ⋆

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Abstract: A novel passivity-based sliding mode controller for active suspension system with uncertainties is presented in this paper to decrease vibration and increase robustness. Based on the characteristics of the suspension system, interconnection and damping assignment passivity-based control approach is used to construct integral sliding surface. By elaborately designing desired interconnection matrix, damping matrix and energy function, an original integral sliding surface is completed and a subcontroller is obtained to achieve sliding mode dynamic. A simplified form of subcontroller is provided when some of designable parameters are chosen properly. Combining the subcontroller with a nonlinear component, passivity-based sliding mode controller is created. Robustness analysis is given and it is proofed that the integral sliding surface is robustly stable and can be reached. Simulation under three cases circumstances validates the effect of the proposed method.

1. INTRODUCTION

Because vehicles often suffer from vibration caused by uneven ground, vehicle suspensions play important roles in keeping vehicles to have satisfactory performances. Though all of passive /semi-active /active suspension systems have been investigated since 1970s, active suspensions attract much more attention[1],[2]. Many researchers have shown that active suspension can effectively isolate road-induced vibration, reduce unwell shake and noise in the vehicle body, improve ride comfort [3]–[9]. Due to some of performance requirements in vehicles are contradictory, such as decreasing suspension acceleration and tyre displacement at same time, and restricting suspension vibration while keeping active suspension system to be stable, many investigations are developed to deal with those confliction and a various of control methods are introduced into active suspension system, including LQ optimal control [1], robust control [3]–[5], adaptive control, fuzzy control and sliding mode control [6]–[9].

Because suspension systems are inevitable confronted with different loads, while aging problem is unavoidable and unmodeled dynamics are inescapable, it is necessary to take the uncertainties of the suspension systems into consideration. As an effective control method with strong robustness, sliding mode control (SMC) strategy has been applied to active suspension systems recently. [6] proposed an adaptive fuzzy sliding mode controller to suppress the sprung mass position oscillation. By taking suspension system as a plant with two loops, [7] introduced a dynamic sliding-mode controller in the outer loop so that to alleviate the discontinuous jump in the inner loop. In [8], a complex controller design algorithm was presented for an active suspension system based on adaptive, fuzzy and SMC approaches. All of these three work was based on linear sliding surface, thus the dynamic performance on sliding mode is quite normal. On the basis of a traditional integral sliding surface, an adaptive sliding mode controller was given in [9].

In this paper, a novel integral sliding surface will be created to improve the sliding mode dynamic of suspension system by passivity-based control (PBC) concept. PBC is a design methodology to achieve stabilization by rendering the system passive with respective to a desired storage function and injecting damping [10]. In [11], [12], Ortega et.al developed Interconnection and Damping Assignment Passivity-based Control (IDA-PBC). One of distinguishing benefits of IDA-PBC is that the closed-loop energy function can be obtained via solving a partial differential equation which is as a result of selecting desired interconnections matrix and damping matrix. Since IDA-PBC be proposed, many theoretical extensions and practical applications have been reported [10]–[14]. Therefore, the PBC method is a ideal option to be employed to construct sliding surface for active suspension systems.

The purpose of this paper is to decrease vibration induced by uneven road and to increase robustness when suspension system suffers from various uncertainties, caused by stiffness variation, dumping perturbation, mess changing and so on. Meanwhile, reducing input consumption is also expected. Hence, according to SMC principle, the closed-

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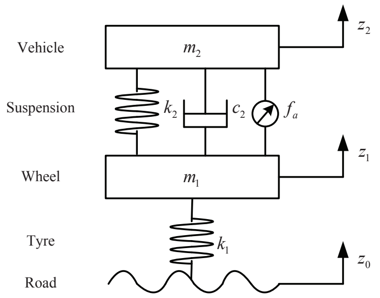

Fig. 1. Mechanical model of quarter vehicle active suspension system

loop active suspension system design has two steps: firstly, a passivity-based integral sliding surface is created to make sliding mode possess favorable performance, for the sake of analyzing the characteristics of suspension system elaborately and selecting desired interconnection matrix, damping matrix and ideal energy function carefully; secondly, an appropriate passivity-based sliding mode controller with low input consumption is constructed to force the states of suspension system to origin along with the sliding surface no matter of the influence of uncertainties. After obtaining approving controller, a sufficient condition is given to guarantee the closed-loop active suspension system is robustly stable.

