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Distributed Tracking for Multiple Lagrangian Systems Using Only Position Measurements. ⋆

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Abstract: This paper investigates the distributed tracking control problem for multiple Lagrangian systems under a general directed graph where only a portion of the agents have access to the desired time-varying trajectory. To overcome the problem that only positions are measured, a observer is designed to estimate the velocity for each follower. By employing the estimated states, the distributed observer-based controller is proposed using only position measurements. Furthermore, the condition for the distributed tracking problem on the directed graph is derived, such that the tracking errors and observer errors semi-globally converge to zero. Finally, simulation examples are provided to show the effectiveness of the proposed control algorithms.

Keywords: Multi-agent systems, Distributed control, Coordinated tracking, Lagrangian system, Velocity observer

1. INTRODUCTION

Distributed cooperative control of multi-agent systems has received considerable attention from various scientific communities in recent years due to the broad applications, such as autonomous underwater vehicles, sensor networks, and unmanned aerial vehicles (Ren and Beard [2005]; Li et al. [2010]). To the best of our knowledge, linear systems described by single/double integrators are focused on in most existing results on the distributed control problem (Hong et al. [2008]; Cao et al. [2013]). However, the dynamics of a large class of mechanical systems, including autonomous vehicles, walking robots and robotic manipulators are much more complicated than those linear models in practice (Sarlette et al. [2009]; Ren and Cao [2011]). Thus, it is significant and nontrivial to study the distributed coordination problem for multiple Lagrangian systems which can represent the motion of a great many mechanical systems.

Recently, some researchers have focused on the distributed cooperative control problems of the networked Lagrangian systems. The containment control problem were concerned in (Meng et al. [2010]) and (Mei et al. [2012]). The convergence of the followers' states to the dynamic convex hull was achieved with the aid of a distributed sliding-mode estimator and a non-singular sliding surface in (Meng et al. [2010]). While, the results in (Meng et al. [2010]) are only suitable in the case that the graph associated with the followers is undirected. In (Mei et al. [2012]), containment control problem was realized under directed graph which is more general than that in (Meng et al. [2010]). The problem of coordinated tracking to a reference trajectory for networked Lagrangian systems has been addressed in the following literatures. Under the assumption that the interaction topology is undirected, a distributed adaptive controller in (Dong [2011]) was designed to track the reference trajectory for multiple mechanical systems. The work in (Chen and Lewis [2011]) extended the topology requirements in (Dong [2011]) to directed graph containing a spanning tree by using neural network. However, the position and velocity tracking errors for each vehicle can only be guaranteed to be cooperatively uniformly ultimately bounded (UUB). Reference (Mei et al. [2011]) presented a class of model-independent sliding mode control law for networked Lagrangian systems with a dynamic leader such that the tracking errors converge to zero asymptotically at the cost of utilizing two-hop communication information.

It should be noted that the full state information are required to implement the proposed control algorithms in the above references. Nevertheless, in practice, velocity measurements may not always be measurable due to the strict constraints on the cost and space for installing the devices. Motivated by these facts, a distributed consensus algorithm was proposed for Lagrangian system in (Ren [2009]) to deal with the problem that the agents have no access to the velocity information. Differing from the undirected topology condition in Ren [2009], the authors in (Chen and Lewis [2012]) focused on distributed tracking

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problem for networked mechanical systems under directed graph. However, all mechanical systems synchronize to the reference system with bounded residual errors under the given control approach. The work in (Mei et al. [2012]) was extended by (Mei et al. [2013]), where the distributed containment control law was designed for multiple Lagrangian systems under a directed graph. Though the neighbors' velocity information is not used in (Mei et al. [2013]), the leaders are required to keep stationary and the velocity information of the follower itself is necessary.

In this paper, the distributed tracing control problem is discussed under a directed communication topology, where each agent has Lagrangian dynamics and only a subset of the followers have access to the time-varying desired trajectory. To overcome the absence of the velocity measurements, a novel observer is constructed to estimate the unavailable states. Based on the estimated states together with the position measurements, the distributed controller is designed such that the followers could track the leader asymptotically. The main contribution of this paper is that the coordinated tracking problem for Lagrangian systems is solved using the minimum amount of information with the aid of elaborately designed observer. It is worth mentioning that the observer and the control law proposed in this paper can be implemented without any velocity information, which is more general than the requirements in other existing literatures such as (Mei et al. [2013]).

2. PRELIMINARIES

A group of n mechanical systems labeled as agents 1 to n are considered as followers. The dynamics of the ith agent is described by Euler-Lagrange equation as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \quad i = 1, \cdots, n \qquad (1)$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of coriolis and centrifugal torques, $G_i(q_i)$ is the vector of gravitational torques, and $\tau_i \in \mathbb{R}^p$ is the vector of control torque on agent i.

