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Robust H_∞ Fault-Tolerant Control for Linear Systems with Fast Adaptive Fault Estimation ⋆

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Abstract: The problem of adaptive fault-tolerant control for linear systems with time-varying actuator faults including outage and loss of effectiveness is investigated in this paper. After presenting a time-varying actuator fault model, a novel adaptive observer is designed so that the fault parameter can be estimated fast. Here, the inevitable fault estimation error, as well as the exogenous disturbance, is regarded as a part of the combined disturbance to be attenuated. Then, based on the Lyapunov stability theory and the estimated fault parameters, some sufficient conditions for designing the adaptive robust H_∞ fault-tolerant controller are presented in the framework of linear matrix inequalities (LMIs), which can guarantee that the closed-loop system is asymptotically stable and robust for both time-varying fault and exogenous disturbance. Finally, the effectiveness and applicability of the proposed method is illustrated by a linearized longitudinal motion equation of the F-18 aircraft.

Keywords: Fault-tolerant control, time-varying actuator failure, fault estimator, fast adaptation, robust control, linear matrix inequalities(LMIs).

1. INTRODUCTION

With the increasing requirements for high performance, modern control systems have become more and more complex. Component failures, such as actuator faults, are usually inevitable and often cause performance degradation or even instable (Steinberg (2005)). Therefore, designing fault-tolerant control (FTC) systems, which make the systems operate in safe conditions and have required performance whenever components of the system are healthy or faulted, has received extensive attention in the past few decades, see Chen (2013), Mahmoud (2003), Ye and Yang (2006), and Zolghadri and Henry (2013).

Generally, FTC can be achieved in two ways: passive and active approaches. The former utilizes robust control technique to design a feedback control law with fixed gain to make the faulty system robust for possible system faults (Liao (2002); Pujol (2007); Zhang and Wang (2007). The passive fault-tolerant controller is usually easy to be designed, but it may result in limited fault tolerant capability.

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Compared with passive approach, the structure or the parameters of the controllers designed with active approach are adjustable according to the online fault information. There are many active approaches, such as Ding (2002), Cieslak (2008) and Maki (2004). For active fault-tolerant control schemes, it can be seen that fault estimation is a key link. In Zhang and Jiang (2001), a Kalman filter is used to estimate the efficiency of the actuators. Polycarpou (2001) uses the learning method based on neural network to estimate fault information. In Shen (2013), an adaptive law is given to estimate fault parameters which are used to design the robust controller online. Among the above-mentioned methods, adaptive technique has been paid much attention for its adapting ability to deal with the parametric uncertainties and structural variations of the systems. In Yang and Ye (2010), a novel reliable controller is proposed, which is updated adaptively to reduce the effect of actuator fault using the on-line fault estimation. Motivated by this, Zuo, Ho, and Wang (2010) and Chen (2013) extended the above idea to singular systems and Markovian jumping systems. This works have further shown that the reliable controller design method based on on-line fault estimation is effective to attenuate actuator degradation. However, the algorithms given by the above works are suitable for the constant fault case and cannot be simply extended to the time-varying fault case.

In practical application, the faults are usually time-varying and sometimes even be fast time-varying. In Zhang and

Jiang (2009), the active fault-tolerant control is studied to deal with the time-varying actuator fault, but it is assumed that the fault is piecewise constant. Shen (2012) investigates the fault-tolerant control problem for Takagi-Sugeno (T-S) fuzzy systems based on fault diagnosis and estimation to reduce the effect of actuator time-varying faults. It can been seen that there may exists error between real fault parameter and the estimation value in the estimation process, especially for the fast time-varying fault. If the estimation error is ignored, the effect of adaptive control may be discounted. Therefore, it is necessary to consider the fault estimation error when designing fault-tolerant controller, which motivates this paper.

This paper investigates the problem of designing robust adaptive fault-tolerant controller for linear systems with time-varying actuator faults. Compared with some existing work, the main contributions of this paper are as follows: Firstly, for time-varying actuator fault, a novel adaptive fault estimation architecture is presented so that the fault parameters can be estimated fast and smoothly. Secondly, the estimation error is regarded as a part of the combined disturbance to be attenuated when the fault-tolerant controller is design. Thus, the closed-loop system is robust for fault estimation error as well as the exogenous disturbance. Thirdly, the adaptive fault estimator and the robust controller can be designed independently.

