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Leaderless Consensus in Networks of Flexible–Joint Manipulators

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Abstract: This work studies the leaderless consensus problem in networks composed of non-identical flexible-joint robot manipulators. Using standard functional analysis, i.e., Barbălat's Lemma, it is established that a simple control law provides a solution to the leaderless consensus problem. The network is modeled as an undirected graph and the interconnection can exhibit variable time-delays. The proposed controller consists of two different terms, one that dynamically compensates the robot gravity and another which ensures the desired consensus objective. This last term is a simple Proportional plus damping scheme. Simulations, using a network with nine 2-degrees of freedom manipulators, are provided to support the theoretical contributions of this work.

Keywords: Network of Robots; Flexible Arms; Decentralized Control; Time-Delays.

1. INTRODUCTION

Using a distributed controller, the leaderless consensus control objective is to reach an agreement between certain coordinates of interest of the network of EL-systems. The solutions to these problems have recently attracted the attention of the research community in different fields, such as biology, physics, control theory and robotics. For *fully-actuated* systems, refer to (Olfati-Saber et al., 2007; Scardovi and Sepulchre, 2009; Ren, 2008), for solutions with linear agents, and to (Yu et al., 2011; Scardovi et al., 2009; Stan and Sepulchre, 2007; Zhao et al., 2009; Nuño et al., 2011), for solutions with different classes of nonlinear agents. The practical applications of the theoretical results of this work span different areas such as underwater and space exploration (underwater cultural heritage recovery, coordination of clusters of satellites, synchronization of spacecrafts), hazardous environments (search and rescue missions, military operations), and service robotics (commercial cleaning, material handling, furniture assembly, etc.).

Consensus of networks of EL-systems without time-delays has been considered in (Ren, 2009; Mei et al., 2011) using simple proportional controllers together with filtered velocities. Actuator saturation is also considered in (Ren, 2009). The work of Nuño et al. (2011) proposes an adaptive controller for EL-systems that solves the consensus problem with constant time-delays. Further results are those in (Liu and Chopra, 2012) and in (Hatanaka et al., 2012), which consider the consensus problem in Cartesian space with constant delays. Recently, in (Nuño et al., 2012) it has been proved that networks composed by non-identical EL-systems with variable time–delays can reach a consensus, using Proportional plus damping injection (P+d) controllers, provided enough damping is injected. The work of Munz et al. (2011) provides a robust controller for the leaderless consensus for multiple fully-actuated nonlinear systems with relative degree two. It should be underscored that, all these previous results deal with *fully–actuated* EL-systems (fully–actuated robots). However, in diverse applications, including space and surgical robots, the use of thin, lightweight and cable-driven manipulators is increasing. These systems exhibit joint or link flexibility and hence they are *under-actuated* mechanical systems. See (Tavakoli and Howe, 2009; Morita et al., 2007; Mahvash and Dupont, 2008) for the application of linearized flexible-joint manipulators in teleoperation systems. Furthermore, it must be noted that, as it has been shown by Tavakoli and Howe (2009), the lumped (linear) dynamics of a flexible link is identical to the (linear) dynamics of a flexible joint.

Even for single under-actuated systems, proposing stabilizing controllers has been a non-trivial task due, in particular, to the presence of the gravity term that cannot be canceled. Hence, the need to search for solutions that hinged upon elaborate Lyapunov analysis to indirectly dominate gravity while ensuring the convergence to the desired equilibrium. A first solution to the regulation control problem for single flexible-joint robots with full state measurement has been proposed by Tomei (1991). It has been later extended to the case with only joint position measurements by Ailon and Ortega (1993); Kelly (1993). In (Kelly et al., 1994; Ortega et al., 1998) an interpretation of these controllers in terms of energy-shaping is given. The recent works of Luca and Flacco (2010, 2011) propose a dynamic gravity cancelation controller which aims at providing better performance than the previous regulation schemes (Luca et al., 2005).

