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Event-Triggered Least Squares Fault Estimation with Stochastic Nonlinearities ⋆

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Abstract: In this paper, the event-triggered least squares state and fault estimation problem is investigated for a class of systems with stochastic nonlinearities. An event-triggered scheme is properly proposed whose main idea is to transmit the measurement output to a remote estimator only when a specified event condition is violated and an event is triggered. A filter is designed so as to minimize an upper bound of the filtering error covariance with event-triggered measurement transmissions and additive stochastic nonlinearities. By solving two sets of discrete matrix equations, the desired filter parameters are calculated recursively and thus the method is applicable for online computation. Both the state and fault estimation problems are handled within the same framework using the least squares method. A numerical simulation is exploited to illustrate the effectiveness of the proposed algorithm.

1. INTRODUCTION

For decades, the state/fault estimation problem has been a research focus as one of the fundamental problems in the control and signal processing areas. The traditional Kalman filter, which serves as an optimal filter for linear systems, minimizes the filtering error covariance at each time step. To deal with more comprehensive systems (e.g. systems with nonlinearities or uncertainties), a great number of results have been reported to improve the performance of Kalman filter, such as extended Kalman filter (Reif et al. [1999]), mixed H_2/H_∞ filter (Xie et al. [2004]), etc.

Recently, event-triggered estimation and control problems have gained an increasing research interest and much effort has been made to address the problems (Tabuada [2007], Wang et al. [2011], Dimarogonas et al. [2012], Hu et al. [2012]). In an event-triggered measurement scheme, the measurement output is transmitted to a remote estimator only when a specified event condition is violated in an event generator. An estimator that could cope with event-triggered measurement transmission is much needed to save data transfer and processing power, especially in wireless networked systems with limited energy resources. The event-triggered optimal filtering problem in the least squares sense has stirred some initial research interests (Sijs et al. [2009], Wu et al. [2013]). These results have utilized the probability density functions of states and innovations conditional on measurements to calculate the posteriori probability density function of states and obtain the exact minimum mean-squared estimation error. However, when the system model is relatively complicated, the conditional probability density functions may be unavailable or overcomplex, which would limit the applicabilities of the proposed algorithms. Therefore, there is a practical need to address the event-triggered least squares filtering problem with less prior knowledge than exact probability density functions (e.g. only the means and covariances of the disturbances).

The analysis problem for nonlinear systems has been extensively studied for decades, since nonlinearities are inevitably encountered in various industrial systems, which may lead to undesirable dynamic behaviors. In many real-world systems, nonlinear disturbances may occur in a random way, because of sudden environment changes, random failure and repairs of components, intermittent transmission congestion, etc (Shen et al. [2011]). Such a kind of nonlinearities can be referred to as stochastic nonlinearities or randomly occurring nonlinearities. There have been some results on filtering problem with stochastic nonlinearities (Dong et al. [2011], Shen et al. [2011], Ma et al. [2011]), most of which have focused on conservative robust H_∞ filtering schemes without considering the estimation performances such as the error covariances. On the other hand, the least squares filtering problem with stochastic nonlinearities has not been sufficiently investigated yet (Hu et al. [2012, 2013]), not to mention the event-triggered case. This constitutes the main motivation of our present work.

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In this article, the joint least squares state and fault estimation problem is addressed for a class of systems with event-triggered measurement transmissions as well as additive stochastic nonlinearities. In the proposed event-triggered scheme, the measurement output is transmitted to a remote estimator only when a specified event condition is violated in the event generator. A filter is designed recursively that guarantees the minimization of an upper bound of the filtering error covariance at each time step. The main contributions of the paper are outlined as follows: 1) a comprehensive system model is put forward that covers event-triggered measurement transmission and additive stochastic nonlinearities; 2) additive faults and system states are simultaneously estimated to facilitate both the fault isolation and the state estimation problems; and 3) the proposed algorithm can be carried out recursively and thus applicable for online computation.

2. PROBLEM FORMULATION

Consider the following discrete-time nonlinear stochastic system:

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k + h(x_k, \alpha_k) + D_k w_k + F_k f_k, \\ y_k = C_k x_k + g(x_k, \beta_k) + E_k v_k, \quad x_0 = x_0, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state; $y_k \in \mathbb{R}^m$ is the measurement output; $u_k \in \mathbb{R}^l$ is the control input; $f_k \in \mathbb{R}^s$ is the additive fault; $w_k \in \mathbb{R}^p$ and $v_k \in \mathbb{R}^q$ are the process noise and the measurement noise, respectively. The noise sequences are independent zero-mean Gaussian sequences, with $\mathbb{E}\{w_k w_k^T\} = W_k$, $\mathbb{E}\{v_k v_k^T\} = V_k$. A_k, B_k, C_k, D_k, E_k and F_k are known matrices with appropriate dimensions.

