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Filtering and Identification of Stochastic Risk Premium for Electricity Spot Price Models

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Abstract: Starting from the simple model for the spot price which is set as the jump augmented Vasicek model, we construct a factor model of the electricity futures as the stochastic hyperbolic systems with jumps. Representing the main spike phenomena of the electricity spot price from one observed futures data by proxy, the filtering of factor process and the related stochastic risk premium are formulated in a Gaussian frame work. After serving the likelihood functional, the systems parameter estimation problem is solved.

Keywords: Electricity Spot, Risk premium, Hyperbolic system, MLE, Kalman filter, Jump process

1. INTRODUCTION

From the recent deregulation of electricity markets, the electricity is quoted almost as any other commodity. Noting that the power prices present a higher volatility than equity prices, the mathematical model for electricity spot behaviors is required for pricing of electricity-related options, risk management and others. The special feature of electricity is that one can not store electricity but there are many other features which distinguish electricity from other commodities.

In Fig.1, the spot price (a *day-ahead* market) is shown.

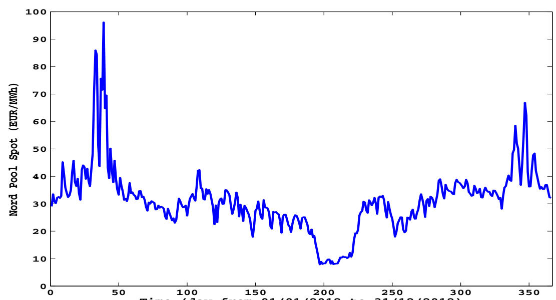

Fig. 1. Nord Pool electricity spot price (day ahead implicit auction market)

From this figure, we observe the special behaviors of electricity spot,i.e., many spikes frequently and seasonal effect. Instead of modeling this process from the fundamentals of supply and demand, the simple mathematical model for this spot price is proposed and leads to calibrate the model parameters and price the options by using the system theoretical approach. Along this line Schwartz and Smith Schwartz and Smith [2000] proposed a two-factor diffusion model and the system parameters are estimated from M.L.E. (Maximum likelihood estimate) by using Kalman filter. In spite of the mathematically elegant derivation of the futures prices, which are the observation data, one need to add *ad hoc* observation noise in order to derive the Kalman filter. This assumption has been made by numerous authors, either in the commodity or interest rate markets, see Elliott and Hyndman [2007]. The additional noise in the observation has been interpreted to take into account bid-ask spread, price limits, non-simultaneity of the observations, or errors in the data. The argument is clearly forced and unconvincing. By using the idea proposed by Aihara and Bagchi [2010], we approach the modeling differently. In our setup, on the one hand, the added measurement noise is built in the model. On the other hand, the modeling of the correlation structure between the futures (observation) is a natural component of our formulation. Hence the model parameters can be calibrated through the derived likelihood functional without any *ad hoc* observation noise. See Aihara et al. [2009] for the detailed identification procedure.

However in these works, the important spikes in the electricity prices are not included, because including jumps [1] means giving up on the closed-form estimator like Kalman filter. In this paper, we do not use the filtering theory for jump-diffusion processes by using martingale theory in van Schuppen [1977]. Fortunately, for the term structure model in the electricity problem, we can represent the jump

[1] The closed-form formulae for forwards and options are possible even for the jump-diffusion and Levy processes in Benth et al. [2008].

process for the spike phenomena by using one observation component and transform the Non-Gaussian estimation problem into the Gaussian frame work with correlated noises.

