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RNGA Loop Pairing Criterion for Multivariable Systems Subject to a Class of Reference Inputs ⋆

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Abstract: This paper aims to generalize the loop interaction measurement, relative normalized gain array (RNGA), for multivariable systems regarding a class of reference inputs. In the existing studies, RNGA loop pairing criterion is analyzed and widely utilized on the basis of detailed assumption of step reference input. For multivariable systems under step, ramp and other general types of set-point changes, the general loop pairing technique is put forward, and the average residence time is calculated in terms of first order plus delay time and second order plus delay time processes. The analysis results show RNGA based control-loop configuration is independent of input signals, and available to multivariable systems for various reference inputs. Several examples are employed to demonstrate the effectiveness and universality of the pairing approach of this paper.

Keywords: Multivariable systems; Loop pairing; Relative normalized gain array; Interaction measurement.

1. INTRODUCTION

In multivariable control structure design, the key procedure, which is especially important for decentralized control, is the control configuration selection or the input-output pairing problem (Grosdidier and Morari (1986), Chen and Seborg (2002), Khaki-Sedigh and Moaveni (2009), Bao et al. (2007), Liu and Gao (2012), Wittenmark and Salgado (2002)). An improper loop pairing of manipulated variables and controlled variables may lead to closed-loop instability or deteriorate closed-loop performance. Therefore, ahead of the design of the controller, guidance for the selection of control configuration will be benefited to minimize interactions among control loops and achieve better control performance. In the past years, various techniques for control-loop configuration are available in the literatures.

A commonly and widely recognized technique in industry for selecting the best input-output pairing is the relative gain array (RGA) method, which was originally presented by Bristol (1966). Only considering steady state of the process make it very simple in calculation for interaction measurement. Yet not sensitive to time constants, delays and even more importantly dynamic information, dynamic RGA (DRGA) latter was introduced to use the transfer function model at all frequencies to substitute the steady-state gain matrix (Witcher and McAvoy (1977), Tung and Edgar (1981)). To overcome the defection of assuming perfect control at all frequency, Avoy et al. (2003) proposed an improved controller-dependent DRGA approach with the known dynamic process model. Desiring to keep the simplicity of RGA and take steady-state and dynamic information of the process model into consideration, Xiong et al. (2005) proposed a new concept of effective gain array (ERGA) based control configuration for multivariable systems. And Naini et al. (2009) ameliorated ERGA method with effective relative energy array (EREA).

Since the calculation of ERGA and EREA, to a great extent, relies on the critical frequency of the transfer function of each loop, two ways defined the critical frequency will generate different control structure configuration. He et al. (2009) defined the relative normalized gain array (RNGA) to provide a less calculating and optimal pairing decision in practical applications, which describes the effects of processes information in a more intuitional and comprehensive way. However, RNGA loop pairing criterion proposed in He et al. (2009) was limited to multivariable systems under step reference input, which makes RNGA based control configuration only suitable for industrial processes under step inputs. However, other set-point changes appear even more often than step changes in industrial practice. Therefore, it is important to put forward a general loop pairing technique available to multivariable systems for various reference inputs, in order to avoid adverse effects caused by abrupt step changes.

Based on the aforementioned motivation, in this paper, we generalize RNGA pairing criterion to multivariable systems subject to a class of reference inputs. To be specific, we will present a general loop pairing technique for multivariable systems under step, ramp and other general

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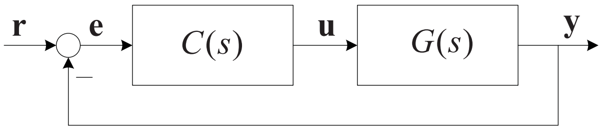

Fig. 1. Block diagram of decentralized control system

types of reference inputs. What is more, compared with other interaction measurement methods, it is an optimal loop pairing decision for not only step changes, but also other types of reference inputs.

