Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Non-certainty equivalence adaptive tracking control for hypersonic vehicles

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Abstract: The design of a novel non-certainty equivalence adaptive control system for hypersonic vehicles is presented, based on the immersion and invariance (I&I) theory. The interest here is to achieve a robust trajectory tracking of the reference commands in spite of large parameter uncertainties. The architecture of the whole control system is constructed by decomposing the vehicle model into the velocity subsystem and the flight path angle (FPA) subsystem. A backstepping design procedure is applied to the cascaded FPA subsystem. Three non-certainty equivalence controllers, each consisting of a control module and an I&I based parameter estimator, are designed respectively for the velocity subsystem and the two steps of the FPA subsystem, which suffer from parameter uncertainties. First-order filters are implemented in all the design of the three controllers to avoid solving certain partial differential inequality in I&I adaptive control. Stability analysis is presented using Lyapunov theory and asymptotical convergence of the tracking errors to zero is accomplished. Simulation results with rapid and accurate command tracking show the effectiveness and robustness of the proposed control system.

Keywords: Hypersonic vehicles, Non-certainty equivalence principle, Adaptive control, Immersion and invariance, Tracking control.

1. INTRODUCTION

Hypersonic vehicles (HSVs) have drawn much attention during the past decades for their significant potential in both military and civil applications. Despite years of research, it is extensively recognized that more effective and reliable design methods, as well as major advances in propulsion and materials, are required for the development of a full scale operational HSV (Bolender, 2009). The well-known peculiarity of the vehicle dynamics, like strong coupling between propulsive and aerodynamic effects, structural flexibility, large nonlinearities and modeling uncertainties, presents diverse challenges for the nonlinear control problem associated with HSVs (Bolender, 2009; Fidany, Mirmirani, & Ioannou, 2003; Kuipers, Mirmirani, Ioannou, & Huo, 2007).

Many attempts have been applied to this problem. Gain scheduling is a very common practice in flight control design. However, this method becomes time-consuming for the large flight envelope and complex plant characteristics of HSVs. This necessitates the development of nonlinear flight control techniques. Robust outer-loop controllers are usually combined with the inner-loop feedback linearization to solve this problem (Parker, Bolender, & Doman, 2007; Sigthorsson et al., 2008). Also, some nonlinear adaptive control methods have been employed, such as adaptive sliding mode control(X. Hu, Wu, Hu, & Gao, 2012; H. Xu, Mirmirani, & Ioannou, 2004; Zong, Wang, Tian, & Tao, 2013), fuzzy adaptive control (Lian, Shi, Ren, & Shao, 2012; Wang, Jiang, & Wu, 2013), and neural network control (B. Xu, Wang, Sun, & Shi, 2012). In these cases, the key problem is that the global stability of the closed-loop system or the convergence of the tracking error usually cannot be guaranteed (Farrell, Sharma, & Polycarpou, 2005). Fiorentini, Serrani, Bolender, and Doman (2009), Poulain, Piet-Lahanier, and Serre (2010) and Ji, Zong, and Zeng (2012) employ the adaptive backstepping theory to deal with model uncertainties while also obtaining these guarantees. However, all the aforementioned adaptive control solutions are based upon the classical certainty-equivalence (CE) principle (Ioannou & Sun, 1996), whose parameter update laws are established by perfect cancelation of terms in the time-derivative of Lyapunov function which is carefully chosen. This only ensures boundedness of the estimation errors, but the influence on the transient response of the whole system resulting from the estimation error's dynamics has not been accounted for. A thorough survey of the shortcomings of CE-based adaptive control is given in Seo and Akella (2008), Astolfi, Karagiannis, and Ortega (2008) and Karagiannis and Astolfi (2008). Recently, a novel adaptive control approach named immersion and invariance (I&I), which is non-CE-based, has been proposed (Astolfi, et al., 2008; Astolfi & Ortega, 2003). This approach does not require knowledge of a Lyapunov function and can assign appointed stable dynamics to the estimation error. It has been applied to control system design of aerial vehicles and missiles (J. Hu & Zhang, 2013; Kobayashi & Takahashi, 2009; Lee & Singh, 2010). However, the application of this approach relies upon the solution of a partial differential inequality (PDE) (Astolfi, et al., 2008; Astolfi & Ortega, 2003), which imposes limitations on multivariable systems. Some modifications have been developed for this approach to avoid this problem. One is to add a dynamic scaling factor in the estimator dynamics (Astolfi, et al., 2008; Ji, et al., 2012). Yet the construction of over-parameterized estimators, which are needed to achieve the desired modularity, and the selection of the dynamics of the scaling factors, both make the design procedure complex. This limitation can also be removed by implementing first-order filters (Seo & Akella, 2008, 2009). Filters for the regressor terms, i.e., the gain matrix of the unknown parameter vector, are constructed to sidestep the solution of PDE, which makes this problem more tractable.

Among the numerous challenges encountered in designing control systems for HSVs, the presence of uncertainties in aerodynamic parameters might be one of the most severe. This paper focuses on this problem. A non-CE adaptive control system based on I&I is presented, under the condition that all the aerodynamic parameters are not known (except the sign of control input coefficients). It is organized as follows. In Section 2, the longitudinal dynamics of a generic hypersonic vehicle are presented and the control objectives are defined. The non-CE adaptive control law based on I&I is derived in Section 3, along with analysis of the closed-loop system stability. Simulation results are shown in Section 4, and the conclusions are contained in Section 5.

