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An adaptive "quasi" repetitive controller for the fundamental frequency estimation of periodic signals

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Abstract: A repetitive control scheme for asymptotic tracking of the fundamental frequency of periodic signals is presented. The method uses an adaptive orthogonal signals generator based on a second order generalized integrator for identifying the fundamental frequency of the signal with accuracy depending on the adaptive gain. The robustness of the repetitive controller is established by investigating its sensitivities with respect to an inaccurate estimation of the period.

Keywords: Repetitive control, periodic signals, internal model principle, orthogonal signals generator.

1. INTRODUCTION

Most of the classical fundamental frequency estimators do not assume a model for the periodic signal(s), i.e., they are classified as non-parametric methods. A popular approach has been to compare the observed signal with a delayed version of the self-same signal by a similarity measure. The rationale behind doing this is that, when the delay corresponds to the reciprocal of the fundamental frequency, the similarity measure should be maximized since the desired signal is periodic. Some of the first methods utilizing this approach used the autocorrelation function Rabiner (1977) and the average magnitude difference function Ross et al. (1974) as similarity measures. More recent variants of methods using the delay approach can be found in Combescure et al. (1982); Medan et al. (1991); Tolonen et al. (2000). Another subclass of non-parametric fundamental frequency estimators is based on peak detection. These methods exploits the fact that the peaks of, e.g., the time-series representation Gold et al. (1969) or the cepstrum Noll (1967) of the observed signal should appear in fixed intervals, where the length of the intervals can be mapped to a fundamental frequency estimate. A third approach to non-parametric fundamental frequency estimation is based on the harmonic product spectrum. In methods based on this approach, the spectrum at the fundamental frequency and multiples thereof are multiplied for different candidate fundamental frequencies Noll (1969); Schroeder (1968). Then, the fundamental frequency estimate is obtained from the maximizer of the so-called harmonic product spectrum. For an overview of the above and other non-parametric fundamental frequency estimators, see, e.g., Hess (1983). While the non-parametric methods are intuitively sound, they are often relying on several heuristics and suffer from poor resolution. To tackle these issues, research in parametric fundamental frequency estimators has attracted considerable attention in the recent years. In general, the parametric estimators can be divided into three groups of methods Christensen et al. (2009):

- statistical methods,
- subspace methods,
- filtering methods.

In the statistical methods, the likelihood or probability of the fundamental frequency is maximized possibly under some noise assumptions (e.g., the noise being white and Gaussian). Examples of maximum likelihood and maximum a posteriori probability approaches can be found in Christensen et al. (2008a, 2009). Moreover, examples of other Bayesian approaches can be found in Cemgil et al. (2006); Davy et al. (2006); Godsill et al. (2002). The statistical methods do often provide efficient estimates, however, they are rather computationally demanding. This has motivated research in other groups of parametric methods such as the subspace methods. The subspace methods utilize the fact that the space spanned by the observed signal covariance matrix can be divided into two subspaces spanning the signal and the noise subspaces, respectively. The properties of these subspaces can then be exploited for various estimation and identification tasks Krim et al. (1996); Vary et al. (2006); Viberg et al. (1991). A third group of parametric fundamental frequency estimators is the filtering methods. The idea behind these methods is to design a filter that passes a periodic signal undistorted and apply it on the observed signal. More specifically, the filter is designed such that it passes the harmonics of the periodic signal while the noise is attenuated Moorer (1974); Medan et al. (1991). Some optimal filtering based methods were proposed recently Christensen et al. (2008b, 2011). In these methods, the filters are designed to pass the desired periodic signal undistorted, while minimizing the filter output power. This paper presents a new and alternative approach to the fundamental frequency estimation of a periodic signal based on a repetitive control (**RC**) methodology. The signal with unknown period is considered as a periodic disturbance acting on a system which is an approximate linearization of a derivative block. The addition of a pole to the ideal derivative block filters the signal before differentiating reducing the effect of noise. A modified RC is then adaptively tuned to cancel the effect of the periodic signal on the system output. The adaption scheme to tune the period of RC makes use of an adaptive orthogonal signals generator based on second order generalized integrator, namely **OSG-SOGI**, see Fedele et al. (2009a,b); Fedele (2012a); Fedele et al. (2012b,c, 2013) and the references therein. OSG-SOGI is used to estimate the period of the signal with a predefined level of uncertainty and to provide the correct input signal to the frequency estimator system. In fact, once the periodic components are sufficiently suppressed in the output system, excitation of the estimation algorithm is lost, resulting in a bad estimate of the period. Therefore internal signals of OSG-SOGI are used to correct power the frequency estimator system. RC is then used to refine such an estimate reducing the harmonic components at the output of the derivative block. At this stage, the input to the frequency estimator system is a purely sinusoid with the same period of the periodic signal. Other approaches for frequency estimation in case of periodic signals with known components number and/or bounded high frequency noise term can be found also in Bobtsov et al. (2012); Marino et al. (2002); Wu et al. (2003); Xia (2002). The paper is organized as follows: Section 2 presents the proposed scheme with the quasi repetitive controller; the characteristics of the OSG-SOGI are discussed in Section 3. The robustness of the method is established in Section 4. Section 5 presents some numerical simulations while the last section is devoted to conclusions.