The reference system, namely, the leader, is represented as

$$\ddot{q}_d(t) = f(t, q_d(t), \dot{q}_d(t)) \qquad (2)$$

where $f : \mathbb{R} \times \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}^p$ is a uniformly continuously differentiable vector-valued function, q_d ($\dot{q}_d$ and $\ddot{q}_d$, respectively) is the position, (velocity and acceleration, respectively) of the leader.

Assumption 1. The desired trajectory $q_d(t)$ is a bounded smooth signal with bounded derivative, i.e., $\|q_d(t)\| \le \gamma_p$, $\|\dot{q}_d(t)\| \le \gamma_v$, where γ_p and γ_v are positive constants.

In the following, we use $M(q) \triangleq \mathrm{diag}[M_1(q_1), \cdots, M_n(q_n)]$, $C(q,\dot{q}) \triangleq \mathrm{diag}[C_1(q_1,\dot{q}_1), \cdots, C_n(q_n,\dot{q}_n)]$, $G(q) \triangleq [G_1^T(q_1), \cdots, G_n^T(q_n)]^T$ as the vector form of $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$, $G_i(q_i)$, respectively.

For (1), the following properties are held. (Kelly et al. [2005]; Spong et al. [2006])

(1) For any i, there exist positive constants k_m and k_M such that $0 < k_m I_p \le M_i(q_i) \le k_M I_p$.
(2) $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew symmetric, i.e.

$$\xi^T[\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)]\xi = 0, \quad \forall \xi \in R^n \qquad (3)$$

(3) For all $q_i, x, y, z \in \mathbb{R}^p$, $C_i(q_i,\dot{q}_i)$ in (1) satisfies

$$C_i(q_i,x)y = C_i(q_i,y)x \qquad (4)$$

$$C_i(q_i,x+y)z = C_i(q_i,x)z + C_i(q_i,y)z \qquad (5)$$

Notations: The superscript T means transpose for real matrices. Let $\mathbf{1}_n$ denote the $n \times 1$ column vector with all entries equal to one. I_n represents the identity matrix of dimension n. $\|\cdot\|$ denotes the Euclidean norm.

We use $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ to represent the interactions among the agents 1 to n with the node set $\mathcal{V} \triangleq \{1, \cdots, n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The edge (i,j) denotes that the agent i transmits information to agent j in a directed graph, but not vice versa. In an undirected graph, an edge $(i,j) \in \mathcal{E}$ if agent i and j can receive information from each other. Here, it is assumed that there is no loop in the graph, i.e., $(i,i) \notin \mathcal{E}$. If an edge $(i,j) \in \mathcal{E}$, then we call node i is a neighbor of node j. Thus, the neighbor set of agent i is defined as $\mathcal{N}_i \triangleq \{j|(j,i) \in \mathcal{E}\}$. The root is a node that has directed paths to all the other nodes in a directed graph. A directed tree contains exactly one root and every other node has only one parent. A directed tree is called a directed spanning tree if it consists of all the nodes in a graph. A directed graph contains a directed spanning tree as long as one of its subgraphs is a directed spanning tree. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined such that $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Define the weighted in-degree of node i as $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$.

Definition 1. (Chen and Lewis [2012]). The auxiliary variables of position and velocity errors for systems i are defined as

$$s_i \triangleq q_i - \frac{1}{d_i + b_i}\Big(\sum_{j=1}^{n} a_{ij}q_j + b_i q_d\Big) \triangleq q_i - q_{si} \qquad (6)$$

$$\dot{s}_i \triangleq \dot{q}_i - \frac{1}{d_i + b_i}\Big(\sum_{j=1}^{n} a_{ij}\dot{q}_j + b_i \dot{q}_d\Big) \triangleq \dot{q}_i - \dot{q}_{si} \qquad (7)$$

where $b_i > 0$ if the leader is the neighbor of agent i and $b_i = 0$ otherwise.