2. HYPERSONIC VEHICLE MODEL

Ignoring the flexibility effects of the body structure and assuming a flat Earth, the longitudinal dynamics of HSVs can be described as(H. Xu, et al., 2004)

$$\begin{aligned}
&\dot V = (T\cos\alpha - D)/m - \mu\sin\gamma/\gamma^2\\
&\dot h = V\sin\gamma\\
&\dot\gamma = (L + T\sin\alpha)/(mV) - (\mu - V^2 r)\cos\gamma/(Vr^2)\\
&\dot\theta = Q\\
&\dot Q = M/I_{yy}
\end{aligned}\tag{1}$$

where V, h, γ, θ, Q are the flight velocity, altitude, flight-path angle (FPA), pitch angle, and pitch rate respectively. $\alpha = \theta - \gamma$ is the angle of attack. The thrust T, lift L, drag D, and moment M are given by

$$\begin{aligned}
&L = qSC_{L\alpha}\alpha\\
&D = qS(C_{D\alpha^2}\alpha^2 + C_{D\alpha}\alpha + C_{D0})\\
&T = qS(C_{T\beta}\beta + C_{T0})\\
&M = qS\overline{c}[C_{M\alpha^2}\alpha^2 + C_{M\alpha}\alpha + C_{M0} + c_e(\delta_e - \alpha)\\
&\quad + \overline{c}Q(C_{MQ\alpha^2}\alpha^2 + C_{MQ\alpha}\alpha + C_{MQ0})/(2V)]
\end{aligned}\tag{2}$$

where q is the dynamic pressure, S is the reference area and $\overline{c}$ is the reference length. The control inputs are the elevator deflection δ_e and the fuel equivalence ratio β.

In general, there are various uncertainties in hypersonic vehicle dynamics, and how to design a control system to achieve robust velocity and altitude tracking in the presence of such uncertainties is the control objective. Here, all aerodynamic coefficients (C_i) in (2) are assumed to be unknown, except the sign of $C_{T\beta}, C_{L\alpha}$ and c_e (they are both positive). $[V,\gamma]$ are selected as output rather than $[V,h]$ as was done in numerous references (Fiorentini, et al., 2009; Sigthorsson, et al., 2008; H. Xu, et al., 2004). Compared with the output choice of h, the choice of γ is more appropriate for the inner loop, i.e., the attitude controller. Moreover, there is no uncertainty but a strictly accurate bijection between h and γ according to the second equation of (1). So the altitude command, h_c , can be accurately transformed into FPA command, γ_c . With this output selection, the FPA command to be tracked is generated from the altitude command by

$$\gamma_c = -\arctan(k_h(h - h_c) + k_i\int(h - h_c)dt)\tag{3}$$

where $k_h > 0$ and $k_i > 0$.

3. ADAPTIVE CONTROL SYSTEM DESIGN

According to (1), the longitudinal motion of HSV can be divided into two parts, the velocity and the FPA subsystems. Each subsystem can be controlled separately. The fuel equivalence ratio is used directly to control the thrust, and hence velocity, whereas the elevator deflection is for the FPA. A backstepping design procedure is applied to the FPA subsystem as it has a cascaded structure. Three steps are needed for completing the design of the FPA subsystem. Moreover, three I&I based non-CE adaptive controllers are designed. One is for the velocity subsystem, and the others for the two steps of the FPA subsystem, which suffer from parameter uncertainties.

3.1 Non-CE adaptive controller design for the velocity subsystem

Define the velocity tracking error as $\tilde V = V - V_c$. Using (1) and (2) yields

$$\dot{\tilde V} = C_{T\beta}(u_V + \boldsymbol{\Phi}_V{}^T\boldsymbol{P}_V) - k_V\tilde V\tag{4}$$

where $u_V = qS\beta\cos\alpha/m$, $k_v > 0$, $\boldsymbol{P}_V$ is the vector of unknown parameters and $\boldsymbol{\Phi}_V$ is the nonlinear regressor of $\boldsymbol{P}_V$, which are defined respectively as

$$\begin{aligned}
&\boldsymbol{\Phi}_V = [\frac{qS}{m}\cos\alpha, -\frac{qS}{m}\alpha^2, -\frac{qS}{m}\alpha, -\frac{qS}{m}, -(\frac{\mu\sin\gamma}{r^2} + \dot V_c - k_V\tilde V)]^T\\
&\boldsymbol{P}_V = (C_{T\beta})^{-1}[C_{T0}, C_{D\alpha^2}, C_{D\alpha}, C_{D0}, 1]^T, C_{T\beta} > 0
\end{aligned}$$

