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A Distributed Algorithm for Energy Optimization in Hydraulic Networks

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Abstract: An industrial case study in the form of a large-scale hydraulic network underlying a district heating system is considered. A distributed control is developed that minimizes the aggregated electrical energy consumption of the pumps in the network without violating the control demands. The algorithm is distributed in the sense that all calculations are implemented where the necessary information is available, including both parameters and measurements. A communication network between the pumps is implemented for global optimization. The local implementation of the algorithm means that the system becomes a Plug & Play control system as most commissioning can be done during the manufacture of the pumps. Only information on the graph-structure of the hydraulic network is needed during installation.

Keywords: District Heating Systems, Plug & Play Process Control, Optimization, Over-actuated systems.

1. INTRODUCTION

An industrial system distributed over a network is studied. The system is a large-scale hydraulic network underlying a district heating system with an arbitrary number of end-users. A distributed control is developed that minimizes the aggregated electrical energy consumption of the pumps in the network without violating the control demands at the end-users. The regulation problem addressed is distributed pressure setting of district heating systems, where multiple pumps are distributed across the network, as proposed in Bruus et al. (2004) for widening the range of district heating systems.

Similar networks and models arise for instance in mine ventilation networks and cardiovascular systems. These classes of systems are the motivation for the works Hu et al. (2003), Koroleva and Krstić (2005), Koroleva et al. (2006), where nonlinear adaptive controllers are proposed to deal with the presence of uncertain parameters.

The control of the proposed district heating systems is treated in Jensen and Wisniewski (2013), De Persis et al. (2013) and De Persis and Kallesøe (2011), amongst others. In the latter, where the system model is derived, it is shown that the system is over-actuated. Here this over-actuation is used for energy optimal control. The chosen algorithm design ensures that all necessary model information can be embedded in the pumps during production, meaning that the pumps and their control can be plugged into the hydraulic network, without further commissioning, except for information on the network graph, which can easily be obtained by the commissioning personnel. This is in accordance with the philosophy behind Plug & Play Process Control, Stoustrup (2009).

As stated before, the hydraulic networks treated in this work are over-actuated, which is the reason why the energy optimization is possible. Over-actuated systems and their control are discussed both in theory and in practice, for example in flight applications, ship applications, and car safety control Boskovic et al. (2002), Lue et al. (2004), Tønnås and Johansen (2008), Laine and Andreasson (2007), Zaccarian (2007). These papers deal with the control problem as a centralized control problem. In the systems considered here it is natural to distribute the control and optimization problem between the pumps in the hydraulic network. Distributed optimization is handled for example in Rantzer (2009), Nedić and Ozdaglar (2009), Nedić et al. (2010).

The paper starts by introducing the system model and the optimization problem in Section 2. In Section 3, convexity of the optimization problem is proven, and the algorithm for the energy optimization is derived in Section 4. Section 5 presents experimental results obtained on a laboratory setup, which emulates a small district heating system. Section 6 comprises concluding remarks.

2. PROBLEM FORMULATION

A district heating system with distributed pumping is considered. The model of this system is derived in De Persis and Kallesøe (2011), where it is shown that the model has the following structure

$$\begin{aligned} J\dot{q} &= f(B^T q) + \Delta h_e + F\Delta h_b \\ y_i &= \mu_i(q_i)\ ,\ i = 1, 2, \ldots, n\ , \end{aligned} \tag{1}$$

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where $q \in \mathbb{R}^n$ is a vector of free variables coinciding with the flows through the valves at the end-users; $f \in C^1$ describes the natural damping in the system; B is an n-by-m fundamental cycle matrix of the system graph; $\Delta h_e \in \mathbb{R}^n$ is the vector of pressures delivered by the pumps at the end-users; $\Delta h_b \in \mathbb{R}^k$ is the vector of pressures delivered by the so called booster pumps; $\mu_i(q_i)$ is the pressure drop across the i^{th} end-user valve. The vectors Δh_e and Δh_b form the control inputs to the system.

It is possible to prove that asymptotic output regulation of the system (1) can be obtained with a PI-controller of the form

$$\begin{aligned} \dot{\xi} &= -K(y-r) \\ u &= \xi - N(y-r) \ , \end{aligned} \tag{2}$$

where $K > 0$ and $N > 0$ are diagonal; $r > 0$ is constant and

$$u = \Delta h_e + F\Delta h_b \ . \tag{3}$$

A proof can be found in De Persis et al. (2013).