The vector form of the error dynamics can be written as

$$s = ((L + B) \otimes I_p)(q - \mathbf{1}_n \otimes q_d) \qquad (8)$$

$$\dot{s} = ((L + B) \otimes I_p)(\dot{q} - \mathbf{1}_n \otimes \dot{q}_d) \qquad (9)$$

where $s = [s_1^T, s_2^T, \cdots, s_n^T]^T$, $q = [q_1^T, q_2^T, \cdots, q_n^T]^T$,

$$L = \begin{bmatrix} \frac{d_1}{d_1+b_1} & -\frac{a_{12}}{d_1+b_1} & \cdots & \cdots & -\frac{a_{1n}}{d_1+b_1} \\ -\frac{a_{21}}{d_2+b_2} & \frac{d_2}{d_2+b_2} & \cdots & \cdots & -\frac{a_{2n}}{d_2+b_2} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ -\frac{a_{n1}}{d_n+b_n} & \cdots & \cdots & -\frac{a_{n(n-1)}}{d_n+b_n} & \frac{d_n}{d_n+b_n} \end{bmatrix},$$

and $B = diag\{\frac{b_1}{d_1+b_1}, \cdots, \frac{b_n}{d_n+b_n}\}$.

Lemma 2. (Khoo et al. [2009]; Chen and Lewis [2012]). If the graph has a spanning tree with the leader as the root, then $s = 0$ and $\dot{s} = 0$ if and only if $q = \mathbf{1}_n \otimes q_d$ and $\dot{q} = \mathbf{1}_n \otimes \dot{q}_d$.

3. MAIN RESULTS

3.1 Observer Design

Before moving on, the following auxiliary variables are needed.

The position “estimation” error is given as

$$\tilde{q}_i = \hat{q}_i - q_i \tag{10}$$

Accordingly, the velocity estimation error is

$$\dot{\tilde{q}}_i = \dot{\hat{q}}_i - \dot{q}_i \tag{11}$$

And, the combined estimation error is defined as

$$\eta_i = \dot{\tilde{q}}_i + \alpha\tilde{q}_i \tag{12}$$

where α is a positive constant to balance the velocity and position convergence rate.

To cope with the problem that only the position measurement of each follower is available, a velocity observer for ith system is specially designed as follows, in which only the local position information is used.

$$\begin{aligned}\dot{\hat{q}}_i &= w_i - k_i\tilde{q}_i \\ \dot{w}_i &= M_i(q_i)^{-1}[\tau_i - C_i(q_i,\dot{\hat{q}}_i)(\dot{\hat{q}}_i + \alpha\tilde{q}_i) - G_i(q_i)] - l_i\tilde{q}_i\end{aligned} \tag{13}$$

where $k_i > \alpha$, $l_i = \alpha(k_i - \alpha)$ are user-chosen constant scalars, w_i is the intermediate variable.

Remark 1. Motivated by the idea that the observer is designed to make the storage function decrease in (Malgari and Drissen [2012]), where the tracking problem for single robot is considered. Hence, the desired reference trajectory is always available to the certain follower, while the case has been extended to multi-agent system in this paper, where not all the followers can get the reference trajectory.

Choose the sub Lyapunov function candidate

$$V_0 = \frac{1}{2}\sum_{i=1}^{n}\eta_i^T M_i(q_i)\eta_i + \frac{1}{2}\sum_{i=1}^{n}\tilde{q}_i^T\tilde{q}_i \tag{14}$$

Taking the time derivative of V_0 along (11)-(12) gives

$$\begin{aligned}\dot{V}_0 &= \frac{1}{2}\sum_{i=1}^{n}\eta_i^T\dot{M}_i(q_i)\eta_i + \sum_{i=1}^{n}\eta_i M_i(q_i)\dot{\eta}_i + \sum_{i=1}^{n}\tilde{q}_i^T\dot{\tilde{q}}_i \\ &= \frac{1}{2}\sum_{i=1}^{n}\eta_i^T\dot{M}(q_i)\eta_i + \sum_{i=1}^{n}\eta_i^T M_i(q_i)(\ddot{\tilde{q}}_i + \alpha\dot{\tilde{q}}_i) \\ &\quad + \sum_{i=1}^{n}\tilde{q}_i^T(\eta_i - \alpha\tilde{q}_i)\end{aligned} \tag{15}$$

From the observer (13), one has

$$M_i(q_i)\dot{w}_i = \tau_i - C_i(q_i,\dot{\hat{q}}_i)(\dot{\hat{q}}_i + \alpha\tilde{q}_i) - G_i(q_i) - l_iM_i\tilde{q}_i \tag{16}$$

and

$$M_i(q_i)\ddot{\hat{q}}_i = M_i(q_i)\dot{w}_i - k_iM_i(q_i)\dot{\tilde{q}}_i \tag{17}$$

Substituting equation (16) into (17), we can get

$$\begin{aligned}M_i(q_i)\ddot{\hat{q}}_i = &\ \tau_i - C_i(q_i,\dot{\hat{q}}_i)(\dot{\hat{q}}_i + \alpha\tilde{q}_i) - G_i(q_i) \\ &- l_iM_i\tilde{q}_i - k_iM_i(q_i)\dot{\tilde{q}}_i\end{aligned} \tag{18}$$