Since $\boldsymbol{P}_V$ is not known, its estimate is needed for synthesis. To overcome the problem of solving PDE which is essential for I&I based estimators, first-order filters are introduced here. The filtered signals are generated by using

$$\dot{\tilde V}_f = -\lambda\tilde V_f + \tilde V\tag{5}$$

$$\dot{\boldsymbol{\Phi}}_{Vf} = -\lambda\boldsymbol{\Phi}_{Vf} + \boldsymbol{\Phi}_V\tag{6}$$

$$\dot u_{Vf} = -\lambda u_{Vf} + u_V\tag{7}$$

where $\lambda > 0$ and the variables with subscript f denote filtered signals. It should be pointed out that u_{Vf} is only for the purpose of stability analysis and will not be used for the implementation of control law. From (5), we have $\lim_{t\to\infty}\tilde V = 0$ if $\lim_{t\to\infty}\tilde V_f = 0$. Differentiating (5) and using (4), (6), and (7), we have

$$\begin{aligned}
&\ddot{\tilde V}_f - C_{T\beta}(\dot u_{Vf} + \dot{\boldsymbol{\Phi}}_{Vf}{}^T\boldsymbol{P}_V) + k_V\dot{\tilde V}_f\\
&\qquad = -\lambda[\dot{\tilde V}_f - C_{T\beta}(u_{Vf} + \boldsymbol{\Phi}_{Vf}{}^T\boldsymbol{P}_V) + k_V\tilde V_f]
\end{aligned}\tag{8}$$

It is easily recognized the solution of (8) can be expressed as

$$\dot{\tilde{V}}_f = C_{T\beta}(u_{Vf} + \boldsymbol{\Phi}_{Vf}{}^T \boldsymbol{P}_V) - k_V \tilde{V}_f + \varepsilon_V(t) \tag{9}$$

where $\varepsilon_V(t)$ is an exponentially decaying term. Ignoring $\varepsilon_V(t)$ (Seo & Akella, 2009) yields the filtered error dynamics

$$\dot{\tilde{V}}_f = C_{T\beta}(u_{Vf} + \boldsymbol{\Phi}_{Vf}{}^T \boldsymbol{P}_V) - k_V \tilde{V}_f \tag{10}$$

Now define the estimate of $\boldsymbol{P}_V$ as $\hat{\boldsymbol{P}}_V + \boldsymbol{\delta}_V(\tilde{V}_f, \boldsymbol{\Phi}_{Vf})$. The estimate error can then be written as

$$\boldsymbol{Z}_V = \hat{\boldsymbol{P}}_V + \boldsymbol{\delta}_V(\tilde{V}_f, \boldsymbol{\Phi}_{Vf}) - \boldsymbol{P}_V \tag{11}$$

where $\hat{\boldsymbol{P}}_V$ is generated by an update law $\dot{\hat{\boldsymbol{P}}}_V = \boldsymbol{\omega}(\tilde{V}_f, \boldsymbol{\Phi}_{Vf})$, and $\boldsymbol{\delta}_V$ is a judiciously chosen nonlinear vector function. Note that this is very different from the traditional adaptive control design based on CE principle for the estimate, $\hat{\boldsymbol{P}}_V$, generated by the update law, is not applied directly in the controller, but treated as only a partial estimate. The additional term, $\boldsymbol{\delta}_V$, in the definition of the estimation allows construction of the error dynamics (Astolfi, et al., 2008; Astolfi & Ortega, 2003), which will be justified later.

In view of (10), the stabilizing signal u_{Vf} can be specified as

$$u_{Vf} = -\boldsymbol{\Phi}_{Vf}{}^T(\hat{\boldsymbol{P}}_V + \boldsymbol{\delta}_V) \tag{12}$$

Substituting (12) into (10) yields

$$\dot{\tilde{V}}_f = -C_{T\beta} \boldsymbol{\Phi}_{Vf}{}^T \boldsymbol{Z}_V - k_V \tilde{V}_f \tag{13}$$

Consider the design of the parameter estimator. The error dynamics are given by differentiating (11) and using (13) as

$$\dot{\boldsymbol{Z}}_V = \dot{\hat{\boldsymbol{P}}}_V + \frac{\partial \boldsymbol{\delta}_V}{\partial \tilde{V}_f}(-C_{T\beta}\boldsymbol{\Phi}_{Vf}{}^T \boldsymbol{Z}_V - k_V \tilde{V}_f) + \frac{\partial \boldsymbol{\delta}_V}{\partial \boldsymbol{\Phi}_{Vf}} \dot{\boldsymbol{\Phi}}_{Vf} \tag{14}$$

So the update law can be chosen as

$$\dot{\hat{\boldsymbol{P}}}_V = (\partial \boldsymbol{\delta}_V / \partial \tilde{V}_f) k_V \tilde{V}_f - (\partial \boldsymbol{\delta}_V / \partial \boldsymbol{\Phi}_{Vf}) \dot{\boldsymbol{\Phi}}_{Vf} \tag{15}$$

Substituting (15) into (14) yields

$$\dot{\boldsymbol{Z}}_V = -(\partial \boldsymbol{\delta}_V / \partial \tilde{V}_f) C_{T\beta} \boldsymbol{\Phi}_{Vf}{}^T \boldsymbol{Z}_V \tag{16}$$

The nonlinear vector function $\boldsymbol{\delta}_V(\tilde{V}_f, \boldsymbol{\Phi}_{Vf})$ is now chosen to ensure the $\boldsymbol{Z}_V$ dynamics have stable behavior. One choice is

$$\partial \boldsymbol{\delta}_V / \partial \tilde{V}_f = r_V \boldsymbol{\Phi}_{Vf} \tag{17}$$

where $r_V > 0$. Substituting (17) into (16) yields

$$\dot{\boldsymbol{Z}}_V = -r_V C_{T\beta} \boldsymbol{\Phi}_{Vf} \boldsymbol{\Phi}_{Vf}{}^T \boldsymbol{Z}_V \tag{18}$$