As K and N are diagonal the controllers only rely on information available locally at each end-user pump. On the other hand, the controller is affecting both the end-user pump and booster pump pressures. Evaluating the connection between the control output and the pump pressures (3), it is recognised that full control can be obtained by using Δh_e only. This means that the structure of the control can be depicted as shown in Fig. 1. Therefore as long as Δh_b is piecewise constant and Δh_e is feasible in a neighbourhood around $\Delta h_e = -f(B^T q^*) - F\Delta h_b$, then the equilibrium point of the system

$$\begin{aligned} J\dot{q} &= f(B^T q) + \xi - N(\mu(q) - r) + F\Delta h_b \\ \dot{\xi} &= -K(\mu(q) - r) \end{aligned} \tag{4}$$

is globally asymptotically stable. The goal of this work is to choose Δh_b such that the steady state power consumption of the pumps in the system is minimized. In other words, at the desired equilibrium $r = \mu(q)$ we seek values of Δh_b such that $P = \sum_{i=1}^n P_i^e + \sum_{j=1}^k P_j^b$ is minimized. Here P_i^e is the power consumption of the i^{th} end-user pump and P_j^b is the power consumption of the j^{th} booster pump. This means that the following minimization problem should be solved

$$\min_{\Delta h_b} P = \min_{\Delta h_b} \left(\sum_{i=1}^n P_i^e + \sum_{j=1}^k P_j^b \right)$$

$$\begin{aligned} \text{Subject to: } J\dot{q} &= f(B^T q) + \xi - N(\mu(q) - r) + F\Delta h_b \\ \dot{\xi} &= -K(\mu(q) - r) \\ 0 &\le \Delta h_e \le \kappa_e \\ 0 &\le \Delta h_b \le \kappa_b \ . \end{aligned}$$

It is assumed that the control (2) is fast enough to ensure that $r = \mu(q)$ almost all the time. The reference r is assumed to be constant, which implies that $\dot{q} = \dot{\xi} = 0$ can be assumed almost all the time. These assumptions are similar to traditional assumptions in cascaded control. Using these assumptions, the minimization problem is simplified to a static optimization problem;

$$\min_{\Delta h_b} P = \min_{\Delta h_b} \left(\sum_{i=1}^n P_i^e + \sum_{j=1}^k P_j^b \right) \tag{5a}$$

$$\begin{aligned} \text{Subject to: } \quad 0 &= f(B^T q^*) + \xi^* + F\Delta h_b \\ \xi^* &= \Delta h_e \\ r &= \mu(q^*) \\ 0 &\le \Delta h_e \le \kappa_e \\ 0 &\le \Delta h_b \le \kappa_b \ . \end{aligned} \tag{5b}$$

Note that $q_i^* = \mu_i^{-1}(r_i)$ is constant as long as r_i is constant as $\mu_i(\cdot)$ is a continuous function.

Since $f(B^T q^*)$ is a constant vector, the equality constraint in (5b) is affine in Δh_b and thus define a convex set, say C_1. Also, it is immediately seen that the inequality constraints are affine hence they form a convex set, say C_2. We need to ensure that the problem is feasible, that is, $C_1 \cap C_2 \neq \emptyset$. To this end, it is observed that $q_i^* > 0$ since $r_i > 0$ and $\mu_i(\cdot)$ is strict monotonically increasing. Furthermore, he following lemma is needed

Lemma 1. De Persis and Kallesøe (2011) Under the network Assumptions 1-3 in De Persis and Kallesøe (2011), $q \in \mathbb{R}_+^n$ implies $-f(B^T q) \in \mathbb{R}_+^n$.

From De Persis and Kallesøe (2011) it is known that the entries of the matrix F in (5) belongs to the set $\{0, 1\}$, $f(0) = 0$ and $f(\cdot) \in C^1$, hence it follows from Lemma 1 that there exists $R_i > 0$ for $i = 1, 2, \ldots, n$ and Δh_e, Δh_b, such that for $0 \le r_i \le R_i$ the constraints of (5) are fulfilled, since $\Delta h_e + F\Delta h_b > 0$ if $\Delta h_e > 0$ and $\Delta h_b > 0$. This means that C_1 and C_2 has a non-empty intersection for properly chosen r. It is worth noting that in practice the problem is always feasible, since the network is designed to accommodate the control objective at the end-users.

