Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Discrete Time-Varying Internal Model-Based Control of a Novel Parallel Kinematics Multi-Axis Servo Gantry

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Abstract: This paper presents an internal model-based approach for trajectory tracking in the discrete time-varying setting. A newly designed parallel kinematics servo gantry is well suited for high precision contouring at high speed ranges. The discrete time-varying internal model-based control is developed for controlling the servo gantry system to track complicated trajectories generated by linear time-varying systems. Based on a novel parallel time-varying internal model structure, a low order robust discrete time-varying stabilizer is synthesized. The proposed tracking control architecture is deployed on the novel parallel kinematics servo gantry system to achieve high precision contour tracking performance for frequency-varying signals generated by high order time-varying autonomous system.

Keywords: Mechatrnics; Tracking Control; Robust Control

1. INTRODUCTION

With the rapid increase of advanced machatronics, the design of a motion stage and high precision tracking control are key enabling techniques for improving the trajectory and contour accuracy. There are wild applications in high precision machine tools [1, 6], and energy based direct writing technology [4, 5].

Among these applications, tracking control algorithm also play important role to achieve high performance especially for complicated trajectories high speed ranges. Note that significant efforts have been devoted to various aspects of tracking control theory in the past several decades. As one of the most investigated approaches, the internal model-based control method has emerged as a fundamental technique for tracking and/or rejecting periodic signals generated by autonomous systems. Although the internal model-based control theory for LTI (Linear Time-Invariant) systems has been well established [3], the design for LTV (Linear Time-Varying) systems remains open, due to the fundamental challenges of constructing a time-varying internal model to render the error-zeroing subspace invariant, and a robust time-varying stabilizer. We refer to [11] and the reference therein for some general results of internal model-based design for LTV systems.

More recently, a systematic design method for a time-varying internal model unit has been provided in [9, 12] in both input/output and state-space settings. However, the implementations of the above method still face great challenge lies in a low-order robust stabilizer design. To tackle this difficulty some attempts have been made via LPV (Linear Parameter-Varying) based approaches in continuous time settings, for example [13, 7, 14]. Due to the obvious benefits to reduce the computational burdens and avoid numerical issues, it is desirable to design the internal model-based control for tracking sophisticated signals in a discrete time setting. Very recently, discrete time tracking controller designs have been proposed in [8, 15], which are non-trivial extension of the results in continuous time settings [7, 14].

In the present work, we investigate the discrete time-varying internal model-based control by resorting to the recently developed parallel structure for time-varying in-

⋆ We would like to acknowledge the support from the NSFC (Grant no. 61004004, 61327003, and 51005205), Specialized Research Fund for the Doctoral Program of Higher Education (Grant no. 20100002120043), SRF for ROCS SEM, and China Fundamental Research Funds for the Central Universities (Grant no. 10062013YWF13-ZY-68).

ternal model control [14], which can be considered as the counter part of the continuous time results in [14]. The robust performance of the stabilizer against disturbances is also discussed. The proposed tracking algorithm is then deployed on a newly developed parallel kinematics X-Y servo gantry system for complex trajectory (by high order exosystem) tracking and high precision contouring.

The rest of the paper is organized as follows: In Section 2, we present the modeling of the novel parallel kinematics servo gantry system. In Section 3, the controller structure and some preliminaries on a time-varying internal model design are briefly discussed. In Section 4, the discrete time-varying robust stabilizer design is discussed based on the parallel connection with the internal model unit. The experimental results are given in Section 5 to demonstrate the performance of proposed control algorithm on the designed gantry system, followed by conclusions.

2. DESIGN AND MODELING OF A NOVEL HIGH SPEED PARALLEL KINEMATICS SERVO GANTRY

Due to the inherent unbalanced axial dynamics of serial configuration motion stages, we in this work consider a newly designed parallel kinematics gantry shown in Figure 2 (Parallel Motion Stage). Its major advantages lie in symmetric configuration and balanced axial dynamics (the detailed design is referred to [16]). Therefore by design the servo gantry is suitable to achieve high speed and high precision contour performance.