According to (17), $\boldsymbol{\delta}_V$ is obtained. Substituting it into (15) gives $\dot{\hat{\boldsymbol{P}}}_V$. Then the stabilizing signal u_{Vf} can be derived from (12). To complete the design, we still need to recover the actual control signal u_V. Using (7) and substituting $\dot{\hat{\boldsymbol{P}}}_V$ and $\dot{\boldsymbol{\delta}}_V$ yields

$$u_V = -\boldsymbol{\Phi}_V{}^T(\hat{\boldsymbol{P}}_V + \boldsymbol{\delta}_V) - \boldsymbol{\Phi}_{Vf}{}^T[r_V k_V \boldsymbol{\Phi}_{Vf} \tilde{V}_f + r_V \boldsymbol{\Phi}_{Vf}(-\lambda \tilde{V}_f + \tilde{V})] \tag{19}$$

So the control input β can be obtained from $u_V = qS\beta\cos\alpha / m$.

3.2 Non-CE adaptive controller design for the FPA subsystem

Since the FPA subsystem has a cascaded structure, a backstepping design procedure is applied to the construction of the controller. The whole construction will be completed in three steps.

Step 1: Define the tracking error of FPA as $\tilde{\gamma} = \gamma - \gamma_c$, where γ_c is determined by (3). Using (1) and (2) yields

$$\dot{\tilde{\gamma}} = C_{L\alpha}(u_\gamma + \Phi_\gamma P_\gamma) - k_\gamma \tilde{\gamma} + qSC_{L\alpha}\tilde{\theta}/(mV) \tag{20}$$

where $u_\gamma = qS(\theta_c - \gamma)/(mV)$, $k_\gamma > 0$ and $\tilde{\theta} = \theta - \theta_c$ with θ_c a stabilizing control signal yet to be determined. The nonlinear regressor Φ_γ and the unknown parameter P_γ are

$$\Phi_\gamma = -(\mu - V^2 r)\cos\gamma/(Vr^2) - \dot{\gamma}_c + k_\gamma \tilde{\gamma}$$

$$P_\gamma = C_{L\alpha}{}^{-1}, C_{L\alpha} > 0$$

Note that the thrust T is ignored in (20) since $T\sin\alpha << L$. Similar to the velocity subsystem, we introduce filtered signals as

$$\dot{\tilde{\gamma}}_f = -\lambda \tilde{\gamma}_f + \tilde{\gamma} \tag{21}$$

$$\dot{\Phi}_{\gamma f} = -\lambda \Phi_{\gamma f} + \Phi_\gamma \tag{22}$$

$$\dot{u}_{\gamma f} = -\lambda u_{\gamma f} + u_\gamma \tag{23}$$

$$\dot{\tilde{\theta}}_f = -\lambda \tilde{\theta}_f + \tilde{\theta} \tag{24}$$

Differentiating (21) and using (20), (22), (23) and (24) yields

$$\dot{\tilde{\gamma}}_f = C_{L\alpha}(u_{\gamma f} + \Phi_{\gamma f} P_\gamma) - k_\gamma \tilde{\gamma}_f + qSC_{L\alpha}\tilde{\theta}_f/(mV) \tag{25}$$

Let the estimate of P_γ be $\hat{P}_\gamma + \delta_\gamma(\tilde{\gamma}_f, \Phi_{\gamma f})$. The estimate error can then be written as

$$Z_\gamma = \hat{P}_\gamma + \delta_\gamma(\tilde{\gamma}_f, \Phi_{\gamma f}) - P_\gamma \tag{26}$$

In view of (25), the filtered control signal $u_{\gamma f}$ is specified as

$$u_{\gamma f} = -\Phi_{\gamma f}(\hat{P}_\gamma + \delta_\gamma) \tag{27}$$

Substituting (27) into (25) yields

$$\dot{\tilde{\gamma}}_f = -C_{L\alpha}\Phi_{\gamma f} Z_\gamma - k_\gamma \tilde{\gamma}_f + qSC_{L\alpha}\tilde{\theta}_f/(mV) \tag{28}$$

Now the design of the estimator is discussed. Differentiating (26) and using (28), we have

$$\begin{aligned}\dot{Z}_\gamma = \dot{\hat{P}}_\gamma + (\partial\delta_\gamma/\partial\tilde{\gamma}_f)[-C_{L\alpha}\Phi_{\gamma f} Z_\gamma - k_\gamma \tilde{\gamma}_f +{}& \\ qSC_{L\alpha}\tilde{\theta}_f/(mV)] + (\partial\delta_\gamma/\partial\Phi_{\gamma f})\dot{\Phi}_{\gamma f}&\end{aligned} \tag{29}$$

In view of (29), the update law $\dot{\hat{P}}_\gamma$ can be chosen as

$$\dot{\hat{P}}_\gamma = (\partial\delta_\gamma/\partial\tilde{\gamma}_f)k_\gamma\tilde{\gamma}_f - (\partial\delta_\gamma/\partial\Phi_{\gamma f})\dot{\Phi}_{\gamma f} \tag{30}$$

