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Robust PI based set-point learning control for batch processes subject to time-varying uncertainties and load disturbance

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Abstract: Based on the proportional-integral (PI) closed-loop control widely used in industrial engineering practice, a robust iterative learning control (ILC) method is proposed for industrial batch processes subject to time-varying uncertainties and load disturbance. An important merit is that the proposed ILC design is independent of the PI tuning which maintains the closed-loop system stability, owing to that the ILC updating law is implemented through adjusting the set-point of the closed-loop system and adding a feedforward control signal to the plant input along the batch-to-batch direction. Using the robust H infinity control objective, a robust discrete-time PI tuning algorithm is given in terms of the plant state-space model description with norm-bounded time-varying uncertainties. For the batch-to-batch direction, a robust ILC updating law is developed based on the two-dimensional (2D) control system theory, which is capable of perfect output tracking against repetitive type load disturbance. An illustrative example from the literature is adopted to demonstrate the effectiveness and merits of the proposed ILC method.

1. INTRODUCTION

It has become appealing to develop robust ILC methods to deal with time-varying uncertainties occurring in a cycle or cycle-to-cycle (batchwise) uncertainties, because many batch processes, e.g., pharmaceutical crystallization, are slowly varying from batch to batch, while repeating fundamental dynamic response characteristics (Seborg et al., 2003; Nagy et al., 2008). As surveyed by Wang et al. (2009), most of the existing references have been devoted to time-invariant linear or nonlinear batch processes. The developed robust ILC methods have been in general classified into two types, one is called direct-type that means the ILC design integrates the feedback control (responsible for closed-loop stability and no steady output deviation) and the feedforward control (responsible for the set-point tracking) through the identical closed-loop controller, and another is called indirect-type which implies that either the feedback or the feedforward control could be implemented through different controllers which may be designed relatively independent.

For the direct-type ILC, the traditional proportional-integral-derivative (PID) controller is mostly used to execute the integrated control for both the set-point tracking and closed-loop stabilization, e.g., the P-type ILC (Xiong and Zhang, 2003), the PI-type ILC (Shi et al., 2005), the PD-type ILC (Mi et al., 2005), the PID-type ILC (Ruan et al., 2008). The achievable robustness and output tracking performance, however, have not yet been fully explored (Tayebi, 2007). Based on a two-dimensional (2D) state-space description of a batch process and using the linear quadratic optimal control criterion in combination with the robust control theory, full-order controller matrices were used to develop robust direct-type ILC methods to accommodate for a variety of process uncertainties (Liu and Wang, 2012), but at the expense of controller complexity and computation effort.

For the indirect-type ILC, the control structure is typically composed of two loops, one loop constructed in terms of a conventional controller like PID, and another loop used for adjusting the set-point or the process input. Based on the internal model control (IMC) structure, a set-point learning design was proposed (Liu et al., 2010) to robustly track the set-point profile against the process input delay uncertainty. Based on the conventional PID control structure, a parallel learning-type PID was added to improve the set-point tracking performance (Tan et al., 2007). The robust stability condition of a learning-type set-point design in terms of a PI control loop was analyzed by Wang et al. (2012). The achievable tracking performance of an indirect-type ILC scheme was assessed by computing the minimum output variance bound (Chen and Kong, 2009).

Here, a set-point learning type ILC design is proposed based on the widely used PI control structure to cope with time-varying process uncertainties and load disturbance. With a state-space model description of the process together with norm-bounded uncertainties, a robust PI tuning algorithm is first given in terms of the H infinity control objective, which is primarily responsible for maintaining the closed-loop system robust stability. Then, an ILC scheme consisting of the learning type controllers to adjust the set-point and the process input is proposed to realize robust tracking against time-varying uncertainties and load disturbance. It is therefore a merit that the PI tuning and the ILC design can be made relatively independent of each other. By establishing sufficient conditions in terms of linear matrix inequality (LMI) constraints for holding robust stability of the PI control loop and robust convergence of the ILC scheme, respectively, both the PI and ILC controllers are formulated along with an adjustable robust H infinity performance level.

Throughout this paper, the following notations are used: $\Re^{n\times m}$ denotes a $n\times m$ real matrix space. For any matrix $P\in\Re^{n\times m}$, $P>0$ (or $P\geq 0$) means P is a positive (or semipositive) definite symmetric matrix, where the symmetric elements are indicated by '*'. P^T denotes the transpose of P. Denote by $diag\{\bullet\}$ a block-diagonal matrix. The identity or zero vector (or matrix) is denoted by $\boldsymbol{I}$ or $\mathbf{0}$.

2. PROBLEM FORMULATION

To study a batch process with time-varying uncertainties from cycle to cycle, the following observable canonical discrete-time state-space model description is considered,

$$\begin{cases} x(t+1,k+1)=[A_{\mathrm{m}}+\Delta\tilde{A}(t,k+1)]x(t,k+1)+ \\ \qquad [B_{\mathrm{m}}+\Delta\tilde{B}(t,k+1)]u(t,k+1)+\omega(t,k+1) \\ y(t,k+1)=Cx(t,k+1),\ \ 0\leq t\leq T_{\mathrm{p}}; \\ x(0,k+1)=x(0),\ \ k=0,1,\cdots. \end{cases} \tag{1}$$

