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Pursuit Formation Control Scheme for Double-Integrator Multi-Agent Systems

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Abstract: Pursuit formation control of double-integrator Multi-Agent Systems (MASs) is considered in this paper. To achieve a formation, a hierarchical control scheme is proposed in two layers. In the first layer, in a leaderless architecture, each agent pursues another one in a cyclic topology to achieve a regular polygon formation around a centroid, namely, pursuit centroid, and in the second layer, the agents agree on a pursuit centroid around which they pursue each other. Despite existing cyclic pursuit approaches in the literature which are based on agents with kinematic models and constant forward speeds, the proposed approach is based on agents with double-integrator dynamics without speed constraints. Numerical simulations for a team of six agents confirm the accuracy of the proposed control scheme.

Keywords: Formation control, mobile agents, double-integrators, cyclic pursuit.

1. INTRODUCTION

Control of mobile autonomous agents is an interesting topic of research in the area of control theory and robotics. Motivated by recent developments in communication networks, electronics, sensors technology, and computing science, decentralized control of MASs recently has found many applications such as surveillance, search and rescue missions, maintenance, monitoring missions, and so on. These systems provide us opportunities to use simple, small, and cheap agents instead of employing a sophisticated and large agent, and they are more reliable, robust, flexible, and precise for doing tasks (Ghommam et al. [2011], Merino et al. [2012], Palunko et al. [2012]).

Formation control is a challenging problem in the area of MASs expressed as maintaining relative positions between agents in a desired geometric pattern. Regarding communication topologies associated with MASs, two classes of approaches can be considered in formation control, namely, *acyclic* and *cyclic*. In acyclic approaches, all agents are connected via a directed topology in which there are no any loops. Therefore, the stability analysis of these approaches is reduced to two agents: one agent as a follower which tracks another one as a leader. Hence, the ease of implementation and stability analysis is the main characteristic of these approaches. However, since no loops exist in the associated communication topology, no feedbacks are sent from the followers to their leaders (Lin et al. [2013], Mesbahi and Hadaegh [2001]). In some applications, to increase the degree of precision in formation keeping, the agents track a virtual leader as a reference signal. In other words, they track a relative position with respect to the trajectory of the virtual leader, and the formation will be kept rigid during maneuvers with high precision. However, the virtual leader cannot find its trajectory autonomously, and therefore this issue decreases the degree of autonomy in the MAS (Rezaee et al. [In Press], Tan and Lewis [1996]). On the other hand, in a cyclic approach, all the agents are connected via a topology such that each agent is a follower of other agents and no independent leaders exist in the MAS.

Cyclic pursuit is a swarming behavior in decentralized MASs based on cyclic communication topologies. This scenario is inspired by living organisms in groups such as beetles and ants to increase the chance of finding foods or avoiding predators which has found a lot of engineering applications. Indeed, the rotational motion around a centroid increases their searching capability (Behroozi and Gagnon [1975], Marshall and Tsai [2011]).

For the first time, cyclic pursuit was mathematically studied in Klamkin and Newman [1971], Behroozi and Gagnon [1975], and Behroozi and Gagnon [1979], and recently it is applied for formation control of MASs. For instance, cyclic formation control of single-integrator kinematics in regular patterns around a centroid was studied in Lin et al. [2004] in which the centroid was obtained from an agreement problem. However, first-order kinematics cannot model a large number of agents in practice. Hence, considering single-integrator kinematics restricted its applications. That approach was extended for unicycle kinematics in Marshall et al. [2004] and Marshall and Tsai [2011]. In those papers, each agent pursued another one in a regular polygon formation while all the agents were rotating on a circle around a centroid. Since the agents were considered with constant forward speeds, in the case of identical speeds, they rotated after achieving a regular polygon formation. In this condition, the direction of the rotation was determined based on the initial relative information of the agents. Moreover, the constant forward speeds of the unicycles decreased the performance of the agents to converge to a desired formation. A similar approach was proposed in Marshall et al. [2006] in which each agent speed was proportional to the distance from the next agent. Those approaches were extended in Sinha and Ghose [2005] for non-holonomic first-order agents with different constant forward speeds. Therefore, since the speeds were not identical, the agents rotated on orbits with different radiuses, and in Ramirez-Riberos et al. [2010] and Ramirez-Riberos et al. [2009] cyclic pursuit in double-integrator agents was studied. However, in those approaches, it was not feasible to control the pursuit angular velocity. In other words, the pursuit angular velocity just depended on the number of agents, and when the formation radius was large, the agents linear velocities became large which may not be practical.