The functions $h(x_k, \alpha_k)$ and $g(x_k, \beta_k)$ represent the stochastic nonlinearities. $\alpha_k \in \mathbb{R}$ and $\beta_k \in \mathbb{R}$ are independent zero-mean Gaussian noise sequences. The nonlinearities have the following first moment:

$$\mathbb{E}\left\{ \begin{bmatrix} h(x_k, \alpha_k) \\ g(x_k, \beta_k) \end{bmatrix} \middle| x_k \right\} = 0, \tag{2}$$

and the covariance given by

$$\mathbb{E}\left\{ \begin{bmatrix} h(x_k, \alpha_k) \\ g(x_k, \beta_k) \end{bmatrix} \begin{bmatrix} h(x_j, \alpha_j) \\ g(x_j, \beta_j) \end{bmatrix}^T \middle| x_k \right\} = \begin{cases} 0, & \text{if } k \neq j, \\ \begin{bmatrix} \Pi_1 x_k^T \Omega_1 x_k & 0 \\ 0 & \Pi_2 x_k^T \Omega_2 x_k \end{bmatrix}, & \text{if } k = j, \end{cases} \tag{3}$$

where Ω_i and Π_i $(i \in \{1, 2\})$ are known matrices with appropriate dimensions.

Implementing a given state feedback controller $u_k = K_k x_k$ to system (1) and defining $\bar{x}_k = \left[x_k^T, f_k^T\right]^T$, we consider the following stochastic nonlinear closed-loop system:

$$\begin{cases} \bar{x}_{k+1} = \bar{A}_k \bar{x}_k + \bar{h}(\bar{x}_k, \alpha_k) + \bar{D}_k w_k, \\ y_k = \bar{C}_k \bar{x}_k + \bar{g}(\bar{x}_k, \beta_k) + E_k v_k, \end{cases} \tag{4}$$

where

$$\bar{A}_k = \begin{bmatrix} A_k + B_k K_k & F_k \\ 0 & I \end{bmatrix}, \ \bar{D}_k = \begin{bmatrix} D_k \\ 0 \end{bmatrix}, \ \bar{C}_k = [\, C_k \ 0 \,],$$

$$\bar{h}(\bar{x}_k, \alpha_k) = \left[h^T(x_k, \alpha_k), 0\right]^T, \ \bar{g}(\bar{x}_k, \beta_k) = g(x_k, \beta_k).$$

Moreover, (2) and (3) can be easily rewritten as follows, respectively:

$$\mathbb{E}\left\{ \begin{bmatrix} \bar{h}(\bar{x}_k, \alpha_k) \\ \bar{g}(\bar{x}_k, \beta_k) \end{bmatrix} \middle| \bar{x}_k \right\} = 0, \tag{5}$$

and

$$\mathbb{E}\left\{ \begin{bmatrix} \bar{h}(\bar{x}_k, \alpha_k) \\ \bar{g}(\bar{x}_k, \beta_k) \end{bmatrix} \begin{bmatrix} \bar{h}(\bar{x}_j, \alpha_j) \\ \bar{g}(\bar{x}_j, \beta_j) \end{bmatrix}^T \middle| \bar{x}_k \right\} = \begin{cases} 0, & \text{if } k \neq j, \\ \begin{bmatrix} \bar{\Pi}_1 \bar{x}_k^T \bar{\Omega}_1 \bar{x}_k & 0 \\ 0 & \Pi_2 \bar{x}_k^T \bar{\Omega}_2 \bar{x}_k \end{bmatrix}, & \text{if } k = j, \end{cases} \tag{6}$$

where

$$\bar{\Pi}_1 = \begin{bmatrix} \Pi_1 & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{\Omega}_1 = \begin{bmatrix} \Omega_1 & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{\Omega}_2 = \begin{bmatrix} \Omega_2 & 0 \\ 0 & 0 \end{bmatrix}.$$