where t and k denotes time and batch indices, respectively. and $k+1$ indicates the current batch (or cycle). $x(t,k+1)\in\Re^{n_x}$ denote the state variables, $u(t,k+1)\in\Re^{n_u}$ the control inputs, $y(t,k+1)\in\Re^{n_y}$ the process outputs. $\Delta\tilde{A}(t,k+1)$ and $\Delta\tilde{B}(t,k+1)$ denotes time-varying uncertainties from cycle to cycle, which is practically specified as $\Delta\tilde{A}(t,k+1)=\Delta\bar{A}_1\tilde{\Theta}_1(t)\Delta\bar{A}_2$, $\Delta\tilde{B}(t,k+1)=\Delta\bar{B}_1\tilde{\Theta}_2(t)\Delta\bar{B}_2$, where $\Delta\bar{A}_1$, $\Delta\bar{A}_2$, $\Delta\bar{B}_1$, and $\Delta\bar{B}_2$ are constant matrices, and $\tilde{\Theta}_i^T(t)\tilde{\Theta}_i(t)\leq\boldsymbol{I}$, $i=1,2$ for $0\leq t\leq T_{\mathrm{p}}$. Denote by T_{p} the time period of each cycle, and $x(0)$ is the initial resetting condition of each cycle. Practically, other process uncertainties such as from input actuator and output measurement may also be lumped into $\Delta\tilde{A}(t,k+1)$ and $\Delta\tilde{B}(t,k+1)$ for analysis.

The control objective is to determine a control law such that the system output(s) can track the desired output profile (or target output trajectory) as close as possible against the process uncertainties and/or load disturbance.

To facilitate the ILC design along the batchwise direction, we define the output tracking error in the current cycle as

$$e(t,k+1)\triangleq \mathrm{Y}_{\mathrm{r}}(t)-y(t,k+1) \tag{2}$$

where $\mathrm{Y}_{\mathrm{r}}(t)$ denotes the desired output profile, and $y(t,k+1)$ the real output in the current cycle. Correspondingly, the time integral of $e(t,k+1)$ is denoted by $\sum e_{\mathrm{s}}(t,k+1)$, i.e.

$$\Sigma e(t,k+1)=\sum_{i=0}^{t}e(i,k+1),\ 0\leq t\leq T_{\mathrm{p}} \tag{3}$$

By comparison, we define the set-point tracking error in the current cycle by

$$e_{\mathrm{s}}(t,k+1)\triangleq \mathrm{y}_{\mathrm{s}}(t,k+1)-y(t,k+1) \tag{4}$$

where $y_{\mathrm{s}}(t,k+1)$ denotes the set-point input in the current cycle, which is different with $\mathrm{Y}_{\mathrm{r}}(t)$ in that it is adjusted in real time for tracking $\mathrm{Y}_{\mathrm{r}}(t)$.

The time integral of $e_{\mathrm{s}}(t,k+1)$ is denoted by $\sum e_{\mathrm{s}}(t,k+1)$. It follows that

$$\sum e_{\mathrm{s}}(t,k+1)=\sum e_{\mathrm{s}}(t-1,k+1)+e_{\mathrm{s}}(t,k+1),\ 0\leq t\leq T_{\mathrm{p}} \tag{5}$$

Moreover, we define a batchwise error function by

$$\delta f(t,k+1)\triangleq f(t,k+1)-f(t,k) \tag{6}$$

where f may denote x, y_{s}, u, e, e_{s}, or ω.

It follows from (1) by using (2), (4), and (6) that

$$e(t,k+1)=e(t,k)-C\delta x(t,k+1) \tag{7}$$

$$\begin{aligned}\delta x(t+1,k+1)&=[A_{\mathrm{m}}+\Delta\tilde{A}(t,k+1)]\delta x(t,k+1)\\&\quad+[B_{\mathrm{m}}+\Delta\tilde{B}(t,k+1)]\delta u(t,k+1)+\varpi(t,k+1)\end{aligned} \tag{8}$$

where

$$\begin{aligned}\varpi(t,k+1)&=[\Delta\tilde{A}(t,k+1)-\Delta\tilde{A}(t,k)]x(t,k)\\&\quad+[\Delta\tilde{B}(t,k+1)-\Delta\tilde{B}(t,k)]u(t,k)+\delta\omega(t,k+1)\end{aligned} \tag{9}$$

It is obvious that $\omega(t,k+1)\neq 0$ for any non-repeatable parameter uncertainties and load disturbance.

Based on the conventional PI control structure, the proposed ILC scheme is shown in Fig.1,

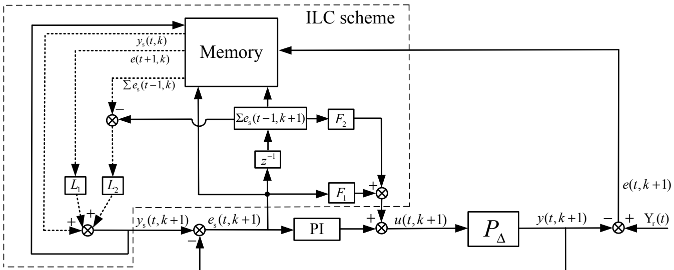

Fig. 1. Block diagram of the proposed PI based ILC scheme

where the learning controllers, L_1 and L_2 are used to adjust the set-point command, i.e.