Substituting $\dot{\hat{P}}_\gamma$ into (29) yields

$$\dot{Z}_\gamma = (\partial\delta_\gamma/\partial\tilde{\gamma}_f)[-C_{L\alpha}\Phi_{\gamma f} Z_\gamma + qSC_{L\alpha}\tilde{\theta}_f/(mV)] \tag{31}$$

Now for guaranteeing a stable dynamics of Z_γ, we select

$$\partial\delta_\gamma/\partial\tilde{\gamma}_f = r_\gamma \Phi_{\gamma f} \tag{32}$$

Substituting (32) into (31) gives

$$\dot{Z}_\gamma = -r_\gamma C_{L\alpha}\Phi_{\gamma f}\Phi_{\gamma f} Z_\gamma + qSr_\gamma C_{L\alpha}\Phi_{\gamma f}\tilde{\theta}_f/(mV) \tag{33}$$

δ_γ and $\hat{P}_\gamma$ can be derived from (32) and (30) sequentially. Then, from (27), $u_{\gamma f}$ is obtained. Using (23) and substituting $\dot{\delta}_\gamma$ and $\dot{\hat{P}}_\gamma$, u_γ can be written as

$$u_\gamma = -\Phi_\gamma(\hat{P}_\gamma + \delta_\gamma) - \Phi_{\gamma f}[r_\gamma k_\gamma \Phi_{\gamma f}\tilde{\gamma}_f + r_\gamma \Phi_{\gamma f}(-\lambda\tilde{\gamma}_f + \tilde{\gamma})] \quad (34)$$

So the stabilizing control signal θ_c can be obtained from $u_\gamma = qS(\theta_c - \gamma)/(mV)$.

Step 2: Now θ_c is the new variable to be tracked. Using the fourth equation of (1), the dynamics of $\tilde{\theta}$ can be written as

$$\dot{\tilde{\theta}} = u_\theta + \Phi_\theta - k_\theta\tilde{\theta} + \tilde{Q} \quad (35)$$

where $u_\theta = Q_c$, $k_\theta > 0$, $\tilde{Q} = Q - Q_c$ and $\Phi_\theta = -\dot{\theta}_c + k_\theta\tilde{\theta}$. Filtered signals are introduced as

$$\dot{\Phi}_{\theta f} = -\lambda\Phi_{\theta f} + \Phi_\theta \quad (36)$$

$$\dot{u}_{\theta f} = -\lambda u_{\theta f} + u_\theta \quad (37)$$

$$\dot{\tilde{Q}}_f = -\lambda\tilde{Q}_f + \tilde{Q} \quad (38)$$

Differentiating (24) and using (35), (36), (37), and (38) yields

$$\dot{\tilde{\theta}}_f = u_{\theta f} + \Phi_{\theta f} - k_\theta\tilde{\theta}_f + \tilde{Q}_f \quad (39)$$

In view of (39), the filtered control signal $u_{\theta f}$ is specified as

$$u_{\theta f} = -\Phi_{\theta f} \quad (40)$$

Substituting (40) into (39) yields

$$\dot{\tilde{\theta}}_f = -k_\theta\tilde{\theta}_f + \tilde{Q}_f \quad (41)$$

Using (36), (37) and (40), u_θ can be written as

$$u_\theta = -\Phi_\theta \quad (42)$$

So the stabilizing control signal Q_c can be obtained from $u_\theta = Q_c$.

Step 3: Now the elevator deflection δ_e should be designed so that the pitch rate tracking error $\tilde{Q}$ can asymptotically converge to zero. Substituting the expressions of M and $C_M(\alpha)$, $C_M(\delta_e)$, $C_M(Q)$ into the fifth equation of (1) yields

$$\dot{\tilde{Q}} = c_e(u_Q + \boldsymbol{\Phi_Q}^T\boldsymbol{P_Q}) - k_Q\tilde{Q} - \tilde{\theta} \quad (43)$$

where $u_Q = qS\bar{c}\delta_e/I_{yy}$, $k_Q > 0$, the nonlinear regressor $\boldsymbol{\Phi_Q}$ and the vector of unknown parameters $\boldsymbol{P_Q}$ are

$$\boldsymbol{\Phi_Q} = [qS\bar{c}\alpha^2/I_{yy}, qS\bar{c}\alpha/I_{yy}, qS\bar{c}/I_{yy}, qSQ\bar{c}^2\alpha^2/(2VI_{yy}), qSQ\bar{c}^2\alpha/(2VI_{yy}), qSQ\bar{c}^2/(2VI_{yy}), k_Q\tilde{Q} - \dot{Q}_c + \tilde{\theta}]^T$$

$$\boldsymbol{P_Q} = (c_e)^{-1}[C_{M\alpha^2}, C_{M\alpha} - c_e, C_{M0}, C_{MQ\alpha^2}, C_{MQ\alpha}, C_{MQ0}, 1]^T, c_e > 0$$