In this paper, the following transmission architecture is considered: the measurement output of (4) is sampled at each time step by the sampler, and then transmitted to the event generator. The current measurement is released by the generator when the current measurement y_{k+j} and the previously transmitted measurement y_k satisfy the following inequality:

$$(y_{k+j} - y_k)^T (y_{k+j} - y_k) > \sigma, \tag{7}$$

where σ is a predefined positive scalar. If (7) is satisfied, the current measurement is forwarded to a Zero-Order Hold (ZOH). Considering the characteristic of ZOH, the real estimator input $\tilde{y}_k$ can be written as

$$\tilde{y}_k = y_{k_i}, k \in \{k_i, k_i + 1, \cdots, k_{i+1} - 1\}, \tag{8}$$

where $k_0, k_1, \cdots$ are assumed to be the release times under the strategy (7).

For system (4), an estimator of the following structure is proposed:

$$\hat{x}_{k+1|k} = \bar{A}_k \hat{x}_{k|k}, \tag{9}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + \Xi_{k+1} \left(\tilde{y}_{k+1} - \bar{C}_k \hat{x}_{k+1|k} \right), \tag{10}$$

where $\hat{x}_{k+1|k} \in \mathbb{R}^n$ and $\hat{x}_{k|k} \in \mathbb{R}^n$ are the one-step prediction and the estimate of $\bar{x}_k$ at time step k with $\hat{x}_{0|0} = \left[x_0^T, 0^T\right]^T$, respectively, and Ξ_{k+1} is the parameter to be designed.

The model (4) is put forward in the paper so as to cater for the nonlinearities that occur in a random way. In fact, such stochastic nonlinearities could encompass large quantities of nonlinearities including the state multiplicative noises. The proposed transmission condition (7), which means that the current measurement is transmitted only when it changes significantly, can lead to lower traffic requirement and more efficient resource utilization. Meanwhile, the estimation results would have some robustness to small variations in the system under such a strategy. In an extreme situation, if σ=0, then all the measurements would be transmitted, and it reduces to traditional time-triggered transmission.

Denote the prediction error and filtering error by $e_{k+1|k} = \bar{x}_{k+1} - \hat{x}_{k+1|k}$, and $e_{k+1|k+1} = \bar{x}_{k+1} - \hat{x}_{k+1|k+1}$, respectively, and their covariances by $P_{k+1|k} = \mathbb{E}\left\{e_{k+1|k} e_{k+1|k}^T\right\}$ and $P_{k+1|k+1} = \mathbb{E}\left\{e_{k+1|k+1} e_{k+1|k+1}^T\right\}$. The goal of the paper is to design a recursive filter in the form of (9) and (10) for system (4) such that an upper bound of the filtering error covariance $P_{k+1|k+1}$ can be provided and minimized.

3. MAIN RESULTS

Firstly, by the definitions of $P_{k+1|k}$ and $P_{k+1|k+1}$, the two covariances would be obtained in the sequel, respectively.

Theorem 1. $P_{k+1|k}$ obeys the following recursion relation:

$$P_{k+1|k} = \bar{A}_k P_{k|k} \bar{A}_k^T + \bar{\Pi}_1 \mathrm{tr}\left\{\mathbb{E}\left\{\bar{x}_k \bar{x}_k^T\right\}\bar{\Omega}_1\right\} + \bar{D}_k W_k \bar{D}_k^T. \tag{11}$$

Proof. Based on (4) and (9), we have

$$e_{k+1|k} = \bar{A}_k e_{k|k} + \bar{h}(\bar{x}_k, \alpha_k) + \bar{D}_k w_k. \tag{12}$$

Noticing that the stochastic nonlinearity $\bar{h}(\bar{x}_k, \alpha_k)$ and the additive noise w_k are zero-mean, (11) follows directly from (6) and (12).

Theorem 2. $P_{k+1|k+1}$ satisfies the following equation:

$$\begin{aligned} P_{k+1|k+1} =& (I - \Xi_{k+1}\bar{C}_{k+1})P_{k+1|k}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\ &+ \Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})(\tilde{y}_{k+1} - y_{k+1})^T\right\} \\ &\times \Xi_{k+1}^T + \Xi_{k+1}\Pi_2 \mathrm{tr}\left\{\mathbb{E}\left\{\bar{x}_{k+1}\bar{x}_{k+1}^T\right\}\bar{\Omega}_2\right\}\Xi_{k+1}^T \\ &+ \Xi_{k+1}E_{k+1}V_{k+1}E_{k+1}^T\Xi_{k+1}^T - \Xi_{k+1} \\ &\times \mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})e_{k+1|k}^T\right\}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\ &- (I - \Xi_{k+1}\bar{C}_{k+1})\mathbb{E}\left\{e_{k+1|k}(\tilde{y}_{k+1} - y_{k+1})^T\right\} \\ &\times \Xi_{k+1}^T. \end{aligned} \tag{13}$$