$$y_{\mathrm{s}}(t,k+1)=y_{\mathrm{s}}(t,k)+L_1e(t+1,k)+L_2\delta\sum e_{\mathrm{s}}(t-1,k+1) \tag{10}$$

where $y_{\mathrm{s}}(t,k)$ denotes the set-point input in the previous cycle, and $e(t+1,k)$ the one-step ahead output error in the previous cycle. It follows from (4), (5), and (6) that

$$\delta e_{\mathrm{s}}(t-1,k+1)=e_{\mathrm{s}}(t-1,k+1)-e_{\mathrm{s}}(t-1,k) \tag{11}$$

$$\delta\sum e_{\mathrm{s}}(t,k+1)=\delta\sum e_{\mathrm{s}}(t-1,k+1)+\delta e_{\mathrm{s}}(t,k+1) \tag{12}$$

In Fig. 1, the feedforward controllers, F_1 and F_2, are used to adjust the process input, i.e.

$$u(t,k+1)=u_{\mathrm{PI}}(t,k+1)+F_1e_{\mathrm{s}}(t,k+1)+F_2\sum e_{\mathrm{s}}(t-1,k+1) \tag{13}$$

where u_{PI} is the PI control output.

It is seen from (13) and Fig.1 that the ILC scheme (in the dash-line box) is relatively independent of the PI control loop. Therefore, both of them can be designed separately, as detailed in the following two sections.

3. ROBUST PI TUNING

According to the process state-space description in (1), by omitting the batch index for brevity due to its irrelevance to the PI tuning in the proposed control scheme shown in Fig. 1, a PI control law is generally expressed in the following form,

$$u_{\mathrm{PI}}(t)=k_{\mathrm{p}}e(t)+k_{\mathrm{i}}\sum e(t) \tag{14}$$

where k_{p} and k_{i} are the proportional and integral parameters of PI, respectively.

By introducing an auxiliary state variable, $\sum e(t)$, we establish the augmented control system description,

$$\begin{cases}\begin{bmatrix}x(t+1)\\ \sum e(t)\end{bmatrix}=(\begin{bmatrix}\tilde{A} & \mathbf{0}\\ -C & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}x(t)\\ \sum e(t-1)\end{bmatrix}+\begin{bmatrix}\tilde{B}\\ \mathbf{0}\end{bmatrix}u(t)+\begin{bmatrix}\boldsymbol{I}\\ \mathbf{0}\end{bmatrix}\omega(t)\\ y(t)=\begin{bmatrix}C & \mathbf{0}\end{bmatrix}\begin{bmatrix}x(t)\\ \sum e(t-1)\end{bmatrix}\end{cases} \tag{15}$$

where $\tilde{A}=A_{\mathrm{m}}+\Delta\tilde{A}(t)$ and $\tilde{B}=B_{\mathrm{m}}+\Delta\tilde{B}(t)$.

Let

$$\hat{k}_{\mathrm{p}}=k_{\mathrm{p}}+k_{\mathrm{i}} \tag{16}$$

Substituting (14) and (16) into (15) yields the augmented system,

$$\begin{cases}\begin{bmatrix}x(t+1)\\ \sum e(t)\end{bmatrix}=\begin{bmatrix}\tilde{A}-\tilde{B}\hat{k}_{\mathrm{p}}C & \tilde{B}k_{\mathrm{i}}\\ -C & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}x(t)\\ \sum e(t-1)\end{bmatrix}+\begin{bmatrix}\boldsymbol{I}\\ \mathbf{0}\end{bmatrix}\omega(t)\\ y(t)=\begin{bmatrix}C & \mathbf{0}\end{bmatrix}\begin{bmatrix}x(t)\\ \sum e(t-1)\end{bmatrix}\end{cases} \tag{17}$$

For tuning the PI controller to maintain the control system robust stability, the H infinity control objective is adopted, i.e.

$$\|e(t)\|_2<\gamma_{\mathrm{PI}}\|\omega(t)\|_2 \tag{18}$$

where γ_{PI} denotes the robust performance level.

To achieve the H infinity control objective, we give the following theorem,

Theorem 1: *The PI control system in (17) subject to time-varying uncertainties shown in (1) is guaranteed robustly stable with a H infinity control performance level,* γ_{PI}*, if there exist* $P_{11}>0$*,* $P_{22}>0$*, matrices* P_{12}*,* R_1*,* R_2*, and positive scalars* ε_1*,* ε_2*, such that the following LMI holds,*

$$\begin{bmatrix}-P+\varepsilon_1\Phi_{\mathrm{A1}}\Phi_{\mathrm{A1}}^T+\varepsilon_2\Phi_{\mathrm{B1}}\Phi_{\mathrm{B1}}^T & \Gamma & D_{\mathrm{g}} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ * & -P & \mathbf{0} & PH^TC^T & P\Phi_{\mathrm{A2}}^T & P\Phi_{\mathrm{B2}}^T\\ * & * & -\gamma_{\mathrm{PI}}\boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ * & * & * & -\gamma_{\mathrm{PI}}\boldsymbol{I} & \mathbf{0} & \mathbf{0}\\ * & * & * & * & -\varepsilon_1\boldsymbol{I} & \mathbf{0}\\ * & * & * & * & * & -\varepsilon_2\boldsymbol{I}\end{bmatrix}<0 \tag{19}$$

where $D_{\mathrm{g}}=[\boldsymbol{I}\quad \mathbf{0}]^T$, $H=[\boldsymbol{I}\quad \mathbf{0}]$, $\Phi_{\mathrm{A1}}=[\Delta\overline{A}_1^T,\ \mathbf{0}]^T$, $\Phi_{\mathrm{A2}}=[\Delta\overline{A}_2P_{11},\ \Delta\overline{A}_2P_{12}]$, $\Phi_{\mathrm{B1}}=[\Delta\overline{B}_1^T,\ \mathbf{0}]^T$, $\Phi_{\mathrm{B2}}=[\Delta\overline{B}_2R_1,\ \Delta\overline{B}_2R_2]$,