The remainder of this paper is organized as follows: Section 1 shows the considered active suspension system with uncertainties. Section 2 produces the main results, including the constructing of passivity-based integral sliding mode surface, the obtaining of passivity-based sliding mode controller and the robustness analysis. Simulation results are shown in Section 3. Section 4 draws the conclusions of this paper.

2. PROBLEM FORMULATION

In this paper, a kind of quarter vehicles are considered, as shown in Fig. 1.

In the figure, m_2 is sprung mass, m_1 is unsprung mass; k_2 and c_2 are the coefficients of stiffness and damping of the suspension system, respectively; k_1 stands for compressibility of the pneumatic tyre; z_2 and z_1 are the displacements of the sprung and unsprung masses, respectively; z_0 is vertical ground displacements caused by road unevenness; and f_a is the active input force of the suspension system.

The ideal dynamic equations of the sprung and unsprung masses are

$$\begin{aligned} &m_2\ddot{z}_2 + c_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) = f_a \\ &m_1\ddot{z}_1 + c_2(\dot{z}_1 - \dot{z}_2) + k_2(z_1 - z_2) + k_1(z_1 - z_0) = -f_a \end{aligned} \tag{1}$$

Let $q = [z_2, z_1]^{\mathrm{T}}$, $p = [\dot{z}_2, \dot{z}_1]^{\mathrm{T}}$, $x = [q^{\mathrm{T}}, p^{\mathrm{T}}]^{\mathrm{T}}$ denotes states, $u = f_a$ represents input. Then (1) can be rewritten to

$$\begin{aligned} \dot{x} &= Ax + Bu + B_w z_0 \\ &= \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} + \begin{bmatrix} 0_{2\times 1} \\ b_2 \end{bmatrix} u + \begin{bmatrix} 0_{2\times 1} \\ b_{w2} \end{bmatrix} z_0 \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_2}{m_2} & \frac{k_2}{m_2} & \frac{-c_2}{m_2} & \frac{c_2}{m_2} \\ \frac{k_2}{m_1} & \frac{-k_2 - k_1}{m_1} & \frac{c_2}{m_1} & \frac{-c_2}{m_1} \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} \frac{-k_2}{m_2} & \frac{k_2}{m_2} \\ \frac{k_2}{m_1} & \frac{-k_2 - k_1}{m_1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_2} \\ \frac{-1}{m_1} \end{bmatrix}, B_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_1}{m_1} \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} \frac{-c_2}{m_2} & \frac{c_2}{m_2} \\ \frac{c_2}{m_1} & \frac{-c_2}{m_1} \end{bmatrix}, b_2 = \begin{bmatrix} \frac{1}{m_2} \\ \frac{-1}{m_1} \end{bmatrix}, b_{w2} = \begin{bmatrix} 0 \\ \frac{k_1}{m_1} \end{bmatrix}$$

with $I_{2\times 2}$ is identity matrix with appropriate dimension, $0_{2\times 2}$ and $0_{2\times 1}$ are zero matrices with appropriate dimensions.

Obviously, the masses m_i, stiffness coefficients k_i $(i = 1, 2)$ and dumping coefficients c_2 of active suspension system are inevitable to suffer from perturbation and unmodeled nonlinearities dynamics, the quarter vehicle active suspension system with uncertainties is described as

$$\begin{aligned} \dot{x} &= (A + \Delta A)x + (B + \Delta B)u + (B_w + \Delta B_w)z_0 \\ &= Ax + Bu + d \end{aligned} \tag{2}$$

where $\Delta A, \Delta B, \Delta B_w$ are the uncertainties of A, B, B_w, respectively. $d = \Delta Ax + \Delta Bu + \Delta B_w z_0$ represents dumped uncertainties.