2. PRELIMINARIES

Consider the following decentralized control multivariable system in Fig. 1. r, e, u and y are the vectors of set-points, feedback errors, manipulated variables and controlled variables, respectively. $C(s)$ represents a decentralized controller, and $G(s)$ is denoted as

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \cdots & \cdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \quad (1)$$

Let

$$g_{ij}(s) = g_{ij}(0)\bar{g}_{ij}(s),\ i,j = 1,2,...,n \quad (2)$$

where $g_{ij}(0)$ and $\bar{g}_{ij}(s)$ are the steady state gain and the normalized transfer function of $g_{ij}(s)$, respectively. To take into account both steady-state gain and the dynamic information of the plant, the normalized gain $k_{N,ij}$ for a particular transfer function $g_{ij}(s)$ is defined as

$$k_{N,ij} = \frac{g_{ij}\,(0)}{\bar{\tau}_{ar,ij}} \quad (3)$$

where $\bar{\tau}_{ar,ij}$ is the average residence time of $\bar{g}_{ij}(s)$, which is equal to the accumulation of the difference between the expected and the real outputs of the process (Astrom (1995)).

Similar to the definition of RGA Bristol (1966), by replacing the steady-state gain matrix with the normalized gain matrix with elements $k_{N,ij}$, relative normalized gain array (RNGA) can be calculated by

$$\Phi = K_N \otimes K_N^{-T} \quad (4)$$

where $\otimes$ is the Hadamard product and $K_N = [k_{N,ij}]_{n\times n} = G(0) \odot T_{ar}$, $G(0)$ is the steady-state gain matrix of $G(s)$, $T_{ar} = [\tau_{ar,ij}]_{n\times n}$ and $\odot$ indicates element-by-element division.

Once RNGA is obtained, the best loop pairing can be conformed by selecting pairing loop elements in RNGA closest to one in the premise of all paired RGA elements are positive.The more loop pairing elements in RNGA close to one means the smaller interaction with other loop. Another important Niederlinski Index (Niederlinski (1971)) should be ensured to be positive simultaneously, which provides a necessary stability condition for the already paired system. Above procedures for loop pairing are called RGA-NI-RNGA rules. However, the average residence time for RNGA in He et al. (2009) is specified to step inputs, which makes the control configuration rule is merely applicable for multivariable processes just responded to step inputs in practice. In the following section, we will generalize this pairing criterion to the processes responded to inputs of ramp and other general types of set-point changes.

3. RNGA FOR DIFFERENT SET-POINT CHANGES

To analyze and design the control system conveniently, most processes in practice are frequently modeled as the lower order models, such as the first order plus delay time (FOPDT) and the second order plus delay time (SOPDT). This section derives the normalized gains of FOPDT and SOPDT for multivariable process subject to step, ramp and other general types of set-point changes.

3.1 Step Changes

Literature (He et al. (2009)) has calculated the normalized gains of FOPDT and SOPDT under step changes. However, the normalized gain of critical damping ($\zeta = 1$) for SOPDT is omitted. Here for the sake of completeness, we obtain the normalized gain for critical damping.

The transfer function of SOPDT process is given as

$$g(s) = \frac{k}{as^2 + bs + 1}e^{-\theta s} = k\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}e^{-\theta s} \quad (5)$$

where $a = 1/\omega_n^2$ and $b = 2\zeta/\omega_n$.

When $\zeta = 1$, the transient transfer function given in equation (5) can be rewritten as

$$\begin{aligned} \bar{Y}(s) = \bar{g}(s)r(s) &= \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)}e^{-\theta s} \\ &= \frac{e^{-\theta s}}{s} + \frac{-e^{-\theta s}}{s+\omega_n} + \frac{-\omega_n e^{-\theta s}}{(s+\omega_n)^2} \end{aligned} \quad (6)$$

Then, from the inverse Laplace transform, step response of $\bar{g}(s)$ is as follows:

$$\bar{y}(t) = \begin{cases} 0 & 0 \le t < \theta \\ 1 - e^{-\omega_n(t-\theta)} - \omega_n(t-\theta)e^{-\omega_n(t-\theta)} & \theta \le t < \infty \end{cases} \quad (7)$$

Subsequently, the average residence time $\bar{\tau}_{ar}$ can be obtained as

$$\begin{aligned} \bar{\tau}_{ar} &= \int_0^\infty [r(t) - \bar{y}(t)]dt \\ &= \int_0^\theta 1dt + \int_\theta^\infty e^{-\omega_n(t-\theta)} + \omega_n(t-\theta)e^{-\omega_n(t-\theta)}dt \\ &= \theta + 1/\omega_n + 1/\omega_n = \theta + 2\zeta/\omega_n \end{aligned} \quad (8)$$

Thus, the normalized gain of SOPDT at $\zeta = 1$ is

$$k_N = k/(2\zeta/\omega_n + \theta) = k/(\theta + b) \quad (9)$$

which is in accordance with the case of $\zeta > 1$ and $0 < \zeta < 1$.