In this paper,we choose the spot price dynamics as the jump-diffusion model proposed in Duffie et al. [2000]. According to the idea in Aihara and Bagchi [2010], we construct the arbitrary free model of the term structure including jump-diffusion processes in Sec. 2. In the electricity market, the averaged-type forward and futures contracts are observed and are used as the observation data for calibrating system parameters. After presenting this observation dynamics in Sec. 3, we derive the closed form filter for estimating the whole term structure and obtain the likelihood functional. To derive this filter, we choose one component of observation as the proxy for the spike process of the spot price, say $y_o(t)$. Reconstructing the spikes from $y_o(t)$ and plugging this into observation and system equations, the filtering problem with jumps is converted to the Gaussian frame work in Sec. 4. For solving the filtering problem, we need to work under the real world measure. Hence a suggested in Carmona and Ludkovski [2004], the stochastic market price of risk is introduced as the linear Ito equation. This implies that our extended state including the stochastic market price of risk is still Gaussian, and the filtering algorithm still works with the explicit form of likelihood functional. In the final section, some numerical examples are demonstrated.

2. SPOT RATE MODEL WITH JUMP

We consider the short rate dynamics of the jump augmented Vasicek model;

$$dr(t) = \kappa(\bar{r} - r(t))dt + \sigma_r dw_r(t) + \int_R \nu p(d\nu, dt), \quad (1)$$

where w_r is a standard BMP which is independent of the Poisson random measure p and the compensated Poisson measure q_c is is given by

$$q_c(d\nu, dt) = p(d\nu, dt) - (\lambda^+ \psi^P(d\nu) + \lambda^- \psi^M(d\nu))dt \quad (2)$$

and where λ^+ (λ^-) denotes the positive jump (negative jump) time intensity and ψ^P (ψ^M) is a distribution of the positive (negative) jump size.

Remark: It is possible to represent the random Poisson measure p as the compound Poisson processes:

$$\int_R \nu p(d\nu, dt) = J^P(t;\psi^P)dN(t;\lambda^+) + J^M(t;\psi^M)dN(t;\lambda^-),$$

where $J^{\cdot}(t;\psi^{\cdot})$ denotes the jump size with identically distributed law $\psi^{\cdot}$ and $N(t,\lambda^{\cdot})$ is a counting process with parameter $\lambda^{\cdot}$.

Now by using above model, the spot price shown in Fig.1 is given by

$$S(t) = \exp(r(t) + se(t)), \quad (3)$$

where $se(t)$ denotes the seasonality function. This function is determined *a-priori* from the historical data by using FFT as used in Aihara et al. [2009], Imreizeeq [2011]. For the data shown in Fig.1, the seasonality function is given by

$$\begin{aligned} se(t) &= 0.1526\cos(2\pi 5.5 \times 10^{-3}t + 2.12) \\ &+0.1641\cos(2\pi 8.2 \times 10^{-3}t - 1.18) \\ &+0.1739\cos(2\pi 16.4 \times 10^{-3}t + 1.36). \end{aligned}$$

The obtained shape is shown in Fig. 2

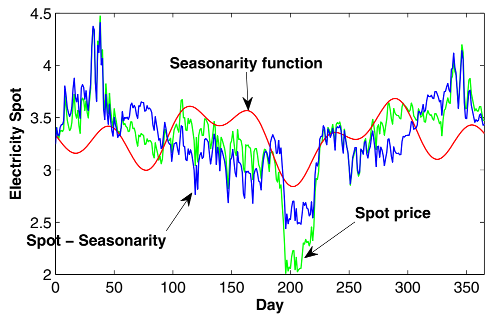

Fig. 2. Nord Pool electricity spot log price with seasonality function

In Table 1, the identified frequencies (cycle/day) by using FFT are summarized with corresponding periods.