The rest of this paper is organized as follows. Section 2 presents the system model with actuator fault. In Section 3, a novel adaptive estimator is designed to estimate the fault parameters fast and smoothly. Then, based on the estimated parameters, the design method of adaptive robust controller is presented in Section 4. Finally, the simplified F-18 aircraft model is used to illustrate the effectiveness of the proposed design method.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following class of continuous-time linear system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_1\omega(t) \\ y(t) = Cx(t) + Du(t) + D_1\omega(t) \\ z(t) = C_z x(t) + D_z u(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $y(t) \in R^p$ is the measured output,$z(t) \in R^q$ is the regulated output. $\omega(t) \in L_2[0,\infty)$ is the exogenous disturbance, and it is assumed that $\omega(t)$ is norm bounded, that is $||\omega(t)|| \le \bar{\omega}$. A, B, C, D, B_1, D_1, C_z, D_z are known constant matrices.

In this work, the fault model of actuator degradation is assumed as follows:

$$u_i^F(t) = (1-\rho_i(t))u_i(t), 0 \le \underline{\rho}_i \le \rho_i(t) \le \bar{\rho}_i \tag{2}$$

where $\rho_i(t)$ is a time-varying fault parameter. $u_i^F(t)$ denotes the ith actuator has failed. $\underline{\rho}_i$ and $\bar{\rho}_i$ are the lower and upper bounds of $\rho_i(t)$, respectively. At the same time, it is assumed that there exist the known positive constants $\bar{\bar{\rho}}_i$ such that $|\dot{\rho}_i(t)| \le \bar{\bar{\rho}}_i$.

Set $\rho(t) = (\rho_1(t), \rho_2(t), \cdots, \rho_m(t))^T$, the uniform actuator fault model is defined as follows:

$$u^F(t) = (I - \Lambda(\rho(t)))u(t) \tag{3}$$

The dynamics(1) with actuator faults can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + B(I-\Lambda(\rho(t)))u(t) + B_1\omega(t) \\ y(t) = Cx(t) + D(I-\Lambda(\rho(t)))u(t) + D_1\omega(t) \\ z(t) = C_z x(t) + D_z(I-\Lambda(\rho(t)))u(t) \end{cases} \tag{4}$$

The objective of this paper is to stabilize system (1) with a robust H_∞ controller and meet the required performance despite actuator fault and external disturbance.

3. FAULT PARAMETER ESTIMATION

In this section, a novel adaptive fault estimation algorithm is proposed to estimate the actuator fault.

The considered observer is as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B(I-\Lambda(\hat{\rho}(t)))u(t) + L(y(t)-\hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) + D(I-\Lambda(\hat{\rho}(t)))u(t) \end{aligned} \tag{5}$$

where $\hat{\rho}(t)$ is the estimated value of $\rho(t)$ at time t, which means $\hat{\rho}_i(t)$ is the estimation of $\rho_i(t)$.

Let $e(t) = x(t) - \hat{x}(t)$, the fault estimation error $\widetilde{\rho}(t) = \rho(t) - \hat{\rho}(t)$, $\widetilde{\rho}_f(t) = \rho(t) - \hat{\rho}_f(t)$, then the error dynamics can be obtained as

$$\begin{aligned} \dot{e}(t) = (A-LC)e(t) - (B-LD)\Lambda(u(t))\widetilde{\rho}(t) \\ + (B_1 - LD_1)\omega(t) \end{aligned} \tag{6}$$