The corresponding nominal system of (2) is

$$\dot{x} = f(x) + g(x)u \tag{3}$$

$$f(x) = \begin{bmatrix} p \\ A_{21}q + A_{22}p \end{bmatrix} \triangleq \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} \tag{4}$$

$$g(x) = B = \begin{bmatrix} 0_{2\times 1} \\ b_2 \end{bmatrix} \tag{5}$$

Assumption 1: The parameters perturbations of active suspension system are bounded, namely,$|\Delta m_i| \leq \delta_{m_i} m_i$, $|\Delta k_i| \leq \delta_{k_i} k_i$, $|\Delta c_2| \leq \delta_{c_2} c_2$, where $\delta_{m_i} > 0, \delta_{k_i} > 0$,(i=1,2), $\delta_{c_2} > 0$ are known constants.

3. PASSIVITY-BASED SLIDING MODE ACTIVE SUSPENSION CONTROL

According to sliding mode control theory, a sliding surface with desired performance should be created first of all, and then a suitable control law is required to drive states to origin along with the sliding surface. In the following, we will create a suitable sliding surface on the basis of passivity-based control theory and then obtain the corresponding sliding mode controller.

3.1 Passivity-based integral sliding surface design

Design a integral sliding surface for the considered active suspension system as follows,

$$s(t) = \sigma x(t) - \sigma x(0) - \sigma \int_0^t F_d(x) \nabla H_d(x) d\tau \tag{6}$$

where σ is chosen according to linear sliding mode design theory[15], and $F_d(x)$, $H_d(x)$ are selected as

$$F_d(x) = J_d(x) - R_d(x), \quad H_d(x) = \frac{1}{2}x^{\mathrm{T}}Px,$$
$$J_d(x) = \begin{bmatrix} 0 & q_{12} & q_{13} & 0 \\ -q_{12} & 0 & 0 & q_{24} \\ -q_{13} & 0 & 0 & 0 \\ 0 & -q_{24} & 0 & 0 \end{bmatrix}, P = \begin{bmatrix} P_{11} & 0 & 0 & 0 \\ 0 & P_{22} & 0 & 0 \\ 0 & 0 & P_{33} & 0 \\ 0 & 0 & 0 & P_{44} \end{bmatrix}$$
$$R_d(x) = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & q_{33} & 0 \\ 0 & 0 & 0 & q_{44} \end{bmatrix}, \tag{7}$$

with $\nabla = \frac{\partial^{\mathrm{T}}}{\partial x}\|\cdot\|$ is vector differential operator, and

$$\begin{aligned} & q_{ii} > 0, (i = 2, 3, 4), \quad P_{ii} > 0, (i = 1, 2, 3, 4) \\ & q_{11} = \frac{q_{22}P_{22}^2 z_1^2}{P_{11}^2 z_2^2}, \quad q_{12} = \frac{q_{22}P_{22}z_1}{P_{11}z_2} \\ & q_{13} = P_{33}^{-1}, \quad q_{24} = P_{44}^{-1} \end{aligned} \tag{8}$$

Theorem 1. The sliding surface (6) of system (3), is stable robustly to the equilibrium $x_\star = 0$ under control law

$$\begin{aligned} u_0 = & [c_2\dot{z}_2 + (m_1q_{44}P_{44} - c_2)\dot{z}_1 + k_2z_2 \\ & + (m_1P_{22}P_{44}^{-1} - k_2)z_1 - k_1z_1] \\ & + [(c_2 - m_2q_{33}P_{33})\dot{z}_2 - c_2\dot{z}_1 \\ & + (k_2 - m_2P_{11}P_{33}^{-1})z_2 - k_2z_1] \end{aligned} \tag{9}$$

Proof: According to (7) and (8), there exists

$$J_d(x) + J_d^{\mathrm{T}}(x) = 0, \quad R_d(x) = R_d^{\mathrm{T}}(x) \geq 0 \tag{10}$$
$$F_d(x) = \begin{bmatrix} -q_{11} & q_{12} & q_{13} & 0 \\ -q_{12} & -q_{22} & 0 & q_{24} \\ -q_{13} & 0 & -q_{33} & 0 \\ 0 & -q_{24} & 0 & -q_{44} \end{bmatrix} \triangleq \begin{bmatrix} F_{d_1}(x) \\ F_{d_2}(x) \\ F_{d_3}(x) \\ F_{d_4}(x) \end{bmatrix} \tag{11}$$
$$\nabla H_d(x) = Px, \quad \nabla^2 H_d(x) = P, \quad P = P^{\mathrm{T}} > 0 \tag{12}$$
$$\nabla H_d(x_\star) = 0, \quad \nabla^2 H_d(x_\star) > 0 \tag{13}$$

where $x_\star$ is the equilibrium of system (3).