2. PROBLEM STATEMENT AND PROPOSED APPROACH

This paper addresses the problem of estimating the fundamental frequency ω_c of a periodic signal $d(t)$ which can be expressed in the form of Fourier series:

$$d(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_c t) + b_n \sin(n\omega_c t)\right), \quad (1)$$

where n denotes the harmonic index, and $a_n, b_n (n = 1, 2, ...)$ are the Fourier coefficients of the nth harmonic; a_0 is the signal dc component. Even if finite harmonics are generally substituted for the infinite harmonics in practice, here no assumptions are made on the largest harmonic index, i.e. the method does not require the number of harmonics to run properly.

A possible approach to the tracking and/or disturbance rejection problem is to use the internal model principle which states that for asymptotically reject a disturbance, a model of the disturbance generating system should be included in the feedback loop. The idea discussed in this paper is to use a “quasi” repetitive controller (**QRC**) to cancel the effect of a periodic unknown disturbance $d(t)$ at the output of a linear block which is the approximate linearization of a derivative block (see Fig. 1). The Frequency estimator system (**FES**) tunes the QRC with an estimate of the fundamental signal period. In particular, it uses an adaptive orthogonal signals generator based on a second order generalized integrator, namely OSG-SOGI, to estimate the fundamental frequency with a predefined level of uncertainty depending on the parameter γ as it will be shown in the next section. OSG-SOGI also provides an input signal to FES which asymptotically tends to a pure sinusoid with frequency equal to the fundamental of $d(t)$. In this case, FES refines the estimate of the frequency by converging asymptotically to the unknown value.

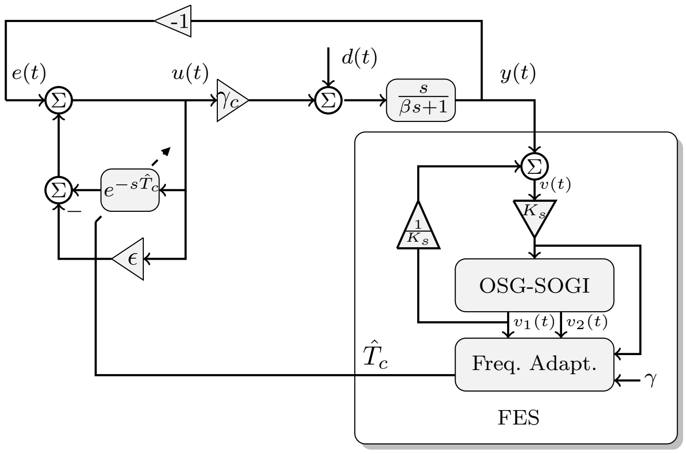

Fig. 1. Quasi repetitive control scheme.

The transfer function from $e(t)$ to $u(t)$ is

$$R(s) = \frac{1}{1 - e^{-s\hat{T}_c} + \epsilon}. \quad (2)$$

For frequencies $\omega = 2\pi k/\hat{T}_c$ the magnitude of the denominator of $R(s)$ is equal to ϵ. Clearly, if ϵ tends to zero then the gain of the transfer function tends to infinite, obtaining the classical repetitive controller whose principle is to have infinite loop gain at the harmonics of the disturbance. For $\epsilon \neq 0$ it is possible to choose the (finite) gain at the harmonics frequencies, from which the term “quasi” repetitive controller (see Fig. 2, where QRC is set with $\hat{T}_c = 1$).