In this paper, a hierarchical control scheme for cyclic pursuit of double-integrator MASs in regular polygon formations is proposed in two layers. In the first layer (kinematics level), the desired velocity vector of each agent is designed under which the agents keep relative angles around a centroid such that a regular polygon formation with cyclic pursuit is achieved. Since the MAS is decentralized, it is supposed that the pursuit centroid of the agents is not predefined. Therefore, in the second layer (dynamics level), the agents control inputs are designed such that while satisfying the kinematics level formulation, the agents reach agreement on a pursuit centroid around which they pursue each other. In summary, the main contributions of the paper can be listed as follows:

- The proposed formation control scheme is based on double-integrator dynamics. Therefore, it is more practical than existing approaches in the literature based on first-order kinematics and unicycles with constant forward speeds.
- It is feasible to control the cyclic pursuit direction and angular velocity.

The following notations are considered throughout this paper. $\mathbb{R}$ expresses the set of real numbers, $\mathbb{R}_+$ and $\mathbb{R}_-$ present the set of positive and negative real numbers, respectively, $\mathbf{I}_n$ is an $n \times n$ identity matrix, $\mathbf{0}_{n\times m}$ is an $n \times m$ matrix with zero entries, $\otimes$ denotes the Kronecker product, sgn(.) expresses the sign function, $\text{diag}(\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_n)$ is a block diagonal matrix composed of matrices $\mathbf{M}_1$, $\mathbf{M}_2, \cdots$, and $\mathbf{M}_n$, $\|.\|_2$ presents the magnitude of a vector, v_0 denotes the initial value of a variable v, and eig(.) denotes the eigenvalues of a matrix. Moreover, if we consider $\mathbf{C}_1, \mathbf{C}_2, \cdots$, and $\mathbf{C}_n \in \mathbb{R}$, $\text{circ}(\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_n)$ is called a circulant matrix which is defined as follows:

$$\text{circ}(\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_n) = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 & \cdots & \mathbf{C}_n \\ \mathbf{C}_n & \mathbf{C}_1 & \cdots & \mathbf{C}_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_2 & \mathbf{C}_3 & \cdots & \mathbf{C}_1 \end{bmatrix}. \quad (1)$$

The matrix defined in (1) can be specified by the set $\mathbf{C}_1, \mathbf{C}_2, \cdots$, and $\mathbf{C}_n$ which appear in the first row, and other rows are formed by its previous row that is right shifted with wrapping around. Moreover, if $\mathbf{C}_i, i \in \{1, 2, \cdots, n\}$ is an $m \times m$ matrix, then (1) is called a block circulant matrix.

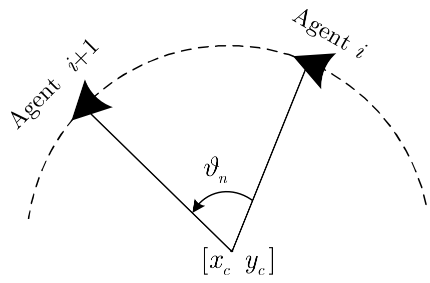

Fig. 1. Cyclic pursuit configuration in which each agent pursues another one with relative degree ϑ_n where the motion direction of the agents are depicted by arrows.

The paper outline is as follows. Regular polygon formation control in kinematics level is studied in Section 2. Dynamics level control for agents agreement on a pursuit centroid while satisfying the kinematics level formulation is presented in Section 3. Simulation results are provided in Section 4, and the paper ends with conclusions in Section 5.

2. CYCLIC PURSUIT IN KINEMATICS LEVEL

Consider a team of n agents which the ith agent position is denoted by $[x_i \ y_i], i \in \{1, 2, \cdots, n\}$. The objective is to design the ith agent desired velocity vector, $[\dot{x}_i \ \dot{y}_i]$, for cyclic pursuit in a regular polygon formation with angular velocity $\varpi \in \mathbb{R}$ around a centroid $[x_c \ y_c]$. To achieve this goal, a leaderless architecture is proposed such that each agent pursues another agent with a relative angle ϑ_n on a circle with centroid $[x_c \ y_c]$. In other words, as shown in Fig. 1, each agent keeps a relative angle ϑ_n from another one such that a regular polygon formation is rotating around the centroid. This angle depends on the number of the agents; therefore, if we suppose that each agent pursues its front agent, a regular polygon formation is achieved if $\vartheta_n = \text{sgn}(\varpi)\frac{2\pi}{n}$. Therefore, the desired position of the ith agent can be stated based on the position of the $i+1$th agent (modulo n) as follows:

$$\begin{bmatrix} x_i^d \\ y_i^d \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \end{bmatrix} + \begin{bmatrix} \cos\vartheta_n & \sin\vartheta_n \\ -\sin\vartheta_n & \cos\vartheta_n \end{bmatrix} \begin{bmatrix} x_{i+1} - x_c \\ y_{i+1} - y_c \end{bmatrix} \quad (2)$$

where $\begin{bmatrix} x_i^d & y_i^d \end{bmatrix}$ is the desired position of the ith agent to achieve a regular polygon formation around the centroid. Moreover, the agent should rotate while it is keeping the desired formation defined in (2). The following lemma proposes a velocity vector for rotating around the centroid.