Suppose the $g^\perp(x)$ is a full-rank left annihilator of $g(x)$, i.e., $g^\perp(x)g(x) = 0$, according to (5), one of optical candidates of $g^\perp(x)$ can be

$$g^\perp(x) = \begin{bmatrix} \gamma_1 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 \end{bmatrix}$$

with $\gamma_1 > 0$, $\gamma_2 > 0$. From (3), (5) and (7), it yields

$$g^\perp(x)[f(x) - F_d(x)\nabla H_d(x)] = \begin{bmatrix} \gamma_1(\dot{z}_2 - F_{d_1}(x)\nabla H_d(x)) \\ \gamma_2(\dot{z}_1 - F_{d_2}(x)\nabla H_d(x)) \end{bmatrix}$$

Since

$$\dot{z}_2 - F_{d_1}(x)\nabla H_d(x) = \dot{z}_2 + q_{11}P_{11}z_2 - q_{12}P_{22}z_1 - q_{13}P_{33}\dot{z}_2$$
$$\dot{z}_1 - F_{d_2}(x)\nabla H_d(x) = \dot{z}_1 + q_{12}P_{11}z_2 - q_{22}P_{22}z_1 - q_{24}P_{44}\dot{z}_1$$

Because of (8), it is clear that the following equation (14) is held.

$$g^\perp(x)[f(x) - F_d(x)\nabla H_d(x)] = 0 \tag{14}$$

According to Interconnection and Damping Assignment Passivity-based Control (IDA-PBC) theory [11],[12], when conditions (10), (12), (13) and (14) are held, there exists a controller u_0

$$u_0 = -\{[g^{\mathrm{T}}(x)g(x)]^{-1}g^{\mathrm{T}}(x)\}[f(x) - F_d(x)\nabla H_d(x)] \tag{15}$$

such that the closed-loop nominal system (3) is asymptotically stable to the equilibrium $x_\star = 0$, and the system can be transformed to port-controlled hamiltonian (PCH) form

$$\dot{x} = [J_d(x) - R_d(x)]\nabla H_d(x) \tag{16}$$

From (2) and (5), it is obviously $[g^{\mathrm{T}}(x)g(x)]^{-1}g^{\mathrm{T}}(x) = [0, 0, m_2, -m_1]$, thus according to (4), (11) and (12), (15) can be written to

$$\begin{aligned} u_0 &= -[m_2, -m_1]\begin{bmatrix} f_3(x) - F_{d_3}(x)\nabla H_d(x) \\ f_4(x) - F_{d_4}(x)\nabla H_d(x) \end{bmatrix} \\ &= [c_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) - k_1z_1] \\ &\quad - m_1(-q_{24}P_{22}z_1 - q_{44}P_{44}\dot{z}_1) \\ &\quad - [-c_2(\dot{z}_2 - \dot{z}_1) - k_2(z_2 - z_1)] \\ &\quad + m_2(-q_{13}P_{11}z_2 - q_{33}P_{33}\dot{z}_2) \end{aligned}$$

Based on (8), yields

$$\begin{aligned} u_0 = & [c_2\dot{z}_2 + (m_1q_{44}P_{44} - c_2)\dot{z}_1 + k_2z_2 \\ & + (m_1P_{44}^{-1}P_{22} - k_2)z_1 - k_1z_1] \\ & + [(c_2 - m_2q_{33}P_{33})\dot{z}_2 - c_2\dot{z}_1 \\ & + (k_2 - m_2P_{33}^{-1}P_{11})z_2 - k_2z_1] \end{aligned}$$

namely (9). Therefore, under the controller (9), (16) can be substituted into (6) and the sliding surface (6) is transformed to

$$s(t) = \sigma x(t) - \sigma x(0) - \sigma\int_0^t \dot{x}d\tau$$

and the derivative of $s(t)$ is $\dot{s}(t) = \sigma\dot{x}(t) - \sigma\dot{x}(t)$. Obviously, $s(t) = 0$ and $\dot{s}(t) = 0$ are held for nominal system (3).