Introducing filtered signals as

$$\dot{\boldsymbol{\Phi}}_{\boldsymbol{Qf}} = -\lambda\boldsymbol{\Phi_{Qf}} + \boldsymbol{\Phi_Q} \quad (44)$$

$$\dot{u}_{Qf} = -\lambda u_{Qf} + u_Q \quad (45)$$

Differentiating (38) and using (24), (43), (44) and (45) gives

$$\dot{\tilde{Q}}_f = c_e(u_{Qf} + \boldsymbol{\Phi_{Qf}}^T\boldsymbol{P_Q}) - k_Q\tilde{Q}_f - \tilde{\theta}_f \quad (46)$$

Define the estimate of $\boldsymbol{P_Q}$ as $\hat{\boldsymbol{P}}_{\boldsymbol{Q}} + \boldsymbol{\delta}_Q(\tilde{Q}_f, \boldsymbol{\Phi_{Qf}})$. The estimate error can then be written as

$$\boldsymbol{Z_Q} = \hat{\boldsymbol{P}}_{\boldsymbol{Q}} + \boldsymbol{\delta}_Q(\tilde{Q}_f, \boldsymbol{\Phi_{Qf}}) - \boldsymbol{P}_Q \quad (47)$$

In view of (46), the filtered control signal u_{Qf} is chosen as

$$u_{Qf} = -\boldsymbol{\Phi_{Qf}}^T(\hat{\boldsymbol{P}}_{\boldsymbol{Q}} + \boldsymbol{\delta_Q}) \quad (48)$$

Substituting (48) into (46) yields

$$\dot{\tilde{Q}}_f = -c_e\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q} - k_Q\tilde{Q}_f - \tilde{\theta}_f \quad (49)$$

Consider now the design of the parameter estimator. Differentiating (47) and using (49), we have

$$\dot{\boldsymbol{Z}}_{\boldsymbol{Q}} = \dot{\hat{\boldsymbol{P}}}_{\boldsymbol{Q}} + \frac{\partial\boldsymbol{\delta_Q}}{\partial\tilde{Q}_f}(-c_e\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q} - k_Q\tilde{Q}_f - \tilde{\theta}_f) + \frac{\partial\boldsymbol{\delta_Q}}{\partial\boldsymbol{\Phi_{Qf}}}\dot{\boldsymbol{\Phi}}_{\boldsymbol{Qf}} \quad (50)$$

The update law can be chosen as

$$\dot{\hat{\boldsymbol{P}}}_{\boldsymbol{Q}} = (\partial\boldsymbol{\delta_Q}/\partial\tilde{Q}_f)(k_Q\tilde{Q}_f + \tilde{\theta}_f) - (\partial\boldsymbol{\delta_Q}/\partial\boldsymbol{\Phi_{Qf}})\dot{\boldsymbol{\Phi}}_{\boldsymbol{Qf}} \quad (51)$$

Substituting (51) into (50) yields

$$\dot{\boldsymbol{Z}}_{\boldsymbol{Q}} = -(\partial\tilde{Q}_f/\partial\tilde{Q}_f)c_e\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q} \quad (52)$$

For ensuring stable stability of the $\boldsymbol{Z_Q}$ dynamics, we select

$$\partial\boldsymbol{\delta_Q}/\partial\tilde{Q}_f = r_Q\boldsymbol{\Phi_{Qf}} \quad (53)$$

where $r_Q > 0$. Substituting (53) into (52) yields

$$\dot{\boldsymbol{Z}}_{\boldsymbol{Q}} = -r_Q c_e\boldsymbol{\Phi_{Qf}}\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q} \quad (54)$$

$\boldsymbol{\delta_Q}$ and $\dot{\hat{\boldsymbol{P}}}_{\boldsymbol{Q}}$ can be derived from (53) and (51) sequentially. Then the filtered control signal u_{Qf} can be obtained from (48). Using (45) and substituting $\dot{\boldsymbol{\delta}}_{\boldsymbol{Q}}$ and $\dot{\hat{\boldsymbol{P}}}_{\boldsymbol{Q}}$, u_Q can be written as

$$u_Q = -\boldsymbol{\Phi_Q}^T(\hat{\boldsymbol{P}}_{\boldsymbol{Q}} + \boldsymbol{\delta_Q}) - \boldsymbol{\Phi_{Qf}}^T[r_Q k_Q\boldsymbol{\Phi_{Qf}}\tilde{Q}_f + r_Q\boldsymbol{\Phi_{Qf}}(-\lambda\tilde{Q}_f + \tilde{Q})] \quad (55)$$

So the actual control input δ_e can be obtained from $u_Q = qS\bar{c}\delta_e/I_{yy}$.

3.3 Closed-loop stability analysis

Consider the Lyapunov candidate function

$$\begin{aligned} &W(\tilde{V}_f, \boldsymbol{Z_V}, \tilde{\gamma}_f, Z_\gamma, \tilde{\theta}_f, \tilde{Q}_f, \boldsymbol{Z_Q}) \\ &= (C_{T\beta}^{-1}\tilde{V}_f^2 + r_V^{-1}k_V^{-1}\boldsymbol{Z_V}^T\boldsymbol{Z_V} + C_{L\alpha}^{-1}\tilde{\gamma}_f^2 + r_\gamma^{-1}k_\gamma^{-1}Z_\gamma^2 \\ &\quad + \eta c_e^{-1}\tilde{\theta}_f^2 + \eta c_e^{-1}\tilde{Q}_f^2 + \eta r_Q^{-1}k_Q^{-1}\boldsymbol{Z_Q}^T\boldsymbol{Z_Q})/2 \end{aligned} \quad (56)$$