Proof. From (4) and (10), it follows that

$$e_{k+1|k+1} = e_{k+1|k} - \Xi_{k+1}\left(\tilde{y}_{k+1} - \bar{C}_k \hat{x}_{k+1|k}\right). \tag{14}$$

Adding a zero term $\Xi_{k+1}y_{k+1} - \Xi_{k+1}y_{k+1}$ to the right-hand side of (14), we have

$$\begin{aligned} e_{k+1|k+1} =& (I - \Xi_{k+1}\bar{C}_{k+1})e_{k+1|k} - \Xi_{k+1}\bar{g}(\bar{x}_{k+1}, \beta_{k+1}) \\ &- \Xi_{k+1}E_{k+1}v_{k+1} - \Xi_{k+1}(\tilde{y}_{k+1} - y_{k+1}). \end{aligned} \tag{15}$$

With zero-mean stochastic nonlinearity $\bar{g}(\bar{x}_{k+1}, \beta_{k+1})$ and additive noise v_{k+1}, $P_{k+1|k+1}$ can be written as:

$$\begin{aligned} P_{k+1|k+1} =& (I - \Xi_{k+1}\bar{C}_{k+1})P_{k+1|k}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\ &+ \Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})(\tilde{y}_{k+1} - y_{k+1})^T\right\} \\ &\times \Xi_{k+1}^T + \Xi_{k+1}E_{k+1}\mathbb{E}\left\{v_{k+1}v_{k+1}^T\right\}E_{k+1}^T\Xi_{k+1}^T \\ &+ \Xi_{k+1}\mathbb{E}\left\{\bar{g}(\bar{x}_{k+1}, \beta_{k+1})\bar{g}^T(\bar{x}_{k+1}, \beta_{k+1})\right\}\Xi_{k+1}^T \\ &- (I - \Xi_{k+1}\bar{C}_{k+1})\mathbb{E}\left\{e_{k+1|k}(\tilde{y}_{k+1} - y_{k+1})^T\right\} \\ &\times \Xi_{k+1}^T - \Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})e_{k+1|k}^T\right\} \\ &\times (I - \Xi_{k+1}\bar{C}_{k+1})^T. \end{aligned} \tag{16}$$

Then with (6) and $\mathbb{E}\{v_{k+1}v_{k+1}^T\} = V_{k+1}$, (13) can be obtained directly. The proof is complete.

In Theorem 1 and Theorem 2, the exact recursion relations of $P_{k+1|k}$ and $P_{k+1|k+1}$ have been obtained. However, the terms which are related to $(\tilde{y}_{k+1} - y_{k+1})$, are very difficult to calculate. This results from the factor that the outputs are restricted not only by the measurement formulation (4) – but also by the transmission strategy (7). In other words, the exact covariances of prediction error and filtering error are dependent on whether the current measurement is transmitted or not. To obtain exact covariances of prediction error and filtering error, we need to calculate the posteriori probability density function of the states based on the probability density functions of states and innovations conditional on measurements. When the system dynamics is relatively complicated, for example, the system contains some stochastic nonlinearities as (4), the conditional probability density functions might be difficult to calculate, and the exact error covariances might be over-complicated or even unavailable. A seemingly nature way is to find an upper bound of the filtering error covariance, and then minimize the bound by appropriately designing the filter gain at each time step. This way, the conditional probability density functions will be no longer needed.

Before proceeding further, the following lemma is to be introduced.

Lemma 3. (Hu et al. [2012]) For any two vectors $x, y \in \mathbb{R}^n$, the following inequality holds

$$xy^T + yx^T \le \varepsilon xx^T + \varepsilon^{-1}yy^T, \tag{17}$$

where $\varepsilon > 0$ is a scalar.

With Lemma 3, an approach is proposed in the following theorem to determine the filter gain such that an upper bound of the filtering error covariance is minimized.