The transfer function between $d(t)$ and $y(t)$ is

$$W_{dy}(s) = \frac{s(1 - e^{-s\hat{T}_c} + \epsilon)}{(\beta s + 1)(1 - e^{-s\hat{T}_c} + \epsilon) + \gamma_c s}, \quad (3)$$

with $\beta > 0$, and it can be rewritten as

$$W_{dy}(s) = \frac{\frac{1}{\gamma_c}(1 - e^{-s\hat{T}_c} + \epsilon)}{1 + L(s)}, \quad (4)$$

where

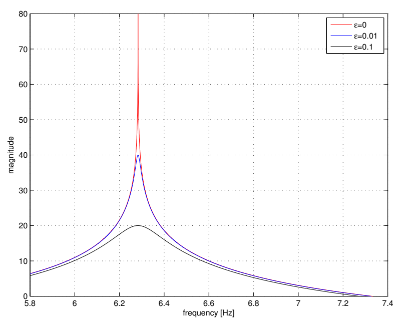

Fig. 2. Magnitude frequency response of QRC.

$$L(s) = \frac{1}{\gamma_c}\frac{\beta s + 1}{s}(1 - e^{-s\hat{T}_c} + \epsilon). \tag{5}$$

Proposition 1. The function $\gamma_c L(\jmath\omega)$ has no intersections with the negative real axis.

Proof It must be proved that the system

$$\beta\omega\left[1 + \epsilon - \cos(\omega\hat{T}_c)\right] + \sin(\omega\hat{T}_c) = -\alpha^2\omega, \tag{6}$$

$$\beta\omega\sin(\omega\hat{T}_c) + \cos(\omega\hat{T}_c) - (1+\epsilon) = 0, \tag{7}$$

with $\alpha \in \mathbb{R}$, has no solutions $\forall\omega > 0$.

From (7)

$$\omega\sin(\omega\hat{T}_c) = \frac{1}{\beta}\left[1 + \epsilon - \cos(\omega\hat{T}_c)\right] \tag{8}$$

which substituted in (6) gives

$$\left(\beta\omega^2 + \frac{1}{\beta}\right)\left[1 + \epsilon - \cos(\omega\hat{T}_c)\right] = -\alpha^2\omega. \tag{9}$$

The proof follows since $1 + \epsilon - \cos(\omega\hat{T}_c) > 0$ for all $\omega > 0$. □

Corollary 1. The closed loop system with transfer function $W_{dy}(s)$ is asymptotically stable.

Proof By Proposition 1, the Nyquist diagram of $L(s)$ has no intersection with the negative real axis and then it does not encircle the critical point $-1 + \jmath 0$. □

3. FREQUENCY ESTIMATOR SYSTEM

The OSG-SOGI block in Fig. 1 is a second-order filter characterized by a resonant frequency ω_s and a gain K_s. The frequency estimator system is then composed by an OSG-SOGI system and a frequency adaptive block capable to tune the resonant frequency ω_s

OSG-SOGI provides two orthogonal signals $v_1(t)$ and $v_2(t)$, according to the following differential equations

$$\dot{x}_1(t) = x_2(t), \tag{10}$$

$$\dot{x}_2(t) = -\omega_s^2 x_1(t) - K_s\omega_s x_2(t) + K_s v(t) \tag{11}$$

with $v(t)$ as input and outputs defined as $v_1(t) = K_s\omega_s x_2(t)$ and $v_2(t) = K_s\omega_s^2 x_1(t)$. For ω_s constant, signals $v_1(t)$ and $v_2(t)$ are the outputs of linear time-invariant systems with transfer functions