$$P=\begin{bmatrix}P_{11} & P_{12}\\ * & P_{22}\end{bmatrix},\ \Gamma=\begin{bmatrix}A_{\mathrm{m}}P_{11}+B_{\mathrm{m}}R_1 & A_{\mathrm{m}}P_{12}+B_{\mathrm{m}}R_2\\ -CP_{11}+P_{12}^T & -CP_{12}+P_{22}\end{bmatrix}$$

by parameterizing

$$[-\hat{k}_{\mathrm{p}}C\quad k_{\mathrm{i}}]=[R_1\quad R_2]P^{-1} \tag{20}$$

Proof: Define the following Lyapunov-Krasovskii inequality of state energy to guarantee the asymptotic stability of the closed-loop system shown in (17),

$$V_P[\hat{x}(t+1)]-V_P[\hat{x}(t)]<-(\gamma_{\mathrm{PI}}^{-1}\|e(t)\|_2^2-\gamma_{\mathrm{PI}}\|\omega(t)\|_2^2) \tag{21}$$

where $\hat{x}(t)=[x^T(t),\ \sum e(t-1)^T]^T$.

Considering that $e(t)=-Cx(t)$ by letting $\Upsilon_{\mathrm{r}}(t)=0$, and $x(t)=[\boldsymbol{I},\ \mathbf{0}]\hat{x}(t)$, we have

$$e(t)=-CH\hat{x}(t) \tag{22}$$

By substituting (17) into (21), we obtain

$$\zeta^T\Xi_1\zeta<0 \tag{23}$$

where $\zeta=[\hat{x}^T(t),\ \omega^T(t)]^T$, $D_{\mathrm{g}}=[\boldsymbol{I}\quad 0]^T$, and

$$\tilde{A}_{\mathrm{g}}=\begin{bmatrix}\tilde{A}-\tilde{B}\hat{k}_{\mathrm{p}}C & \tilde{B}k_{\mathrm{i}}\\ -C & \boldsymbol{I}\end{bmatrix} \tag{24}$$

$$\Xi_1=\begin{bmatrix}\tilde{A}_{\mathrm{g}}^T\\ D_{\mathrm{g}}^T\end{bmatrix}P\begin{bmatrix}\tilde{A}_{\mathrm{g}} & D_{\mathrm{g}}\end{bmatrix}-\begin{bmatrix}P-\gamma_{\mathrm{PI}}^{-1}H^TC^TCH & \mathbf{0}\\ * & \gamma_{\mathrm{PI}}\boldsymbol{I}\end{bmatrix} \tag{25}$$

By the Schur complement, it can be derived that (23) is guaranteed by

$$\begin{bmatrix}-P & \tilde{\Gamma} & D_{\mathrm{g}} & \mathbf{0}\\ * & -P & \mathbf{0} & PH^TC^T\\ * & * & -\gamma_{\mathrm{PI}}\boldsymbol{I} & \mathbf{0}\\ * & * & * & -\gamma_{\mathrm{PI}}\boldsymbol{I}\end{bmatrix}<0 \tag{26}$$

where

$$\tilde{\Gamma}=\begin{bmatrix}\tilde{A}P_{11}-\tilde{B}\hat{k}_{\mathrm{p}}CP_{11}+\tilde{B}k_iP_{12}^T & \tilde{A}P_{12}-\tilde{B}\hat{k}_{\mathrm{p}}CP_{12}+\tilde{B}k_iP_{22}\\ -CP_{11}+P_{12}^T & -CP_{12}+P_{22}\end{bmatrix} \tag{27}$$

Note that $\tilde{\Gamma}$ can be reformulated as

$$\tilde{\Gamma}=\Gamma+\Phi_{\mathrm{A1}}\tilde{\Theta}_1(t)\Phi_{\mathrm{A2}}+\Phi_{\mathrm{B1}}\tilde{\Theta}_2(t)\Phi_{\mathrm{B2}} \tag{28}$$

where $R_1=-\hat{k}_{\mathrm{p}}CP_{11}+k_iP_{12}^T$ and $R_2=-\hat{k}_{\mathrm{p}}CP_{12}+k_iP_{22}$ in Γ.

The following lemma is used herein for analysis.

Lemma 1 *(Wang et al., 1992)**:** Let* A*,* D*,* E*, and* F *be real matrices of appropriate dimensions with* $\|F\|\le 1$*, the following inequality holds for any scalar* $\varepsilon>0$*,*

$$DFE+E^TF^TD^T<\varepsilon DD^T+\varepsilon^{-1}E^TE \tag{29}$$

Using Lemma 1 and the Schur complement, it can be seen that (26) is guaranteed by (19) in Theorem 1. □

Hence, the PI parameters can be retrieved from (16) and (20).

To achieve good robust control performance, the PI controller can be determined by performing the following optimization program,

$$\underset{\Delta\tilde{A}(t),\ \Delta\tilde{B}(t)}{\text{Minimize}}\ \gamma_{\mathrm{PI}} \tag{30}$$

In fact, a smaller value of γ_{PI} leads to a more aggressive control action and vice versa. Therefore, a trade-off should be made for tuning the PI controller.