The literature on the control of networks of *under-actuated* EL-systems is scarce, since, in this case the number of inputs is strictly less than the degrees-of-freedom and designing a controller is far more complicated. Few remarkable exceptions are (Nair and Leonard, 2008; Lee, 2012; Avila-Becerril and Espinosa-Pérez, 2012) and, more recently, (Avila-Becerril et al., 2014; Nuño et al., 2013). In (Nair and Leonard, 2008) the Controlled–Lagrangian technique is employed to solve the leaderless consensus in networks without delays. (Lee, 2012) presents a backstepping controller for the centroid formation control of multiple thrust propelled vehicles in the Special Euclidean space of dimension three for strongly-connected and balanced graphs without interconnection time-delays. Via a full-state feedback controller, for general directed graphs with constant time-delays, and under the assumption that the initial conditions are known, (Avila-Becerril and Espinosa-Pérez, 2012) propose the first solutions to the leaderless consensus problem. (Avila-Becerril et al., 2014) find the solution to the leaderless consensus problem eliminating the assumption of the knowledge of the initial conditions of (Avila-Becerril and Espinosa-Pérez, 2012). The authors previous work (Nuño et al., 2013) deal with the leaderless consensus problem for flexible-joint manipulators assuming that the effects of the gravity forces are absent from the EL-dynamics.

Under the assumption that the interconnection graph is undirected, and inspired by the work of Luca and Flacco (2010, 2011), this paper proposes a novel controller composed of a dynamic gravity cancelation term together with a simple P+d controller which provides a Globally Asymptotically Stable (GAS) solution to the leaderless consensus problem. Furthermore, the proposed scheme is robust to bounded interconnection variable time-delays. Simulations, using a network with nine 2-degrees of freedom manipulators, are provided to support the theoretical contributions of this work.

2. FLEXIBLE–JOINT ROBOT MANIPULATORS

This work considers networks composed of N non-identical, flexible–joint robot manipulators with n-DOF. Directly actuated, revolute joints robots are assumed and the simplified model for flexibility of Spong et al. (2005) is adopted. The dynamics of each i-th manipulator is given by

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{S}_i(\mathbf{q}_i - \boldsymbol{\theta}_i) = \mathbf{0}_n$$
$$\mathbf{J}_i\ddot{\boldsymbol{\theta}}_i + \mathbf{S}_i(\boldsymbol{\theta}_i - \mathbf{q}_i) = \boldsymbol{\tau}_i \quad (1)$$

where $\mathbf{q}_i \in \mathbb{R}^n$ is the link angular position and $\boldsymbol{\theta}_i \in \mathbb{R}^n$ is the joint (motor) angular position. The matrix $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, the matrix $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ describes the Coriolis and centrifugal effects (defined via the Christoffel symbols of the first kind), the vector $\mathbf{g}_i(\mathbf{q}_i)$ is the gravity force, the matrix $\mathbf{J}_i \in \mathbb{R}^{n\times n}$ is the motor inertia at the joints, which is symmetric and positive definite, the matrix $\mathbf{S}_i \in \mathbb{R}^{n\times n}$ is the joint stiffness which is also symmetric and positive definite and the vector $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control input. The subindex $i \in \bar{N} := \{1,\ldots,N\}$.

Dynamics (1) possess some important and well-known properties and thus they are used throughout this paper (Spong et al., 2005; Kelly et al., 2005).

P1. $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric and there exist $\lambda_{mi}, \lambda_{Mi} > 0$ such that $0 < \lambda_{mi}\mathbf{I}_n \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_{Mi}\mathbf{I}_n < \infty$.
P2. The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ is skew-symmetric.
P3. There exists $k_{gi} > 0$ such that $\left|\frac{\partial \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i}\right| \leq k_{gi}$.

3. MODELING THE INTERCONNECTION

The interconnection of the N agents is modeled using the Laplacian matrix $\mathbf{L} := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, whose elements are defined as

$$\ell_{ij} = \begin{cases} \sum_{j\in\mathcal{N}_i} w_{ij} & i = j \\ -w_{ij} & i \neq j \end{cases} \quad (2)$$

where $w_{ij} > 0$ if $j \in \mathcal{N}_i$ and $w_{ij} = 0$ otherwise. $\mathcal{N}_i$ is the set of agents transmitting information to the i-th robot.

In order to ensure that the interconnection forces are generated by the gradient of a potential function, the following assumption is used in this paper:

A1. The robot interconnection graph is *undirected* and *connected*.

Note that, by construction, $\mathbf{L}$ has a zero row sum, *i.e.*, $\mathbf{L}\mathbf{1}_N = \mathbf{0}_N$. Moreover, Assumption **A1**, ensures that $\mathbf{L}$ is symmetric, has a single zero-eigenvalue and that the rest of the spectrum of $\mathbf{L}$ has positive real parts. Thus, rank$(\mathbf{L}) = N - 1$ (Olfati-Saber and Murray, 2004).

The information exchange between the i-th and the j-th agent is subject to a variable time-delay, denoted $T_{ji}(t) \geq 0$. It is assumed that the time-delays satisfy the following:

A2. The variable time-delay $T_{ji}(t)$ has a known upper bound $^*T_{ji}$, i.e., $0 \leq T_{ji}(t) \leq {}^*T_{ji} < \infty$, and its first and second time-derivatives are bounded.