Also, it can be easily obtained from the system model (1) that

$$M_i(q_i)\ddot{q}_i = \tau_i - C_i(q_i,\dot{q}_i)\dot{q}_i - G_i(q_i) \tag{19}$$

In the light of (12), we have

$$M_i(q_i)\ddot{\tilde{q}}_i = M_i(q_i)\ddot{\hat{q}}_i - M_i(q_i)\ddot{q}_i \tag{20}$$

Then, replacing the two terms on the right side of equation (20) with equation (18) and (19), one has

$$\begin{aligned}M_i(q_i)\ddot{\tilde{q}}_i = &\ C_i(q_i,\dot{q}_i)\dot{q}_i - C_i(q_i,\dot{\hat{q}}_i)(\dot{\hat{q}}_i + \alpha\tilde{q}_i) \\ &- k_iM_i\dot{\tilde{q}}_i - l_iM_i\tilde{q}_i\end{aligned} \tag{21}$$

Substituting equation (21) into (15), and replacing $\dot{q}_i$ with $\dot{q}_i = \dot{\hat{q}}_i + \alpha\tilde{q}_i - \eta_i$, we have

$$\begin{aligned}\dot{V}_0 = &\ \frac{1}{2}\sum_{i=1}^{n}\eta_i^T\dot{M}_i(q_i)\eta_i - \sum_{i=1}^{n}\eta_i^T C_i(q_i,\dot{q}_i)\eta_i \\ &+ \sum_{i=1}^{n}\eta_i^T C_i(q_i,\dot{q}_i)\dot{\hat{q}}_i + \alpha\sum_{i=1}^{n}\eta_i^T C_i(q_i,\dot{q}_i)\tilde{q}_i \\ &- \sum_{i=1}^{n}\eta_i^T C_i(q_i,\dot{\hat{q}}_i)\dot{\hat{q}}_i - \alpha\sum_{i=1}^{n}\eta_i^T C_i(q_i,\dot{\hat{q}}_i)\tilde{q}_i \\ &- \sum_{i=1}^{n}(k_i - \alpha)\eta_i^T M_i(q_i)\eta_i - \alpha\sum_{i=1}^{n}\tilde{q}_i^T\tilde{q}_i \\ &+ \sum_{i=1}^{n}\tilde{q}_i^T\eta_i\end{aligned} \tag{22}$$

Then, by using property (3), the crossing terms in (22) follows that

$$\begin{aligned}&C_i(q_i,\dot{q}_i)\dot{\hat{q}}_i + \alpha C_i(q_i,\dot{q}_i)\tilde{q}_i - C_i(q_i,\dot{\hat{q}}_i)\dot{\hat{q}}_i - \alpha C_i(q_i,\dot{\hat{q}}_i)\tilde{q}_i \\ &= -C_i(q_i,\dot{\hat{q}}_i)(\eta_i - \alpha\tilde{q}_i) - \alpha C_i(q_i,\dot{\hat{q}}_i)\tilde{q}_i \\ &= -C_i(q_i,\dot{\hat{q}}_i)\eta_i + \alpha C_i(q_i,\dot{\hat{q}}_i)\tilde{q}_i - \alpha C_i(q_i,\dot{\hat{q}}_i)\tilde{q}_i\end{aligned} \tag{23}$$

Defining the variable $x \triangleq \{\dot{\tilde{q}}_i, e_i, \dot{e}_i\}^T$, where $e_i = q_i - q_d$, $\dot{e}_i = \dot{q}_i - \dot{q}_d$ and the set Ω_r :

$$\Omega_r \triangleq \left\{x : |\frac{1}{2}\dot{\tilde{q}}_i^T\dot{\tilde{q}}_i + \frac{1}{2}e_i^T e_i + \frac{1}{2}\dot{e}_i^T\dot{e}_i \le r,\ \ \forall t \ge 0\right\} \tag{24}$$

For $x \in \Omega_r$, it can be deduced that $\dot{\hat{q}}_i$ and $\dot{q}_i$ are bounded. we note that for $C_i(q_i,\dot{\hat{q}}_i)\zeta$ and $C_i(q_i,\dot{q}_i)\zeta$ are both zero at $\zeta = 0$. From the locally Lipschitz property of the coriolis and centrifugal torque, there exist positive constants ϵ_1 and ϵ_2 depending only upon r and γ_v such that

$$\|C_i(q_i,\dot{\hat{q}}_i)\zeta\| \le \epsilon_1\|\zeta\| \ \text{ and } \ \|C_i(q_i,\dot{q}_i)\zeta\| \le \epsilon_2\|\zeta\|. \tag{25}$$