3.2 Ramp Changes

Let the set-point be the following ramp signal:

$$r(t) = \begin{cases} at, & 0 \le t < T \\ aT, & T \le t < \infty \end{cases} \tag{10}$$

Here, the slope of the ramp signal is a, which alternatively takes a positive or negative real value for the time being. T is the time duration for the ramp signal to increase original constant value into the final one. Decomposing $r(t)$ into two parts, i. e., $r_1(t)$ and $r_2(t)$.

$$r(t) = r_1(t) + r_2(t) \tag{11}$$

where $r_1(t) = a\,t,\ 0 \le t < \infty$, $r_2 = \begin{cases} 0, & 0 \le t < T \\ a\,(t-T)\,, & T \le t < \infty \end{cases}$.

The Laplace transform of $r(t)$ is

$$r(s) = r_1(s) + r_2(s) = \frac{a}{s^2} + \frac{-a}{s^2}e^{-Ts} \tag{12}$$

3.2.1 Normalized Gain of FOPDT Model

Give the FOPDT transfer function as

$$g(s) = \frac{k}{\tau s + 1}e^{-\theta s} \tag{13}$$

From equations (12) and (13), we reach the Laplace transform of the output response of $\bar{g}(s)$ as

$$\bar{Y}(s) = \frac{a}{\tau}\left(\frac{e^{-\theta s}}{s^2(s+1/\tau)} - \frac{e^{-(\theta+T)s}}{s^2(s+1/\tau)}\right) \tag{14}$$

The inverse Laplace transform of $\bar{Y}(s)$ is

$$\bar{y}(t) = \begin{cases} 0, & 0 \le t < \theta \\ a(\tau e^{-((t-\theta)/\tau)} + t - \theta - \tau), & \theta \le t < \theta + T \\ a(\tau e^{-(t-\theta)/\tau} - \tau e^{-(t-\theta-T)/\tau} + T), & \theta + T \le t < \infty \end{cases} \tag{15}$$

It needs to discuss two cases of $\theta \le T$ and $\theta > T$. For $\theta \le T$, it has

$$r(t) - \bar{y}(t) = \begin{cases} at, & 0 \le t < \theta \\ a(-\tau e^{-((t-\theta)/\tau)} + \theta + \tau), & \theta \le t < T \\ a(-\tau e^{-((t-\theta)/\tau)} + \theta + \tau + T - t), & T \le t < \theta + T \\ a(-\tau e^{-((t-\theta)/\tau)} + \tau e^{-((t-\theta-T)/\tau)}), & \theta + T \le t < \infty \end{cases} \tag{16}$$

then,

$$\begin{aligned} \bar{\tau}_{ar} &= \int_0^\infty [r(t) - \bar{y}(t)]dt \\ &= \int_0^\theta atdt + \int_\theta^T a(-\tau e^{-((t-\theta)/\tau)} + \theta + \tau)dt + \\ &\int_T^{\theta+T} a(-\tau e^{-((t-\theta)/\tau)} + \theta + \tau + T - t)dt + \\ &\int_{\theta+T}^\infty a(-\tau e^{-((t-\theta)/\tau)} + \tau e^{-((t-\theta-T)/\tau)})dt \\ &= aT(\theta + \tau) \end{aligned} \tag{17}$$

For $\theta > T$, the following equation holds

$$r(t) - \bar{y}(t) = \begin{cases} at, & 0 \le t < T \\ aT, & T \le t < \theta \\ a(-\tau e^{-((t-\theta)/\tau)} + \theta + \tau + T - t), & \theta \le t < \theta + T \\ a(-\tau e^{-((t-\theta)/\tau)} + \tau e^{-((t-\theta-T)/\tau)}), & \theta + T \le t < \infty \end{cases} \tag{18}$$

then,

$$\begin{aligned} \bar{\tau}_{ar} &= \int_0^\infty [r(t) - \bar{y}(t)]dt = \int_0^T atdt + \int_T^\theta aTdt + \\ &\int_\theta^{\theta+T} a(-\tau e^{-((t-\theta)/\tau)} + \theta + \tau + T - t)dt + \\ &\int_{\theta+T}^\infty a(-\tau e^{-((t-\theta)/\tau)} + \tau e^{-((t-\theta-T)/\tau)})dt \\ &= aT(\theta + \tau) \end{aligned} \tag{19}$$