4. PROPOSED CONTROLLER WITH DYNAMIC GRAVITY COMPENSATION

The control objective is to ensure that all EL-agents link positions reach a common consensus point, while the generalized velocities asymptotically converge to zero, *i.e.* for all $i \in \bar{N}$ and any $\mathbf{q}_c \in \mathbb{R}^n$,

$$\lim_{t\to\infty} \mathbf{q}_i(t) = \mathbf{q}_c, \quad \lim_{t\to\infty} \dot{\mathbf{q}}_i(t) = \mathbf{0}. \quad (3)$$

To achieve such objective and inspired by the exact gravity cancelation scheme of (Luca and Flacco, 2010, 2011), let us define a new variable $\mathbf{x}_i \in \mathbb{R}^n$ as

$$\mathbf{x}_i := \boldsymbol{\theta}_i - \mathbf{S}_i^{-1}\mathbf{g}_i(\mathbf{q}_i). \quad (4)$$

Using (4) and defining the controller

$$\boldsymbol{\tau}_i = \bar{\boldsymbol{\tau}}_i + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{J}_i\mathbf{S}_i^{-1}\ddot{\mathbf{g}}_i(\mathbf{q}_i) - d_i\dot{\mathbf{x}}_i, \quad (5)$$

where $d_i \in \mathbb{R}_{>0}$ is the damping gain and $\bar{\boldsymbol{\tau}}_i \in \mathbb{R}^n$ is the interconnection controller term that will be defined later, the flexible-joint robot manipulator dynamics (1) can be written as

$$\begin{aligned}\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i) &= \mathbf{0}_n \\ \mathbf{J}_i\ddot{\mathbf{x}}_i + d_i\dot{\mathbf{x}}_i + \mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i) &= \bar{\boldsymbol{\tau}}_i\end{aligned}$$

Let us now define $\bar{\boldsymbol{\tau}}_i$ as the interconnection control term given by

$$\bar{\boldsymbol{\tau}}_i = -k_i \sum_{j\in\mathcal{N}_i} w_{ij}\Big(\mathbf{x}_i - \mathbf{x}_j\big(t - T_{ji}(t)\big)\Big), \tag{6}$$

where $k_i \in \mathbb{R}_{>0}$ is the proportional gain.

The closed-loop system (1), (5) and (6) is

$$\begin{aligned}\ddot{\mathbf{q}}_i &= -\mathbf{M}_i^{-1}(\mathbf{q}_i)\Big[\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i)\Big] \\ \ddot{\mathbf{x}}_i &= -\mathbf{J}_i^{-1}\Big[d_i\dot{\mathbf{x}}_i + \mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i)\Big] \\ &\quad -k_i\mathbf{J}_i^{-1}\sum_{j\in\mathcal{N}_i} w_{ij}\Big(\mathbf{x}_i - \mathbf{x}_j\big(t - T_{ji}(t)\big)\Big).\end{aligned} \tag{7}$$

Before presenting the solution to the consensus problem let us state the following lemma, which is used in the proof of the main results and has been borrowed from (Nuño et al., 2009).

Lemma 1. For any vector signals $\mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\alpha > 0$, the following inequality holds

$$-\int_0^t \mathbf{y}^\top(\sigma)\int_{\sigma-T(\sigma)}^{\sigma} \mathbf{z}(\theta)d\theta d\sigma \leq \frac{\alpha}{2}\|\mathbf{y}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{z}\|_2^2.$$

◇

Proposition 1. Consider a network of nonidentical flexible-joint manipulators of the form (1). Assume that the agents are interconnected through a communication graph with different variable time-delays characterized by Assumptions **A1** and **A2**. Then, the control law (5)-(6) ensures that (3) holds provided that the controller gains are set according to

$$2d_i > k_i\ell_{ii}\alpha_i + k_i\sum_{j\in\mathcal{N}_i} w_{ij}\frac{{}^*T_{ji}^2}{\alpha_j}, \quad \forall i \in \bar{N} \tag{8}$$

where α_i is any positive constant. ◇

Proof. Every i-th closed-loop system (7) exhibits the following energy function

$$\mathcal{E}_i := \mathcal{K}_i(\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i) + \mathcal{U}_i(\mathbf{q}_i, \mathbf{x}_i),$$

where $\mathcal{K}_i$ is the kinetic energy, given by

$$\mathcal{K}_i = \frac{1}{2}\Big[\dot{\mathbf{q}}_i^\top\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i + \dot{\mathbf{x}}_i^\top\mathbf{J}_i\dot{\mathbf{x}}_i\Big],$$

and $\mathcal{U}_i$ is the potential energy stored in the virtual link between the $\mathbf{x}_i$-coordinate and the link position, such that

$$\mathcal{U}_i(\mathbf{q}_i, \mathbf{x}_i) = (\mathbf{x}_i - \mathbf{q}_i)^\top\mathbf{S}_i(\mathbf{x}_i - \mathbf{q}_i).$$