The performance demand of the above designed mechanism is as follows: The platform has a workspace of $15 \times 15mm^2$. The system is required to track high-frequency signals up to 50 Hz in each axis, hence the platform can reach to the maximum velocity of 750 mm/s.

2.1 Analytical results

The XY parallel servo gantry consists of six linear guideways. To meet the design requirements of working frequency of 50 Hz, we need to avoid resonances in the frequencies range of interest. For the analysis of the dynamic behaviors, a linear spring was widely used to simulate the contact feature of the ball between the carriage and guide way.

We present the modeling of the servo gantry system below. The main parts of the servo gantry that impacts dynamic characters is the internal contact of the linear guideway. The mathematical model of each contact structure is assumed as a spring-mass system, where the rail and carriage are treated as rigid bodies and connect a series of linear spring elements with adequate axial stiffness. We use 22 groups of perpendicular distribution of the linear springs to simulate one guideway and each degree of freedom are simplified to a two-degree-mass-spring system.

From Lagrangian model we have the motion equations corresponding to yaw degree of freedom, i.e. the torque in yaw direction of the motion stage:

$$\begin{aligned} &J_{1z}\ddot{\psi}_1 + K_c\psi_1 \sum_{j=5}^{8}\sum_{i=1}^{N_b} \bar{l}_{ji}^2 \\ &-K_c\psi_2 \sum_{i=1}^{N_b}[(\bar{l}_{5i} - l_b)^2 + (\bar{l}_{6i} - l_b)^2] = 0\,, \end{aligned} \tag{1}$$

Table 1. Nomenclature

Symbol	Definition
K_c	Contact stiffness
N_b	The number of contacted rolling ball
J_{1z}	The rotational inertia of the motion stage (MS) in yaw direction
J_{2z}	The rotational inertia of the decoupling components (DC) in yaw direction
$\bar{l}_{ji}$	The distance between the i-th rolling ball of the j-th guide way of MS (DC) and the center of the mass of DC (MS)
l_b	The distance between MS and DC
ψ_i	The yawing angle of MS and DC

and the torque in yaw direction of the decoupling components:

$$\begin{aligned} &J_{2z}\ddot{\psi}_2 + K_c\psi_2 \sum_{j=1}^{6}\sum_{i=1}^{N_b} \bar{l}_{ji}^2 \\ &-K_c\psi_1 \sum_{i=1}^{N_b}[(\bar{l}_{5i} - l_b)^2 + (\bar{l}_{6i} - l_b)^2] = 0\,. \end{aligned} \tag{2}$$

By substituting the geometric parameters into above equations, we obtain natural frequency in yaw as 121.2 Hz (see [16] in details).

2.2 Dynamical modeling

The major dynamics of the servo gantry system consists of the current amplifier and the mechanical part. For the current amplifier, two custom built linear power amplifier modules with bandwidth 2000 Hz are employed to generate driving currents, which is well above the frequency range of the motion control. Hence the dynamics from control voltage to the amplifier current can be assumed by a DC gain. The mechanical part of each axis can be characterized by:

$$\ddot{x}_i + \frac{c_i}{m_i}\dot{x}_i + \frac{k_i}{m_i}x_i = \frac{F_i}{m_i} - \frac{F_{c,i}}{m_i}\,\mathrm{sign}(\dot{x}_i) + \bar{\Delta}_i\,,$$

where $i = x, y$, and m_i, c_i, k_i represents the moving mass, the equivalent damping coefficient, and the equivalent stiffness in i axis respectively, and $F_{c,i}$ (for compensate the nonlinear parts of the damping) and F_i are Coulomb viscous friction and the driving force of the VCM (voice coil motor) in i axis respectively, and $\bar{\Delta}_i$ represents unmodeled lumped nonlinear dynamics such as magnetic lag, eddy current loss, and other disturbances in i axis, but the Coulomb viscous friction is far greater than the impact of the modeling accuracy of $\bar{\Delta}_i$.