Lemma 1. (Rezaee and Abdollahi [2014]) In the following state space equations, the trajectory of the states with initial values $[x_0 \ y_0]$ rotates on a circle with center $[x_c \ y_c]$ and radius $\| [x_0 - x_c \ y_0 - y_c] \|_2$ with angular velocity ϖ:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & -\varpi \\ \varpi & 0 \end{bmatrix} \begin{bmatrix} x - x_c \\ y - y_c \end{bmatrix}.$$

By inspiring from (2) to provide a regular polygon formation and Lemma 1 to rotate around a centroid, Theorem 1 proposes the agents desired velocity vectors for cyclic pursuit in regular polygon formations. At first, let us state the following lemma which our proof is based on.

Lemma 2. (Davis [1994]) If we define the ith of n roots of unit by $\omega = e^{\frac{2\pi}{n}\mathrm{j}}$ in which $\mathrm{j} = \sqrt{-1}$, the block circulant matrix defined in (1) can be stated as follows:

$$\mathrm{circ}(\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_n) = \mathbf{F}_n^* \mathrm{diag}(\mathbf{D}_1, \mathbf{D}_2, \cdots, \mathbf{D}_n)\mathbf{F}_n$$

where $\mathbf{F}_n^*\mathbf{F}_n = \mathbf{I}_n$, and

$$\begin{bmatrix}\mathbf{D}_1\\ \mathbf{D}_2\\ \vdots\\ \mathbf{D}_n\end{bmatrix} = \sqrt{n}(\mathbf{F}_n^* \otimes \mathbf{I}_m)\begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\\ \vdots\\ \mathbf{C}_n\end{bmatrix}$$

where $\mathbf{F}_n$ denotes an $n \times n$ Fourier matrix which is defined as follows:

$$\mathbf{F}_n^* = \frac{1}{\sqrt{n}}\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1}\\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(n-1)}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \cdots & \omega^{(n-1)(n-1)}\end{bmatrix}.$$

Based on Lemma 2, the following theorem is stated.

Theorem 1. Consider a team of n mobile agents. Keeping a regular polygon formation around a centroid $[x_c \ y_c]$, each agent pursues its front agent with angular velocity ϖ, if the ith agent velocity vector is defined as follows:

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix} = \begin{bmatrix}-\lambda & -\varpi\\ \varpi & -\lambda\end{bmatrix}\begin{bmatrix}x_i - x_c\\ y_i - y_c\end{bmatrix} + \begin{bmatrix}\lambda\cos\vartheta_n & \lambda\sin\vartheta_n\\ -\lambda\sin\vartheta_n & \lambda\cos\vartheta_n\end{bmatrix}\begin{bmatrix}x_{i+1} - x_c\\ y_{i+1} - y_c\end{bmatrix} \quad (3)$$

where $\lambda \in \mathbb{R}_+$.

Proof. Considering (3), if we define $\mathbf{x}_i = [x_i - x_c \ y_i - y_c]^\top$ one can get:

$$\dot{\mathbf{x}}_i = \begin{bmatrix}-\lambda & -\varpi\\ \varpi & -\lambda\end{bmatrix}\mathbf{x}_i + \begin{bmatrix}\lambda\cos\vartheta_n & \lambda\sin\vartheta_n\\ -\lambda\sin\vartheta_n & \lambda\cos\vartheta_n\end{bmatrix}\mathbf{x}_{i+1} \quad (4)$$

which can be written for the whole MAS as follows:

$$\begin{bmatrix}\dot{\mathbf{x}}_1\\ \dot{\mathbf{x}}_2\\ \vdots\\ \dot{\mathbf{x}}_n\end{bmatrix} = \mathbf{C}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \vdots\\ \mathbf{x}_n\end{bmatrix}$$

where $\mathbf{C} = \mathrm{circ}(\mathbf{C}_1, \mathbf{C}_2, \mathbf{0}_{2\times 2}, \cdots, \mathbf{0}_{2\times 2})$ and

$$\mathbf{C}_1 = \begin{bmatrix}-\lambda & -\varpi\\ \varpi & -\lambda\end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix}\lambda\cos\vartheta_n & \lambda\sin\vartheta_n\\ -\lambda\sin\vartheta_n & \lambda\cos\vartheta_n\end{bmatrix}.$$