Combining (23) and (25), $\dot{V}_0$ can be upper bounded by

$$\begin{aligned}\dot{V}_0 \le &-\Big[M_m(\min_i k_i - \alpha) - \epsilon_1\Big]\|\eta\|^2 - \alpha\|\tilde{q}\|^2 \\ &+ (1 + \alpha\epsilon_1 + \alpha\epsilon_2)\|\eta\|\|\tilde{q}\|\end{aligned} \tag{26}$$

3.2 Control Law Design and Stability Analysis

Based on the estimated states, the distributed control law using only position measurements is proposed as

$$\tau_i = M_i(q_i)(\hat{\ddot{q}}_{si} + s_i) + C_i(q_i,\dot{\hat{q}}_i)(\hat{\dot{q}}_{si} - s_i) - \beta_i(\hat{\dot{s}}_i + s_i) + G_i \tag{27}$$

where β_i are positive constant, $\hat{\ddot{q}}_{si}, \hat{\dot{q}}_{si}$ and $\hat{\dot{s}}_i$ are the estimates of the variables $\ddot{q}_{si}$, $\dot{q}_{si}$ and $\dot{s}_i$, respectively.

Accordingly, from (6) and (7), they have the following forms

$$\begin{aligned}\ddot{\hat{q}}_{si} &= \frac{1}{d_i+b_i}(\sum_{j=1}^{n} a_{ij}\ddot{\hat{q}}_j + b_i\ddot{q}_d)\\ \dot{\hat{q}}_{si} &= \frac{1}{d_i+b_i}(\sum_{j=1}^{n} a_{ij}\dot{\hat{q}}_j + b_i\dot{q}_d)\\ \hat{s}_i &= \dot{\hat{q}}_i - \dot{\hat{q}}_{si}\end{aligned} \tag{28}$$

Theorem 3. Suppose that the communication topology among the $n+1$ agents contains a directed spanning tree with the leader as the root and no loop exists. Under assumption 1, the tracking and velocity observation errors of multiple Lagrangian systems (1) are semi-globally converge to zero using the distributed observer (13) and control law (27), if parameters α, k_i, β_i are selected such that

$$\alpha > 0 \tag{29}$$

$$\min_i k_i > \frac{1}{4M_m\alpha}(1+\alpha\epsilon_1+\alpha\epsilon_2)^2 + \frac{\epsilon_1}{M_m} + \alpha \tag{30}$$

$$\min_i \beta_i > \frac{\phi_1^2[M_m(\alpha-\min_i k_i)+\epsilon_1] - \phi_1\phi_2[1+\alpha(\epsilon_1+\epsilon_2)] - \alpha\phi_2^2}{4\alpha[M_m(\min_i k_i-\alpha)-\epsilon_1]-(1+\alpha\epsilon_1+\alpha\epsilon_2)^2} \tag{31}$$

where ϕ_1 and ϕ_2 are defined (39).

Proof. Consider the sub-Lyapunov function candidate

$$V_1 = \frac{1}{2}\sum_{i=1}^{n}\xi_i^T M_i(q_i)\xi_i \tag{32}$$

where $\xi_i = \dot{s}_i + s_i$.

Before moving on, some preparations are firstly made to facilitate the calculation of $\dot{V}_1(t)$. On one hand,

$$\begin{aligned}M_i\ddot{\hat{q}}_{si} - M_i\ddot{q}_{si} &= \frac{M_i}{d_i+b_i}(\sum_{j\in\mathcal{N}_i} a_{ij}\ddot{\hat{q}}_j + b_i\ddot{q}_d - \sum_{j\in\mathcal{N}_i} a_{ij}\ddot{q}_j - b_i\ddot{q}_d)\\ &= \frac{M_i}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}(\ddot{\hat{q}}_j - \ddot{q}_j)\end{aligned} \tag{33}$$

Further, it follows from the observer (13) and the system model (1) that

$$\begin{aligned}&\ddot{\hat{q}}_j - \ddot{q}_j\\ &= \dot{w}_j - k_j\dot{\hat{q}}_j - \ddot{q}_j\\ &= M_j^{-1}\big[\alpha C_j(q_j,\dot{q}_j)\tilde{q}_j - C_j(q_j,\dot{q}_j+\dot{\hat{q}}_j)\eta_j\big]\\ &\quad -k_j\eta_j + \alpha^2\tilde{q}_j\end{aligned} \tag{34}$$