$$F_1(s) = \frac{K_s^2\omega_s s}{s^2 + K_s\omega_s s + \omega_s^2}, \tag{12}$$

$$F_2(s) = \frac{K_s^2\omega_s^2}{s^2 + K_s\omega_s s + \omega_s^2}. \tag{13}$$

$F_1(s)$ and $F_2(s)$ represent second order filters with a bandwidth depending on the gain K_s and a resonant frequency equal to ω_s. In particular, $F_2(s)$ presents second order low-pass filtering characteristics with static gain K_s and F_1 behaves as a second order band-pass filter with no attenuation and no phase shift at the resonant frequency. If K_s decreases, the bandwidth of the filter $F_1(s)$ becomes narrower resulting a heavy filtering, nevertheless this entails a slowdown on the dynamic response of the system increasing oscillations and the stabilization time. In Figs. 3 - 4 the Bode diagrams of the $F_1(s)$ and $F_2(s)$ filters are reported respectively with the same resonant frequency $\omega_s = 100$ rad/sec and with different values of gain K_s.

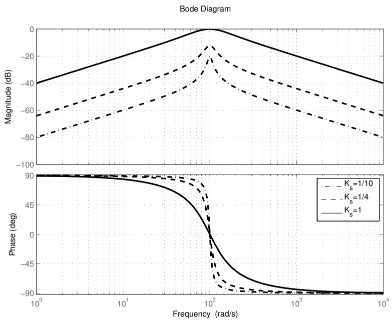

Fig. 3. Bode diagrams of $F_1(s)$ for $K_s = 1/10, 1/4, 1$.

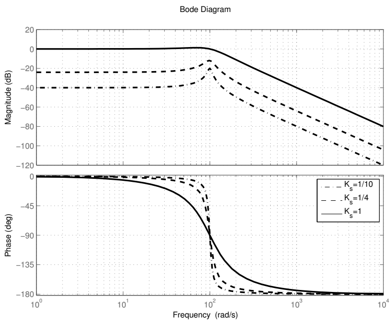

Fig. 4. Bode diagrams of $F_2(s)$ for $K_s = 1/10, 1/4, 1$.

Thus, for a signal

$$v(t) = A_v\sin(\omega_c t + \phi_v), \tag{14}$$

$v_1(t)$ and $v_2(t)$ converge exponentially to

$$v_{1\infty}(t) = m_1 A_v\sin(\omega_c t + \phi_v + \phi), \tag{15}$$

$$v_{2\infty}(t) = -m_2 A_v\cos(\omega_c t + \phi_v + \phi), \tag{16}$$

where

$$m_1 = \frac{K_s^2\omega_s\omega_c}{\sqrt{(\omega_s^2 - \omega_c^2)^2 + K_s^2\omega_s^2\omega_c^2}}, \tag{17}$$

$$m_2 = m_1\frac{\omega_s}{\omega_c}, \tag{18}$$

$$\phi = sign\,(\omega_s - \omega_c)\,\frac{\pi}{2} - \arctan\frac{K_s\omega_s\omega_c}{\omega_s^2 - \omega_c^2} \tag{19}$$

and the $sign(\cdot)$ function is defined as

$$sign(x) = \begin{cases} 1 & \text{iff } x \geq 0, \\ -1 & \text{otherwise.} \end{cases} \tag{20}$$

These results imply that, for sinusoidal excitation and constant ω_s, the IS-OSG-SOGI generates two orthogonal sinusoidal signals $v_1(t)$ and $v_2(t)$. Further, if the input frequency is equal to ω_s ($\omega_c \equiv \omega_s$), the IS-OSG-SOGI generates sin/cos waves that have the same magnitude as $v(t)$ and with $v_1(t)$ in phase with the input signal. In order to determine the unknown frequency ω_c, the resonant frequency ω_s can be adapted according to the following differential equation

$$\dot{\omega}_s = -\gamma K_s\omega_s\left(K_s v(t) - v_1(t)\right)v_2(t) \tag{21}$$

where $\gamma > 0$ is the adaptation gain. In Fedele et al. (2009a), a similar adaptive law has been proposed where the input $v(t)$ is not pre-scaled by the term K_s. However, the analysis in Fedele et al. (2009a) assumes a pure sinusoidal signal as input for the OSG-SOGI. As stated in the next Theorem, the convergence properties of the adapted law (21) are still valid in a neighborhood of the nominal one even if an harmonic signal is assumed in input.