Note that $F_i = K_{F_i}I_i$, where K_{F_i} is the force constant and I_i is the coil current of each axis. Also notice that the Coulomb viscous friction term $F_{c,i}$ can be experimentally compensated. Therefore the system dynamics from the control voltage to the position measurement can be characterized by the following transfer functions:

$$G_i(s) = \frac{X_i(s)}{I_i(s)}\frac{I_i(s)}{U_i(s)} = \frac{K_{F_i}K_{ui}/m_i}{s^2 + (c_i/m_i)s + k_i/m_i}\,. \tag{3}$$

The natural frequency of each axis can be experiemntally obtained as $f_x = 115.1$ Hz and $f_y = 120.5$ Hz, which agree well with the theoretical ones (121.2 Hz). Then the discrete-time representation of (3) reads as

$$G_i(z) = \frac{z^{-1}b_1^i + z^{-2}b_0^i}{1 + z^{-1}a_1^i + z^{-2}a_0^i}\,, \tag{4}$$

where a_j^i, and b_j^i, $i = X,\ Y$ axis, $j = 0,\ 1$ are readily obtained by the discretization.

3. CONTROLLER STRUCTURE AND PRELIMINARIES

The discrete time-varying tracking control of the above gantry system can be formulated in the following form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \\ e(k) &= y(k) + r(k)\,, \end{aligned} \tag{5}$$

with plant state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}$, output $y \in \mathbb{R}$, reference $r \in \mathbb{R}$, and regulated error $e \in \mathbb{R}$ satisfying:

Assumption 3.1. The triplet $(A,\ B,\ C)$ is controllable and observable.

The reference $r(k)$ to be tracked is generated by an LTV autonomous system, called by exosystem, of the form

$$\begin{aligned} w(k+1) &= S(k)w(k) \\ r(k) &= Q(k)w(k) \end{aligned} \tag{6}$$

with exogenous state $w \in \mathbb{R}^\rho$. The exosystem under consideration is characterized by the following assumption:

Assumption 3.2. The trajectories $w(k)$ in the forward and backward directions of time are stable in the sense of Lyapunov, and the pair $(Q(\cdot),\ S(\cdot))$ is uniformly observable.

The goal is to achieve bounded closed-loop trajectories and $\lim_{k\to\infty} e(k) = 0$.

Remark 3.1. The above formulation represents a typical tracking problem for a wild class motion control applications where the output y of the actuation system is required to track a reference r which is periodic with respect to rotational angle or linear position but not periodic to temporal variable due to a varying speed. In this circumstance the reference r can be treated as an output of an autonomous time-varying system (see for instance [13] for detailed explanation). It is worth noting that either plant model (5) or exosystm (6) is time-varying, the internal model is time-varying.

The problem of *asymptotically tracking* complicated signals generated by the time-varying exosystem (6) has yet to be completely solved due to its difficulty of time-varying internal model and robust stabilizer. Towards a real-time implementation orientated solution, a novel controller architecture is proposed in Figure 1, which consists of a time-varying internal model unit and a time-varying robust stabilizer.

And the design of the time-varying internal model can be constructed by a two-step [9, 12]. Along this thread, the design of a simple and low order stabilizer for the resulting time-varying systems remains a great challenge. Aiming at resolving this difficulty, we consider a novel compensator structure where the internal model unit and the stabilizer are interconnected in parallel (see Figure 1). The detailed design of the time-varying internal model can be referred to [12] (see also the illustrative example in Section 5 for the reader's convenience).