In the following a method for converting the constrained problem (5) to an unconstrained one is described. The purpose for this is to enable the design of an on-line gradient search algorithm. The tool chosen here is softening the inequality constraints by the use of penalty functions, see for instance Fletcher (1975).

Softing constraints The inequality constraints are softened by including penalty functions in the objective function as shown below

$$\min_{\Delta h_b} \tilde{P} = \min_{\Delta h_b} \left(\sum_{i=1}^n \left(P_i^e + s_i^e(\Delta h_{e,i})\right) + \sum_{j=1}^k \left(P_j^b + s_j^b(\Delta h_{b,j})\right) \right) \tag{6a}$$

$$\begin{aligned} \text{Subject to: } \quad 0 &= f(B^T q^*) + \Delta h_e + F\Delta h_b \\ q^* &= \mu(r)^{-1} \ , \end{aligned} \tag{6b}$$

where $s_i^e(\cdot)$ and $s_i^b(\cdot)$ are additional terms included to penalize violation of the inequality constraints. One possible implementation of these terms is given in (7),

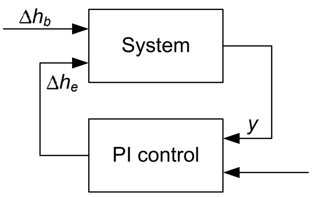

Fig. 1. A block diagram of the control structure.

$$s_i^*(x) = \begin{cases} \kappa(x-\underline{x})^2 \ , & x \le \underline{x} \\ 0 & , \underline{x} \le x \le \overline{x} \\ \kappa(x-\overline{x})^2 \ , & \overline{x} \le x \end{cases} , \ * = e, b \tag{7}$$

where $\underline{x}$ and $\overline{x}$ are the maximum and minimum allowed pressures of the i^{th} pump, and $\kappa > 0$ is a gain. Furthermore, note that the equality constraints of (5) are reformulated. In the following two sections, the convexity of the optimization problem (6) will first be analysed, followed by the development of an optimization algorithm that can be distributed between the pumps.

3. CONVEXITY

In this section it is shown that the problem (6) is a convex program for systems of the form (1). This in turn means that the minimum of the objective function exists and the set of minimizers is convex, (see for instance (Jensen et al., 2014, Thm 26)). Recall the definition of a convex program

Definition 1. Let C be a non-empty convex set in $\mathbb{R}^n$, and let $f : C \to \mathbb{R}$ be a convex function on C. Then,

$$\min_{x\in C} f(x) \tag{8}$$

is said to be a convex program.

The objective function in (6) is formed by the power consumption of the pumps in the network and the penalty functions. From the definition of the penalty functions in (7) it is seen that they are convex.

To derive convexity of the power function, a model of the pump operation is necessary. In Kallesøe (2005) it is shown that centrifugal pumps can be modelled by two polynomials for pump speeds $\omega_i \ge 0$ and pump flows $q_i \ge 0$.

$$\Delta h_i(q_i, \omega_i) = -a_{h2,i}q_i^2 + a_{h1,i}q_i\omega_i + a_{h0,i}\omega_i^2 \tag{9a}$$

$$T_i(q_i, \omega_i) = -a_{t2,i}q_i^2 + a_{t1,i}q_i\omega_i + a_{t0,i}\omega_i^2 \ , \tag{9b}$$

where $\Delta h_i(\cdot)$ is the pressure delivered by the i^{th} pump, $T_i(\cdot)$ is the shaft torque, and $a_{h2,i}$, $a_{h1,i}$, $a_{h0,i}$, $a_{t2,i}$, $a_{t1,i}$, $a_{t0,i}$ are constant parameters describing the i^{th} pump. The power consumption $P_i(\cdot)$ of the pump is given by

$$P_i(q_i, \omega_i) = \omega_i T_i(q_i, \omega_i) + P_0, \tag{10}$$

and calculable from the pump model described in (9). In (10), P_0 is a constant term describing the idle consumption of the pump due to the control hardware on the pump. Due to physical constraints of the pump, the following can be assumed on the parameters (see Kallesøe (2005))