From equation 2, the normalized gain of FOPDT under ramp input is calculated as

$$k_N = k/(aT(\theta + \tau)) \tag{20}$$

3.2.2 Normalized Gain of SOPDT Model

From equations (5) and (12), the Laplace transform of the output response for SOPDT reaches

$$\bar{Y}(s) = \frac{\omega_n^2 e^{-\theta s}}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} - \frac{\omega_n^2 e^{-(\theta+T)s}}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} \tag{21}$$

Consider two cases: $\theta \le T$ and $\theta > T$, and when $0 < \zeta < 1$, $\zeta = 1$ and $\zeta > 1$, the normalized gain of SOPDT under ramp input is calculated as

$$\bar{\tau}_{ar} = aT(\theta + 2\zeta/\omega_n) \tag{22}$$

From the results derived by the conditions of $\theta \le T$ and $\theta > T$, the normalized gain of SOPDT under ramp input is given as

$$k_N = k/(aT(2\zeta/\omega_n + \theta)) = k/(aT(b + \theta)) \tag{23}$$

Hence, RNGA of multivariable system with elements of FOPDT or SOPDT under ramp inputs is

$$\begin{aligned} \Phi &= K_N \otimes K_N^{-T} = (1/(aT) \cdot \bar{K}_N) \otimes (aT \cdot \bar{K}_N^{-T}) \\ &= \bar{K}_N \otimes \bar{K}_N^{-T} \end{aligned} \tag{24}$$

where $\bar{K}_N$ is the matrix with elements being $k/(\tau + \theta)$ or $k/(b + \theta)$.

Remark 1: From equation (24) it can be seen that the obtained RNGA is only related with steady-state gain, time constant and time delay of FOPDT and SOPDT, i.e. the parameters of model rather than input signal.

3.3 A General-type Changes

Suppose the set-point $r(t)$ changes in a general-type path from one steady value to another, which is shown in Fig. 2.

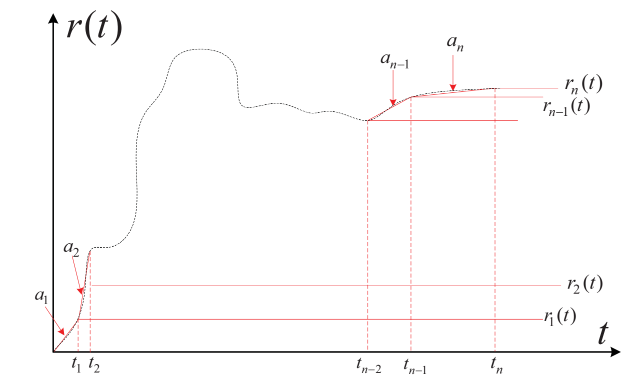

Fig. 2. Decomposition of general type of set-point changes

Approximate the general type change to a series of ramp signals, which is written in the form of

$$r(t) \approx r_1(t) + r_2(t) + \cdots + r_n(t) \tag{25}$$

where $r_i(t) = \begin{cases} 0, & 0 \le t < t_{i-1} \\ a_i(t - t_{i-1}), & t_{i-1} \le t < t_i \\ a_i(t_i - t_{i-1}), & t_i \le t < \infty \end{cases}$

From equation (25), it is ready to obtain the average residence time.

$$\begin{aligned} \bar{\tau}_{ar} &= \int_0^\infty [r(t) - \bar{y}(t)]dt \approx \int_0^\infty [r_1(t) - \bar{y}_1(t)]dt + \\ &\int_0^\infty [r_2(t) - \bar{y}_2(t)]dt + \cdots + \int_0^\infty [r_n(t) - \bar{y}_n(t)]dt \\ &= (a_1(t_1 - t_0) + \cdots + a_n(t_n - t_{n-1}))T(\tau + \theta) \end{aligned} \tag{26}$$

The normalized gain of FOPDT derived from above equation is

$$k_N = k/((a_1(t_1 - t_0) + \cdots + a_n(t_n - t_{n-1})) \cdot (\tau + \theta)) \tag{27}$$

Analogously, that of SOPDT is

$$k_N = k/((a_1(t_1 - t_0) + \cdots + a_n(t_n - t_{n-1})) \cdot (b + \theta)) \tag{28}$$