On the other hand,

$$\begin{aligned}&C_i(q_i,\dot{\hat{q}}_i)(\dot{\hat{q}}_{si}-s_i) - C_i(q_i,\dot{q}_i)(\dot{q}_{si}-s_i)\\ &= C_i(q_i,\dot{\hat{q}}_i)\dot{\hat{q}}_{si} - C_i(q_i,\dot{\hat{q}}_i)\dot{q}_{si} + C_i(q_i,\dot{\hat{q}}_i)\dot{q}_{si} - C_i(q_i,\dot{q}_i)\dot{q}_{si}\\ &\quad -C_i(q_i,\dot{\hat{q}}_i)s_i + C_i(q_i,\dot{q}_i)s_i\\ &= \frac{C_i(q_i,\dot{\hat{q}}_i)}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_j-\alpha\tilde{q}_j) - C_i(q_i,\dot{\hat{q}}_i)(\dot{s}_i+s_i)\\ &\quad +C_i(q_i,\dot{q}_i)(\eta_i-\alpha\tilde{q}_i)\end{aligned} \tag{35}$$

Thus, the time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= \frac{1}{2}\sum_{i=1}^{n}\xi_i^T\dot{M}_i\xi_i + \sum_{i=1}^{n}\xi_i^T M_i\dot{\xi}_i\\ &= \frac{1}{2}\sum_{i=1}^{n}\xi_i^T\dot{M}_i\xi_i - \sum_{i=1}^{n}\xi_i^T C_i(q_i,\dot{q}_i)\xi_i + \sum_{i=1}^{n}\xi_i^T\big[\tau_i -\\ &\quad C_i(q_i,\dot{q}_i)(\dot{q}_{si}-s_i) - M_i\ddot{q}_{si} - G_i(q_i) + M_i\dot{s}_i\big]\end{aligned} \tag{36}$$

Then, substituting the control law (27) into (36) and combining (33)-(35), it yields

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{n}\xi_i^T\frac{M_i}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}\Big\{M_j^{-1}\big[\alpha C_j(q_j,\dot{q}_j)\tilde{q}_j\\ &\quad -C_j(q_j,\dot{q}_j+\dot{\hat{q}}_j)\eta_j\big] - k_j\eta_j + \alpha^2\tilde{q}_j\Big\}\\ &\quad +\sum_{i=1}^{n}\xi_i^T\Big\{\frac{C_i(q_i,\dot{\hat{q}}_i)}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_j-\alpha\tilde{q}_j)\\ &\quad -C_i(q_i,\dot{\hat{q}}_i)(\dot{s}_i+s_i) + C_i(q_i,\dot{q}_i)(\eta_i-\alpha\tilde{q}_i)\Big\}\\ &\quad -\sum_{i=1}^{n}\xi_i^T(\Lambda_i-M_i)\xi_i - \sum_{i=1}^{n}\xi_i^T\Lambda_i\big[\eta_i-\alpha\tilde{q}_i\\ &\quad -\frac{1}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_j-\alpha\tilde{q}_j)\big]\end{aligned} \tag{37}$$

where we have used the fact that

$$\begin{aligned}\hat{s}_i &= \dot{\hat{q}}_i - \dot{\hat{q}}_{si} = \dot{q}_i - \dot{q}_{si} + (\dot{\hat{q}}_i - \dot{q}_i) + (\dot{q}_{si} - \dot{\hat{q}}_{si})\\ &= \dot{s}_i + \eta_i - \alpha\tilde{q}_i - \frac{1}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_j-\alpha\tilde{q}_j)\end{aligned} \tag{38}$$

Recalling (25), for $x \in \Omega_r$, we can obtain the following inequality after merging the similar terms.

$$\begin{aligned}\dot{V}_1 &\le -\big[\min_i\beta_i - M_M - (\epsilon_1+\epsilon_2)\big]\|\xi\|^2 + \alpha\Big[\frac{\sqrt{n}M_M}{M_m}\\ &\quad +\sqrt{n}M_M\alpha + \epsilon_1\sqrt{n} + \epsilon_2 + \max_i\beta_i\overline{\sigma}(\mathcal{L}+B)\Big]\|\xi\|\|\tilde{q}\|\\ &\quad +\Big[\frac{\sqrt{n}M_M}{M_m}(\epsilon_1+\epsilon_2) + \sqrt{n}M_M\max_i k_i + \epsilon_1\sqrt{n} + \epsilon_2\\ &\quad +\alpha\max_i\beta_i\overline{\sigma}(\mathcal{L}+B)\Big]\|\xi\|\|\eta\|\\ &\triangleq -\big[\min_i\beta_i - M_M - (\epsilon_1+\epsilon_2)\big]\|\xi\|^2 + \phi_1\|\xi\|\|\tilde{q}\|\\ &\quad +\phi_2\|\xi\|\|\eta\|\end{aligned} \tag{39}$$

where $\overline{\sigma}(X)$ denotes the maximum singular value of the matrix X.