Theorem 1. If there exists a symmetric positive-definite matrix X, positive-definite diagonal matrix Υ,Υ_f, real matrices L and $W > 0$ with appropriate dimensions, and positive scalars τ and ε, such that the following conditions hold:

$$\begin{aligned} &X(A-LC) + (A-LC)^T X \\ &+\varepsilon X(B_1 - LD_1)(B_1-LD_1)^T X < -W \end{aligned} \tag{7}$$

and $\hat{\rho}_i$ is to be generated through the following adaptive law:

$$\begin{aligned} \dot{\hat{\rho}}_i(t) &= Proj\{F_i\} \\ &= \begin{cases} 0, & \begin{matrix} \hat{\rho}_i(t) \ge \bar{\rho}_i, \ and \ F_i > 0 \\ or \ \ \hat{\rho}_i(t) \le \underline{\rho}_i \ and \ F_i < 0 \end{matrix} \\ F_i, & otherwise, \quad i = 1,2,\cdots,m \end{cases} \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{\hat{\rho}}_{fi}(t) &= Proj\{F_{fi}\} \\ &= \begin{cases} 0, & \begin{matrix} \hat{\rho}_{fi}(t) \ge \bar{\rho}_i, \ and \ F_{fi} > 0 \\ or \ \ \hat{\rho}_{fi}(t) \le \underline{\rho}_i \ and \ F_{fi} < 0 \end{matrix} \\ F_{fi}, & otherwise, \quad i = 1,2,\cdots,m \end{cases} \end{aligned} \tag{9}$$

where $F = -\Upsilon[\Lambda(u(t))(B-LD)^T Xe(t) + \tau(\hat{\rho}(t) - \hat{\rho}_f(t))]$, $F_f = \Upsilon_f(\hat{\rho}(t) - \hat{\rho}_f(t))$, and F_i,F_{fi} are the ith row of F and F_f, then the state of system (6) is semiglobally uniformly ultimately bounded, which means $\widetilde{\rho}(t)$, $\widetilde{\rho}_f(t)$ and the state are converging to a small neighborhood of zero asymptotically.

Proof. Choose the following function

$$V(t) = e^T(t)Xe(t) + \widetilde{\rho}^T(t)\Upsilon^{-1}\widetilde{\rho}(t) + \tau\widetilde{\rho}_f^T(t)\Upsilon_f^{-1}\widetilde{\rho}_f(t) \tag{10}$$

Differentiating V and substitute (7)- (9) into (11) yields

$$\begin{aligned} \dot{V} \le& -e^T(t)We(t) + \varepsilon^{-1}\omega(t)^T\omega(t) \\ &+ 2\sum_{i=1}^{m} r_i\widetilde{\rho}_i(t)\dot{\rho}_i(t) + 2\tau\sum_{i=1}^{m} r_{fi}\widetilde{\rho}_{fi}(t)\dot{\rho}_i(t) \end{aligned} \tag{11}$$

where r_i, and r_{fi} are the ith main diagonal elements of Υ^{-1} and Υ_f^{-1}, respectively.

Since $0 \le \underline{\rho}_i \le \hat{\rho}_i(t) \le \bar{\rho}_i \le 1$ and $0 \le \underline{\rho}_i \le \hat{\rho}_{fi}(t) \le \bar{\rho}_i \le 1$ which can be guaranteed by (8) and (9), and $|\dot{\rho}_i(t)| \le \bar{\bar{\rho}}_i$, then we have

$$\widetilde{\rho}_i(t)\dot{\rho}_i(t) \le -\widetilde{\rho}_i^2(t) + (\bar{\rho}_i - \underline{\rho}_i)(\bar{\rho}_i - \underline{\rho}_i + \bar{\bar{\rho}}_i) \quad (12)$$

$$\widetilde{\rho}_{fi}(t)\dot{\rho}_i(t) \le -\widetilde{\rho}_{fi}^2(t) + (\bar{\rho}_i - \underline{\rho}_i)(\bar{\rho}_i - \underline{\rho}_i + \bar{\bar{\rho}}_i) \quad (13)$$

With $||\omega(t)|| \le \bar{\omega}$, the following inequality can be obtained.

$$\dot{V} \le -\lambda_0 V(t) + \mu \quad (14)$$

where $\lambda_0 = min\{\frac{\lambda_{min}(W)}{\lambda_{max}(X)}, 2\}$, $\mu = \varepsilon^{-1}\bar{\omega}^2 + 2\sum_{i=1}^{m}(r_i + \tau r_{fi})(\bar{\rho}_i - \underline{\rho}_i)(\bar{\rho}_i - \underline{\rho}_i + \bar{\bar{\rho}}_i)$.