Theorem 1. Let us assume a T_c-periodic input, with $T_c = \frac{2\pi}{\omega_c}$, $v(t) = A_1 \sin(\omega_c t) + A_2 \cos(\omega_c t) + n(t)$, where $n(t)$ is an arbitrary function that has no frequency component at ω_c. Then the adaptive law (21) has an equilibrium point at an $\mathcal{O}(\gamma)$ neighborhood of the input frequency ω_c.

Proof The proof is reported in Appendix A. □

4. SENSITIVITY OF THE QUASI REPETITIVE CONTROL

Here the sensitivity of the signal $v(t)$ with respect to the inaccurate estimate of the fundamental frequency ω_c is analyzed Steinbuch (2002); Tsao et al. (1998). Let us consider the transfer function between $d(t)$ and $v(t)$:

$$W_{dv}(s) = \frac{s(1 - e^{-s\hat{T}_c} + \epsilon)(s^2 + K_s\omega_s s + \omega_s^2)}{\left[(\beta s + 1)(1 - e^{-s\hat{T}_c} + \epsilon) + \gamma_c s\right](s^2 + \omega_s^2)}. \quad (22)$$

From Theorem 1 it follows that, after the transient, the OSG-SOGI resonant frequency ω_s is $\omega_s = \omega_c + \delta\omega$, where $\delta\omega$ is $\mathcal{O}(\gamma)$. Therefore $\delta\omega$ can be made arbitrarily small by an opportune choice of γ. We would like that the $W_{dv}(s)$ acts as a filter that passes the fundamental at ω_c and attenuates the other harmonics at $k\omega_c$ with $k \geq 2$. Therefore we investigate the filter response when $\hat{T}_c = 2\pi/(\omega_c + \delta\omega)$ and $s = \jmath k\omega_c$. Substitution of these values into Eq. (22) and Taylor expansion with $\delta\omega \to 0$ give

$$|W_{dv}(\jmath k\omega_c)| = c_0 + c_1\delta\omega + \mathcal{O}(\delta\omega^2), \quad (23)$$

where

$$c_0 = \frac{\epsilon k\omega_c}{(k^2-1)\sqrt{\frac{\epsilon^2 + (\beta\epsilon+\gamma_c)^2 k^2\omega_c^2}{(k^2-1)^2 + K_s^2 k^2}}}, \quad (24)$$

$$c_1 = \frac{2\epsilon k^3\gamma_c\omega_c\pi\sqrt{(k^2-1)^2 + K_s^2k^2}}{(k^2-1)\left[\epsilon^2 + (\beta\epsilon+\gamma_c)^2k^2\omega_c^2\right]^{3/2}} + \\ + \frac{\epsilon K_s^2 k^3}{(k^2-1)\sqrt{\left[(k^2-1)^2 + K_s^2k^2\right]\left[\epsilon^2 + (\beta\epsilon+\gamma_c)^2k^2\omega_c^2\right]}}. \quad (25)$$

Remark 1. It is straightforward to note that other terms in the Taylor expansion, as c_0 and c_1, contains $k^2 - 1$ in the denominator. This means that $|W_{dv}(\jmath k\omega_c)|$ is infinite only for $k = 1$. Therefore the OSG-SOGI input $v(t)$ is actually constituted by a single sinusoid at the fundamental frequency of the periodic signal and then its adaptive regulation makes the resonant frequency converging to the correct value.

5. NUMERICAL SIMULATIONS

In this section two numerical simulations are reported to show the characteristics of the proposed method.