4. ROBUST ILC DESIGN

To develop a robust ILC method, we construct a 2D system model to describe the dynamics of both the time and batchwise directions, for the purpose of analyzing the 2D stability against process uncertainties and load disturbance.

Consider a 2D Roesser's system (Kaczorek, 1985),

$$\begin{cases}\begin{bmatrix}x^h(i+1,j)\\ x^v(i,j+1)\end{bmatrix}=\begin{bmatrix}A_{11}+\Delta A_{11} & A_{12}+\Delta A_{12}\\ A_{21}+\Delta A_{21} & A_{22}+\Delta A_{22}\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}+\omega(i,j)\\ y(i,j)=\begin{bmatrix}C_1 & C_2\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}\\ i,j=0,1,2,\cdots.\end{cases} \tag{31}$$

where $x^h\in\Re^{n_1}$ is the horizontal state vector, $x^v\in\Re^{n_2}$ the vertical state vector, y the system output. The boundary condition of the Roesser's system is denoted by $\hat{x}(t) = \left[[x^h(0,j)]^T,\ [x^v(i,0)]^T \right]^T$.

Lemma 2 *(Wang et al., 2012)**:** If there exist matrices $P_1 > 0$* and $P_2 > 0$ *such that the following LMI holds*

$$\tilde{A}^T P \tilde{A} - P < 0 \tag{32}$$

where

$$\tilde{A} = \begin{bmatrix} A_{11} + \Delta A_{11} & A_{12} + \Delta A_{12} \\ A_{21} + \Delta A_{21} & A_{22} + \Delta A_{22} \end{bmatrix},\ P = diag\{P_1,\ P_2\}$$

then the 2D Roesser's system in (31) with $\omega = 0$ is asymptotically stable. In addition, if $x^h(0,j) \equiv 0$, there exists a positive scalar $\rho \in (0,1)$ such that

$$\sum_{i=0}^{I_0} V_{P_2}[x^v(i,j+1)] < \rho \sum_{i=0}^{I_0} V_{P_2}[x^v(i,j)],\ \forall j, I_0 > 0,\ \forall x^v(i,0) \tag{33}$$

According to the proposed ILC scheme shown in Fig.1, it follows from (4), (6), (10) and (12) that

$$\delta e_s(t,k+1) = L_1 e(t+1,k) + L_2 \delta \sum e_s(t-1,k+1) - C\delta x(t,k+1) \tag{34}$$

$$\delta \sum e_s(t,k+1) = L_1 e(t+1,k) + (\boldsymbol{I} + L_2)\delta \sum e_s(t-1,k+1) - C\delta x(t,k+1) \tag{35}$$

Substituting the PI control law of (14) into (13), we obtain

$$u(t,k+1) = (k_p + k_i + F_1) e_s(t,k+1) + (k_i + F_2) \sum e_s(t-1,k+1) \tag{36}$$

Correspondingly, it follows that

$$\delta u(t,k+1) = (k_p + k_i + F_1) \delta e_s(t,k+1) + (k_i + F_2) \delta \sum e_s(t-1,k+1) \tag{37}$$

Substituting (34), (35), and (37) into (8), we obtain

$$\begin{aligned} \delta x(t+1,k+1) = {} & [\tilde{A} - \tilde{B}(k_p + k_i + F_1)C]\delta x(t,k+1) + \tilde{B}(k_p + k_i + F_1) L_1 e(t+1,k) \\ & + \tilde{B}[(k_p + k_i + F_1)L_2 + k_i + F_2]\delta \sum e_s(t-1,k+1) + \varpi(t,k+1) \end{aligned} \tag{38}$$

Consequently, the output error prediction can be derived as

$$\begin{aligned} e(t+1,k+1) & = e(t+1,k) - C\delta x(t+1,k+1) \\ & = [-C\tilde{A} + C\tilde{B}(k_p + k_i + F_1)C]\delta x(t,k+1) + [\boldsymbol{I} - C\tilde{B}(k_p + k_i + F_1)L_1] e(t+1,k) \\ & \quad - C\tilde{B}[(k_p + k_i + F_1)L_3 + k_i + F_2]\delta \sum e_s(t-1,k+1) - C\varpi(t,k+1) \end{aligned} \tag{39}$$

Therefore, a 2D system description of the proposed ILC scheme can be formulated by

$$\begin{cases} \begin{bmatrix} \delta x(t+1,k+1) \\ \delta \sum e_s(t,k+1) \\ e(t+1,k+1) \end{bmatrix} = \tilde{\Psi} \begin{bmatrix} \delta x(t,k+1) \\ \delta \sum e_s(t-1,k+1) \\ e(t+1,k) \end{bmatrix} + D_w \varpi(t) \\ \varsigma(t,k+1) = G \begin{bmatrix} \delta x(t,k+1) \\ \delta \sum e_s(t-1,k+1) \\ e(t+1,k) \end{bmatrix} \end{cases} \tag{40}$$

where $G = [\boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{I}]$, $D_w = [\boldsymbol{I} \quad \boldsymbol{0} \quad -C^T]^T$,

$$\tilde{\Psi} = \begin{bmatrix} \tilde{A} - \tilde{B}(k_p + k_i + F_1)C & \tilde{B}[(k_p + k_i + F_1)L_2 + k_i + F_2] & \tilde{B}(k_p + k_i + F_1)L_1 \\ -C & \boldsymbol{I} + L_2 & L_1 \\ -C\tilde{A} + C\tilde{B}(k_p + k_i + F_1)C & -C\tilde{B}[(k_p + k_i + F_1)L_2 + k_i + F_2] & \boldsymbol{I} - C\tilde{B}(k_p + k_i + F_1)L_1 \end{bmatrix}$$