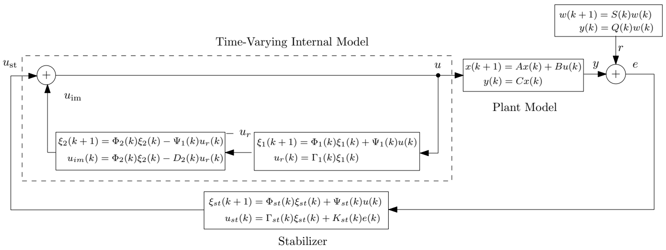

Fig. 1. The block diagram of a parallel connected time-varying internal model-based controller.

4. DISCRETE LTV ROBUST STABILIZER

With the internal model unit available, the remaining task is to design a time-varying stabilizer for the augmented system (the plant model and time-varying internal model). Note that the interconnection of the plant model (5) and the internal models is not controllable as

$$\tilde{\xi}_1(k+1) = A_{\mathrm{o}}\,\tilde{\xi}_1(k)$$

with $\tilde{\xi}_1 = \xi_1 - x$. Therefore the interconnected system is stabilizable only if the above system is asymptotically stable. Since the plant (actuator) is design by ourselves, we can assume that the plant model is already stabilized.

Assumption 4.1. Suppose that system $x(k+1) = A_{\mathrm{o}}\,x(k)$ is asymptotically stable.

The augmented system to be stabilized reads as

$$\begin{pmatrix} \xi_2(k+1) \\ x(k+1) \end{pmatrix} = \begin{pmatrix} \Phi_2(k) & -\Psi_2(k)C_{\mathrm{o}} \\ B_{\mathrm{o}}\Gamma_2(k) & A_{\mathrm{o}} - B_{\mathrm{o}}D_2(k)C_{\mathrm{o}} \end{pmatrix} \begin{pmatrix} \xi_2(k) \\ x(k) \end{pmatrix} + \begin{pmatrix} 0 \\ B_{\mathrm{o}} \end{pmatrix} u_{\mathrm{st}}(k) \tag{7}$$

where quadruplet $(\Phi_2(\cdot),\ \Psi_2(\cdot),\ \Gamma_2(\cdot),\ D_2(\cdot))$ is in controller canonical form by design.

It is worth noting that if one considers a serial interconnection between the internal model and stabilizer, the order of the dynamic stabilizer is of $\dim(x) + \dim(\xi_2)$. The rest of the paper shows that a low order discrete time-varying stabilizer of $\dim(x)$ can be designed by leveraging on the proposed parallel compensator structure.

4.1 A low order time-varying stabilizer

We start of providing the following lemma on stabilization of system (7).

Lemma 4.1. If the above assumptions hold, a sufficient condition of stabilizing system (7) is to stabilize the following system

$$\begin{aligned} x_{\mathrm{o}}(k+1) &= A(k)x_{\mathrm{o}} + B(k)u_{\mathrm{st}}(k) \\ y(k) &= (1\ 0\ \cdots\ 0)\ x_{\mathrm{o}}(k)\,, \end{aligned} \tag{8}$$

where

$$A(k) = \begin{pmatrix} -\alpha(k) & \begin{matrix} I \\ 0 \end{matrix} \end{pmatrix}, \quad B(k) = B\,,$$

with $\alpha(k)$ collecting the coefficients of the first column of exosystem in its observer canonical form $S_{\mathrm{o}}(k)$.

The proof is omitted due to the space limit.

Remark 4.1. The above lemma shows that by leveraging on the proposed structure, the stabilization of the augmented system (7) can be reduced to a system with order of dim (x). Moreover the resulting system to be stabilized is of observer canonical form, which further simplifies the observer design.