Evaluating $\dot{\mathcal{E}}_i$ along (7) and using Property **P2**, yields

$$\dot{\mathcal{E}}_i = -d_i|\dot{\mathbf{x}}_i|^2 - k_i\sum_{j\in\mathcal{N}_i} w_{ij}\dot{\mathbf{x}}_i^\top\Big(\mathbf{x}_i - \mathbf{x}_j\big(t - T_{ji}(t)\big)\Big).$$

Now, the scaled total energy plus the potential energy in the interconnection is given by

$$\mathcal{E} := \sum_{i\in\bar{N}}\left(\frac{1}{k_i}\mathcal{E}_i + \frac{1}{4}\sum_{j\in\mathcal{N}_i} w_{ij}|\mathbf{x}_i - \mathbf{x}_j|^2\right). \tag{9}$$

Using the fact that

$$\mathbf{x}_j - \mathbf{x}_j\big(t - T_{ji}(t)\big) = \int_{t-T_{ji}(t)}^t \dot{\mathbf{x}}_j(\theta)d\theta,$$

and using $\dot{\mathcal{E}}_i$ returns

$$\dot{\mathcal{E}} = -\sum_{i\in\bar{N}}\left[\frac{d_i}{k_i}|\dot{\mathbf{x}}_i|^2 + \sum_{j\in\mathcal{N}_i} w_{ij}\dot{\mathbf{x}}_i^\top\int_{t-T_{ji}(t)}^t \dot{\mathbf{x}}_j(\theta)d\theta\right].$$

Since $\mathcal{E}$ does not qualify as a *bona fide* Lyapunov-Function and in order to establish this proof a similar procedure as in (Nuño et al., 2013) is used. For, let us first integrate $\dot{\mathcal{E}}$, from 0 to t. This yields

$$\begin{aligned}\mathcal{E}(t) - \mathcal{E}(0) = &-\sum_{i\in\bar{N}}\frac{d_i}{k_i}\int_0^t |\dot{\mathbf{x}}_i(\sigma)|^2 d\sigma \\ &-\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} w_{ij}\int_0^t \dot{\mathbf{x}}_i^\top(\sigma)\int_{\sigma-T_{ji}(\sigma)}^{\sigma}\dot{\mathbf{x}}_j(\theta)d\theta d\sigma.\end{aligned}$$

Applying Lemma 1 to the double integral term, with $\alpha_i > 0$, yields

$$\begin{aligned}\mathcal{E}(t) - \mathcal{E}(0) \leq &-\sum_{i\in\bar{N}}\frac{d_i}{k_i}\|\dot{\mathbf{x}}_i\|_2^2 + \\ &+\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} w_{ij}\left(\frac{\alpha_i}{2}\|\dot{\mathbf{x}}_i\|_2^2 + \frac{{}^*T_{ji}^2}{2\alpha_j}\|\dot{\mathbf{x}}_j\|_2^2\right).\end{aligned}$$

Recalling that $\ell_{ii} = \sum_{j\in\mathcal{N}_i} w_{ij}$ then it holds that

$$\begin{aligned}\mathcal{E}(t) - \mathcal{E}(0) \leq &-\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} w_{ij}\left(\frac{d_i}{k_i\ell_{ii}} - \frac{\alpha_i}{2}\right)\|\dot{\mathbf{x}}_i\|_2^2 \\ &+\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} w_{ij}\frac{{}^*T_{ji}^2}{2\alpha_j}\|\dot{\mathbf{x}}_j\|_2^2,\end{aligned}$$

which can be further written as

$$\mathcal{E}(t) + \mathbf{1}_N^\top\boldsymbol{\Psi}\mathrm{col}\Big(\|\dot{\mathbf{x}}_1\|_2^2, \cdots, \|\dot{\mathbf{x}}_N\|_2^2\Big) \leq \mathcal{E}(0)$$