Example 1. A square signal with starting frequency 10 rad/sec and duty-cycle of 50% is here considered. The signal is affected by a white noise with normal distribution $\mathcal{N}(0, 0.01)$. The simulation time is 40 sec with a sampling time equal to $4 \cdot 10^{-5}$ sec. At time $t = 20$ sec, a frequency step occurs passing from 10 rad/sec to 12 rad/sec. The OSG-SOGI parameters are $K_s = 1$, $\gamma = 10^{-3}$. The parameter ϵ is chosen as 10^{-6} while $\gamma_c = 10^{-1}$ and $\beta = 0.1$. The method is compared with a bank of five SOGIs as in Fedele et al. (2009a) and the estimation approach in Wu et al. (2003), namely **MS** and **WB** methods, respectively. In SOGIs-bank, only the first SOGI block is adapted with an adaptive gain equal to 1, while the others are tuned with multiple frequencies. The gain $K_s = 1$ is set for all blocks. WB approach is set up with $g_m = g_\phi = 10$, $g_w = 1.75$ and $K_f = 2.5$. The same number of component blocks, as in MS, has been considered for WB method. All the estimators start with an initial frequency condition equal to 75.4 rad/sec. Parameters are chosen in order to guarantee that all methods present the same settling-time approximatively, in the first sub-interval. WB performs better in steady-state conditions. However its convergence time becomes too slow if the initial condition is not close enough to the frequency to be estimated. The frequency estimate of the proposed method, namely ω_s^{RC}, MS estimate, namely ω_s^{MS}, and WB one, namely ω_s^{WB}, are depicted in Fig. 5.

3	5	7	9	11	13	15	17	19
5%	6%	5%	1.5%	3.5%	3%	0.5%	2%	1.5%

Table 1. Harmonic order and amplitude expressed as percentage of the first harmonic.

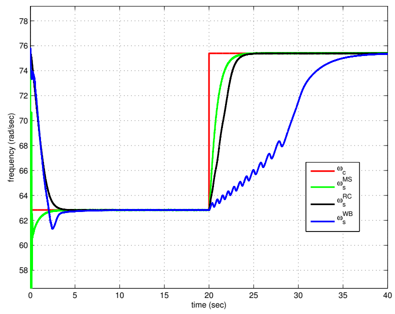

Fig. 5. Example 1. Fundamental frequency estimation.

Example 2. In this example a 50Hz sinusoidal signal is distorted with 9 odd harmonics. Each component of the signal has a random phase uniformly distributed in $[0, 2\pi]$. The fundamental amplitude is $230\sqrt{2}$ while the other harmonics amplitudes are expressed as a percentage of the first harmonic amplitude as reported in Table I. In this example the observation time is 1 sec with a sampling time equal to 10^{-5} sec. The initial value of the OSG-SOGI resonant frequency is $\omega_s(0) = 377$ rad/sec, $\gamma = 10^{-6}$ while the other parameters are the same of the previous example. The frequency estimate is depicted in Fig. 6.

6. CONCLUSIONS

In this paper a novel method to estimate the fundamental frequency of a periodic signal has been presented. An adaptive orthogonal signals generator is coupled with a quasi repetitive controller to mitigate the effects of the harmonics in the frequency estimation process. After a

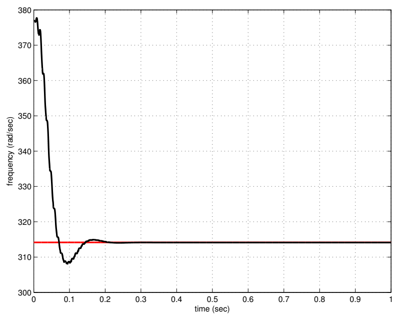

Fig. 6. Example 2. Fundamental frequency estimation.

transient the orthogonal signals generator is excited by a pure sinusoidal signal at the fundamental frequency since the transfer function between the periodic signal and the input to the orthogonal signals generator is a quasi resonant filter with bandwidth centered at the frequency of interest.

Appendix A. PROOF OF THEOREM 1

The proof is based on the one reported in Mojiri et al. (2004, 2007a,b). Let us consider the system defined by (10)-(11) plus the frequency adaptation (21). Note that such a system has a periodic orbit for pure nominal sinusoidal input at the frequency ω_c, i.e. $v_n(t) = A_1 \sin(\omega_c t) + A_2 \cos(\omega_c t)$. By taking into account such facts, the following change of variables has been considered

$$\begin{pmatrix} x_1 \\ x_2 \\ \omega_s \end{pmatrix} = \begin{pmatrix} -\sin(\omega_c t) & -\cos(\omega_c t) & 0 \\ -\omega_c \cos(\omega_c t) & \omega_c \sin(\omega_c t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \omega_s \end{pmatrix}. \quad (A.1)$$