Table 1. The largest frequencies and the corresponding period of Nord Pool spot log data by using FFT

Frequency (cycle/day)	Period (in day)
5.5 $\times 10^{-3}$	183 (days)
8.2 $\times 10^{-3}$	122 (days)
16.4 $\times 10^{-3}$	61 (days)

3. ELECTRICITY MODELL

By a basic no-arbitrage argument it follows that the price of a futures contract $F(t, T-t)$ which has payoff $S(T)$ at future time T equals to

$$F(t, T-t) = E\{S(T)|\mathcal{F}_t\},$$

with respect to the risk neutral measure. Hence we can write the futures price as

$$F(t, T-t) = \exp\{A(t, T-t) + B(t, T-t)r(t)\}, \quad (4)$$

where A and B satisfy deterministic equations. (See in Duffie et al. [2000] for details.) Although this model is mathematically elegant, it is not consistent with the *forward curve* as stated in Carmona and Ludkovski [2004]. From the systems identification view points, the observation futures data is added to the artificial observation noises. In order to avoid this ambiguity, we add the extra noise in (4) as used in Aihara and Bagchi [2010]. This noise represents the model errors from the basic property of $r(t)$. This will mean that the corresponding futures price should be given by a slight perturbation of (4),i.e.,

$$\begin{aligned} F(t, T-t) = \exp\{&A(t, T-t) + B(t, T-t)r(t) \\ &+ \int_0^t \sigma dw(s, T-s)\}, \end{aligned} \quad (5)$$

where we use the same symbols in (4) and

$$\int_0^t \sigma dw(s, T-s) = \sum_{k=1}^{\infty} \int_0^t \sigma \frac{1}{\lambda_k} e_k(T-s) d\beta_k(s), \quad (6)$$

and where $e_k(\cdot)$ is a sequence of differentiable functions forming an orthonormal basis in $L^2(0,T^*)$ [2] and $\{\beta_k(t)\}$ are mutually independent Brownian motion processes with $\sum \frac{1}{\lambda_k^2} < \infty$,i.e.,

$$\sigma^2 E\{w(t,x_1)w(t,x_2)\} = tq(x_1,x_2)$$

and

$$\int_0^{T^*} q(x,x)dx = \sum_{k=1}^{\infty} \frac{\sigma^2}{\lambda_k^2} < \infty.$$

We set f as

$$f(t,x) = A(t,x) + B(x)r(t) + \int_0^t \sigma dw(s, x+t-s), \quad (7)$$

the futures contracts $F(t, T-t)$ becomes

$$F(t, T-t) = \exp(f(t, T-t)) \quad (8)$$

with $F(T,0) = \exp(f(T,0)) = S(T)$. Now we derive the explicit forms of A and B so that $F(t, T-t)$ is a $\mathcal{F}_t$ martingale in the risk neutral measure. (see Appendix A.) Applying the results by Aihara and Bagchi [2010], we get

Theorem 1. The explicit form of (7) is a solution of

$$\begin{aligned} df(t,x) = &\frac{\partial f(t,x)}{\partial x}dt - \tilde{q}_J(x)dt + e^{-\kappa x}\{\sigma_r dw_r(t) \\ &+ \int_R \nu q_c(d\nu, dt)\} + \sigma dw(t,x) \end{aligned} \quad (9)$$

$$\begin{aligned} f(0,x) = &\bar{r}(1-e^{-\kappa x}) + \frac{\sigma_r^2}{2\kappa}(1-e^{-2\kappa x}) + \frac{1}{2}\int_0^x q(z,z)dz \\ &+se(x) + \int_0^x (\lambda^+ C^P(z) + \lambda^- C^M(z))dz + e^{-\kappa x} r(0), \end{aligned} \quad (10)$$

where

$$\tilde{q}_J(x) = \sigma_r^2 e^{-2\kappa x} + \frac{1}{2}q(x,x) + (\lambda^+ C^P(x) + \lambda^- C^M(x))$$

and

$$C^{\bullet}(x) = \int_R \exp(e^{-\kappa x}\nu)\psi^{\bullet}(d\nu) - 1. \quad (11)$$