Thus, RNGA of multivariable system with elements of FOPDT or SOPDT under general-type inputs is

$$\begin{aligned} \Phi &= \left(1/(a_1(t_1 - t_0) + \cdots + a_n(t_1 - t_0)) \cdot \bar{K}_N\right) \otimes \\ &\quad \left((a_1(t_1 - t_0) + \cdots + a_n(t_1 - t_0)) \cdot \bar{K}_N^{-T}\right) \\ &= \bar{K}_N \otimes \bar{K}_N^{-T} \end{aligned} \tag{29}$$

Remark 2: Equation (29) gives the evidence that RNGA is derived as $k/(\tau + \theta)$ of FOPDT and $k/(b + \theta)$ of SOPDT for processes in case of responses being steps, ramps and general-types of set-points. Further, the analysis results demonstrate that RNGA based control configuration is independent of various reference inputs, and only related to the parameters of process model. From Luo et al. (2012), the conception of RNGA generalized by this paper is also applicable to non-square multivarialbe processes with unequal number of inputs and outputs.

4. CASE STUDY

4.1 Example 1

Consider a two-input and two-output process (He et al. (2009)),

$$G(s) = \begin{bmatrix} \dfrac{5e^{-s}}{100s+1} & \dfrac{e^{-4s}}{10s+1} \\ \dfrac{-5e^{-4s}}{10s+1} & \dfrac{5e^{-s}}{100s+1} \end{bmatrix}$$

The RGA value is obtained as

$$\Lambda = \begin{bmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{bmatrix}$$

From the result of RGA, diagonal pairing is preferred for Example 1.

According to equation (29), RNGA is easily calculated as

$$\begin{aligned} \Phi &= \bar{K}_N \otimes \bar{K}_N^{-T} \\ &= \begin{bmatrix} 5/101 & 1/14 \\ -5/14 & 5/101 \end{bmatrix} \otimes \begin{bmatrix} 5/101 & 1/14 \\ -5/14 & 5/101 \end{bmatrix}^{-T} \\ &= \begin{bmatrix} 0.0877 & 0.9123 \\ 0.9123 & 0.0877 \end{bmatrix} \end{aligned}$$

From the RGA-NI-RNGA rules, the off-diagonal values in the RNGA matrix are close to one, which means the pairing $y_1 - u_2/y_2 - u_1$ is the preferred one with smaller interactions between each loop. In ERGA method (Xiong et al. (2005)), if the critical frequency is determined by the frequency where the phase of frequency response is negative π, ERGA method selects diagonal pairing, which has been listed in literature (He et al. (2009)). Controller settings of diagonal and off-diagonal designed by the decentralized IMC-PID controller tuning methods (He et al. (2005)) are given in Table 1 with the controller taking the form of $C_i = k_{Pi}(1 + 1/\tau_{Ii}s)$.

To illustrate which is the optimal pairing of diagonal and off-diagonal scenarios, references of all control loops change one-by-one using signals of ramp and triangle wave. The integral absolute error (IAE) is used to evaluate the control performance.

$$IAE = \int_0^\infty |e(t)|\, dt = \int_0^\infty |r(t) - y(t)|\, dt$$

The ramp and triangle wave response curves and IAE values are shown in Fig. 3, Fig. 4 and Table 2, respectively, which show that the pairing $y_1 - u_2/y_2 - u_1$ results in superior overall closed-loop performance. That is to say RNGA method is also an optimal loop pairing decision for system under references of ramp and other general types.

Table 1. Controller settings for Example 1

Loop	RGA, ERGA		RNGA	
	k_{Pi}	τ_{Ii}	k_{Pi}	τ_{Ii}
1	0.5	100	1.25	10
2	0.5	100	-0.25	10

4.2 Example 2

Consider a three-input and three-output process (He et al. (2009)) with transfer function matrix given as

$$G(s) = \begin{bmatrix} \dfrac{e^{-9s}}{6s^2+17s+1} & \dfrac{-9e^{-5s}}{s^2+4s+1} & \dfrac{13e^{-3s}}{3s^2+35s+1} \\ \dfrac{-5e^{-13s}}{2s^2+19s+1} & \dfrac{8e^{-2s}}{s^2+33s+1} & \dfrac{7e^{-5s}}{s^2+3s+1} \\ \dfrac{-16e^{-3s}}{s^2+5s+1} & \dfrac{3e^{-7s}}{s^2+14s+1} & \dfrac{e^{-11s}}{3s^2+25s+1} \end{bmatrix}$$