Theorem 4. Let ε be a positive scalar. If the next two equations:

$$Q_{k+1|k} = \bar{A}_k Q_{k|k}\bar{A}_k^T + \Pi_1 \mathrm{tr}\left\{X_k\bar{\Omega}_1\right\} + \bar{D}_k W_k \bar{D}_k^T, \tag{18}$$

$$\begin{aligned} Q_{k+1|k+1} =& (1+\varepsilon)(I - \Xi_{k+1}\bar{C}_{k+1})Q_{k+1|k}(I - \Xi_{k+1} \\ &\times \bar{C}_{k+1})^T + (1+\varepsilon^{-1})\sigma\Xi_{k+1}\Xi_{k+1}^T \\ &+ \Xi_{k+1}\Pi_2 \mathrm{tr}\left\{X_{k+1}\bar{\Omega}_2\right\}\Xi_{k+1}^T \\ &+ \Xi_{k+1}E_{k+1}V_{k+1}E_{k+1}^T\Xi_{k+1}^T, \end{aligned} \tag{19}$$

where

$$X_{k+1} = \bar{A}_k X_k \bar{A}_k^T + \Pi_1 \mathrm{tr}\{X_k\bar{\Omega}_1\} + \bar{D}_k W_k \bar{D}_k^T, \tag{20}$$

with initial conditions $Q_{0|0} \ge 0$ and $X_0 = \bar{x}_0\bar{x}_0^T$, have positive definite solutions. Then, $Q_{k|k}$ is an upper bound of $P_{k|k}$. Meanwhile, if the the filter gain is chosen as

$$\Xi_{k+1} = Z_{k+1}^T Y_{k+1}^{-1}, \tag{21}$$

where

$$\begin{aligned} Y_{k+1} =& (1+\varepsilon)\bar{C}_{k+1}Q_{k+1|k}\bar{C}_{k+1}^T + (1+\varepsilon^{-1})\sigma I \\ &+ \Pi_2 \mathrm{tr}\{X_{k+1}\bar{\Omega}_2\} + E_{k+1}V_{k+1}E_{k+1}^T, \end{aligned} \tag{22}$$

$$Z_{k+1} = (1+\varepsilon)\bar{C}_{k+1}Q_{k+1|k}, \tag{23}$$

then $Q_{k+1|k+1}$ is minimized.

Proof. The conclusions can be obtained by induction. It is already known that $Q_{0|0} > P_{0|0} = 0$. Then, assuming that for $i = 1, 2, \ldots, k$, $P_{i|i} \le Q_{i|i}$, it remains to show that $P_{k+1|k+1} \le Q_{k+1|k+1}$.

With Lemma 3, we have the following inequality:

$$\begin{aligned} &- (I - \Xi_{k+1}\bar{C}_{k+1})\mathbb{E}\left\{e_{k+1|k}(\tilde{y}_{k+1} - y_{k+1})^T\right\}\Xi_{k+1}^T \\ &- \Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})e_{k+1|k}^T\right\}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\ \le& \varepsilon(I - \Xi_{k+1}\bar{C}_{k+1})\mathbb{E}\left\{e_{k+1|k}e_{k+1|k}^T\right\}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\ &+ \varepsilon^{-1}\Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})(\tilde{y}_{k+1} - y_{k+1})^T\right\}\Xi_{k+1}^T. \end{aligned} \tag{24}$$

With the definition of $P_{k+1|k}$, (24) can be written as

$$
\begin{aligned}
&- (I - \Xi_{k+1}\bar{C}_{k+1})\mathbb{E}\left\{e_{k+1|k}(\tilde{y}_{k+1} - y_{k+1})^T\right\}\Xi_{k+1}^T \\
&- \Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})e_{k+1|k}^T\right\}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\
\leq&\varepsilon(I - \Xi_{k+1}\bar{C}_{k+1})P_{k+1|k}(I - \Xi_{k+1}\bar{C}_{k+1})^T \\
&+ \varepsilon^{-1}\Xi_{k+1}\mathbb{E}\left\{(\tilde{y}_{k+1} - y_{k+1})(\tilde{y}_{k+1} - y_{k+1})^T\right\}\Xi_{k+1}^T.
\end{aligned} \tag{25}
$$