4. REAL WORLD DYNAMICS

For applying the filtering for f to the parameter estimation problem, we work in the real world measure. For example, we add a simple risk premium term to (9) . Usually we need to use the measure transformation technique to convert the BMPs and Poisson terms to include the risk coefficients as used in Benth et al. [2008]. In this paper, we simplify the situation that the market price of risk $\Lambda_w(t)$ comes mainly from $w_r(t)$ but this moves stochastically. We set this term as

$$d\Lambda_w(t) = \kappa_\lambda(\bar{\Lambda} - \Lambda_\omega(t))dt + \sigma_\Lambda dw_2(t), \quad (12)$$

where the BMP w_2 is independent of w_r . Now under the real world measure the BMP $\tilde{w}_r(t)$ is represented by

$$w_r(t) = \tilde{w}_r(t) - \int_0^t \Lambda_w(s)ds.$$

[2] T^* denotes the longest future time in mind

Hence our system state $[f(t,x)\ \Lambda_w(t)]$ under the physical measure becomes

$$\begin{cases} df(t,x) = \dfrac{\partial f(t,x)}{\partial x}dt - \tilde{q}_J(x)dt + e^{-\kappa x}\{\sigma_r(-\Lambda_w(t)dt \\ \quad +d\tilde{w}_r(t)) + \displaystyle\int_R \nu q_c(d\nu, dt)\} + \sigma dw(t,x) \\ d\Lambda_w(t) = \kappa_\lambda(\bar{\Lambda} - \Lambda_w(t))dt + \sigma_\Lambda dw_2(t). \end{cases} \quad (13)$$

5. OBSERVATION

Noting that electricity is essentially not storable, the futures contracts are based on the arithmetic averages of the spot prices over a delivery period $[T_0, T]$, given by

$$\frac{1}{T-T_0}\int_{T_0}^T S(\tau)d\tau.$$

Now, for $t < T_0$ we can calculate the futures prices by

$$\begin{aligned} F(t,T_0,T) &= E\{\frac{1}{T-T_0}\int_{T_0}^T S(\tau)d\tau | S(t)\} \\ &= \frac{1}{T-T_0}\int_{T_0}^T F(\tau,t)d\tau \\ &= \frac{1}{T-T_0}\int_{T_0-t}^{T-t} \exp[f(t,x)]dx. \end{aligned} \quad (14)$$

The arithmetically averaged process is not lognormal and in practice we adopt the geometric average as an approximation;

$$F(t,T_0,T) \sim \exp[\frac{1}{T-T_0}\int_{T_0-t}^{T-t} f(t,x)dx]. \quad (15)$$

By using this geometric approximation, the observation data for the futures price is set as

$$y_i(t) = \frac{1}{T-T_0}\int_{\tau_i}^{\tau_i+(T-T_0)} f(t,x)dx,\ \tau_1 < \tau_2 < \cdots < \tau_m. \quad (16)$$

We define

$$H(\cdot) = \frac{1}{T-T_0}[\int_0^{\tau_1+(T-T_0)}(\cdot)dx, \cdots, \int_0^{\tau_m+(T-T_0)}(\cdot)dx]'$$

and

$$H_\delta(\cdot) = [\frac{1}{T-T_0}\int_G (\delta(\eta-(T-T_0+\tau_i)) - \delta(\eta-\tau_i))(\cdot)d\eta]_{m\times 1}.$$

Denoting

$$\boldsymbol{Y}(t) = [y_i(t)]_{m\times 1},$$

we have

$$\begin{aligned} d\boldsymbol{Y}(t) = &H_\delta f(t,\cdot)dt - H(\tilde{q}_J + B(x)\sigma_r\Lambda_w(t))dt \\ &+H[dw_M(t,\cdot)] + H[B\int_R \nu q_c(d\nu, dt)], \end{aligned} \quad (17)$$

where $B(x) = e^{-\kappa x}$ and

$$w_M(t,x) = B(x)\sigma_r w_r(t) + \sigma w(t,x). \quad (18)$$

5.1 Reconstruction of jump process

We choose one yield data for $\tau_0 < \tau_1$, i.e,

$$y_0(t) = \frac{1}{T-T_0}\int_{\tau_0}^{\tau_0+(T-T_0)} f(t,x)dx. \tag{19}$$

By using the following notations,

$$H^0(\cdot) = \frac{1}{T-T_0}\int_0^{\tau_0+(T-T_0)}(\cdot)dx,$$

and

$$H^0_\delta(\cdot) = \frac{1}{T-T_0}\int_G(\delta(\eta-(T-T_0+\tau_0))-\delta(\eta-\tau_0))(\cdot)d\eta,$$