with $\eta > 0$, whose time-derivative along the trajectories of (13), (18), (28), (33), (41), (49) and (54) satisfies

$$\begin{aligned} \dot{W} &= -\boldsymbol{\Phi_{Vf}}^T\boldsymbol{Z_V}\tilde{V}_f - k_V C_{T\beta}^{-1}\left|\tilde{V}_f\right|^2 - k_V^{-1}C_{T\beta}\left|\boldsymbol{\Phi_{Vf}}^T\boldsymbol{Z_V}\right|^2 \\ &\quad - \Phi_{\gamma f}Z_\gamma\tilde{\gamma}_f - k_\gamma C_{L\alpha}^{-1}\left|\tilde{\gamma}_f\right|^2 + qS\tilde{\theta}_f\tilde{\gamma}_f/(mV) - \\ &\quad k_\gamma^{-1}C_{L\alpha}\left|\Phi_{\gamma f}Z_\gamma\right|^2 + \frac{qS}{mV}k_\gamma^{-1}C_{L\alpha}\Phi_{\gamma f}Z_\gamma\tilde{\theta}_f - \eta k_\theta c_e^{-1}\tilde{\theta}_f^2 \\ &\quad - \eta\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q}\tilde{Q}_f - \eta k_Q c_e^{-1}\left|\tilde{Q}_f\right|^2 - \eta k_Q^{-1}c_e\left|\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q}\right|^2 \\ &\le -[\left|\tilde{\gamma}_f\right|, \left|\Phi_{\gamma f}Z_\gamma\right|, \left|\tilde{\theta}_f\right|]\boldsymbol{L}[\left|\tilde{\gamma}_f\right|, \left|\Phi_{\gamma f}Z_\gamma\right|, \left|\tilde{\theta}_f\right|]^T - \frac{1}{2}k_V C_{T\beta}^{-1}\left|\tilde{V}_f\right|^2 \\ &\quad - \frac{1}{2}k_V^{-1}C_{T\beta}\left|\boldsymbol{\Phi_{Vf}}^T\boldsymbol{Z_V}\right|^2 - \frac{1}{2}\eta k_Q c_e^{-1}\left|\tilde{Q}_f\right|^2 - \frac{1}{2}\eta k_Q^{-1}c_e\left|\boldsymbol{\Phi_{Qf}}^T\boldsymbol{Z_Q}\right|^2 \end{aligned} \quad (57)$$

where

$$\boldsymbol{L} = \begin{bmatrix} k_\gamma/(2C_{L\alpha}) & 0 & -qS/(2mV) \\ 0 & C_{L\alpha}/(2k_\gamma) & -qSC_{L\alpha}/(2mVk_\gamma) \\ -qS/(2mV) & -qSC_{L\alpha}/(2mVk_\gamma) & \eta k_\theta/c_e \end{bmatrix}$$

According to (57), $\dot{W}$ is negative-definite if $\boldsymbol{L}$ is a positive matrix. It is apparent that $\boldsymbol{L}$ is positive-definite if the determinant of $\boldsymbol{L}$ is positive. The determinant of $\boldsymbol{L}$ is

$$\det(\boldsymbol{L}) = \eta k_\theta/(4c_e) - (qS)^2 C_{L\alpha}/[4k_\gamma (mV)^2]$$

It is found that selecting $\eta > (\frac{qS}{mV})^2 k_\gamma^{-1} k_\theta^{-1} c_e C_{L\alpha}$ ensures $\det(\boldsymbol{L}) > 0$ .Therefore, $(\tilde{V}_f, \boldsymbol{Z}_V, \tilde{\gamma}_f, Z_\gamma, \tilde{\theta}_f, \tilde{Q}_f, \boldsymbol{Z}_Q) \in \mathcal{L}_\infty$ and $(\tilde{V}_f, \boldsymbol{\Phi}_{Vf}^T \boldsymbol{Z}_V, \tilde{\gamma}_f, \Phi_{\gamma f} Z_\gamma, \tilde{\theta}_f, \tilde{Q}_f, \boldsymbol{\Phi}_{Qf}^T \boldsymbol{Z}_Q) \in \mathcal{L}_2$. Note that the value of η is only needed for the stability analysis, yet not necessary for control law design. Since the terms $(\boldsymbol{\Phi}_{Vf}, \Phi_{\gamma f}, \boldsymbol{\Phi}_{Qf}^T)$ and their time derivates are bounded, it follows that $\lim_{t\to\infty}(\tilde{V}_f, \boldsymbol{\Phi}_{Vf}^T \boldsymbol{Z}_V, \tilde{\gamma}_f, \Phi_{\gamma f} Z_\gamma, \tilde{\theta}_f, \tilde{Q}_f, \boldsymbol{\Phi}_{Qf}^T \boldsymbol{Z}_Q) = \boldsymbol{0}$ by Barbalat's lemma. Finally, we can guarantee $\lim_{t\to\infty}(\tilde{V}, \tilde{\gamma}, \tilde{\theta}, \tilde{Q}) = \boldsymbol{0}$ from (5), (21), (24) and (38).