Substituting (25) into (13), we have

$$
\begin{aligned}
P_{k+1|k+1} \leq&(1+\varepsilon)(I - \Xi_{k+1}\bar{C}_{k+1})P_{k+1|k}(I - \Xi_{k+1} \\
&\times \bar{C}_{k+1})^T + (1+\varepsilon^{-1})\Xi_{k+1}\mathbb{E}\{(\tilde{y}_{k+1} - y_{k+1}) \\
&\times (\tilde{y}_{k+1} - y_{k+1})^T\}\Xi_{k+1}^T + \Xi_{k+1}E_{k+1}V_{k+1} \\
&\times E_{k+1}^T\Xi_{k+1}^T + \Xi_{k+1}\Pi_2\mathrm{tr}\left\{\mathbb{E}\left\{\bar{x}_{k+1}\bar{x}_{k+1}^T\right\}\bar{\Omega}_2\right\} \\
&\times \Xi_{k+1}^T.
\end{aligned} \tag{26}
$$

Based on (7), for any $k \in \mathbb{N}$, we have

$$
(\tilde{y}_k - y_k)(\tilde{y}_k - y_k)^T \leq \sigma I. \tag{27}
$$

Considering (26) and (27), we have

$$
\begin{aligned}
P_{k+1|k+1} \leq&(1+\varepsilon)(I - \Xi_{k+1}\bar{C}_{k+1})P_{k+1|k}(I - \Xi_{k+1} \\
&\times \bar{C}_{k+1})^T + (1+\varepsilon^{-1})\sigma\Xi_{k+1}\Xi_{k+1}^T \\
&+ \Xi_{k+1}\Pi_2\mathrm{tr}\left\{\mathbb{E}\left\{\bar{x}_{k+1}\bar{x}_{k+1}^T\right\}\bar{\Omega}_2\right\}\Xi_{k+1}^T \\
&+ \Xi_{k+1}E_{k+1}V_{k+1}E_{k+1}^T\Xi_{k+1}^T.
\end{aligned} \tag{28}
$$

To proceed further, denote $X_k = \mathbb{E}\left\{\bar{x}_k\bar{x}_k^T\right\}$. From (4), it follows directly that

$$
X_{k+1} = \bar{A}_kX_k\bar{A}_k^T + \Pi_1\mathrm{tr}\{X_k\bar{\Omega}_1\} + \bar{D}_kW_k\bar{D}_k^T,
$$

which is (20).

Based on our assumption that $P_{k|k} \leq Q_{k|k}$, it can be easily verified that,

$$
Q_{k+1|k} - P_{k+1|k} = \bar{A}_k(Q_{k|k} - P_{k|k})\bar{A}_k^T \geq 0. \tag{29}
$$

With (28), (29) and the definition of X_k, we have

$$
\begin{aligned}
P_{k+1|k+1} \leq& (1+\varepsilon)(I - \Xi_{k+1}\bar{C}_{k+1})Q_{k+1|k}(I - \Xi_{k+1} \\
&\times \bar{C}_{k+1})^T + (1+\varepsilon^{-1})\sigma\Xi_{k+1}\Xi_{k+1}^T \\
&+ \Xi_{k+1}\Pi_2\mathrm{tr}\left\{X_{k+1}\bar{\Omega}_2\right\}\Xi_{k+1}^T \\
&+ \Xi_{k+1}E_{k+1}V_{k+1}E_{k+1}^T\Xi_{k+1}^T \\
=& Q_{k+1|k+1}.
\end{aligned}
$$

So far, we have proved that $Q_{k|k}$ is an upper bound of $P_{k|k}$.

Next we are going to show that the filter gain given in (21) minimizes the upper bound $Q_{k+1|k+1}$ at each time step. It follows from (22) and (23) that

$$
\begin{aligned}
Q_{k+1|k+1} =&(1+\varepsilon)Q_{k+1|k} + \Xi_{k+1}Y_{k+1}\Xi_{k+1}^T - Z_{k+1}^T\Xi_{k+1}^T \\
&- \Xi_{k+1}Z_{k+1}.
\end{aligned} \tag{30}
$$

Since $Y_{k+1} = Y_{k+1}^T > 0$, completing the square with respect to Ξ_{k+1} in (30) yields that

$$
\begin{aligned}
Q_{k+1|k+1} =&(\Xi_{k+1} - Z_{k+1}^TY_{k+1}^{-1})Y_{k+1}(\Xi_{k+1} - Z_{k+1}^TY_{k+1}^{-1})^T \\
&- Z_{k+1}^TY_{k+1}^{-1}Z_{k+1} + (1+\varepsilon_1)Q_{k+1|k}.
\end{aligned} \tag{31}
$$

Thus, it is obvious that when $\Xi_{k+1} = Z_{k+1}^TY_{k+1}^{-1}$, $Q_{k+1|k+1}$ is minimized and, in such a case,

$$
Q_{k+1|k+1} = -Z_{k+1}^TY_{k+1}^{-1}Z_{k+1} + (1+\varepsilon)Q_{k+1|k}. \tag{32}
$$

That concludes the proof.