Assumption 1. It is assumed that the parameters of the model (9) fulfil the constraints

$$a_{h2} > 0, \ a_{h0} > 0, \ a_{t2} > 0, \ a_{t1} > 0, \ a_{t0} > 0 \ .$$

The derivative of the pressure delivered by the pump with respect to its rotational speed can be derived from (9) to be

$$\frac{d\Delta h_i}{d\omega_i}(q_i, \omega_i) = a_{h1,i}q_i + 2a_{h0,i}\omega_i. \tag{11}$$

The rotation of the pump is limited to one direction, which means that $\omega_i \ge 0$. Then, from the model (9) and Assumption 1 it is immediately seen that $\Delta h_i > 0$ implies that $\omega_i > 0$ for all q_i since in this case $(a_{h1,i}q_i + a_{h0,i}\omega_i)\omega_i > a_{h2,i}q_i^2 \ge 0$. Furthermore, $(a_{h1,i}q_i + a_{h0,i}\omega_i) > 0$ and $a_{h0,i}\omega_i > 0$, which implies that $0 < a_{h1,i}q_i + a_{h0,i}\omega_i < a_{h1,i}q_i + 2a_{h0,i}\omega_i = \frac{d\Delta h_i}{d\omega_i}(q_i, \omega_i)$. This proves the following lemma.

Lemma 2. For pump pressures $\Delta h_i > 0$, rotational speeds $\omega_i \ge 0$ and flows $q_i \ge 0$, then under Assumption 1

$$\frac{d\Delta h_i}{d\omega_i}(q_i, \omega_i) > 0 \ .$$

Using Assumption 1 it is possible to prove the following proposition.

Proposition 1. Under Assumption 1 the objective function in (6) is convex at the desired reference flow q^*, and with Δh_i within the feasibility set, if the parameter set of the pump model (9) fulfils

$$3a_{t0,i}a_{h1,i}^2 < 4a_{h0,i}(a_{t1,i}a_{h1,i} + a_{t2,i}a_{h0,i}) \tag{12}$$

for every pump $i = 1, \ldots, n + k$.

Sketch of the proof: Utilizing Lemma 2 it can be shown that the second order derivative

$$\frac{d^2P_i}{d\Delta h_i^2}(\Delta h_i) > 0 \ ,$$

where $P_i(\Delta h_i) \equiv P_i(q_i^*, \omega_i)_{\omega_i=\omega_i(\Delta h_i)\equiv\omega_i(q_i^*,\Delta h_i)}$. This shows convexity by the 2nd-order sufficient condition and the fact that a sum of convex function is convex.

Convexity is necessary for the optimization approach proposed in the paper to work. From Proposition 1 convexity can only be stated when (12) is fulfilled. This requirement is easy to check and pumps can even be designed for this. Also, in practise, it seems that almost all centrifugal pumps do fulfil (12).

4. ENERGY OPTIMIZATION

In this section, an algorithm that ensures energy optimal distribution between the pressures provided by the end-user pumps and the booster pumps is considered. An on-line gradient based search algorithm for minimizing the power consumption is derived. This is similar to the approach taken in Jensen et al. (2014) for minimizing the steady state energy consumption of the system. However, in this exposition the true power function of the system is used in the optimization problem, whereas in Jensen et al. (2014) a bi-linear approximation of the power function was used. Also, here the closed loop system does not take the cascaded form described in Jensen et al. (2014), considerably complicating the analysis. The derivative of the objective function in (6) is given by

$$\begin{aligned} \frac{d\tilde{P}}{d\Delta h_b}(\Delta h_b, \Delta h_e) = & \sum_{i=1}^{n} \frac{dP_i^e}{d\Delta h_b}(\Delta h_{e,i}) + \frac{ds_i^e}{d\Delta h_b}(\Delta h_{e,i}) \\ & + \sum_{j=1}^{k} \frac{dP_j^b}{d\Delta h_b}(\Delta h_{b,j}) + \frac{ds_j^b}{d\Delta h_b}(\Delta h_{b,j}) \ , \end{aligned} \tag{13}$$

with $\Delta h_e = -f(B^Tq^*) - F\Delta h_b$. The derivative (13) equals zero at the global optimum due to convexity of the problem. We are seeking a solution, which can be distributed between the pumps. From (13) it is recognized that information on the operation of the end-user pumps is necessary for the optimization, hence a communication protocol between the pumps is needed.