Note that $\varsigma(t,k+1) = e(t+1,k)$ can be viewed as the controlled variable to be minimized against process uncertainties or load disturbance. That is to say, the robust 2D control objective can be determined in terms of a batch process control objective (Liu and Wang, 2012) as

$$J_{BP} = \sum_{t=0}^{N_1 = T_p} \sum_{k=0}^{N_2 \to \infty} (\gamma_{ILC}^{-1} \|\varsigma(t,k+1)\|_2^2 - \gamma_{ILC} \|\varpi(t,k+1)\|_2^2) < 0 \tag{41}$$

By defining $x^v(t,k) = e(t+1,k)$ and

$$x^h(t,k) = \begin{bmatrix} \delta x(t,k+1) \\ \delta \sum e_s(t-1,k+1) \end{bmatrix} \tag{42}$$

the 2D system in (40) can be viewed as a typical Roesser's system in the form of (31).

Hence, analyzing the robust stability of the proposed ILC scheme is equivalent to that of the 2D system in (40). The following theorem is given to assess the robust stability and determine the ILC controllers:

Theorem 2: *The 2D control system in (40) subject to time-varying process uncertainties described by (1) is guaranteed robustly stable with a H infinity control performance level, γ_{ILC}, if there exist $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, matrices $\hat{F}_2$, $\hat{L}_1$, $\hat{L}_2$, and positive scalars ε_1, ε_2, such that the following LMI holds,*

$$\begin{bmatrix} -Q + \varepsilon_1 \Omega_{A1} \Omega_{A1}^T + \varepsilon_2 \Omega_{B1} \Omega_{B1}^T & \Pi & D_w & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & -Q & \boldsymbol{0} & QG^T & P\Omega_{A2}^T & P\Omega_{B2}^T \\ * & * & -\gamma_{ILC}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & -\gamma_{ILC}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -\varepsilon_1 \boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & -\varepsilon_2 \boldsymbol{I} \end{bmatrix} < 0 \tag{43}$$

where $Q = diag\{Q_1,\ Q_2,\ Q_3\}$, $D_g = [\boldsymbol{I} \quad \boldsymbol{0}]^T$, $H = [\boldsymbol{I} \quad \boldsymbol{0}]$,

$\Omega_{A1} = [\Delta\bar{A}_1^T,\ \boldsymbol{0},\ -\Delta\bar{A}_1^T C^T]^T$, $\Omega_{A2} = [\Delta\bar{A}_2,\ \boldsymbol{0},\ \boldsymbol{0}]$, $\Omega_{B1} = [\Delta\bar{B}_1^T,\ \boldsymbol{0},\ -\Delta\bar{B}_1^T C^T]^T$,

$\Omega_{B2} = \left[-\Delta\bar{B}_2(k_p + k_i + F_1)C,\ \Delta\bar{B}_2[(k_p + k_i + F_1)\hat{L}_2 + k_i + \hat{F}_2],\ \Delta\bar{B}_2(k_p + k_i + F_1)\hat{L}_1 \right]$

$$\Pi = \begin{bmatrix} A_m Q_1 - B_m(k_p + k_i + F_1)CQ_1 & B_m(k_p + k_i + F_1)\hat{L}_2 + B_m k_i Q_2 + B_m \hat{F}_2 & B_m(k_p + k_i + F_1)\hat{L}_1 \\ -CQ_1 & Q_2 + \hat{L}_2 & \hat{L}_1 \\ -CA_m Q_1 + CB_m(k_p + k_i + F_1)CQ_1 & -CB_m(k_p + k_i + F_1)\hat{L}_2 - CB_m k_i Q_2 - CB_m \hat{F}_2 & Q_3 - CB_m(k_p + k_i + F_1)\hat{L}_1 \end{bmatrix} \tag{44}$$

by parameterizing $L_1 = \hat{L}_1 Q_3^{-1}$, $L_2 = \hat{L}_2 Q_2^{-1}$, $F_2 = \hat{F}_2 Q_2^{-1}$.

Proof: The robust 2D control objective in (41) can be rewritten as

$$J_{BP} = \sum_{t=0}^{N_1 = T_p} \sum_{k=0}^{N_2 \to \infty} (\gamma_{ILC}^{-1} \|\varsigma(t,k+1)\|_2^2 - \gamma_{ILC} \|\varpi(t,k+1)\|_2^2 + \Delta V) - \sum_{t=0}^{N_1 = T_p} \sum_{k=0}^{N_2 \to \infty} \Delta V < 0$$

where ΔV is a Lyapunov-Krasovskii function used for analysis of 2D asymptotic stability, i.e.

$$\Delta V = V_Q \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1) \end{bmatrix} - V_Q \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \tag{45}$$