Then, it can be obtained that $\frac{d}{dt}(V(t)e^{\lambda_0 t}) \le e^{\lambda_0 t}\mu$, furthermore

$$0 \le V(t) \le \frac{\mu}{\lambda_0} + V(0)e^{-\lambda_0 t} - \frac{\mu}{\lambda_0}e^{-\lambda_0 t} \le \frac{\mu}{\lambda_0} + V(0) \quad (15)$$

Let $\alpha = \frac{\mu}{\lambda_0} + V(0)$, then there exists $|e(t)| \le \sqrt{\frac{\alpha}{\lambda_{min}(X)}}$, $|\widetilde{\rho}_i| \le \sqrt{2\psi\alpha}$, $|\widetilde{\rho}_{fi}| \le \sqrt{2\psi\alpha}$. The proof is completed.

$\tau(\hat{\rho}(t) - \hat{\rho}_f(t))$ is added into adaptive law (8).

4. ROBUST H_∞ FAULT-TOLERANT CONTROLLER DESIGN

Next, we will use the fault estimation information in Section 3 to construct the following controller:

$$u(t) = (K_0 + \sum_{i=1}^{m}\hat{\rho}_i(t)K_{ai})x(t) \quad (16)$$

Let $\bar{I}_i$ denote a square matrix whose (i,i) element is 1 and others are 0. Then, choose

$$\begin{aligned}&\Lambda(\widetilde{\rho}(t))(K_0 + K_a(\hat{\rho}(t)))x(t) = (\bar{I}_1 \;\; \bar{I}_2 \;\; \cdots \bar{I}_m)\bar{\omega}(t)\\ &= (\bar{I}_1 \;\; \bar{I}_2 \;\; \cdots \bar{I}_m)\begin{bmatrix}(K_0 + K_a(\hat{\rho}(t)))x(t)\widetilde{\rho}_1(t)\\ (K_0 + K_a(\hat{\rho}(t)))x(t)\widetilde{\rho}_2(t)\\ \vdots \\ (K_0 + K_a(\hat{\rho}(t)))x(t)\widetilde{\rho}_m(t)\end{bmatrix}\end{aligned} \quad (17)$$

Hence, by substituting the estimation parameter $\hat{\rho}(t)$ and control law (16) into system (4), the system can be rewritten as

$$\begin{cases}\dot{x}(t) = \mathcal{A}_{\hat{\rho}}x(t) + B_F\omega_F(t)\\ z(t) = \mathcal{C}_{\hat{\rho}}x(t) + D_F\omega_F(t)\end{cases} \quad (18)$$

where

$\mathcal{A}_{\hat{\rho}} = A + B(I - \Lambda(\hat{\rho}(t)))K(\hat{\rho}(t)), B_F = [\bar{B} \;\; B_1]$,
$\mathcal{C}_{\hat{\rho}} = C_z + D_z(I - \Lambda(\hat{\rho}(t)))K(\hat{\rho}(t)), D_F = [\bar{D}_z \;\; 0]$,
$\omega_F(t) = [\bar{\omega}^T(t) \;\; \omega^T(t)]^T, \bar{B} = [B\bar{I}_1 \;\; B\bar{I}_2 \;\; \cdots B\bar{I}_m]$,
$\bar{D}_z = [D_z\bar{I}_1 \;\; D_z\bar{I}_2 \;\; \cdots D_z\bar{I}_m]$.

Theorem 2. If there exist matrices $Q > 0$, Y_0, Y_{ai}, $i = 1\cdots m$, constants $\gamma_1 > 0, \gamma_2 > 0$ and a symmetric matrix Θ with

$$\begin{bmatrix}\Theta_{11} & \Theta_{12}\\ \Theta_{21} & \Theta_{22}\end{bmatrix} \quad (19)$$

and $\Theta_{11}, \Theta_{22} \in R^{ml\times ml}$ such that the following inequalities hold:

$$\Theta_{22}^{ii} \le 0, \quad i = 1, \cdots, m \quad (20)$$

with $\Theta_{22}^{ii} \in R^{l\times l}$ is the (i,i) block of Θ_{22}, and for $\delta \in \{\delta = (\delta_1 \cdots \delta_m)^T : \delta_i \in \{\underline{\rho}_i, \bar{\rho}_i\}\}$