The terms $d(P_i^b + s_i^b)(\Delta h_{b,i})/d\Delta h_b$ forms a vector describing the change of the power consumption and penalty in the i^{th} booster pump with respect to pressure delivered by each booster pump. The booster pump pressures Δh_b can be chosen freely as only the end-user pumps are employed for the control, see Fig. 1. Since $P_i^b(\cdot)$ and $s_i^b(\cdot)$ depends only on $\Delta h_{b,i}$ only the term $d(P_i^b + s_i^b)(\Delta h_{b,i})/d\Delta h_{b,i}$ is different from zero. This term is calculable at the i^{th} booster pump.

The terms $d(P_j^e + s_j^e)(\Delta h_{e,i})/d\Delta h_b$ denotes the derivative of the power consumption and penalty of the end-user pumps with respect to the pressure delivered by the booster pumps. The operation of the end-user pumps is affected by the booster pumps as described by the equality constraints in (6b). Recalling our assumption that $q = q^*$, we obtain

$$\Delta h_e = -f(B^T q^*) - F\Delta h_b \quad \Rightarrow \quad \frac{d\Delta h_e}{d\Delta h_b} = -F$$

from which we get

$$\begin{aligned} \frac{dP_j^e}{d\Delta h_b}(\Delta h_{e,j}) &= \frac{dP_j^e}{d\Delta h_{e,j}}(\Delta h_{e,j})\frac{d\Delta h_{e,j}}{d\Delta h_b} \\ &= -\frac{dP_j^e}{d\Delta h_{e,j}}(\Delta h_{e,j})F_j \end{aligned} \tag{14}$$

where $\Delta h_{e,j} = -f_j(B^T q^*) - F_j \Delta h_b$ and F_j is the j^{th} row of F.

The change in power consumption of the i^{th} pump given a change in pressure can be calculated based on the model of the pump presented in (9). The power consumption of the i^{th} pump is calculated from the shaft torque using $P_i(\cdot) = \omega_i T_i(\cdot) + P_0$. Calculating the derivative of $P_i(\cdot)$ with respect to Δh_i the following expression is obtained

$$\begin{aligned} \frac{dP_i}{d\Delta h_i}(\Delta h_i) &= \frac{dP_i/d\omega_i}{d\Delta h_i/d\omega_i}(q_i, \omega_i)\bigg|_{q_i = q_i^*} \\ = \frac{-a_{t2,i}{q_i^*}^2 + 2a_{t1,i}q_i^*\omega_i + 3a_{t0,i}\omega_i^2}{a_{h1,i}q_i^* + 2a_{h0,i}\omega_i}&\bigg|_{\omega_i = \omega_i(\Delta h_i)}, \end{aligned} \tag{15}$$

which can be calculated locally at every pump in the network, assuming that q_i and ω_i are available by either measurements or estimation.

The optimizer In the following the optimization algorithm is presented. The algorithm utilizes the derivatives presented above, which is made available via a communication network.

Let the pressure delivered by the boosting pumps be updated using the following expression

$$\begin{aligned} \Delta\dot{h}_{b,j} = -\gamma\Bigg(&\frac{d(P_j^b + s_j^b)}{d\Delta h_{b,j}}(q_j, \Delta h_{b,j}) \\ &- \sum_{i=1}^{n}\frac{d(P_i^e + s_i^e)}{d\Delta h_{e,i}}(q_i, \Delta h_{e,i})F_{ij}\Bigg) \end{aligned} \tag{16}$$

for some small γ ($\gamma > 0$). Then the equilibrium point of the closed loop system will be such that the power consumption of the pumps is minimal while meeting the control objective $y = \mu(q) = r$.

To realize this, first observe that the closed loop system is given by

$$\begin{aligned} J\dot{q} &= f(B^T q) + \xi - N(\mu(q) - r) + F\Delta h_b \\ \dot{\xi} &= -K(\mu(q) - r) \\ \Delta\dot{h}_b &= -\gamma\Bigg(\frac{d(P^b + s^b)}{d\Delta h_b}(q, \Delta h_b) + \\ &\quad -F^T\frac{d(P^e + s^e)}{d\Delta h_e}(q, \Delta h_e)\Bigg). \end{aligned}$$