where

$$\boldsymbol{\Psi} = \begin{bmatrix}\frac{d_1}{k_1} - \frac{\ell_{11}\alpha_1}{2} & -\frac{a_{12}{}^*T_{21}^2}{2\alpha_1} & \cdots & -\frac{a_{1N}{}^*T_{N1}^2}{2\alpha_1} \\ -\frac{a_{21}{}^*T_{12}^2}{2\alpha_2} & \frac{d_2}{k_2} - \frac{\ell_{22}\alpha_2}{2} & \cdots & -\frac{a_{2N}{}^*T_{N2}^2}{2\alpha_2} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{a_{N1}{}^*T_{1N}^2}{2\alpha_N} & -\frac{a_{N2}{}^*T_{2N}^2}{2\alpha_N} & \cdots & \frac{d_N}{k_N} - \frac{\ell_{NN}\alpha_N}{2}\end{bmatrix}$$

Clearly, if d_i is set according to (8) then there exists $\boldsymbol{\mu} \in \mathbb{R}^n$, defined as $\boldsymbol{\mu} := \boldsymbol{\Psi}^\top\mathbf{1}_N$, such that $\mu_i > 0$, for all $i \in \bar{N}$. Hence $\mathcal{E}(t) + \sum_{i\in\bar{N}}\mu_i\|\dot{\mathbf{x}}_i\|_2^2 \leq \mathcal{E}(0)$.

Thus $\dot{\mathbf{x}}_i \in \mathcal{L}_2$ and $\mathcal{E} \in \mathcal{L}_\infty$. Since $\mathcal{E}$ is positive definite and radially unbounded with regards to $\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i, |\mathbf{x}_i - \mathbf{q}_i|$, $\mathcal{E} \in \mathcal{L}_\infty$ implies that $\dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i, |\mathbf{x}_i - \mathbf{q}_i| \in \mathcal{L}_\infty$ for all $i \in \bar{N}$

and $j \in \mathcal{N}_i$. From the closed-loop system (7), all these bounded signals ensure that $\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$. Invoking Barbalăt's Lemma with $\dot{\mathbf{x}}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$ supports the fact that $\lim_{t\to\infty} \dot{\mathbf{x}}_i(t) = \mathbf{0}_n$.

Now, using (1) and (5) it can be shown that

$$\frac{d}{dt}\ddot{\mathbf{x}}_i = -\mathbf{J}_i^{-1}\left[d_i\ddot{\mathbf{x}}_i + \mathbf{S}_i(\dot{\mathbf{x}}_i - \dot{\mathbf{q}}_i) - \dot{\bar{\boldsymbol{\tau}}}_i\right]. \tag{10}$$

Furthermore, since $\ddot{\mathbf{x}}_i, \dot{\mathbf{x}}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and Assumption **A2** then $\frac{d}{dt}\ddot{\mathbf{x}}_i \in \mathcal{L}_\infty$. This last together with

$$\lim_{t\to\infty}\int_0^t \ddot{\mathbf{x}}_i(\sigma)d\sigma = \lim_{t\to\infty}\dot{\mathbf{x}}_i(t) - \dot{\mathbf{x}}_i(0) = -\dot{\mathbf{x}}_i(0),$$

supports the claim that $\lim_{t\to\infty}\ddot{\mathbf{x}}_i(t) = \mathbf{0}_n$, according to Barbalăt's Lemma.

Note that all signals on the right-hand-side of (10) asymptotically converge to zero, except $\dot{\mathbf{q}}_i$. If it is proved that $\lim_{t\to\infty}\frac{d}{dt}\ddot{\mathbf{x}}_i(t) = \mathbf{0}_n$, then $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}_n$. In fact, (10) and boundedness of $\frac{d}{dt}\ddot{\mathbf{x}}_i, \ddot{\mathbf{x}}_i, \ddot{\mathbf{q}}_i$ together with Assumption **A2** ensure that $\frac{d}{dt}\ddot{\mathbf{x}}_i$ is uniformly continuous, which together with the fact that convergence to zero of $\ddot{\mathbf{x}}_i$ implies that

$$\lim_{t\to\infty}\int_0^t \frac{d}{dt}\ddot{\mathbf{x}}_i(\sigma)d\sigma = \lim_{t\to\infty}\ddot{\mathbf{x}}_i(t) - \ddot{\mathbf{x}}_i(0) = -\ddot{\mathbf{x}}_i(0),$$

ensures that $\lim_{t\to\infty}\frac{d}{dt}\ddot{\mathbf{x}}_i(t) = \mathbf{0}_n$, as needed. Finally, $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}_n$ and $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \dot{\mathbf{x}}_i \in \mathcal{L}_\infty$ ensure, from (7), that $\lim_{t\to\infty}\ddot{\mathbf{q}}_i(t) = \mathbf{0}_n$.