Using the zero boundary conditions from an initial resetting of batch process operation, it can be easily verified that

$$\begin{aligned} \sum_{t=0}^{N_1 = T_p} \sum_{k=0}^{N_2 \to \infty} \Delta V = {} & \sum_{t=0}^{N_1 = T_p} \sum_{k=0}^{N_2 \to \infty} \{ V_{Q_1}[\delta x(t+1,k+1)] - V_{Q_1}[\delta x(t,k+1)] + V_{Q_2}[\delta \sum e_s(t,k+1)] \\ & - V_{Q_2}[\delta \sum e_s(t-1,k+1)] + V_{Q_3}[e(t+1,k+1)] - V_{Q_3}[e(t+1,k)] \\ = {} & \sum_{k=0}^{N_2 \to \infty} V_{Q_1}[\delta x(N_1+1,k+1)] + \sum_{k=0}^{N_2 \to \infty} V_{Q_2}[\delta \sum e_s(N_1,k+1)] + \sum_{t=0}^{N_1} V_{Q_3}[e(t+1,N_2+1)] \\ > {} & 0 \end{aligned}$$

Therefore, a sufficient condition to achieve the control objective in (41) is that

$$\gamma_{ILC}^{-1} \|\varsigma(t,k+1)\|_2^2 - \gamma_{ILC} \|\varpi(t,k+1)\|_2^2 + \Delta V < 0 \tag{46}$$

By substituting the 2D system description in (40) and (45) into (46), we obtain

$$\xi^T \Xi_2 \xi < 0 \tag{47}$$

where $\xi = \left[[x^h(t,k)]^T,\ [x^v(t,k)]^T,\ \varpi^T(t) \right]^T$, and

$$\Xi_2 = \begin{bmatrix} \tilde{\Psi}^T \\ D_w^T \end{bmatrix} Q \begin{bmatrix} \tilde{\Psi} & D_w \end{bmatrix} - \begin{bmatrix} Q - \gamma_{ILC}^{-1} G^T G & \boldsymbol{0} \\ * & \gamma_{ILC} \boldsymbol{I} \end{bmatrix} \tag{48}$$

By the Schur complement, it can be derived that (47) is guaranteed by

$$\begin{bmatrix} -Q & \tilde{\Pi} & D_{\mathrm{w}} & \mathbf{0} \\ * & -Q & \mathbf{0} & QG^T \\ * & * & -\gamma_{\mathrm{ILC}}\boldsymbol{I} & \mathbf{0} \\ * & * & * & -\gamma_{\mathrm{ILC}}\boldsymbol{I} \end{bmatrix} < 0 \tag{49}$$

where $\tilde{\Pi} = \Pi + \Omega_{\mathrm{A1}}\tilde{\Theta}_1(t)\Omega_{\mathrm{A2}} + \Omega_{\mathrm{B1}}\tilde{\Theta}_2(t)\Omega_{\mathrm{B2}}$.

Using Lemma 1 and the Schur complement, it can be seen that (49) is guaranteed by (43) in Theorem 2. □

Note that the feedforward controller, F_1, is prescribed for solving the LMI in (43). To facilitate computing the LMI condition in (43), the choice of F_1 should be made to keep all the eigenvalues of $A_{\mathrm{m}} - B_{\mathrm{m}}(k_{\mathrm{p}} + k_{\mathrm{i}} + F_1)C$ in the unit circle in the z-transfer plane to maintain the asymptotic stability.

To achieve the optimal robust tracking performance, the ILC controllers can be determined by performing an optimization program similar to that shown in (30).

5. ILLUSTRATION

Consider the illustrative example of injection molding studied by Shi et al. (2005) and Wang et al. (2012),

$$y(t,k+1) = \frac{1.239(\pm 5\%)z^{-1} - 0.9282(\pm 5\%)z^{-2}}{1 - 1.607(\pm 5\%)z^{-1} + 0.6086(\pm 5\%)z^{-2}} u(t,k+1) + \omega(t,k+1)$$

which may be reformulated in the state-space form of

$$\begin{cases} x(t+1,k+1) = \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta\tilde{A}\right) x(t,k+1) + \\ \qquad \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \Delta\tilde{B}\right) u(t,k+1) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega(t,k+1) \\ y(t,k+1) = [1,\ 0]\, x(t,k+1) \end{cases}$$

$$\Delta\tilde{A}(t) = \begin{bmatrix} 0.0804\delta(t) & 0 \\ -0.0304\delta(t) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix} \begin{bmatrix} 0.0804 & 0 \\ -0.0304 & 0 \end{bmatrix}$$

$$\Delta\tilde{B}(t) = \begin{bmatrix} 0.062\delta(t) \\ -0.0464\delta(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix} \begin{bmatrix} 0.062 \\ -0.0464 \end{bmatrix}$$

where $\delta(t)$ is a time-varying factor and $|\delta(t)| \le 1$.

By performing the optimization procedure in (30), we obtain the minimal H infinity robust performance level, $\gamma^*_{\mathrm{PI}} = 1.3$. To avoid over aggressive control signal, we take $\gamma_{\mathrm{PI}} = 5$ to solve the LMI condition in (19), obtaining the PI controller parameters, $k_{\mathrm{p}} = 1.29$ and $k_{\mathrm{i}} = 0.034$. For the ILC design, we choose $F_1 = -0.5$ to keep all the eigenvalues of $A_{\mathrm{m}} - B_{\mathrm{m}}(k_{\mathrm{p}} + k_{\mathrm{i}} + F_1)C$ in the unit circle in the z-transfer plane, and then perform an optimization procedure to obtain the minimal H infinity robust performance level, $\gamma^*_{\mathrm{ILC}} = 110$, corresponding to $L_1 = 0.18$, $L_2 = -0.03$, and $F_2 = -0.01$.

The target profile (y_{r}) takes the following form as used in the cited references,

$$Y_{\mathrm{r}} = \begin{cases} 200, & 0 \le t \le 100; \\ 200 + 5(t-100), & 100 < t \le 120; \\ 300, & 120 < t \le T_{\mathrm{p}} = 200. \end{cases}$$

For illustration, the following cases of process uncertainties are tested.