$$\begin{aligned}&\Theta_{11} + \Theta_{12}\Delta(\delta) + (\Theta_{12}\Delta(\delta))^T + \Delta(\delta)\Theta_{22}\Delta(\delta) \ge 0\\ &\begin{bmatrix}U & E\\ E^T & F\end{bmatrix} + G^T\Theta G < 0\end{aligned} \quad (21)$$

where

$$\Delta(\delta) = diag[\delta_1 I_{l\times l}, \cdots, \delta_m I_{l\times l}], \quad l = Dim(U)$$

$$U = \begin{bmatrix}U_{11} & \bar{B} & QC_z^T + Y_0^T D_z^T\\ * & -\gamma_2^2 I & \bar{D}_z^T\\ * & * & -I\end{bmatrix}$$

$$U_{11} = AQ + QA^T + BY_0 + Y_0^T B^T + \frac{1}{\gamma_1^2}B_1B_1^T$$

$$E_i = \begin{bmatrix}-B_iY_0 + BY_{ai} & 0 & -Y_0^T D_{zi}^T + Y_{ai}^T D_z^T\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$F_{ij} = \begin{bmatrix}-B_iY_{aj} - [B_jY_{ai}]^T & 0 & -Y_{aj}^T D_{zi}^T\\ 0 & 0 & 0\\ -D_{zj}Y_{ai} & 0 & 0\end{bmatrix}$$

$Y_0 = K_0Q, \quad Y_{ai} = K_{ai}Q, B_i = B * \bar{I}_i, D_{zi} = D_z * \bar{I}_i$
$E = [E_1, E_2, \cdots, E_m]$,

$$F = \begin{bmatrix}F_{11} & F_{12} & \cdots & F_{1m}\\ F_{21} & F_{12} & \cdots & F_{2m}\\ \vdots & \vdots & \cdots & \vdots\\ F_{m1} & F_{m2} & \cdots & F_{mm}\end{bmatrix}, G = \begin{bmatrix}\begin{bmatrix}I_{l\times l}\\ \vdots\\ I_{l\times l}\end{bmatrix} & 0\\ 0 & I_{ml\times ml}\end{bmatrix}$$

then the controller gain is given by $K(\hat{\rho}(t)) = Y_0Q^{-1} + \sum_{i=1}^{m}\hat{\rho}_i(t)Y_{ai}Q^{-1}$, and $\hat{\rho}_i(t)$ is determined according to the adaptive law (8), which ensure that the system (4) is asymptotically stable with $\omega_F(t) = 0$ and satisfies required H_∞ performance.

Proof. Choose a Lyapunov function as $V(t) = x^T(t)Px(t)$, then

$$\begin{aligned}&\dot{V}(t) + z^T(t)z(t) - \gamma_1^2\omega^T(t)\omega(t) - \gamma_2^2\bar{\omega}^T(t)\bar{\omega}(t)\\ &\le \begin{bmatrix}x(t)\\ \bar{\omega}(t)\end{bmatrix}^T X_F \begin{bmatrix}x(t)\\ \bar{\omega}(t)\end{bmatrix}\end{aligned} \quad (22)$$

where

$$\begin{aligned}&X_F = \begin{bmatrix}X_{F11} & *\\ \bar{B}^TP + \bar{D}_z^T\mathcal{C}_{\hat{\rho}} & \bar{D}_z^T\bar{D}_z - \gamma_2^2 I\end{bmatrix}\\ &X_{F11} = P\mathcal{A}_{\hat{\rho}} + \mathcal{A}_{\hat{\rho}}^TP + \frac{1}{\gamma_1^2}PB_1B_1^TP + \mathcal{C}_{\hat{\rho}}^T\mathcal{C}_{\hat{\rho}}\end{aligned} \quad (23)$$