Since $\mathbf{x}_j\big(t - T_{ji}(t)\big) = \mathbf{x}_j - \int_{t-T_{ji}(t)}^t \dot{\mathbf{x}}_j(\theta)d\theta$ and $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}_n$ then $\int_{t-T_{ji}(t)}^t \dot{\mathbf{x}}_j(\theta)d\theta = \mathbf{0}_n$. Thus, at the equilibrium, it holds that $\mathbf{q}_i = \mathbf{x}_i$ and

$$\sum_{j\in\mathcal{N}_i} w_{ij}(\mathbf{x}_i - \mathbf{x}_j) = \mathbf{0}_n,$$

which by piling up the N vectors $\mathbf{q}_i$ and $\mathbf{x}_i$ as $\mathbf{q} := \mathrm{col}\big(\mathbf{q}_1^\top, \ldots, \mathbf{q}_N^\top\big)$ and $\mathbf{x} := \mathrm{col}\big(\mathbf{x}_1^\top, \ldots, \mathbf{x}_N^\top\big)$, respectively, can be written as $\mathbf{q} = \mathbf{x}$ and $(\mathbf{L} \otimes \mathbf{I}_n)\mathbf{x} = \mathbf{0}_{Nn}$.

This last, together with the Laplacian properties, ensures that the only possible solution to these equations is $\mathbf{q} = \mathbf{x} = \big(\mathbf{1}_N \otimes \mathbf{q}_c\big)$, for any $\mathbf{q}_c \in \mathbb{R}^n$. Hence, for all $i \in \bar{N}$, $\mathbf{q}_i = \mathbf{q}_c$. This concludes the proof. □

Remark 1. If link accelerations are not available for measurement then, in order to implement the proposed controllers, the term $\ddot{\mathbf{g}}_i(\mathbf{q}_i)$ can be algebraically computed as

$$\begin{aligned}\ddot{\mathbf{g}}_i(\mathbf{q}_i) &= -\frac{\partial \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i}\ddot{\mathbf{q}}_i + \sum_{k=1}^n \frac{\partial^2 \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i \partial q_{i_k}}\dot{\mathbf{q}}_i\dot{q}_{i_k}\\ &= -\frac{\partial \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i}\mathbf{M}_i^{-1}\Big[\mathbf{C}_i\dot{\mathbf{q}}_i + \mathbf{S}_i(\mathbf{q}_i - \mathbf{x}_i)\Big]\\ &\quad + \sum_{k=1}^n \frac{\partial^2 \mathbf{g}_i(\mathbf{q}_i)}{\partial \mathbf{q}_i \partial q_{i_k}}\dot{\mathbf{q}}_i\dot{q}_{i_k}.\end{aligned}$$

This algebraic manipulation does not induce an algebraic loop because the relative degree is four.

Remark 2. Since the class of under-actuated EL-systems in (Nuño et al., 2013) is contained within the dynamics (1), it is not surprising that the damping injection condition (8) is the same as in (Nuño et al., 2013).

5. SIMULATIONS

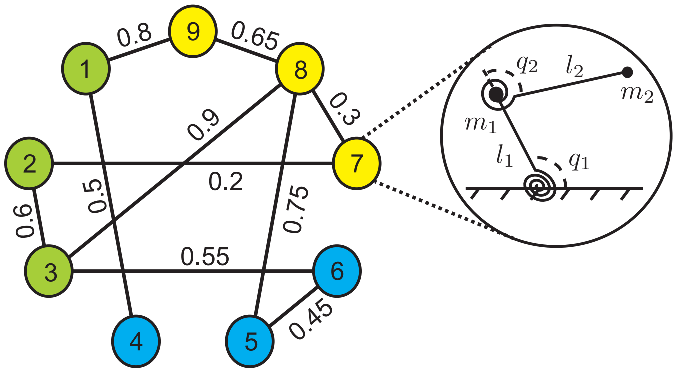

Fig. 1. Weighted network composed of nine 2-DOF nonlinear flexible-joint manipulators with revolute joints. There are three different groups of manipulators and the members of each group are equal.