Notice that system (8) can be split as

$$\begin{aligned} x_1(k+1) &= A_{11}(k)x_1(k) + A_{12}x_b(k) + B_1 u_{\mathrm{st}}(k) \\ x_b(k+1) &= A_{21}(k)x_1(k) + A_{22}x_b(k) + B_2 u_{\mathrm{st}}(k) \\ y(k) &= x_1(k)\,, \end{aligned} \tag{9}$$

where x_1 is the first state of x_{o}, and x_b collects the rest elements of x_{o}, and $A_{11}(k) = -\alpha_{\rho-1}(k)$, $A_{12} = (10\cdots0)$,

$$A_{21}(k) = \begin{pmatrix} -\alpha_{\rho-2}(t) \\ \vdots \\ -\alpha_0(t) \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix},$$

$$B_1 = b_{\rho-1}\,, \quad B_2 = (b_{\rho-2} \ \cdots \ b_0)'\,.$$

In light of the special structure of system (9), and the availability of y for feedback, we introduce a reduced order observer of the state x_b as follows

$$\begin{aligned} \hat{z}(k+1) &= (A_{22} - HA_{12})\hat{z}(k) + (B_2 - HB_1)u_{\mathrm{st}}(k) \\ &\quad + ((A_{22} - HA_{12})H + A_{21} - HA_{11}(k))y(k) \\ \hat{x}_b(k+1) &= \hat{z}(k) + Hy(k)\,, \end{aligned} \tag{10}$$

with $z = x_b - Hx_1$ and $\hat{z} = \hat{x}_b - Hx_1$.

Remark 4.2. The output injection gain H can be chosen as a time-invariant vector by leveraging on the time-invariant observable pair $(A_{12}\,,\, A_{22})$.

With the estimation of $x_b(k)$, we further apply $u_{\mathrm{st}}(k) = K(k)\begin{pmatrix} x_1(k) \\ \hat{x}_b(k) \end{pmatrix}$ to stabilize system (9). That is, make

$$x_{\mathrm{o}}(k+1) = (A(k) + BK(k))x_{\mathrm{o}}(k) \tag{11}$$

asymptotically stable.

In what follows, we discuss the discrete time-varying stabilization. It is known that the feedback gain $K(k)$ can be designed by finding a symmetric matrix $P(k) > 0$ satisfying

$$\begin{pmatrix} Q & (A_{\mathrm{c}}Q)' \\ A_{\mathrm{c}}Q & Q(k+1) \end{pmatrix} > 0\,. \tag{12}$$

where the index k is dropped in the corresponding matrices, and $A_{\mathrm{c}}(k) = A(k) + BK(k)$, $Q = P^{-1}$. Notice that the presence of both indices k and $k+1$ in inequality (12) poses a computational difficulty. A particularly interesting method in the literature is to use a polytope based representation for the description of linear parameter-varying systems. See references [2] and the references therein. Following the same route, we assume:

Assumption 4.2. The terms of $A(k)$ are parameter σ-dependent, and σ belongs to a polytope, i.e.

$$A(k) = A(\sigma(k)) = \sum_{i=1}^{N} \sigma_i(k)A_i\,,$$

where $\sigma_i(k) \geq 0\,,\quad \sum_{i=1}^{N} \sigma_i(k) = 1$, and A_i's are constant matrices.

Accordingly, $A_{\mathrm{c}}(k) = \sum_{i=1}^{N} \sigma_i(k)A_i(k) + Bu(k)$. Note that B is constant in our design,ence we can apply the following result to design the feedback gain $K(k)$.

Lemma 4.2. [2] If there exist symmetric matrices $Q_i > 0$, $Q_j > 0$, and matrices G_i, $\bar{K}_i$, and the following inequality

$$\begin{pmatrix} G_i + G_i' - Q_i & (A_iG_i + B\bar{K}_i)' \\ A_iG_i + B\bar{K}_i & Q_j \end{pmatrix} > 0 \tag{13}$$

for all $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, N$ are feasible, then $K(k) = \bar{K}(k)P(k)$, with $\bar{K}(k) = \sum_{j=1}^{N} \sigma_j(k)\bar{K}_j$ and $P = \sum_{j=1}^{N} \sigma_j(k)P_j$, is the gain to stabilize system (11).

By solving (13), the $(k+1)$-index in matrix Q of (12) can be removed and the feedback gain $K(k)$ can be obtained.

Now we are in position to state the observer-based time-varying stabilizer.