we have

$$\begin{aligned} dy_0(t) &= H^0_\delta f(t,\cdot)dt - H^0(\tilde{q}_J + B(x)\sigma_r\Lambda_w(t))dt \\ &+H^0[dw_M(t,\cdot)] + H^0[B\int_R \nu q_c(d\nu,dt)]. \end{aligned}$$

Hence it is possible to reconstruct the jump process from $y_0(t)$ such that

$$\begin{aligned} \int_0^t\int_R \nu q_c(d\nu,ds) &= \frac{1}{B^0}(y_0(t) - \int_0^t H^0_\delta f ds - H^0 w_M(t,x)) \\ &+\int_0^t \frac{1}{B^0}H^0(\tilde{q}_J + B(x)\sigma_r\Lambda_w(s))ds, \end{aligned} \tag{20}$$

where $B^0 = H^0 B$. Plugging (20) into (13), we have

$$\begin{aligned} df(t,x) &= \frac{\partial f(t,x)}{\partial x}dt - (\tilde{q}_J(x) + B(x)\sigma_r\Lambda_w(t))dt \\ &+dw_M(t,x) + \frac{B(x)}{B^0}\{dy_0(t) - H^0_\delta f dt \\ &+H^0(\tilde{q}_J + B(x)\sigma_r\Lambda_w(t))dt - H^0 dw_M(t,x)\} \quad . \end{aligned} \tag{21}$$

Now for more easy treatment, noting that $w_M - \frac{B}{B^0}H^0 w_M = (1-\frac{B}{B^0}H^0)\sigma w$, we transform the above equation as the robust form for jump term. Define

$$\tilde{f}(t,x) = f(t,x) - \frac{B(x)}{B^0}y_0(t). \tag{22}$$

Hence we get

$$\begin{aligned} d\tilde{f}(t,x) &= (\frac{\partial}{\partial x} - \mathcal{C}_\delta)(\tilde{f}(t,x) + \frac{B(x)}{B^0}y_0(t))dt - (1-\mathcal{C}_0) \\ &\times(\tilde{q}_J(x) + B(x)\sigma_r\Lambda_w(t))dt + (1-\mathcal{C}_0)\sigma dw(t,x), \end{aligned} \tag{23}$$

where

$$\mathcal{C}_\delta = \frac{B(x)}{B^0}H^0_\delta \tag{24}$$

$$\mathcal{C}_0 = \frac{B(x)}{B^0}H^0. \tag{25}$$

5.2 Reconstruction of observed yields

Denoting

$$H^j(\cdot) = \frac{1}{(T-T_0)}\int_{\tau_i}^{\tau_i+(T-T_0)}(\cdot)dx,$$

the original yield $y_j(t)$ becomes

$$\begin{aligned} y_j(t) &= H^j f(t,\cdot) \\ &= H^j\tilde{f}(t,\cdot) + \frac{H^j B}{B^0}y_0(t). \end{aligned} \tag{26}$$

Now we construct the new observation such that

$$\begin{aligned} \tilde{y}_j(t) &= y_j(t) - \frac{H^j B}{B^0}y_0(t), \\ &= H^j\tilde{f}(t,\cdot). \end{aligned} \tag{27}$$