Choose $Q = P^{-1}$, pre- and post-multiplying X_F with

$$\begin{bmatrix}Q & 0\\ 0 & I\end{bmatrix} \quad (24)$$

then,using Schur-complement formula, X_F is rewritten as

$$\begin{aligned}&\begin{bmatrix}\mathcal{A}_{\hat{\rho}}Q + Q\mathcal{A}_{\hat{\rho}}^T + \frac{1}{\gamma_1^2}B_1B_1^T & \bar{B} & Q\mathcal{C}_{\hat{\rho}}^T\\ * & -\gamma_2^2 I & \bar{D}_p^T\\ * & * & -I\end{bmatrix}\\ &= U + \sum_{i=1}^{m}\hat{\rho}_iE_i + (\sum_{i=1}^{m}\hat{\rho}_iE_i)^T + \sum_{i=1}^{m}\sum_{j=1}^{m}\hat{\rho}_i\hat{\rho}_jF_{ij}\end{aligned} \quad (25)$$

Based on Lemma 1 in Yang and Ye (2010), $X_F < 0$ can be obtained when the conditions required in Theorem 2 are met, which means

$$\dot{V}(t) + z^T(t)z(t) - \gamma_1^2\omega^T(t)\omega(t) - \gamma_2^2\bar{\omega}^T(t)\bar{\omega}(t) < 0 \quad (26)$$

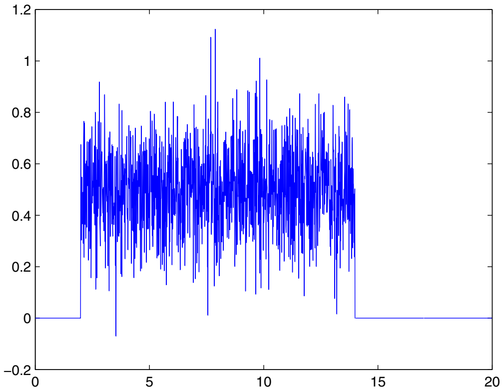

Fig. 1. Exogenous disturbance

When $\bar{\omega}(t) = 0$ and $\omega(t) = 0$, we have $\dot{V}(t) < 0$. Hence, the closed-loop system (4) is asymptotically stable.

Using the initial state $x(0) = 0$ and $V(\infty) > 0$, integrating (26) on $[0, \infty)$ leads to

$$||z(t)||^2 \leq \gamma_1^2 ||\omega(t)||^2 + \gamma_2^2 ||\bar{\omega}(t)||^2 \qquad (27)$$

Therefore, the required performance holds for system (4). The proof is completed.

5. SIMULATION RESULTS

Consider the following linearized longitudinal motion equation of the F-18 in Yang and Ye (2010)

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -0.194 & -0.03593 \\ -19.29 & -3.803 \end{bmatrix} \begin{bmatrix} \delta_E(t) \\ \delta_{PTV}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} \omega(t)$$

$$y(t) = \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \delta_E(t) \\ \delta_{PTV}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega(t)$$

Then, output $z(t)$ is chosen as

$$z(t) = \begin{bmatrix} -1 & 0 \\ 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -0.5 & 0.3 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_E(t) \\ \delta_{PTV}(t) \end{bmatrix}$$

w_d is white noise whose noise power and sample time are 0.1 and 0.01s. The disturbance is given as follows (See Fig. 1):

$$\omega(t) = \begin{cases} 0, & 0 \leq t \leq 2 \\ 0.5 + 0.05 w_d, & 2 < t \leq 14 \end{cases}$$

The actuator fault is supposed as

$$u^F(t) = (I - \Lambda(\rho(t)))u(t)$$

where

$$\rho_1(t) = \begin{cases} 0, & 0 \leq t \leq 2 \\ 0.4, & 2 < t \leq 6 \\ 0.2sin(4t - 24) + 0.4, & 6 < t \leq 12 \\ 0.4 & t > 12 \end{cases}$$

$$\rho_2(t) = \begin{cases} 0, & 0 \leq t \leq 2 \\ 0.3, & 2 < t \end{cases}$$

Choose $\Upsilon = [0.5\ 0; 0\ 2], \tau = 200, \Upsilon_f = [10\ 0; 0\ 40]$. Using standard adaptive law and the adaptive law presented in Theorem 1 respectively, the fault estimations of actuator