Theorem 4.3. If all the above assumptions hold, then the augmented system (7) can be stabilized by the following

$$\begin{aligned} \hat{z}(k+1) &= (A_{22} - HA_{12})\hat{z}(k) + (B_2(k) - HB_1(k))u_{\mathrm{st}}(k) \\ &\quad + ((A_{22} - HA_{12})H + A_{21} - HA_{11}(t))\,e(k) \\ \hat{x}_b(k) &= \hat{z}(k) + He(k) \\ u_{\mathrm{st}}(k) &= K_1(k)x_1(k) + K_2(k)\hat{x}_b(k)\,. \end{aligned} \tag{14}$$

And the gain $K(k) = (K_1(k)\ K_2(k))$ can be explicitly given if matrix inequalities (13) are solvable.

The proof is omitted due to the space limit.

4.2 Robust stabilization

Considering disturbances and model uncertainties existed in reality, we further investigate the robust stabilization as

$$\begin{aligned} d(k) &= \Delta y(k) \\ e(k) &= y(k) + d(k)\,, \end{aligned}$$

and assume that $\|\Delta\| \leq \delta_d$. Note that d represents external disturbances or model uncertainties. More rigorous analysis with respect to robust control against system uncertainties deserves a separate study.

In the above setting, the result in Lemma 4.2 can be extended to the robust case, that is,

Lemma 4.4. If there exist symmetric matrices $Q_i > 0$, $Q_j > 0$, and matrices G_i, $\bar{K}_i$, and the following inequality

$$\begin{pmatrix} -G_i - G_i' + Q_i & \star & \star & \star \\ A_iG_i + B\bar{K}_i & -Q_j & \star & \star \\ CG_i & 0 & -I & \star \\ CG_i & 0 & 0 & -(\gamma^2 - 1)I \end{pmatrix} < 0 \tag{15}$$

for all $i = 1\,, 2\,, \cdots, N$ and $j = 1\,, 2\,, \cdots, N$ are feasible, then $K(k) = \bar{K}(k)P(k)$ with $\bar{K}(k) = \sum_{i=1}^{N} \sigma_i(k)\bar{K}_i$ and $P = \sum_{i=1}^{N} \sigma_i(k)P_i$ is a feedback gain to stabilize (11).

The proof is omitted due to the space limit.

With the above robust stabilization design, the discrete time-varying compensator can be synthesized accordingly.

5. EXPERIMENTS

In this section, we investigate the tracking performance of the newly designed parallel kinematics X-Y servo gantry

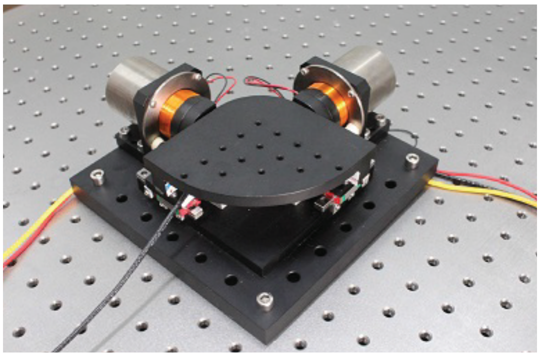

Fig. 2. The parallel kinematics VCM actuated X-Y servo gantry.

system by the proposed control algorithm. The experimental setup is depicted in Figure 2, where two linear optical encoders with 50 nm resolution are adopted for position measurement.