Because of integral sliding mode theory [16], the sliding model surface (6) is stable asymptotically.

Remark 1: According to (8) and (12), (11) can be rewritten to

$$F_d(x) = \begin{bmatrix} -q_{22}(P_{22}P_{11}^{-1}z_1z_2^{-1})^2 & q_{22}(P_{22}P_{11}^{-1}z_1z_2^{-1}) & P_{33}^{-1} & 0 \\ -q_{22}(P_{22}P_{11}^{-1}z_1z_2^{-1}) & -q_{22} & 0 & P_{44}^{-1} \\ -P_{33}^{-1} & 0 & -q_{33} & 0 \\ 0 & -P_{44}^{-1} & 0 & -q_{44} \end{bmatrix}$$

thus, with the controller (9), system (3) can be turn to the PCH form system

$$\dot{x} = F_d(x)\nabla H_d(x) = \begin{bmatrix} \dot{z}_2 \\ -2q_{22}P_{22}z_1 + \dot{z}_1 \\ -P_{33}^{-1}P_{11}z_2 - q_{33}P_{33}\dot{z}_2 \\ -P_{44}^{-1}P_{22}z_1 - q_{44}P_{44}\dot{z}_1 \end{bmatrix} \tag{17}$$

Remark 2: In (11), if $z_2 = 0$ happens, let $z_2 = \xi$ with $\xi = 10^{-6}$, in order to avoid being divided by zero in $F_d(x)$.

Such a replacement is carried out only in (11). Because z_2^{-1} is not appear in (9) and (17), the replacement will not influence the controller u_0 and the sliding surface (6). Therefore, it is feasible to replace z_2 by ξ in (11) when $z_2 = 0$ occurs.

Remark 3: According to IDA-PBC theory, $J_d(x)$ can be view as the desired interconnection matrix, $R_d(x)$ can be taken as desired damping matrix, $H_d(x)$ is ideal energy function and the Lyapunov function of system (3) as well.

Remark 4: Because $J_d(x)$, $R_d(x)$ and $H_d(x)$ are choosing elaborately according to the characteristics of the considered active suspension system, the problem of solving partial differential equations which often brings trouble to conventional passivity-based control [10], [14], is not troublesome in this paper. The relative partial differential equations are convenient to be solved as shown by (10)–(16).

3.2 Passivity-based sliding mode control (PB-SMC)

Consider the suspension system with uncertainties, namely system (2), on the basis of the passivity-based integral sliding surface (6) designed in subsection 2.1, taking u_0 as a subcontroller which is one part of sliding mode controller, and then adding u_0 with a nonlinear subcontroller $u_n = -\rho(\sigma B)^{-1}\mathtt{sgn}(s)$ so to make the sliding surface can be reached, hence the passivity-based sliding mode controller is given as

$$u = u_0 + u_n = u_0 - \rho(\sigma B)^{-1}\mathtt{sgn}(s) \tag{18}$$

Theorem 2. Under the controller (18) and sliding surface (6), the uncertain active suspension closed-loop control system (2) is robustly stable, if the following condition (19) is held.

$$\rho \geq \|\sigma d\| \tag{19}$$

Proof: Consider the derivative of sliding surface (6) and system (2), gives

$$\begin{aligned} \dot{s} &= \sigma\dot{x} - \sigma F_d(x)\nabla H_d(x) \\ &= \sigma[f(x) + Bu + d] - \sigma F_d(x)\nabla H_d(x) \end{aligned} \tag{20}$$

Substituting (18) into (20), yields

$$\dot{s} = \sigma\{f(x)+B[u_0-\rho(\sigma B)^{-1}\mathtt{sgn}(s)]+d\} - \sigma F_d(x)\nabla H_d(x)$$

According to (3), (7), (15) and (16), one can see that under the action of u_0, there exists $f(x) + Bu_0 = F_d(x)\nabla H_d(x)$, then

$$\dot{s} = \sigma[-B\rho(\sigma B)^{-1}\mathtt{sgn}(s) + d]$$

Select Lyapunov function as $V = \frac{1}{2}s^2$, gives $\dot{V} = s\dot{s} = s\sigma[-B\rho(\sigma B)^{-1}\mathtt{sgn}(s) + d] = -\rho s\mathtt{sgn}(s) + s\sigma d$

Because of condition (19), it is clear that $\dot{V} = -\rho s + s\sigma d < 0$ is held when $s > 0$ and $\dot{V} = \rho s + s\sigma d < 0$ is held when $s < 0$. Therefore, $s\dot{s} < 0$ is held. Then, according to sliding mode control theory[15], the states of suspension system can reach the sliding surface (6) and keep on it thereafter. The closed-loop active suspension control system (2) is robustly stable under the control law (18).