By Lemma 2, $\mathbf{C}$ can be stated by its diagonalized form as follows:

$$\mathbf{C} = (\mathbf{F}_n^* \otimes \mathbf{I}_2)\mathrm{diag}(\mathbf{D}_1, \mathbf{D}_2, \cdots, \mathbf{D}_n)(\mathbf{F}_n \otimes \mathbf{I}_2)$$

where $\mathbf{D}_1, \mathbf{D}_2, \cdots$, and $\mathbf{D}_n$ can be stated as:

$$\begin{aligned}\mathbf{D}_1 &= \mathbf{C}_1 + \mathbf{C}_2,\\ \mathbf{D}_2 &= \mathbf{C}_1 + \omega\mathbf{C}_2,\\ \vdots & \ \vdots \ \ \vdots\\ \mathbf{D}_n &= \mathbf{C}_1 + \omega^{n-1}\mathbf{C}_2.\end{aligned} \quad (5)$$

Therefore, the eigenvalues of $\mathbf{C}$ are the set of the eigenvalues of $\mathbf{D}_i, i \in \{1, 2, \cdots, n\}$ which from (5) it follows that:

$$\mathbf{D}_i = \begin{bmatrix}-\lambda(1 - \omega^{i-1}\cos\vartheta_n) & -\varpi + \omega^{i-1}\lambda\sin\vartheta_n\\ -(-\varpi + \omega^{i-1}\lambda\sin\vartheta_n) & -\lambda(1 - \omega^{i-1}\cos\vartheta_n)\end{bmatrix}.$$

Hence, one can get:

$$\mathrm{eig}(\mathbf{C}) = -\lambda(1 - \omega^{i-1}\cos\vartheta_n) \mp \mathrm{j}(-\varpi + \omega^{i-1}\lambda\sin\vartheta_n). \quad (6)$$

To analyze the eigenvalues of $\mathbf{C}$, they should be decomposed into real and imaginary terms. Hence, from (6), since $\omega = e^{\frac{2\pi}{n}\mathrm{j}}$ it can be stated that:

$$\mathrm{eig}(\mathbf{C}) = -\lambda\big(1 - \cos(\sigma_i \mp \vartheta_n)\big) \mp \mathrm{j}\big(-\varpi \mp \lambda\sin(\sigma_i \mp \vartheta_n)\big)$$

where $\sigma_i = \frac{2\pi(i-1)}{n}$. In other words, $\mathrm{eig}(\mathbf{C})$ is the union of the sets $\mathcal{E}_1$ and $\mathcal{E}_2$ where

$$\mathcal{E}_1 = -\lambda\big(1 - \cos(\sigma_i - \vartheta_n)\big) - \mathrm{j}\big(-\varpi - \lambda\sin(\sigma_i - \vartheta_n)\big),$$
$$\mathcal{E}_2 = -\lambda\big(1 - \cos(\sigma_i + \vartheta_n)\big) + \mathrm{j}\big(-\varpi + \lambda\sin(\sigma_i + \vartheta_n)\big).$$

At first, let us analyze $\mathcal{E}_1$. Two conditions are considered to analyze $\mathcal{E}_1$:

i) $i \neq 2$: Since $\vartheta_n = \frac{2\pi}{n}$, in this condition, one can get $\sigma_i - \vartheta_n \neq \{0, 2\pi\}, i \in \{1, 3, 4, \cdots, n\}$. Hence:
$$-1 \leq \cos(\sigma_i - \vartheta_n) < 1,$$
and since $\lambda \in \mathbb{R}_+$ one can get $-\lambda\big(1 - \cos(\sigma_i - \vartheta_n)\big) < 0$, i.e., $\mathcal{E}_1$ contains $n-1$ eigenvalues on the open left half plane.

ii) $i = 2$: In this condition, $\sigma_i - \vartheta_n = 0$; therefore, one can say that $\cos(\sigma_i - \vartheta_n) = 1$ and $\sin(\sigma_i - \vartheta_n) = 0$. Hence, $\mathcal{E}_1 = \mathrm{j}\varpi$, i.e., $\mathrm{j}\varpi$ is the nth eigenvalue in the set $\mathcal{E}_1$.

For $\mathcal{E}_2$, when $i \neq n$, we get the same results as $\mathcal{E}_2$ in a case that $i \neq 2$, and if $i = n$, then $\mathcal{E}_2 = -\mathrm{j}\varpi$. In other words, $-\mathrm{j}\varpi$ is the nth eigenvalue of the set $\mathcal{E}_2$.