The least squares fault and state estimation problem is solved by Theorem 4 for a class of systems subject to event-triggered measurement transmissions and additive stochastic nonlinearities. To deal with the event-triggered measurement transmissions, special effort has been made to calculate an upper bound of the filtering error covariance, which is dependent on the covariances of the stochastic nonlinearities and the threshold in the event generator. By doing so, the traditionally required probability density functions of states and innovations conditional on measurements, which may be complicated or even unavailable for relatively complicated systems, are no longer needed. In other words, we do not need to calculate the posteriori probability density function of states and the exact least squares filtering error covariance, thereby improving the feasibility and robustness of the algorithms at the cost of sacrificing certain accuracy at an acceptable level. It is worth mentioning that, when the measurement is transmitted at each time step (i.e., $\sigma = 0$) and there is no stochastic nonlinearity, the proposed filter can be specialized to the classical Kalman filter. Moreover, the proposed algorithm is suitable for on-line applications, since it is carried out by solving discrete matrix equations.

4. SIMULATION

Consider system (1) with parameters given as follows:

$$
\begin{aligned}
&A_k = \begin{bmatrix} 0.1 & 0 \\ -0.5 & 0.2 \end{bmatrix},\ F_k = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_k = D_k = I, \\
&C_k = \begin{bmatrix} 1 & 0 \end{bmatrix},\ E_k = 1,\ K_k = -0.21I,\ x_0 = [0, 0]^T.
\end{aligned}
$$

Denoting $x_k = \left[x_k^{(1)}, x_k^{(2)}\right]^T$, the stochastic nonlinearities are formulated as:

$$
\begin{aligned}
&h(x_k, \alpha_k) \\
&= \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}\left[0.3\mathrm{sign}(x_k^{(1)})x_k^{(1)}\alpha_k^{(1)} + 0.4x_k^{(2)}\mathrm{sign}(x_k^{(2)})\alpha_k^{(2)}\right], \\
&g(x_k, \beta_k) \\
&=0.3\left[0.3\mathrm{sign}(x_k^{(1)})x_k^{(1)}\beta_k^{(1)} + 0.4x_k^{(2)}\mathrm{sign}(x_k^{(2)})\beta_k^{(2)}\right],
\end{aligned}
$$

where $\alpha_k^{(i)}$ and $\beta_k^{(i)}$ $(i = 1, 2)$ stand for zero-mean uncorrelated Gaussian white noises with unity covariances. Based on the expressions above, it is straightforward to see that:

$$
\mathbb{E}\left\{\begin{bmatrix} h(x_k, \alpha_k) \\ g(x_k, \beta_k) \end{bmatrix}\middle| x_k\right\} = 0,
$$

and

$$
\begin{aligned}
&\mathbb{E}\left\{\begin{bmatrix} h(x_k, \alpha_k) \\ g(x_k, \beta_k) \end{bmatrix}\begin{bmatrix} h(x_j, \alpha_j) \\ g(x_j, \beta_j) \end{bmatrix}^T\middle| x_k\right\} \\
&= \begin{cases} 0, & \text{if } k \neq j, \\ \begin{bmatrix} 0.04 & 0.02 & 0 \\ 0.02 & 0.01 & 0 \\ 0 & 0 & 0.09 \end{bmatrix} x_k^T\begin{bmatrix} 0.09 & 0 \\ 0 & 0.16 \end{bmatrix}x_k, & \text{if } k = j. \end{cases}
\end{aligned}
$$

Other variables are set as: $Q_{0|0} = 20I$, $\sigma = 0.05$, and $\varepsilon = 1$.