According to (14) and (32), the Lyapunov function candidate for the overall observer-controller scheme $V = V_0 + V_1$ satisfies

$$\frac{1}{2}y^T\left(\underbrace{\begin{bmatrix}1&0&0\\0&M_m&0\\0&0&M_m\end{bmatrix}}_{P_1}\right)y \le V(y) \le \frac{1}{2}y^T\left(\underbrace{\begin{bmatrix}1&0&0\\0&M_M&0\\0&0&M_M\end{bmatrix}}_{P_2}\right)y \tag{40}$$

where $y = (\|\tilde{q}\|, \|\eta\|, \|\xi\|)^T$, and it is obvious that P_1 and P_2 are positive definite matrices.

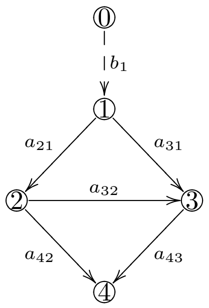

Figure 1. Network topology associated with the leader 0 and four followers. Here, an arrow from i to j denotes agent j can receive information form agent i, $i, j = 0, 1, \cdots, 4$.

Taking the time derivative of $V(y)$ yields

$$\dot{V}(y) = \dot{V}_0 + \dot{V}_1 \leq -y^T Q y \tag{41}$$

where

$$Q = \begin{bmatrix} \alpha & -\frac{1}{2}(1+\alpha\epsilon_1+\alpha\epsilon_2) & -\frac{1}{2}\phi_1 \\ -\frac{1}{2}(1+\alpha\epsilon_1+\alpha\epsilon_2) & M_m(\min_i k_i - \alpha) - \epsilon_1 & -\frac{1}{2}\phi_2 \\ -\frac{1}{2}\phi_1 & -\frac{1}{2}\phi_2 & \min_i \gamma_i - M_M - (\epsilon_1+\epsilon_2) \end{bmatrix}$$

We can get that $Q > 0$ if and only if (29)-(31) hold. Then, it is concluded from Theorem 4.10 in (Khalil [2002]) that $y = \mathbf{0}_3$ is semi-globally exponentially stable. On one hand, it follows from $\xi = \dot{s} + s$ that s and $\dot{s}$ both converge to zero. By Lemma 2, $q_i - q_d \to \mathbf{0}_p$ and $\dot{q}_i - \dot{q}_d \to \mathbf{0}_p$, as $t \to \infty$. On the other hand, we can also obtain η and $\tilde{q}$ converge to zero from (40) and (41), which implies $q_i - \hat{q}_i \to \mathbf{0}_p$ and $\dot{q}_i - \dot{\hat{q}}_i \to \mathbf{0}_p$ as $t \to \infty$, $i = 1, \cdots, n$. Namely, the accuarate estimation can be obtained by using the proposed velocity observer.

4. SIMULATION RESULTS

Numerical simulations are presented in this section to demonstrate the effectiveness of proposed control algorithm and observer. For simplicity, we choose four identical networked two-link manipulators modeled by Euler-Lagrange equation. The readers can refer to Kelly et al. [2005] for details. Let the masses of link 1 and link 2 be, respectively, $m_1 = 0.5kg$, and $m_2 = 0.4kg$, the lengths of link 1 and link 2 be, respectively, $l_1 = 0.4m$, and $l_2 = 0.3m$, the distances of the mass center of link 1 and link 2 between neighbors be, respectively, $l_{c1} = 0.2m$, and $l_{c2} = 0.15m$. In addition, the moments of inertia of link 1 and link 2 are, respectively, $J_1 = 0.0067kg.m^2$, and $J_2 = 0.003kg.m^2$.

To facilitate the comparison with Chen and Lewis [2012], the same interaction topology is chosen as shown in Figure 1, in which the leader is denoted as agent 0. The initial position of the followers are chosen as $q_i(0) = [(\pi/5)i, (\pi/4)i]^T rad$, and the velocity observations of the four followers $\dot{\hat{q}}(0)$ are selected as $[0.1, 0.1]^T rad/s$. Let the reference states of the leader be $q_d(t) = [\sin(t), \cos(t)]^T rad$, and hence the angular velocity be $\dot{q}_d(t) = [\cos(t), -\sin(t)]^T rad/s$. The control parameters are set as $\alpha = 2, \beta_i = 4$, and $k_i = 8$, $i = 1, \cdots, 4$.