Remark 5: Because of the Assumption 1, $\|\sigma d\|$ will be known after σ be set. Hence, the condition (19) can be satisfied.

Remark 6: According to sliding mode theory, chattering may be occur because of the existence of discontinuous term $\mathtt{sgn}(\cdot)$. In order to reduce chattering, there are lots of functions can be used to approximate $\mathtt{sgn}(\cdot)$, such as saturation function $\mathtt{sat}(\cdot)$, arc-tangent function $\mathtt{arctan}(\cdot)$ and so on. In the simulation Section 3, $\frac{2}{\pi}\mathtt{arctan}(\frac{s}{0.001})$ is applied to instead $\mathtt{sgn}(s)$.

Remark 7: From (2), one can find that $A_{21} \neq I_{2\times2}$, $A_{21} \neq A_{21}^{\mathrm{T}}$ and $A_{22} \neq 0_{2\times2}$. Besides, uncertainties are taken into consideration during the system design process. Therefore, the considered system (2) is different from the classical mechanic models which considered in typical IDA-PBC theory. Hence, the method presented in this paper not only has contribution to SMC theory and its application, but also extends IDA-PBC theory and its application as well.

4. SIMULATION

In this section, the simulation will be given on a quarter-car model [17] with parameters are $m_2 = 310kg$, $m_1 = 70kg$, $k_2 = 27.358kN/m$, $k_1 = 309.511kN/m$, $c_2 = 0.984kN\cdot s/m$.

In order to evaluate the performance of the designed closed-loop active suspension system, we consider three typical cases.

Case 1: Consider the case of an isolated bump in an otherwise smooth road surface. The corresponding ground displacement is given by

$$z_0(t) = \begin{cases} \dfrac{A_{z0}}{2}[1-\cos(\dfrac{2\pi v}{L_{z0}}t)], & if\ \ 0 \leq t \leq \dfrac{A_{z0}}{L_{z0}} \\ 0, & if\ \ \ \ t \geq \dfrac{A_{z0}}{L_{z0}} \end{cases}$$

where A_{z0} and L_{z0} are the height and the length of the bump respectively, v is the vehicle forward velocity. Assume the bump is both high and long, with $A_{z0} = 0.5m$, $L_{z0} = 5m$, and $v = 30km/h$. The corresponding road excitation is show in Fig.2.

Case 2: Consider road excitation z_0 as a vibration, which is consistent and typically specified as random process with a ground displacement power spectral density (PSD) of $G_q(f) = 4\pi^2 G_q(n_0)n_0^2 v$, where $G_q(n_0)$ stands for the road roughness coefficient, n_0 is the reference spatial frequency, v is the vehicle forward velocity [3]. Select the road roughness as $G_q(n_0) = 1024\times10^{-6}m^3$, $n_0 = 0.1$, which corresponds to very poor ground, assume $v = 7km/h$.

Case 3: Consider the parameters of suspension system suffer from perturbations

$$\begin{aligned} &\Delta m_2 = 50\%m_2, \Delta k_2 = -30\%k_2, \Delta c_2 = -30\%c_2 \\ &\Delta m_1 = -20\%m_1, \Delta k_1 = -30\%k_1, \end{aligned}$$

Suppose the road roughness as $G_q(n_0) = 256\times10^{-6}m^3$, and $v = 50km/h$. The corresponding road excitation is show in Fig.3.

Let the parameters in the presented PB-SMC controller are $q_{22} = 1$, $q_{33} = 0.5$, $q_{44} = 20$, $P_{11} = 1$, $P_{22} = 1$, $P_{33} = 6$, $P_{44} = 0.075$, $\sigma = [1.0000, -208.1644, 310.6718, 0.1517]$.