Table 2. IAE values of diagonal and off-diagonal for decentralized control of Example 1

Loop	IAE (Ramps)				IAE (Triangle waves)			
	RGA, ERGA		RNGA		RGA, ERGA		RNGA	
1	166.8828	40.5037	65.9784	19.9256	152.6552	53.8929	107.8009	28.786
2	201.874	165.905	100.905	65.9843	268.4837	153.007	143.283	108.335

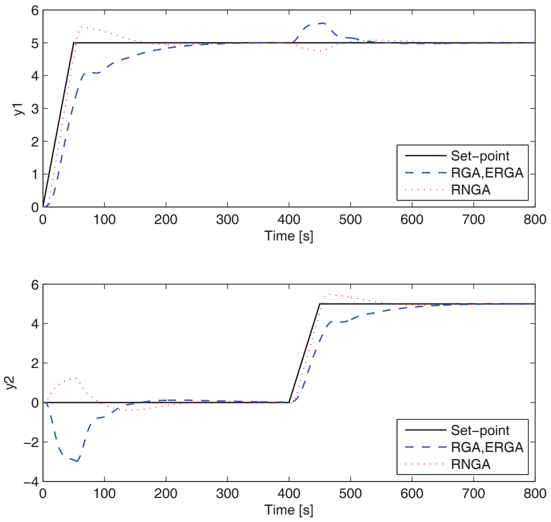

Fig. 3. Simulation results of different pairing for ramp inputs

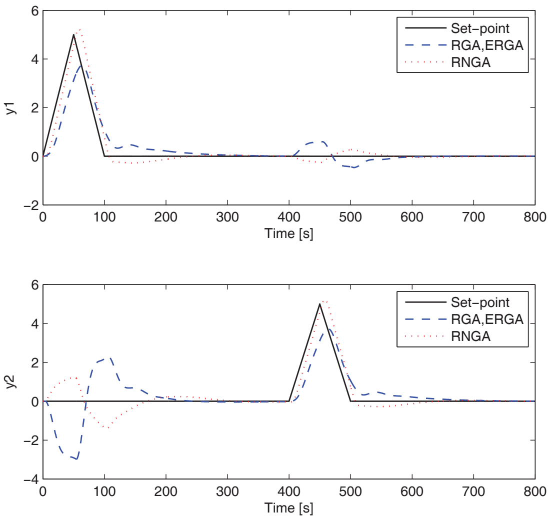

Fig. 4. Simulation results of different pairing for triangle wave inputs

The obtained RGA is given as

$$\Lambda = \begin{bmatrix} -0.0054 & 0.3981 & 0.6073 \\ -0.0992 & 0.6912 & 0.4080 \\ 1.1046 & -0.0893 & -0.0153 \end{bmatrix}$$

In the light of the above result, RGA-NI loop pairing criterion gives off-diagonal pairing as the best one.