4. SIMULATION RESULTS

Simulations have been performed to evaluate the proposed control system. The initial states of HSV are chosen as $[V, h, \gamma, \theta, Q]^T = [4590.3\,m/s, 33528m, 0\deg, 0\deg, 0\deg/s]^T$. The configuration data of HSV in H. Xu, et al. (2004) are used for computation. Note that the nominal aerodynamic parameters are only used in the simulation model, while the control law is derived using their estimates. The simulation model also contains realistic actuators with both position and rate limits. The elevator is modeled as a first order system with a deflection limit of $\pm 20\deg$ and a rate limit of $\pm 50\deg/s$, while the engine is modeled as a second order system with a natural frequency of $20\,rad/s$, a damping factor of 0.7, and a position interval of $[0.05, 1.5]$. To illustrate the robustness of the control system, the control law is performed on the nominal model and a model with parameter uncertainty respectively. A maximum variation within 40% of the nominal aerodynamic coefficients has been considered here. A step altitude and velocity command of 1000m and 100m/s is defined. In both cases, the step command has been filtered through a second-order prefilter to generate the reference trajectory. The initial values of all the estimated parameters are selected as zero which is the worst choice of the initial estimates, yet has been made to further examine the robustness of the control system (Lee & Singh, 2010). The simulation results are shown in Figs. 1~3.

Simulation results given in Fig. 1 show that the proposed control law provides effective and robust tracking performance for the velocity and altitude. The tracking errors in both cases remain small during the entire maneuver and vanish asymptotically. Figure 2 shows smooth tracking of FPA γ , the pitch angle θ and the pitch rate Q relative to the virtual control inputs γ_c , θ_c and Q_c . Note that these states remain within reasonable ranges, which is a necessary requisite for hypersonic vehicles. The deflections of the elevator δ_e and the fuel equivalence ratio β are given in Fig. 3, both of which behave within their physical bounds. It can be seen that there is a short period of saturation for the fuel equivalence ratio β during simulation which results from the −40% deviation of the thrust coefficient.

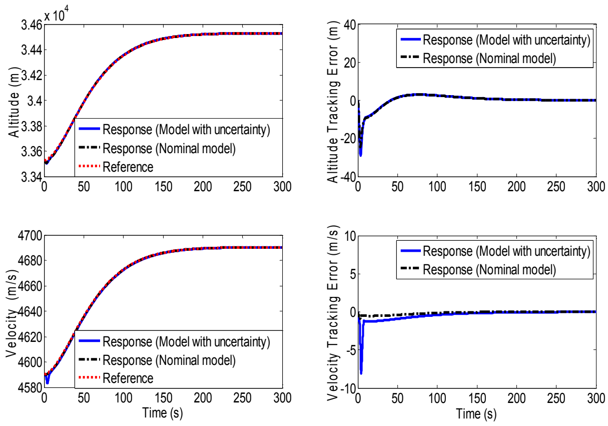

Fig. 1. Response to the altitude and velocity command.

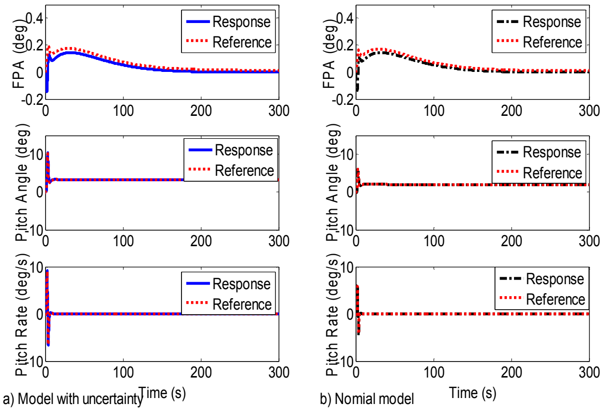

Fig. 2. Tracking performance of FPA, pitch angle and pitch rate: a) Model with uncertainty, b) Nominal model.

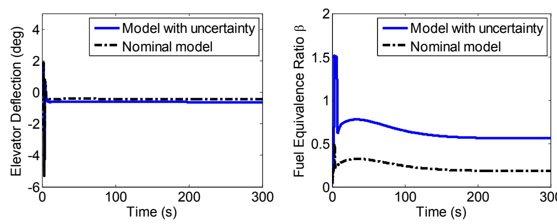

Fig. 3. Control inputs: δ_e and β .

5. CONCLUSIONS

A non-CE adaptive control system based on the immersion and invariance method is designed for the longitudinal HSV dynamics with all the aerodynamic parameters (except the sign of the control input coefficients) unknown in this paper. I&I based parameter estimators are designed for all the unknown parameters. The construction of such parameter estimator is a sum of a partial estimate generated from the update law and an additional nonlinear term. It has been shown that expected dynamic performance of the estimation errors can be achieved by the selection of the addition term. The need for the solution of PDE in I&I adaptive control has been removed by the application of regressor filters. Globally asymptotic stability of the whole closed-loop system is derived using Lyapunov analysis. Efficient and robust tracking of the command trajectories has been illustrated. It should be noted that the proposed approach can also handle other uncertainties.

ACKNOWLEDGMENT

This work is supported by the grants of National Natural Science Foundation of China (No. 61273149, 61203003) and the Innovation Method Fund of China (No. 2012IM010200).

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