The corresponding $F_d(x)\nabla H_d(x)$ is

$$F_d(x)\nabla H_d(x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ -1/6 & 0 & -3 & 0 \\ 0 & -10/3 & 0 & -3/2 \end{bmatrix} x \triangleq \bar{A}x$$

the eigenvalues of $\bar{A}$ are $[-0.0566, -2.9434, -1.7500 + 3.6429i, -1.7500 - 3.6429i]$. Obviously, all of eigenvalues are on the left-half s-plane, which means the stability can be achieved.

To make comparison, we choose traditional integral sliding surface $s = \sigma x - \sigma x(0) - \int_0^t [f(x) + g(x)u(\tau)]d\tau$ and apply reaching law $\dot{s} = -5s - 0.1\mathtt{sgn}(s)$ to construct integral sliding mode controller (ISMC). Similar to the statement in Remark 6, $\mathtt{sgn}(s)$ is replaced by $\frac{2}{\pi}\mathtt{arctan}(\frac{s}{0.001})$ in ISMC.

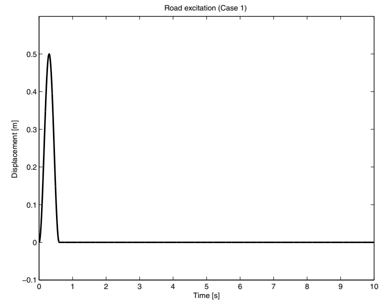

Fig. 2. Road excitation (Case 1)

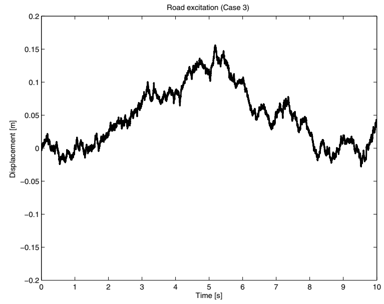

Fig. 3. Road excitation (Case 3)

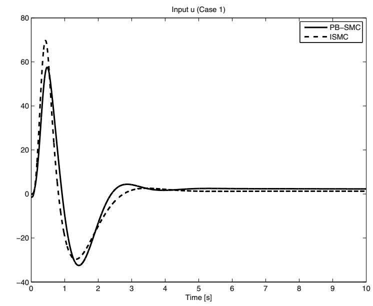

Fig. 4. Input u (Case 1)

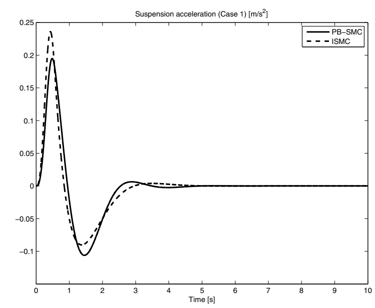

Fig. 5. Suspension acceleration $\ddot{z}_2$ (Case 1)

The simulation results are show in Fig.5–Fig.7. Table.1 shows the improvement percentage of PB-SMC compared with ISMC in the RMS values $RSMy = \sqrt{\frac{\int_0^T y^2 dt}{T}}$, the maximum absolute values $\max|y|$ and the total input consumption $\sum u^2$.

From Fig. 4 and Fig. 5, one can find that under the PB-SMC method, the suspension acceleration $\ddot{z}_2$ and input u have lower peak values. According to Fig. 7 and Fig. 8, it is obvious that under the PB-SMC method, the overall

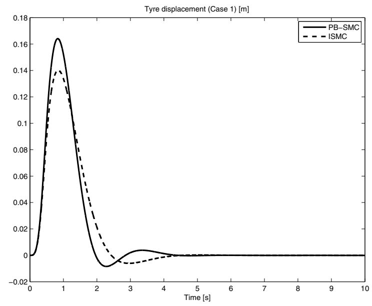

Fig. 6. Tyre displacement z_1 (Case 1)

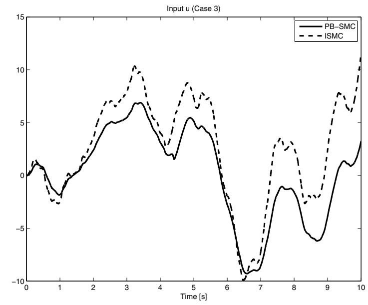

Fig. 7. Input u (Case 3)