Assuming the PI-controller is able to obtain output regulation meaning that $q = q^*$, where $q^* = \mu^{-1}(r)$. Then the following is true

$$\begin{aligned} \Delta h_e = \xi^* &= -f(B^T q^*) - F\Delta h_b \\ \Delta\dot{h}_b &= -\gamma\Bigg(\frac{d(P^b + s^b)}{d\Delta h_b}(q^*, \Delta h_b) + \\ &\quad -F^T\frac{d(P^e + s^e)}{d\Delta h_e}(q^*, \Delta h_e)\Bigg) \end{aligned} \tag{17}$$

Using $\tilde{P}(q^*, \Delta h_b) = P^b(q^*, \Delta h_b) + s^b(\Delta h_b) + P^e(q^*, \Delta h_e) + s^e(\Delta h_e)$,where $\Delta h_e = -f(B^T q^*) - F\Delta h_b$ as a Lyapunov function candidate which fulfils $\tilde{P}(\cdot) > 0$ on the feasibility set defined in (6b). Then it is immediately seen that

$$\begin{aligned} \frac{d}{dt}\tilde{P}(q^*, \Delta h_b) = -\gamma\Bigg|&\frac{d(P^b + s^b)}{d\Delta h_b}(q^*, \Delta h_b) \\ &- F^T\frac{d(P^e + s^e)}{d\Delta h_e}(q^*, \Delta h_e)\Bigg|^2. \end{aligned} \tag{18}$$

From (18) it follows that Δh_b converges to largest invariant set of the reduced system (17) where

$$\frac{dP^b}{d\Delta h_b}(q^*, \Delta h_b) - F^T\frac{dP^e}{d\Delta h_e}(q^*, \Delta h_e) = 0. \tag{19}$$

Since (6) is a convex program, the steady state value of Δh_b is a solution to (6).

Remark 1. Following (Jensen et al., 2014, App. B) it can be shown that there exists gain $\kappa^* > 0$ such that for all $\kappa > \kappa^*$ the objective function in (6) has a local minimum within the feasibility set, say $\mathcal{C}$, of the original optimization problem (5), since the objective function of (5) is continuous and $\mathcal{C}$ is compact.

The gradient $dP^*/d\Delta h_*$ for $* = e, b$ is bounded on the boundary of $\mathcal{C}$ when $q = q^*$. Furthermore, $\mathcal{C}$ is invariant for the system

$$\dot{x} = -\left(\frac{ds^b}{d\Delta h_b}(\Delta h_b) - F^T\frac{ds^e}{d\Delta h_e}(\Delta h_e)\right) \tag{20}$$

by design. Hence, it follows that there exists gain $\kappa^* > 0$ such that for every $\kappa > \kappa^*$, $\mathcal{C}$ is invariant for the reduced system (17). Therefor it follows that for any initial condition $\Delta h_{b,0} \in \mathcal{C}$, $\Delta h_b(t) \in \mathcal{C}$ for every $t > 0$.

The communication structure The gradient of the power $d\tilde{P}(\Delta h_b, \Delta h_e)/d\Delta h_{b,i}$ can be calculated by first calculating (15) locally at end-user- and booster pumps respectively and then (according (14)) communicating the result of the calculation of (15) at the jth end-user pump to the lth booster pump if $F_{jl} \neq 0$. Thus, the only information on the hydraulic network which is necessary is the structure of the matrix F, the entries of which determines which end-user pumps the individual boosting pump needs information from. Assuming that there exist a communication network, the necessary exchange of information is illustrated on a two end-user case in Fig. 2. Here the pressure