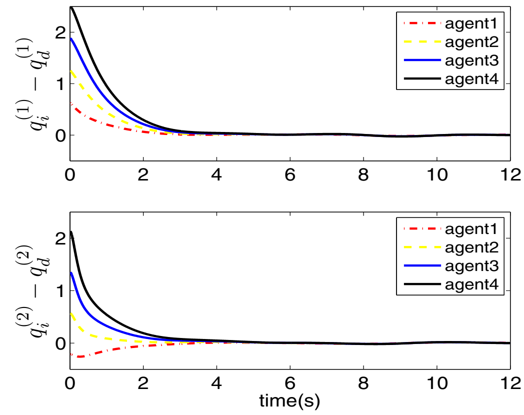

Figure 2. Angle tracking errors between the followers and the leader.

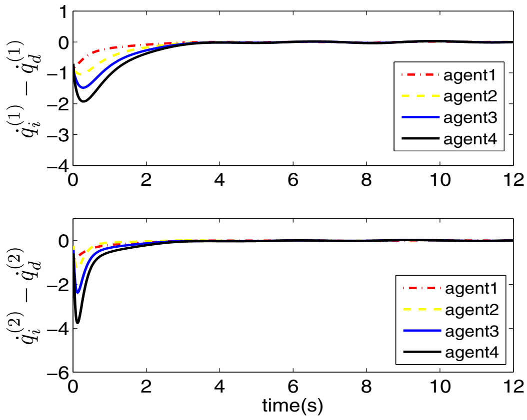

Figure 3. Angular velocity tracking errors between the followers and the leader.

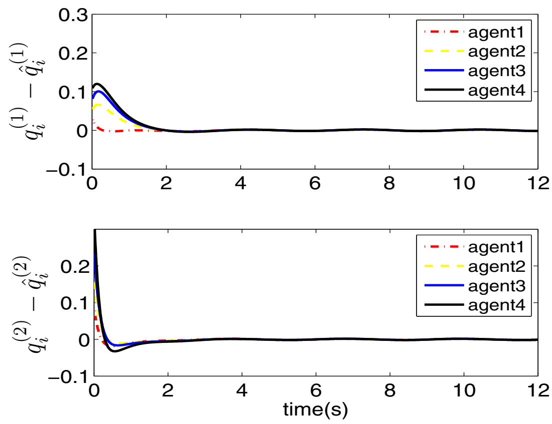

Figure 4. Angle errors between the observations and the true values.

Figure 2-3 illustrate the angle and angular velocity tracking errors during the tracking process. These figures indicated that the tracking errors converge to zero after about two second's regulation. From these figures, it is obvious that the distributed tracking to a dynamic leader is achieved effectively by using the proposed control law.

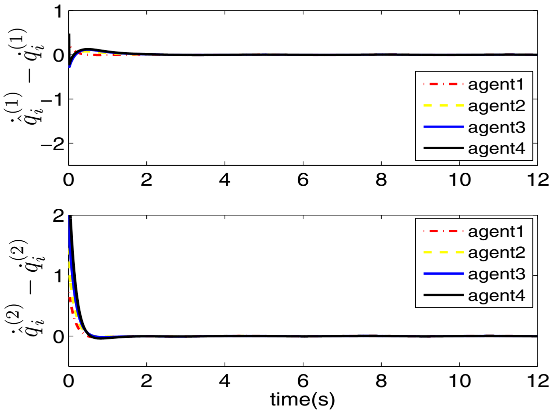

Figure 5. Angular velocity errors between the observations and the true values.

The performance of the observer is presented in Figure 4-5, which show the estimation errors converge to zero asymptotically. These figures imply that accurate estimations of the angle and angular velocity information can be obtained from the observer, which is of great significance for tracking evolution. All the simulation results demonstrate the effectiveness and feasibility of the proposed architecture of the observer-based controller.

In comparison with the results that the tracking errors are uniformly ultimately bounded in Chen and Lewis [2012], we realized more preferable tracking performance with lower control gains under the same topology condition.

5. CONCLUSION

The distributed tracking problem of multiple Lagrangian systems using only position measurements is studied under a directed graph. A novel observer is elaborately designed for each follower to cope with the unavailability of velocity information. Moreover, the distributed control strategy using only local interaction information is introduced. Sufficient conditions that guarantee the stability of the system are provided and the effectiveness of the proposed observer-based control law is verified by simulation results. For future research, the authors are interested in the challenging problem that the communication topology is switching with time-delays.

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