Therefore, according to the above-mentioned issues, $\mathbf{C}$ has $2n-2$ eigenvalues on the open left half plane and 2 eigenvalues on $\mp\mathrm{j}\varpi$. Since all the eigenvalues on the open left half plane will not appear in the steady state response (as $t \to \infty$), it means that $\mathbf{x}_i$ has a steady sinusoidal response with frequency $\frac{|\varpi|}{2\pi}$, and since $\mathbf{x}_i = [x_i - x_c \ y_i - y_c]^\top$, the vector $[x_i \ y_i]$ rotates around $[x_c \ y_c]$ with frequency $\frac{|\varpi|}{2\pi}$. In this condition, since each agent steady state has 2 eigenvalues on $\mp\mathrm{j}\varpi$, considering (4) its behavior can be described by one of the following two systems:

i) $[x_i \ y_i]$ rotates around $[x_c \ y_c]$ with angular velocity ϖ; therefore, it can be said that:
$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix} = \begin{bmatrix}0 & -\varpi\\ \varpi & 0\end{bmatrix}\begin{bmatrix}x_i - x_c\\ y_i - y_c\end{bmatrix}. \quad (7)$$

ii) $[x_i \ y_i]$ rotates around $[x_c \ y_c]$ with angular velocity $-\varpi$; therefore, one can get:
$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix} = \begin{bmatrix}0 & \varpi\\ -\varpi & 0\end{bmatrix}\begin{bmatrix}x_i - x_c\\ y_i - y_c\end{bmatrix}. \quad (8)$$

By the first system, considering (3) and (7) one can conclude:

$$\begin{bmatrix}x_i\\ y_i\end{bmatrix} = \begin{bmatrix}x_c\\ y_c\end{bmatrix} + \begin{bmatrix}\cos\vartheta_n & \sin\vartheta_n\\ -\sin\vartheta_n & \cos\vartheta_n\end{bmatrix}\begin{bmatrix}x_{i+1} - x_c\\ y_{i+1} - y_c\end{bmatrix}$$

which implies that while the agents are rotating with angular velocity ϖ around the centroid $[x_c\ y_c]$, each agent reached the desired position defined in (2), and a regular polygon formation is obtained.

By the second system, since $\mathbf{x}_i = [x_i - x_c\ y_i - y_c]^\top$, if (8) is substituted into (4), it can be said that:

$$\begin{bmatrix} -\lambda & 2\varpi \\ -2\varpi & -\lambda \end{bmatrix} \mathbf{x}_i + \begin{bmatrix} \lambda \cos \vartheta_n & \lambda \sin \vartheta_n \\ -\lambda \sin \vartheta_n & \lambda \cos \vartheta_n \end{bmatrix} \mathbf{x}_{i+1} = \mathbf{0}_{2\times 1}$$

which can be written for the whole MAS as follows:

$$\widetilde{\mathbf{C}} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} = \mathbf{0}_{2n\times 1} \qquad (9)$$

where $\widetilde{\mathbf{C}} = \mathrm{circ}(\widetilde{\mathbf{C}}_1, \mathbf{C}_2, \mathbf{0}_{2\times 2}, \cdots, \mathbf{0}_{2\times 2})$ and

$$\widetilde{\mathbf{C}}_1 = \begin{bmatrix} -\lambda & -2\varpi \\ 2\varpi & -\lambda \end{bmatrix}.$$

Similar to $\mathbf{C}$, $\widetilde{\mathbf{C}}$ has $2n-2$ eigenvalues on the open left half plane and 2 eigenvalues on $\mp \mathrm{j}2\varpi$. Therefore, $\det(\widetilde{\mathbf{C}}) \neq 0$, and from (9) one can get:

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} = \mathbf{0}_{2n\times 1}.$$

It means that all the agents have identical initial positions which are not feasible. Therefore, the ith agent behavior, $i \in \{1, 2, \cdots, n\}$, cannot be described by (8). Its behavior should be described by (7) which means that while keeping a regular polygon formation around the centroid $[x_c\ y_c]$, each agent pursues its front agent with angular velocity ϖ, and this proves the theorem. ■

Remark 1. In the proposed approach, it was supposed that $\vartheta = \mathrm{sgn}(\varpi)\frac{2\pi}{n}$. In other words, each agent pursues its front agent. However, a similar procedure can prove the proposed theorem when $\vartheta = -\mathrm{sgn}(\varpi)\frac{2\pi}{n}$.

Therefore, Theorem 1 designs agents desired velocity vectors for cyclic pursuit with angular velocity ϖ while keeping a regular polygon formation. The following example confirms the accuracy of the proposed formation control strategy.