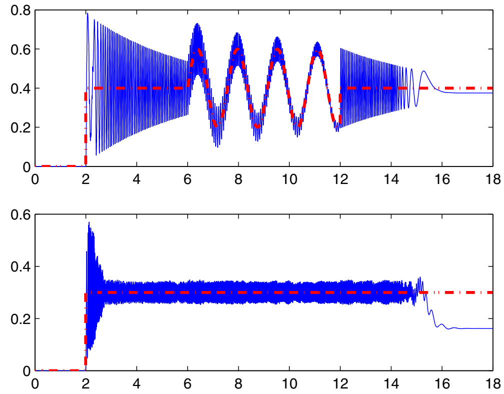

Fig. 2. Faults (dashed) and their estimation (solid) with the standard adaptive law

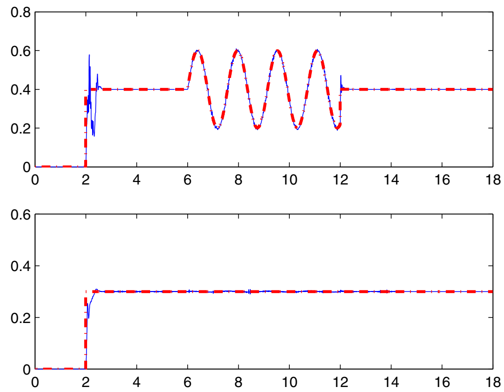

Fig. 3. Faults (dashed) and their estimation (solid) with adaptive law (8)

are shown in Fig.2 and Fig.3. From the Fig.3, it can be seen that the time-varying fault can be estimated fast with small error. Compared with Fig.2, it is easy to find that the adaptive law presented in this paper can generate better estimation of time-varying fault than standard adaptive law, which illustrates the modification term in F works well to avoid the high-frequency oscillations in the adaptive fault update law.

Then, the regulated output $z(t)$ of the closed-loop system using different adaptive estimations are presented in Fig.4 and Fig.5. The actuator fault occurs at $t = 2$, then the adaptive controllers are used to keep the performance and stability of the fault system. Fig.4 and Fig.5 show that different estimations lead to different performance of the closed-loop system. The controller proposed in Theorem 2 makes the fault system have better performance. From Fig. 5, it can be seen that the proposed adaptive controller can reduce the effect of the fault and ensure the stability of the closed-loop system.

Note that, the schemes proposed in Chen (2013); Yang and Ye (2010); Zhang and Jiang (2009) are no longer effective for the above time-varying actuator fault, because the actuator faults are considered as constant and piecewise constant, respectively.

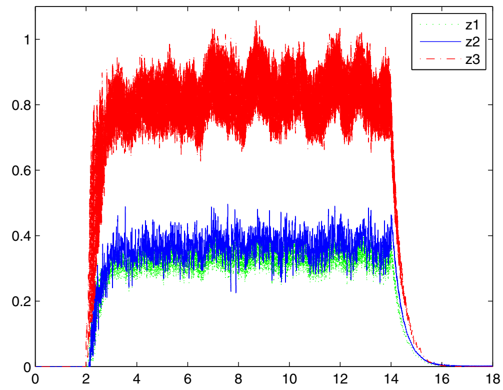

Fig. 4. The output $z(t)$ of the closed-loop system with standard adaptive law

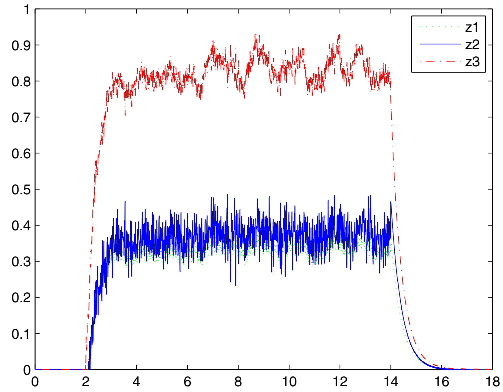

Fig. 5. The output $z(t)$ of the closed-loop system with adaptive law (9)

6. CONCLUSION

In this paper, the problem of adaptive fault-tolerant control for linear systems with time-varying actuator faults is considered. In order to estimated the fault fast, a novel estimator is designed and the adaptive law has a new architecture. Then, in the system dynamic equation, the fault estimation error is regarded as a part of the combined disturbance, and the adaptive fault-tolerant controller is designed so that the closed-loop system is robust for fault estimation error as well as the exogenous disturbance. Simulation results show that the proposed method can enhance the performance of fault estimation and accommodation obviously.

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