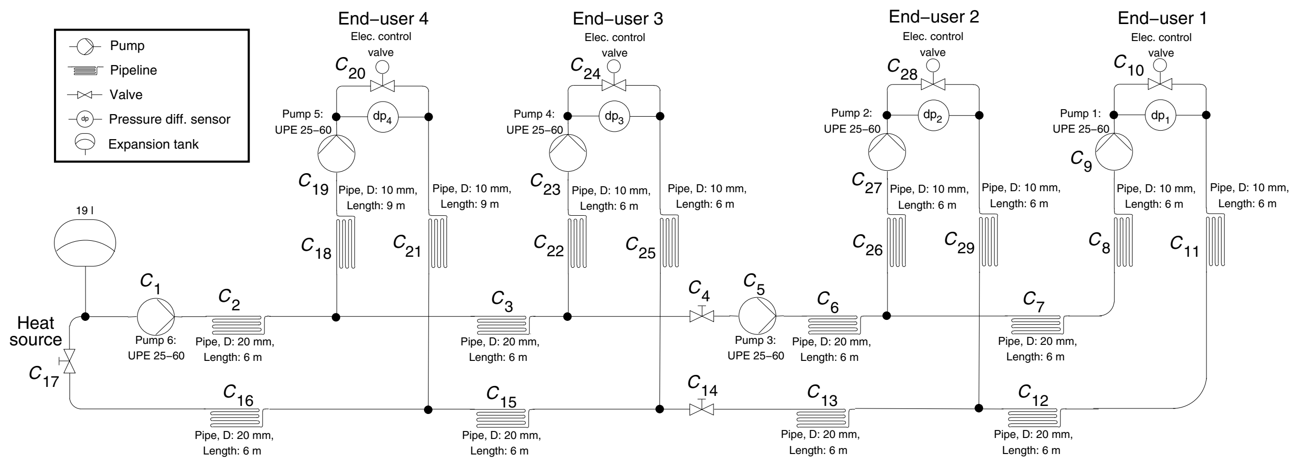

Fig. 3. Sketch of the hydraulic network used in the experiments.

control is located at the two end-user pumps, which also calculates the derivative necessary for the optimization.

5. EXPERIMENTAL RESULTS

In this section, results obtained with the proposed optimization control are presented. These results are obtained on a laboratory setup which emulates the hydraulic dynamics of a district heating system with four end-users. Due to physical constraints imposed by the size of the setup, the system dynamics are 5-10 times faster than what would be expected in a real district heating system. The setup is the same as described in De Persis and Kallesøe (2011), where additional details can be found. The hydraulic network diagram of the setup is illustrated in Fig. 3. This is a system with four end-user pumps, here denoted $\{c_9, c_{19}, c_{23}, c_{27}\}$, and two booster pumps denoted $\{c_1, c_5\}$. The control of each end-user pump is obtained using the controller described in (2), hence the pumps $\{c_9, c_{19}, c_{23}, c_{27}\}$ are controlled such that the pressures $\{dp_1, dp_2, dp_3, dp_4\}$ equals a reference value via a set of PI-controllers. The pumps $\{c_1, c_5\}$ are used for energy optimization, hence they are controlled according to (16). The tests are done in accordance with the communication strategy shown in Fig. 2, meaning that the derivatives used for optimization are calculated at the PI-controllers controlling the pressures and communicated to - and used at - the two optimizers at the booster pumps. The network graph implies in this case that the derivatives from the pumps $\{c_9, c_{19}, c_{23}, c_{27}\}$ are used in the optimizer placed at pump c_1 and the derivatives from the pumps $\{c_9, c_{27}\}$ are used in the optimizer placed at pump c_5.

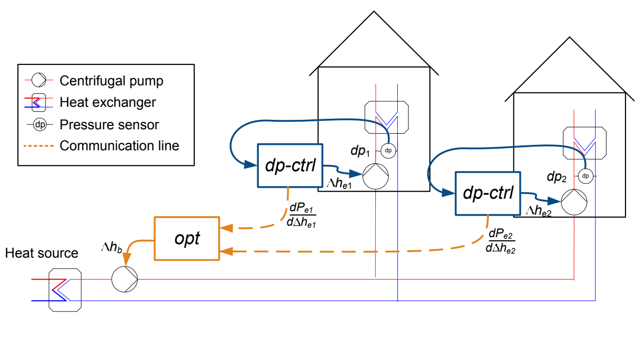

Fig. 2. A sketch of a small District Heating System with distributed optimization control.

The results of the experiment are shown in Fig. 4. The system is started from rest and at time ∼35 [s], all outputs are at the reference value of 0.2 [Bar]. This is achieved mainly by the use of the end-user pumps $\{c_9, c_{27}, c_{19}, c_{23}\}$. At 35 [s] the power consumption of the pumps is ∼75 [W] and is decreasing until the system has converged at 400 [s] where the consumption is ∼36 [W]. This means that for this case, 39 [W] has been saved or approximately 50 %. Furthermore, a step response has been obtained by changing the reference to 0.3 [Bar] at time 600 [s] and back again at time 900 [s]. As it can be seen from the figure, the outputs converge fast to the new reference, and subsequently the optimizing controller also converges to the power optimal distribution of pump pressures.