By means of some numerical simulations, this section shows the consensus performance of the proposed controllers. We consider an undirected, connected and weighted network composed of nine 2-DOF nonlinear flexible-joint manipulators with revolute joints. The inertia matrix, the Coriolis and centrifugal effects matrix and the gravity vector are respectively given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix}\alpha_i + 2\beta_i c_{i_2} & \delta_i + \beta_i c_{i_2}\\ \delta_i + \beta_i c_{i_2} & \delta_i\end{bmatrix},$$

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix}-2\beta_i s_{i_2}\dot{\mathbf{q}}_{i_2} & -\beta_i s_{i_2}\dot{\mathbf{q}}_{i_2}\\ \beta_i s_{i_1}\dot{\mathbf{q}}_{i_2} & 0\end{bmatrix}$$

and

$$\mathbf{g}_i(\mathbf{q}_i) = \mathrm{col}(l_{i_1}(m_{i_1} + m_{i_2})c_{i_1} + g l_{i_2} m_{i_2} c_{i_{12}}, g l_{i_2} m_{i_2} c_{i_{12}}),$$

where c_{i_k}, s_{i_k} are the short notation for $\cos(\mathbf{q}_{i_k})$ and $\sin(\mathbf{q}_{i_k})$; $c_{i_{12}}$ stands for $\cos(q_{i_1} + q_{i_2})$; $\mathbf{q}_{i_k}$ represents the angular position of link k of manipulator i, with $k \in 1, 2$; $\alpha_i = l_{i_2}^2 m_{i_2} + l_{i_1}^2(m_{i_1} + m_{i_2})$, $\beta_i = l_{i_1} l_{i_2} m_{i_2}$ and $\delta_i = l_{i_2}^2 m_{i_2}$, where l_{i_k} and m_{i_k} are the respective lengths and masses of each link and $g = 9.81$ is the acceleration of gravity constant.

The network is composed of three different groups of manipulators, with equal members at each group. The physical parameters are: $m_1 = 4$kg, $m_2 = 2.5$kg and $l_1 = l_2 = 0.5$m, for Agents 1, 2 and 3; $m_1 = 3$kg, $m_2 = 2.5$kg, $l_1 = 0.6$m and $l_2 = 0.5$m for Agents 4, 5 and 6; $m_1 = 3.5$kg, $m_2 = 2$kg, $l_1 = 0.3$m and $l_2 = 0.45$m for Agents 7, 8 and 9. The joint stiffness matrix has been set to $\mathbf{S} = \mathrm{diag}(200, 200)$ and the motor inertia to $\mathbf{J} = \mathrm{diag}(0.5, 0.5)$, for all agents.

Fig. 2 shows the interconnection variable time–delays. These delays emulate an ordinary UDP/IP Internet delay with a normal Gaussian distribution Rossi et al. (2006) and, for simplicity, only three different delays have been employed, namely $T_{j1} = T_{j4} = T_{j7} = T_1(t)$ with mean equal to 0.34 and variance equal to 0.0002; $T_{j2} = T_{j5} = T_{j8} = T_2(t)$ with mean and variance equal to 0.25 and 0.00015, respectively; and $T_{j3} = T_{j6} = T_{j9} = T_3(t)$ with mean: 0.15 and variance: 0.0006. The employed upper bounds of these delays are $^*T_1 = 0.4$s, $^*T_2 = 0.3$s and $^*T_3 = 0.25$s.

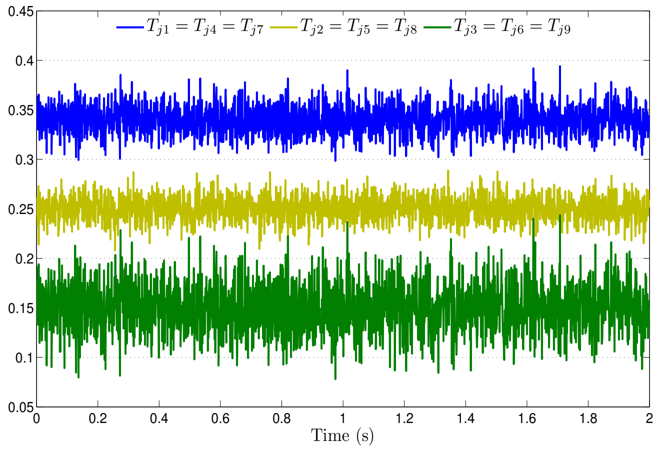

Fig. 2. Emulated UDP/IP Internet delays for the simulations.

The interconnection gains, k_i have all been set to 10. The damping gains are given by: $d_1 = 9.6$, $d_2 = 5.7$, $d_3 = 13.31$, $d_4 = 4$, $d_5 = 8.3$, $d_6 = 6.75$, $d_7 = 4$, $d_8 = 17.4$ and $d_9 = 9.5$. Easy calculations show that these gains satisfy condition (8) with $\alpha_i = 1$ for all $i \in [1, 10]$.