Example 1. Consider a MAS containing six agents with initial positions $[3\ \ -4]$, $[6\ 4]$, $[0\ 7]$, $[-3\ 5]$, $[1\ 2]$, and $[1\ \ -6]$, respectively. By satisfying the desired velocity vector defined in (3), when $[x_c\ y_c] = [0\ 0]$, $\varpi = 0.2$, and $\lambda = 4$, a counter-clockwise cyclic pursuit of the six agents in a hexagonal formation is obtained which is depicted in Fig. 2.

It should be noted that the MAS is decentralized, and to increase the MAS autonomy, the pursuit centroid defined in (3) should not be known a priori. In the next section, while satisfying the kinematics level formulation of Theorem 1, the agents control inputs are designed in order to agree on a pursuit centroid around which a regular polygon formation with cyclic pursuit is achieved.

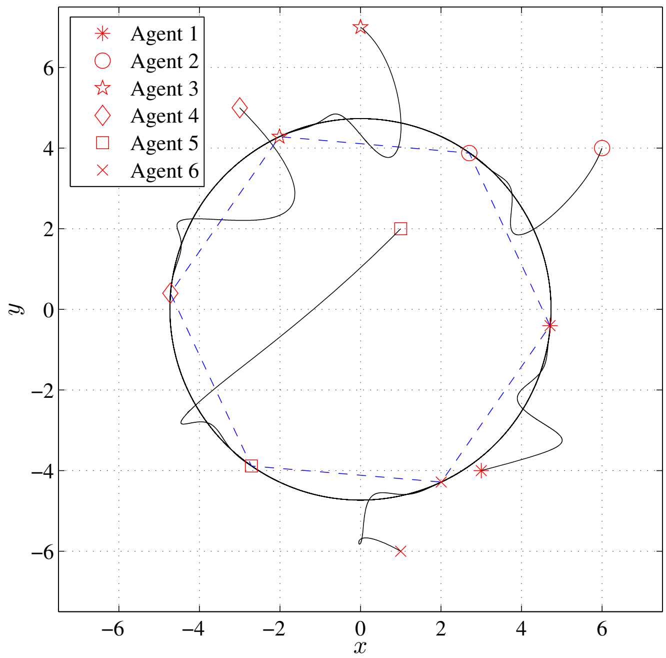

Fig. 2. The cyclic pursuit of the six agents in Example 1.

3. PURSUIT CENTROID AGREEMENT IN DYNAMICS LEVEL

Based on the obtained results is kinematics level in Section 2, this section proposes a dynamics level control strategy to achieve a regular polygon formation of double-integrator MASs with cyclic pursuit around a non-predefined centroid. To achieve this goal, the pursuit centroid of each agent of Theorem 1 is modeled by a single-integrator. Therefore, all the pursuit centroids should agree on a value as the centroid of a regular polygon formation via an agreement strategy. Lemma 3 proposes an agreement strategy for single-integrator kinematics.

Lemma 3. (Marshall et al. [2004]) Consider n single-integrator agents updated as follows:

$$\dot{\xi}_i = \alpha(\xi_{i+1} - \xi_i), i \in \{1, 2, \cdots, n\}$$

where $\xi_i \in \mathbb{R}$ is the ith agent state and $\alpha \in \mathbb{R}_+$. Then, all the states converge to the average of their initial values.

Therefore, inspired by Lemma 3, the following theorem is proposed for cyclic pursuit of double-integrator MASs in a regular polygon formation around a centroid which will be obtained from an agreement strategy.

Theorem 2. Consider the following double-integrator MAS:

$$\begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} = \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix}, i \in \{1, 2, .., n\}$$

where $[x_i\ y_i]$ is the position, and $[u_{xi}\ u_{yi}]$ is the ith agent control input vector. Keeping a regular polygon formation around a non-predefined centroid, each agent pursues its front agent with angular velocity ϖ, if the ith agent control input is as follows:

$$\begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix} = \dot{\Xi}_i + \alpha \left(\begin{bmatrix} \dot{x}_{i+1} \\ \dot{y}_{i+1} \end{bmatrix} - \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} + \Xi_i - \Xi_{i+1} \right) \qquad (10)$$

where $\alpha \in \mathbb{R}_+$, and

$$\Xi_i = \begin{bmatrix} -\lambda & \varpi \\ -\varpi & -\lambda \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} \lambda\cos\vartheta_n & \lambda\sin\vartheta_n \\ -\lambda\sin\vartheta_n & \lambda\cos\vartheta_n \end{bmatrix} \begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix}.$$