The additive fault is set as

$$
f_k = \begin{cases} -1, & \text{if } \quad k \geq 30, \\ 0, & \text{otherwise.} \end{cases}
$$

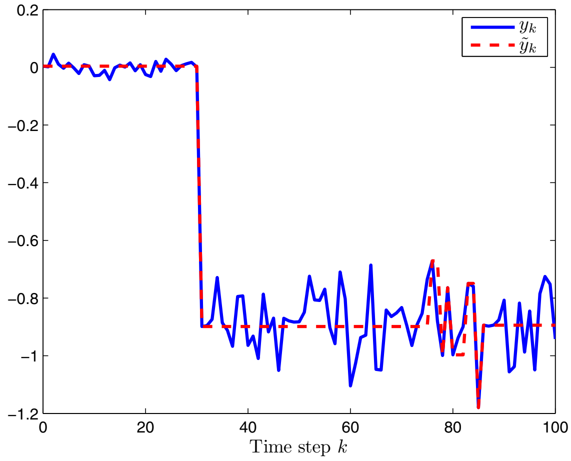

Fig. 1. The actual measurement y_k and transmitted measurement $\tilde{y}_k$

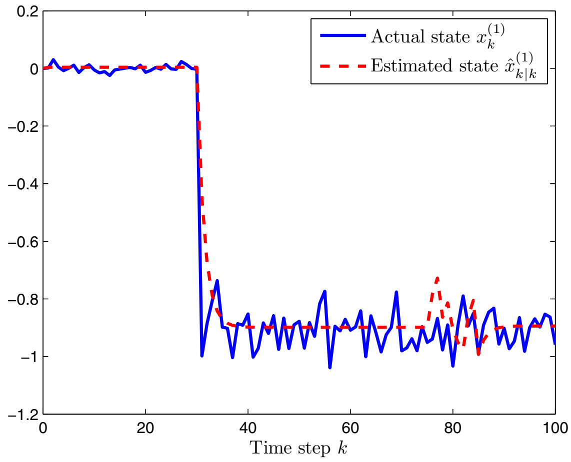

Fig. 2. The state $x_k^{(1)}$ and its estimation

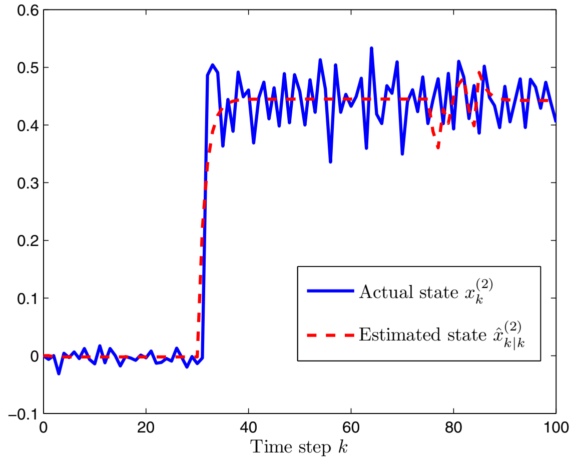

Fig. 3. The state $x_k^{(2)}$ and its estimation

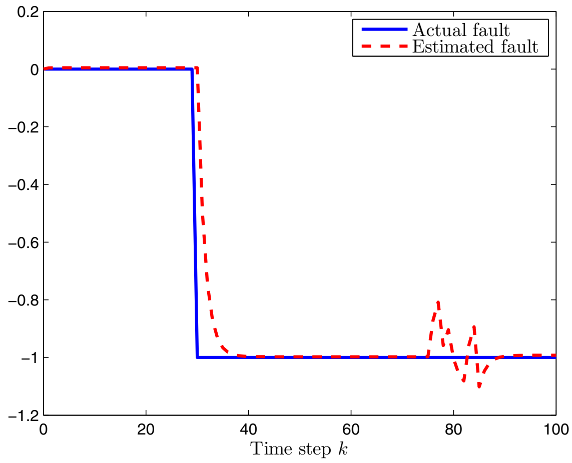

Fig. 4. The fault and its estimation

Fig. 1 depicts the actual measurement and the transmitted measurement. Figs. 2-4 plot the states/fault and their estimations obtained from Theorem 4. It can be seen that, the proposed filter could estimate the states and fault well with event-triggered measurement transmissions and additive stochastic nonlinearities.

5. CONCLUSION

The least squares state and fault estimation problem has been investigated for a class of systems with stochastic nonlinearities and event-triggered measurement transmissions. An event-triggered scheme has been properly proposed where the measurement output is transmitted to a remote estimator only when a specified event condition is violated in an event generator. An appropriate filter gain has been determined to minimize an upper bound of the filtering error covariance with event-based transmissions and additive stochastic nonlinearities at each time step. By solving two sets of discrete matrix equations, the desired filter gain could be calculated recursively, and thus the method is applicable for online computation. Additive faults and system states have been simultaneously estimated in a unified framework. A simulation example has been presented to show the effectiveness of the proposed method.

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