Fig. 3 presents the simulations results for three different scenarios. Column A shows the results when the initial conditions are fixed as

$$\mathbf{q}^\top(0) = [2, 1, 1.5, 2.5, 0.5, 3, -1.5, -1, 3, 0.5, 3.5, -2 \\ -0.5, -2.5, 1, 1.5, -2, 3.5, 2.5, 2]. \tag{11}$$

Column B, depicts the convergence of all agents and, in this case, the initial conditions (11) have been multiplied by 2. Finally, in Column C it can be seen that the EL-systems asymptotically converge to a consensus point and in this case, the initial conditions (11) have been multipied by -2. In all cases, the initial velocities were set to zero, $\boldsymbol{\theta}(0) = \mathbf{q}(0)$. Hence, it can be observed that in the three different sets of initial conditions, the agents asymptotically reach a consensus point and such consensus point changes if the initial conditions change.

In Fig. 4, the stability condition (8) has not been met and it can be observed that the agents do not reach a consensus point. The initial conditions were established as (11). In this case, the controller gains have been set to $k_i = 5$ and $d_i = 1$, for all agents.

6. CONCLUSIONS

Under the assumption that the undirected graph is connected and that interconnection may induce variable time-delays, a globally asymptotically stable solution to the consensus problem for networks composed of nonidentical flexible-joint robot manipulators, is reported in this paper. The proposed controller is composed of two different terms, one that dynamically compensates the link gravity and another which ensures the desired consensus objectives, which is a simple Proportional plus damping scheme.

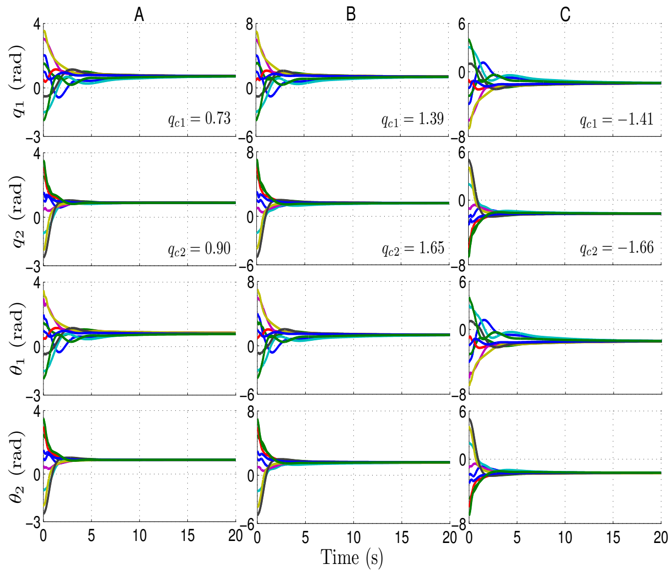

Fig. 3. Simulation results for three different sets of initial conditions.

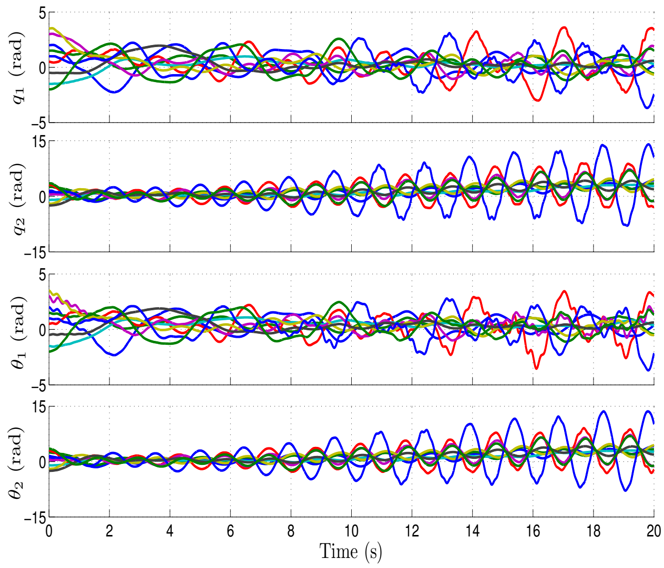

Fig. 4. Simulation results when the damping gains do not satisfy the stability condition.

Using a nine 2-DOF flexible-joint robot network and asymmetric delays in the communication, numerical simulations show the performance of the proposed controller.

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