Proof. The agents desired velocity vectors for cyclic pursuit in a regular polygon formation around a centroid $[x_c\ y_c]$ was proposed in Theorem 1. Now, if the pursuit centroid in Theorem 1 is not pre-defined, a variable pursuit centroid $[x_i^c\ y_i^c]$ can be defined for the ith agent. In this condition, while satisfying the desired velocity vectors designed in Theorem 1, the agents pursuit centroids should reach agreement on a common value. Therefore, considering (3) it can be said that:

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} -\lambda & \varpi \\ -\varpi & -\lambda \end{bmatrix} \begin{bmatrix} x_i - x_i^c \\ y_i - y_i^c \end{bmatrix} + \begin{bmatrix} \lambda\cos\vartheta_n & \lambda\sin\vartheta_n \\ -\lambda\sin\vartheta_n & \lambda\cos\vartheta_n \end{bmatrix} \begin{bmatrix} x_{i+1} - x_i^c \\ y_{i+1} - y_i^c \end{bmatrix}. \quad (11)$$

Note that from Theorem 1, since zero is not an eigenvalue of $\mathbf{C}_1 + \mathbf{C}_2$, $\det(\mathbf{C}_1 + \mathbf{C}_2) \neq 0$. Hence, by (11) one can conclude:

$$\begin{bmatrix} x_i^c \\ y_i^c \end{bmatrix} = (\mathbf{C}_1 + \mathbf{C}_2)^{-1} \left(\Xi_i - \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} \right). \quad (12)$$

To achieve agreement on a pursuit centroid, inspired by Lemma 3, the ith agent pursuit centroid should be updated as follows:

$$\begin{bmatrix} \dot{x}_i^c \\ \dot{y}_i^c \end{bmatrix} = \alpha \left(\begin{bmatrix} x_{i+1}^c \\ y_{i+1}^c \end{bmatrix} - \begin{bmatrix} x_i^c \\ y_i^c \end{bmatrix} \right). \quad (13)$$

Substituting (12) into (13) yields:

$$(\mathbf{C}_1 + \mathbf{C}_2)^{-1} \left(\dot{\Xi}_i - \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} \right) = \alpha (\mathbf{C}_1 + \mathbf{C}_2)^{-1} \left(\Xi_{i+1} - \begin{bmatrix} \dot{x}_{i+1} \\ \dot{y}_{i+1} \end{bmatrix} - \Xi_i + \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} \right)$$

which implies that:

$$\begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} = \dot{\Xi}_i + \alpha \left(\begin{bmatrix} \dot{x}_{i+1} \\ \dot{y}_{i+1} \end{bmatrix} - \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} + \Xi_i - \Xi_{i+1} \right). \quad (14)$$

Therefore, to satisfy the agreement strategy defined in (13), (14) should be satisfied, and the control input defined in (10) guarantees this. Therefore, considering all the above-mentioned issues, a common pursuit centroid will be obtained for (11). On the other hand, considering the results of Theorem 1 for (11), a regular polygon formation of the agents with cyclic pursuit around the centroid with angular velocity ϖ is obtained. Therefore, the proof is completed. ■

Hence, Theorem 2 proposes a control strategy for cyclic pursuit of double-integrator MASs in regular polygon formations. The following section verifies the accuracy of the proposed control strategy via a simulation example.

4. SIMULATION RESULTS

Consider six double-integrator agents to achieve a hexagonal formation with cyclic pursuit. The positions and velocities of the agents are initialized with the values presented in Table 1. Furthermore, let us consider $\varpi = -0.5, \lambda = 1$, and $\alpha = 5$.

Table 1. The Initial Conditions of the Agents

Agent	Position	Velocity
1	$[7\ -8]$	$[0.5\ 1]$
2	$[0.5\ -6]$	$[-0.5\ -1]$
3	$[-1.5\ -1.5]$	$[-1\ 2]$
4	$[1\ 5]$	$[2\ 0]$
5	$[8\ 2]$	$[1\ -2.5]$
6	$[6\ -1]$	$[4\ -1.5]$

By employing the formation control strategy defined in (10), the trajectories of the agents pursuit centroids are depicted in Fig. 3 which show that the agreement is achieved. Moreover, the agents trajectories are depicted in Fig. 4, and Fig. 5 demonstrates the angular velocities of the agents around the agreed centroid. The figures confirm that the agents pursue each other in a hexagonal formation with angular velocity -0.5.