The transformation (A.1) is referred as a *stability preserving map*. It brings the dynamics near the desired periodic orbit and it preserves the stability properties of the original dynamical system Khalil (2002). Note that the transformation matrix in (A.1) and its time-derivative are continuous and bounded on $\mathbb{R}$. Moreover, the determinant of matrix in (A.1) is constantly equal to $-\omega_c, \forall t \in \mathbb{R}$. Applying transformation (A.1) to (10)-(11) and (21), yields the following dynamical system that is equivalent to the original one, in the sense of Lyapunov:

$$\dot{\xi}_1(t) = -\frac{1}{\omega_c} \cos(\omega_c t) \left[c_s(t) + c_c(t) + K_s v(t)\right], \quad (A.2)$$

$$\dot{\xi}_2(t) = \frac{1}{\omega_c} \sin(\omega_c t) \left[c_s(t) + c_c(t) + K_s v(t)\right], \quad (A.3)$$

$$\dot{\omega}_s(t) = \gamma K_s^3 \omega_s^3(t) \left(\sin(\omega_c t)\xi_1(t) + \cos(\omega_c t)\xi_2(t)\right) \times$$
$$\times \left[\omega_c \omega_s(t) \left(\cos(\omega_c t)\xi_1(t) - \sin(\omega_c t)\xi_2(t)\right) + v(t)\right], \quad (A.4)$$

where

$$c_s(t) = -\left[(\omega_c^2 - \omega_s^2(t))\xi_1(t) + K_s \omega_c \omega_s(t) \xi_2(t)\right] \sin(\omega_c t), \quad (A.5)$$

$$c_c(t) = \left[K_s \omega_c \omega_s(t) \xi_1(t) - (\omega_c^2 - \omega_s^2(t))\xi_2(t)\right] \cos(\omega_c t). \quad (A.6)$$

Let us consider $0 < \bar{\epsilon} \ll 1$ a constant parameter and $K_s = \bar{\epsilon}\hat{K}_s$, $\bar{\epsilon}^2\gamma = \hat{\gamma}$ and $\bar{\epsilon}\Omega(t) = (\omega_s^2(t) - \omega_c^2)/\omega_c$.

Therefore, the system (A.2)-(A.4) can be rewritten as:

$$\dot{\xi}_1(t) = -\frac{\bar{\epsilon}}{\omega_c} \cos(\omega_c t) \left[\hat{c}_s(t) + \hat{c}_c(t) + \hat{K}_s v(t)\right], \quad (A.7)$$

$$\dot{\xi}_2(t) = \frac{\bar{\epsilon}}{\omega_c} \sin(\omega_c t) \left[\hat{c}_s(t) + \hat{c}_c(t) + \hat{K}_s v(t)\right], \quad (A.8)$$

$$\dot{\Omega}(t) = \bar{\epsilon}\hat{\gamma}\hat{K}_s^3 \times$$
$$\times \left[\omega_c(\omega_c + \bar{\epsilon}\Omega(t))\right]^{3/2} \left(\sin(\omega_c t)\xi_1(t) + \cos(\omega_c t)\xi_2(t)\right) \times$$
$$\times \left[\omega_c \sqrt{\omega_c(\omega_c + \bar{\epsilon}\Omega(t))} \left(\cos(\omega_c t)\xi_1(t) - \sin(\omega_c t)\xi_2(t)\right) + v(t)\right], \quad (A.9)$$

where

$$\hat{c}_s(t) = \left[\omega_c \Omega(t)\xi_1(t) - \hat{K}_s \omega_c \sqrt{\omega_c(\omega_c + \bar{\epsilon}\Omega(t))}\xi_2(t)\right] \sin(\omega_c t),$$

$$\hat{c}_c(t) = \left[\hat{K}_s \omega_c \sqrt{\omega_c(\omega_c + \bar{\epsilon}\Omega(t))}\xi_1(t) + \omega_c \Omega(t)\xi_2(t)\right] \cos(\omega_c t).$$