Case 1. Time-invariant process uncertainties and repetitive type load disturbance. In this case, $\Delta\tilde{A}(t)$ and $\Delta\tilde{B}(t)$ are assumed to be fixed as their upper bounds. A repetitive type load disturbance is imitated by passing a step change with a magnitude of 150 through a slow transfer function $G_{\mathrm{d}}(z) = (z^{-1} + z^{-2})/(11 - 4z^{-1})$, which is added to the process output at $t = 60$. The tracking results are shown in Fig.2 (a) and (b). It is seen that perfect output tracking is reached through 20 cycles by the proposed method after an initial run of the PI tuning, compared to that of Wang et al. (2012) which used almost 50 cycles to realize perfect tracking. Moreover, there exists no steady output tracking error in each cycle, owing to the use of the integral error information for both the PI and ILC design.

Case 2. Time-varying uncertainties and non-repetitive load disturbance. Assume that the process state transfer matrices becomes time-varying with $|\delta(t)| \le 0.1$, together with non-repetitive type load disturbance, $\omega(t,k+1) = \sin(t + \theta(k))$ where $\theta(k)$ is a random variable uniformly distributed in the range of $[0, 2\pi]$ as assumed by Wang et al. (2012). Since the closed-loop system becomes a stochastic process, we perform 100 Monte Carlo tests, each of which includes 100 cycles. The output tracking error in terms of the following criterion is used for comparison,

$$\mathrm{ATE}(k) = \sum_{t=1}^{T_{\mathrm{p}}} |e(t,k)| / T_{\mathrm{p}}$$

The averaged results of ATE are plotted in Fig.3, together with those of Wang et al. (2012). It is seen that the closed-loop system maintains good robust stability in both the time and batchwise directions by using the proposed ILC method, thus demonstrating that it can be reliably used for robust tracking of the desired output trajectory and on-line optimization against batch-to-batch time-varying process uncertainties and load disturbance.

6. CONCLUSIONS

For industrial batch processes subject to time-varying uncertainties and load disturbance, a robust set-point learning control method has been proposed based on the conventional PI control structure. In the proposed control scheme, either the closed-loop PI controller or the ILC updating law can be designed relatively independent. To cope with time-varying uncertainties, a robust PI design has been given based on the robust H infinity control objective, which is primarily used for maintaining the closed-loop robust stability. For the batchwise direction, an ILC scheme consisting of a learning set-point strategy and a feedforward control added to the process input has been developed based on an equivalent 2D system description of the batch process and the LMI condition formulated in terms of the robust H infinity control objective for robust convergence, which can guarantee no steady output error from the initial cycle and perfect output tracking against repetitive type load disturbance. For the convenience of practical application, only measured output errors of current and previous cycles are used to implement the proposed ILC scheme. The application to an illustrative example from the literature has demonstrated the effectiveness and merits of the proposed ILC method.

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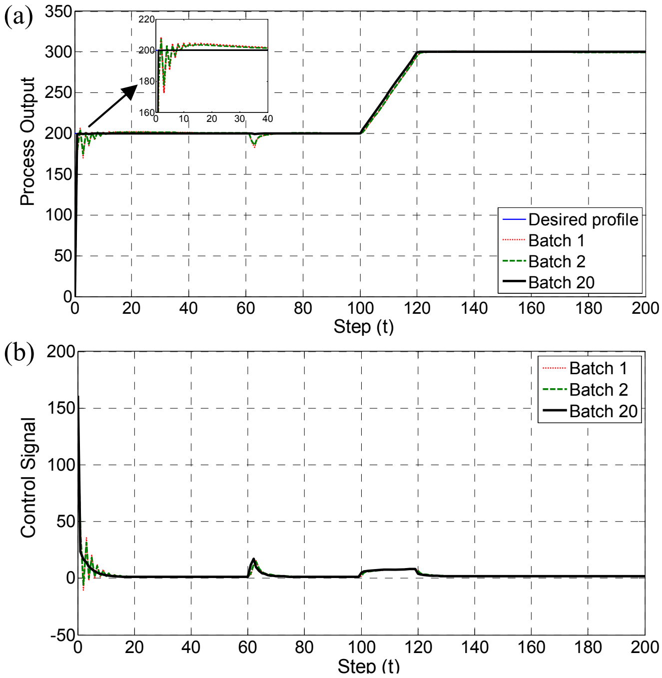

Fig.2. Tracking performance for case 1 with fixed uncertainties and repetitive type load disturbance

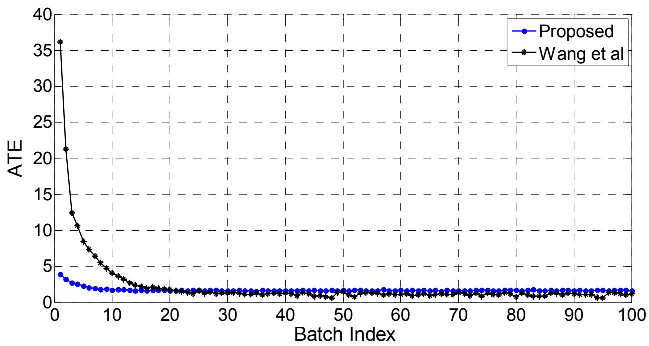

Fig.3. Plot of ATE for case 2