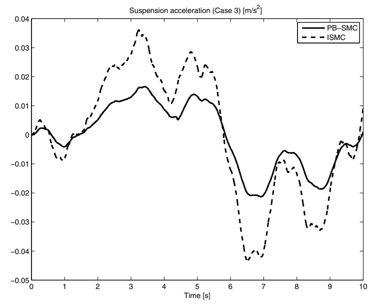

Fig. 8. Suspension acceleration $\ddot{z}_2$ (Case 3)

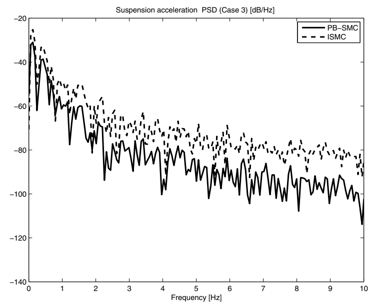

Fig. 9. Suspension acceleration $\ddot{z}_2$ PSD (Case 3)

Table 1. Improvement Percentage of PB-SMC compared with ISMC $\left((1-\frac{PB\_SMC}{ISMC})\times 100\%\right)$

	Case 1	Case 2	Case 3
RMS z_2	9.5304	7.7489	51.4681
RMS z_1	-7.8666	-0.5825	-0.7409
RMS $\ddot{z}_2$	4.7343	4.2401	48.8345
$\max\lvert z_2\rvert$	8.9889	8.2514	51.6568
$\max\lvert z_1\rvert$	-17.1251	-2.0978	-0.7355
$\max\lvert\ddot{z}_2\rvert$	17.4170	3.1368	50.6583
$\sum u^2$	12.5941	14.4626	39.0697

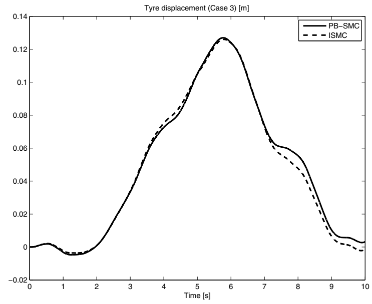

Fig. 10. Tyre displacement z_1 (Case 3)

value of the suspension acceleration $\ddot{z}_2$ and input u are lower than that of in ISMC method. Fig. 9 shows the power spectral density (PSD) of $\ddot{z}_2$ is lower in PB-SMC method for the frequency band 4–8 Hz, which is widely accepted ride comfort closely related frequency range.

Although Fig. 6 indications the peak value of tyre displacement z_1 in Case 1 is a bit higher with PB-SMC than that of by ISMC, and Fig. 10 shows z_1 in Case 3 almost are the same in the two methods, Table.1 illustrate that the proposed PB-SMC method improves all values of RMS z_2, RMS $\ddot{z}_2$, $\max|z_2|$, $\max|\ddot{z}_2|$ and $\sum u^2$ greatly, while increases RMS z_1 and $\max|z_1|$ small, particularly in Case 2 and Case 3.

Therefore, the PB-SMC method optimizes suspension acceleration, has stronger robustness and decreases input consumption, possesses better performance in the whole.

5. CONCLUSIONS

In this paper, a novel passivity-based sliding mode controller for a quarter vehicle active suspension system with uncertainties is proposed. By employing the interconnection and damping assignment passivity-based control approach, and according to the characteristics of suspension system, desired interconnection matrix $J_d(x)$, desired damping matrix $R_d(x)$ and ideal energy function $H_d(x)$ are constructed, and an original passivity-based integral sliding surface is determined. The control law u_0 which is used to achieve sliding mode dynamic, is obtained simultaneously. In order to drive states to the sliding surface and keep on it thereafter, passivity-based sliding mode controller u is created by combining u_0 with a nonlinear term, and the robustness of the closed-loop suspension uncertain system are proofed subsequently. A simplified form of u_0 are given when some of designable parameters are chosen appropriately. Simulation results show that with the presented passivity-based sliding mode controller, the closed-loop active suspension system possesses favorable performance, such as strong robustness, optimized suspension acceleration and lower input consumption.

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