An averaging method is here provided to prove the stability of the system in Eq. (A.7)-(A.9). The averaging method permits to analyze the solutions of a T-periodic dynamical system into the form $\dot{X} = \bar{\epsilon}F(x, t, \bar{\epsilon})$ by investigating the solutions of the *averaged* value $X_{av} = \bar{\epsilon}\frac{1}{T}\int_0^T F(X_{av}, t, 0)dt$ for $T \to \infty$. In particular, averaging results indicate that the periodic orbits of the original system are related to the fixed points of the averaged one which may be much easier to analyze. The input of the system is here assumed to be composed by the nominal part at the frequency ω_c and a periodic term $n(t)$ that has no frequency component at ω_c, i.e, $v(t) = v_n(t) + n(t)$. Note that the input $v(t)$ multiplies T_c-periodic terms ($\sin(\omega_c t)$ and $\cos(\omega_c t)$). Within the assumptions of the averaging analysis, the term $n(t)$ vanishes since $\frac{1}{T_c}\int_0^{T_c} n(t)\sin(\omega_c t)dt = \frac{1}{T_c}\int_0^{T_c} n(t)\cos(\omega_c t)dt = 0$ and, as a consequence, $n(t)$ does not influence the frequency estimator process. The averaged system of (A.7)-(A.9) is then

$$\dot{\xi}_{1_{av}} = \frac{\bar{\epsilon}}{2}\omega_c \left[-\frac{A_2}{\omega_c^2}\hat{K}_s - \hat{K}_s\xi_{1_{av}} - \Omega_{av}\xi_{2_{av}}\right], \quad (A.10)$$

$$\dot{\xi}_{2_{av}} = \frac{\bar{\epsilon}}{2}\omega_c \left[\frac{A_1}{\omega_c^2}\hat{K}_s - \hat{K}_s\xi_{2_{av}} + \Omega_{av}\xi_{1_{av}}\right], \quad (A.11)$$

$$\dot{\Omega}_{av} = \frac{\bar{\epsilon}}{2}\hat{\gamma}\hat{K}_s^3\omega_c^3 \left(A_1\xi_{1_{av}} + A_2\xi_{2_{av}}\right), \quad (A.12)$$

where the notation $(\cdot)_{av}$ indicates the averaged value of the signal of interest and the time dependence is omitted for brevity.

The averaged system has an equilibrium point at $\bar{\xi}_{1_{av}} = -A_2/\omega_c^2$, $\bar{\xi}_{2_{av}} = A_1/\omega_c^2$ and $\omega_{s_{av}} = \omega_c$. Such an equilibrium point is transferred to the origin by the following change of variables $z_1(t) = \xi_{1_{av}}(t) + \frac{A_2}{\omega_c^2}$, $z_2(t) = \xi_{2_{av}}(t) - \frac{A_1}{\omega_c^2}$, $z_3(t) = \Omega_{av}$, obtaining the following differential nonlinear system

$$\dot{z}_1 = -\frac{\bar{\epsilon}}{2}\left(\hat{K}_s\omega_c z_1 + z_2 z_3 + \frac{A_1}{\omega_c^2}z_3\right), \quad (A.13)$$

$$\dot{z}_2 = -\frac{\bar{\epsilon}}{2}\left(\hat{K}_s\omega_c z_2 - z_1 z_3 + \frac{A_2}{\omega_c^2}z_3\right), \quad (A.14)$$

$$\dot{z}_3 = \frac{\bar{\epsilon}}{2}\hat{\gamma}\hat{K}_s^3\omega_c^3 \left(A_1 z_1 + A_2 z_2\right). \quad (A.15)$$

The following candidate Lyapunov function is here proposed $V(z) = z_1^2 + z_2^2 + \frac{1}{\hat{\gamma}\hat{K}_s^3\omega_c^5}z_3^2$. $V(z)$ is a continuously differentiable positive definite function with time-derivative equal to $\dot{V}(z) = -\bar{\epsilon}\hat{K}_s\omega_c(z_1^2 + z_2^2)$. $\dot{V}(z)$ is negative semi-definite in an arbitrary neighborhood of the origin, namely $\mathcal{D}$. The set $\mathcal{S}$ associated with Barbashin-Krasovskii theorem Khalil (2002), is characterized by $\mathcal{S} = \left\{(z_1, z_2, z_3)^T \in \mathcal{D}, z_1 = 0, z_2 = 0\right\}$. The largest invariant set in $\mathcal{S}$ is the origin, hence the zero equilibrium point is asymptotically stable.

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