One benchmark is to let the gantry track swept frequency references. In specific, we adopt the reference signals containing two frequency-varying "harmonics" generated by the following 4-th order discrete exosystem:

$$S(k) = \begin{pmatrix} \cos(\omega_1(k)T_s) & \sin(\omega_1(k)T_s) & 0 & 0 \\ -\sin(\omega_1(k)T_s) & \cos(\omega_1(k)T_s) & 0 & 0 \\ 0 & 0 & \cos(\omega_2(k)T_s) & \sin(\omega_2(k)T_s) \\ 0 & 0 & -\sin(\omega_2(k)T_s) & \cos(\omega_2(k)T_s) \end{pmatrix}$$

$$Q(k) = (1\ 0\ 1\ 0)\ ,$$

where the time-varying terms are induced by $\omega_{i,\min} \leq \omega_i(k) \leq \omega_{i,\max}$, $i = 1, 2$. The corresponding observer canonical form of the above system reads as:

$$S_o(k) = \begin{pmatrix} -\alpha_3(k) & 1 & 0 & 0 \\ -\alpha_2(k) & 0 & 1 & 0 \\ -\alpha_1(k) & 0 & 0 & 1 \\ -\alpha_0(k) & 0 & 0 & 0 \end{pmatrix}, \quad Q_o = (1\ 0\ 0\ 0)\ ,$$

where the explicit expressions of the time-varying coefficients $\alpha_i(k)$, $i = 1, \cdots, 4$ are not shown here due to the space limit. In this study, the angular frequency varies from 6π to 36π, with $\omega_i(k) = \omega_i(0) + T_s\, k\, \Delta\omega_i$, with $\omega_1(0) = 6\pi$, $\omega_2(0) = 8\pi$.

The linear part of system model (3) admits the following discrete state-space representation:

$$A_o = \begin{pmatrix} -a_1 & 1 \\ -a_0 & 0 \end{pmatrix}, \quad B_o = \begin{pmatrix} b_1 \\ b_0 \end{pmatrix}, \quad C_o = (1\ 0)\ ,$$

where the nominal values of coefficients are $a_1 = -f_{11} - f_{22}$, $a_0 = f_{11}f_{22} - f_{12}f_{21}$, $b_1 = g_1$, $b_0 = f_{12}g_2 - f_{22}g_1$, with

$$F = e^{A_s T_s} = \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}, \quad G = (e^{A_s T_s} - I)A_s^{-1}B_s = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix},$$

$$A_s = \begin{pmatrix} -c_x/m_x & 1 \\ -k_x/m_x & 0 \end{pmatrix}, \quad B_s = \begin{pmatrix} 0 \\ K_F K_{ui}/m_x \end{pmatrix}.$$

The sampling time is chosen as $T_s = 5\mathrm{e}-4$ secs.

By design, the internal model subsystem 1 reads as

$$\xi_1(k+1) = A_o\,\xi_1(k) + B_o u(k)$$

and the internal model subsystem 2 reads as

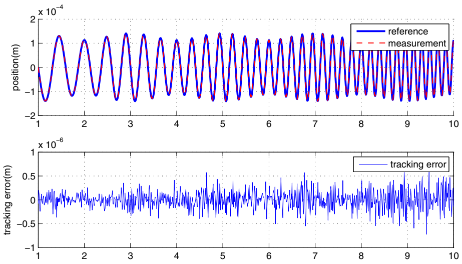

Fig. 3. Tracking references containing 2 varying "harmonics" in experiments.

$$\begin{aligned} \xi_2(k+1) &= \Phi_2(k)\xi_2(k) - \Psi_2 C_o \xi_1(k) \\ u_{\mathrm{im}}(k) &= \Gamma_2(k)\xi_2(k) + D_2(k)[-C_o\xi_1(k)] \end{aligned}$$

where

$$\Phi_2(k) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -q_2(k) \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad D_2(k) = p_3(k)\,,$$

$$\Gamma_2(k) = (p_0(k)\ \ p_1(k-1)\ \ p_2(k-2) - p_3(k)q_2(k))\ ,$$

and $p_i(k)$, $i = 1, \cdots, 3$, and $q_2(k)$ are calculated by solving the following algebraic Sylvester equation:

$$\begin{pmatrix} a_1 & 1 & b_1 & 0 & 0 & 0 \\ a_0 & a_1 & b_0 & b_1 & 0 & 0 \\ 0 & a_0 & 0 & b_0 & b_1 & 0 \\ 0 & 0 & 0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & 0 & 0 & b_0 \end{pmatrix} \begin{pmatrix} 1 \\ q_2(k) \\ p_3(k) \\ p_2(k) \\ p_1(k) \\ p_0(k) \end{pmatrix} = \begin{pmatrix} \alpha_3(k) & 1 \\ \alpha_2(k) & \alpha_3(k) \\ \alpha_1(k) & \alpha_2(k) \\ \alpha_0(k) & \alpha_1(k) \\ 0 & \alpha_0(k) \end{pmatrix} \begin{pmatrix} 1 \\ q_2(k) \end{pmatrix}.$$

The system to be stabilized reads as (8), where

$$A(k) = A(\omega(k)) = \begin{pmatrix} -\alpha_3(k) & 1 & 0 & 0 \\ -\alpha_2(k) & 0 & 1 & 0 \\ -\alpha_1(k) & 0 & 0 & 1 \\ -\alpha_0(k) & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_0 \\ 0 \\ 0 \end{pmatrix}.$$

In this example, the polytopic representation of $A(\omega(k))$ can be written as $A(\omega(k)) = \sum_{i=1}^{8} \sigma_i(k)A_i$. The reduced order observer for state x_o reads as (10), where the output injection gain $H = [h_2\, h_1\, h_0]'$ is invariant. The feedback gain $K(\sigma)$ can be designed by solving (15).

5.1 Experimental results

The same control algorithm is deployed on the servo gantry experimentally, where a dSPACE® 1103 rapid prototyping system is utilized for controller implementation and real time control executions with the sampling rate of $2k$ Hz.

The first experiment is conducted to test the tracking performance of swept frequency references (containing two varying harmonics) for each single axis as depicted in Figure 3, where the tracking error of 0.23 μm in *RMS* (Root Mean Square) value is achieved for the frequency-varying periodic reference with *RMS* value of 71 μm. Note that we use *RMS* value instead of peak-peak value to quantify the performance due to the existence of various disturbances and noises.

Then the performance of tow-axis contouring is tested. As shown in Figure 4, circular contour tracking error is

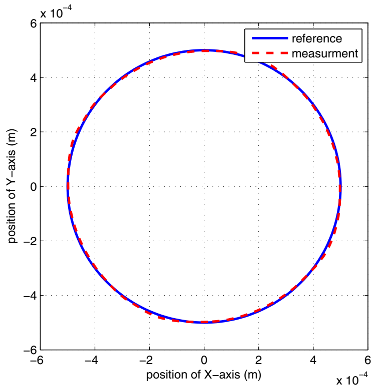

Fig. 4. Tracking a circular contour in experiments.

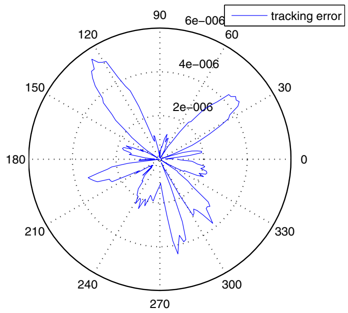

Fig. 5. Tracking error of circle in experiments.

2.1 μm of 500 μm radius, and the contour error can be further illustrated by the cross-axis error plot in the polar coordinates (Figure 5). It can be seen that the contour error is symmetric which agrees well with the design of the paralleled kinematics mechanism.

6. CONCLUSIONS

In this paper, we have proposed a discrete time-varying internal model-based control for tracking complicated references generated by linear time-varying systems. Based on a novel parallel connection structure of the time-varying compensator, we have developed a low-order robust stabilizer in the discrete setting. Then we have deployed the proposed control design on a newly design parallel kinematics servo gantry system. The experimental results demonstrate that by design the parallel servo gantry is well suited for high precision contour tracking and the proposed tracing controller achieves good tracking performance for varying frequency signals in multiple axis. The further improvement of frictions compensation and the implementation of higher order signal references are currently under investigation.

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