The RNGA is obtained as

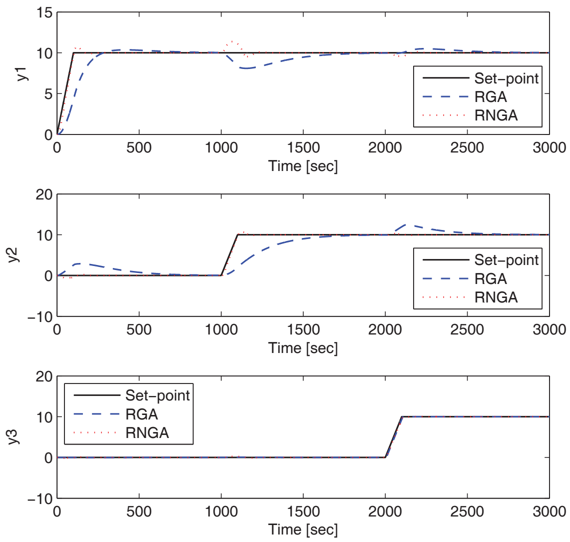

Fig. 5. Simulation results of different pairing for ramp inputs

$$\begin{aligned} \Phi &= \bar{K}_N \otimes \bar{K}_N^{-T} \\ &= \begin{bmatrix} 1/26 & -9/9 & 13/38 \\ -5/32 & 8/35 & 7/8 \\ -16/8 & 3/21 & 1/36 \end{bmatrix} \otimes \begin{bmatrix} 1/26 & -9/9 & 13/38 \\ -5/32 & 8/35 & 7/8 \\ -16/8 & 3/21 & 1/36 \end{bmatrix}^{-T} \\ &= \begin{bmatrix} -0.0024 & 0.9237 & 0.0787 \\ -0.0063 & 0.0829 & 0.9235 \\ 1.0088 & -0.0066 & -0.0022 \end{bmatrix} \end{aligned}$$

From RGA-NI-RNGA based loop pairing rules, the pairing $y_1 - u_2/y_2 - u_3/y_3 - u_1$ should be the best for decentralized control. Decentralized controllers for cases of RNGA and RGA pairing are calculated respectively using the IMC-PID design procedure (He et al. (2005)) with the controller taking the form of $C_i = k_{Pi}(1 + 1/\tau_{Ii}s + \tau_{Di}s)$. The designed controller settings are shown in Table 3.

Table 3. Controller settings for Example 2

Loop	RGA			RNGA		
	k_{Pi}	τ_{Ii}	τ_{Di}	k_{Pi}	τ_{Ii}	τ_{Di}
1	0.0292	35.0	0.0857	-0.0363	4.0	0.2500
2	0.0142	33.0	0.0303	0.0346	3.0	0.3333
3	-0.0515	5.00	0.2000	-0.0518	5.0	0.2000

Response curves for set-point changes of ramp and triangle wave depicted in Fig. 5 and 6 and IAE values listed in Table 4 show that the overall performance of RNGA pairing is significantly better than that of RGA pairing. Once again, the results expound RNGA based control configuration is an optimal loop pairing decision in practical applications.

Table 4. IAE values of RGA and RNGA pairing for decentralized control of Example 2

Loop	IAE (Ramps)						IAE (Triangle waves)					
	RGA			RNGA			RGA			RNGA		
1	821.1	688.1	187.7	110.1	142.6	44	978.4	372.6	103.8	213.3	273.1	81
2	1019.4	1989.5	679.5	63.2	110.3	27.3	555.2	1376.8	453.7	121.8	213.6	53.3
3	6.6	19.1	66.9	15.3	27.4	68.6	4.6	13.2	121	29.8	51.8	127.4

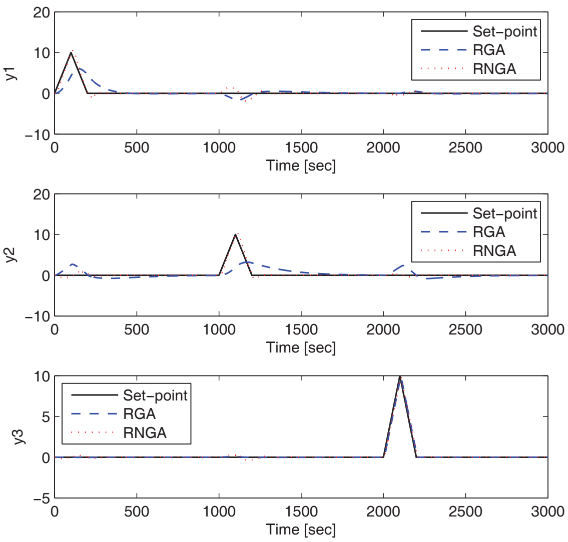

Fig. 6. Simulation results of different pairing for triangle wave inputs

5. CONCLUSIONS

In this paper, a RNGA based loop pairing criterion is analyzed for interaction measurement, which can be well applied to multivariable systems with different reference inputs such as step, ramp and other general types. Both the steady-state and transient information of the process transfer function are investigated to calculate the interactions between each loop. The effectiveness of the method is demonstrated by several examples, which show that the proposed systematical RNGA based loop pairing criterion not only gives more accurate interaction assessment compared to other existing interaction measurement techniques, but also works for general types of reference inputs. Considering most processes in practice are frequently modeled as lower order models, the RNGA method generalized by this paper is only suitable for multivaraible systems with entries of either FOPDT or SOPDT. Future work will generalize RNGA pairing criterion to multivaraible systems with general process model subject to a class of reference inputs.

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