6. CONCLUSION

A distributed approach for optimizing control is derived for a general class of hydraulic networks that forms the backbone in district heating systems. The optimizer is shown to work in all hydraulic networks that fulfil a few assumptions. Therefore the approach forms a Plug & Play optimizing control approach, as only information on the network structure is necessary during installation. All other necessary information can be implemented beforehand as part of an intelligent pump. The approach is exemplified via an experiment on a small scale laboratory setup emulating a system with four end-users.

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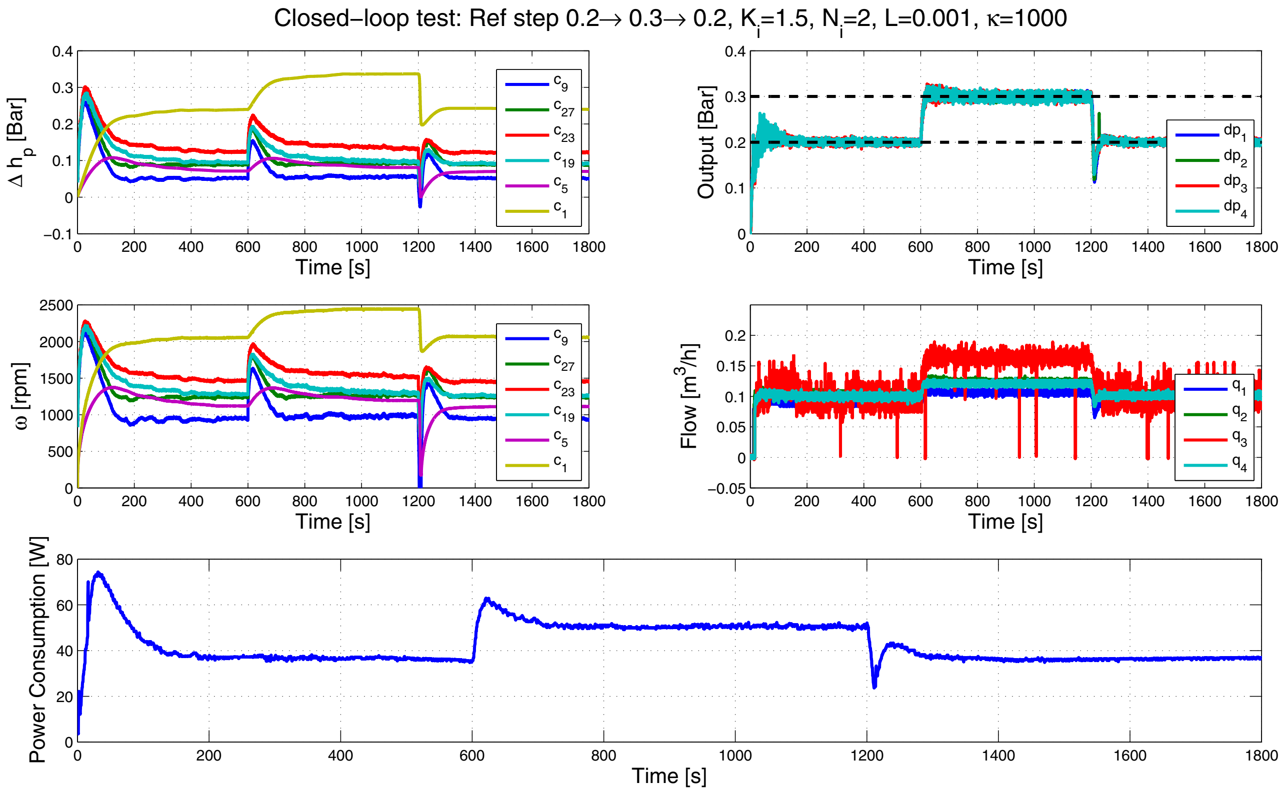

Fig. 4. Example on energy optimization on the system depicted in Fig. 3. A step in the references r_i from 0.2 to 0.3 [Bar] is performed at time 600 [s] and back to 0.2 [Bar] at time 900 [s]. The plots show the signals Δh_i from the controllers and the inputs ω_i to the pumps, the measured differential pressures dp_i and flows q_i and the total electrical power consumption P of the pumps. Note, that the apparent fluctuations in q_3 are due to a hardware issue with the sensor and does not reflect actual fluctuations in the flow.

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