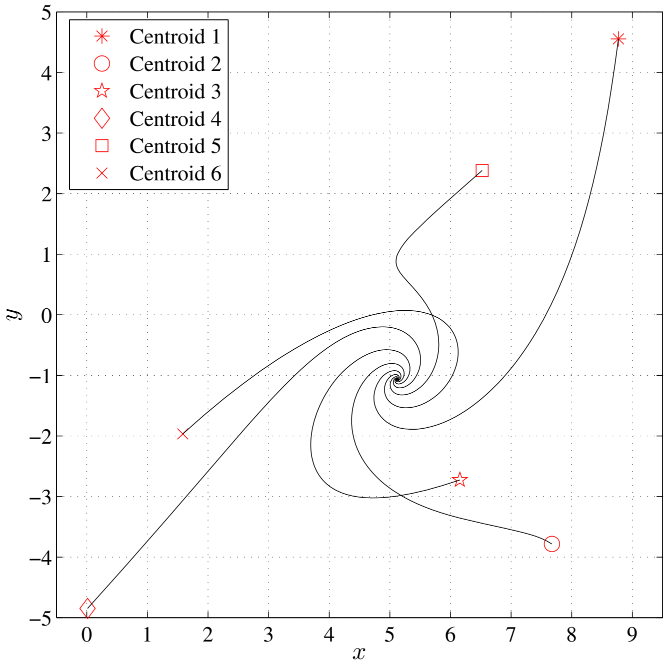

Fig. 3. The agents agreement on a pursuit centroid.

5. CONCLUSIONS

Cyclic pursuit of double-integrator MASs in regular polygon formations was studied in this paper. In a hierarchical architecture, two layers were introduced. In the first layer, the agents desired velocity vectors were designed such that they reached a regular polygon formation with cyclic pursuit around a centroid. Then, in the second layer, the agents agreed on a pursuit centroid around which they pursued each other. Despite existing approaches in the literature for cyclic pursuit of MASs in regular polygon formations, the proposed approach was based on agents with double-integrator dynamics. Moreover, in the proposed approach, it was feasible to control the cyclic pursuit direction and angular velocity.

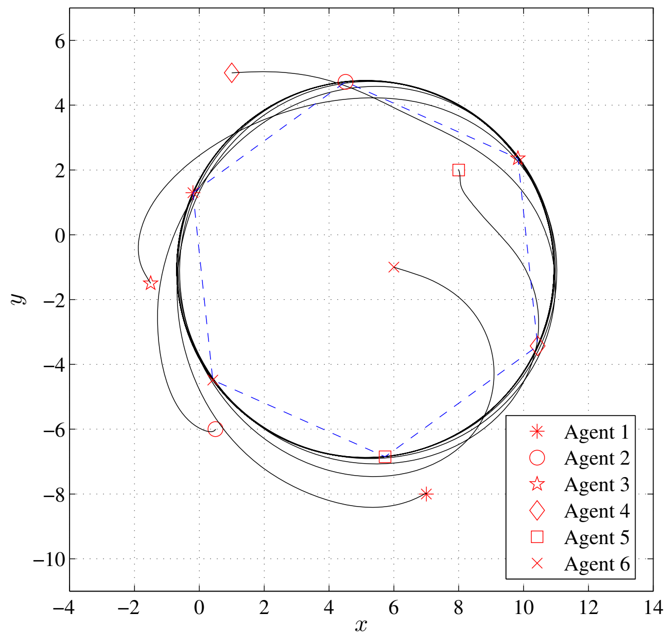

Fig. 4. The hexagonal formation of the agents with cyclic pursuit.

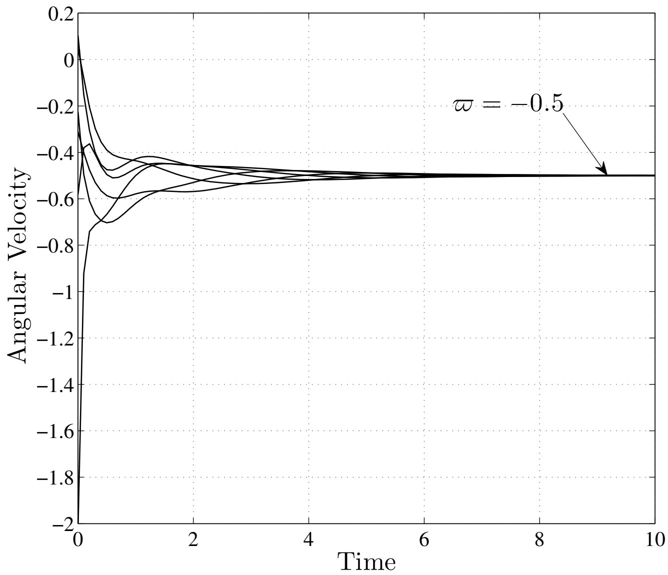

Fig. 5. The agents angular velocities around the agreed centroid.

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