Denoting

$$\tilde{\boldsymbol{Y}}(t) = [\tilde{y}_j(t)]_{m\times 1},$$

and from $(H - H\mathcal{C}_0)B\sigma_r\Lambda_w = HB\sigma_r\Lambda_w - HB\sigma_r\Lambda_w = 0$, we get

$$\begin{aligned} d\tilde{\boldsymbol{Y}}(t) &= (H_\delta - H\mathcal{C}_\delta)\tilde{f}(t,\cdot)dt + (H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0})y_0(t)dt \\ &-(H - H\mathcal{C}_0)\tilde{q}_J dt + (H - H\mathcal{C}_0)\sigma dw(t,x). \end{aligned} \tag{28}$$

6. THE KALMAN FILTER

In (9), Poisson jump processes are included and this is not a usual Kalman filter problem. There are many articles for Non-Gaussian filtering problem by using a martingale approach, e.g. van Schuppen [1977] and however it is still difficult to derive the closed form filtering algorithm. In our situation, the transformed system (23) with the observation (28) do not include jump processes explicitly. Hence our estimation problem is in the Gaussian frame work;

$$\begin{aligned} d\begin{pmatrix} \tilde{f}(t,x) \\ \Lambda_w(t) \end{pmatrix} &= \begin{pmatrix} (\frac{\partial}{\partial x} - \mathcal{C}_\delta) & -(1-\mathcal{C}_0) \\ 0 & -\kappa_\lambda \end{pmatrix}\begin{pmatrix} \tilde{f}(t,x) \\ \Lambda_w(t) \end{pmatrix}dt \\ &+\begin{pmatrix} (\frac{\partial}{\partial x} - \mathcal{C}_\delta)\frac{B}{B^0}y_0 - (1-\mathcal{C}_0)\tilde{q}_J \\ \kappa_\lambda\bar{\Lambda} \end{pmatrix}dt + d\begin{pmatrix} (1-\mathcal{C}_0)w(t,x) \\ w_\Lambda(t) \end{pmatrix} \end{aligned} \tag{29}$$

with

$$\begin{aligned} d\tilde{\boldsymbol{Y}}(t) &= (H_\delta - H\mathcal{C}_\delta\ ,0)\begin{pmatrix} \tilde{f}(t,x) \\ \Lambda_w(t) \end{pmatrix}dt \\ &+((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) - (H - H\mathcal{C}_0)\tilde{q}_J)dt \\ &+(H - H\mathcal{C}_0)dw(t,x). \end{aligned} \tag{30}$$

Denoting

$$\hat{\cdot} = E\{\cdot|\mathcal{Y}_t\},$$

for $\mathcal{Y}_t = \sigma\{\tilde{\boldsymbol{Y}}(s), y_0(s); 0 \le s \le t\}$, we have

$$\begin{aligned} d\hat{\tilde{f}}(t,x) &= (\frac{\partial}{\partial x} - \mathcal{C}_\delta)(\hat{\tilde{f}}(t,x) + \frac{B(x)}{B_0}y_0(t))dt \\ &-(1-\mathcal{C}_0)(B(x)\hat{\Lambda}_w(t) + \tilde{q}_J(x))dt \\ &+\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - H\mathcal{C}_\delta)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}\mathbf{\Phi}^{-1} \\ &\times\left\{d\tilde{\boldsymbol{Y}}(t) - (H_\delta - H\mathcal{C}_\delta\ ,0)\begin{pmatrix} \hat{\tilde{f}}(t,x) \\ \hat{\Lambda}_w(t) \end{pmatrix}dt\right. \\ &\left.-((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) - (H - H\mathcal{C}_0)\tilde{q}_J dt\right\}, \end{aligned} \tag{31}$$

and

$$\begin{aligned} d\hat{\Lambda}_w(t) &= \kappa_\lambda(\bar{\Lambda} - \hat{\Lambda}_w(t)dt \\ &+\tilde{\mathbf{P}}_{\Lambda f}(t)(H_\delta - H\mathcal{C}_\delta)^*\mathbf{\Phi}^{-1} \\ &\times\left\{d\tilde{\boldsymbol{Y}}(t) - (H_\delta - H\mathcal{C}_\delta\ ,0)\begin{pmatrix} \hat{\tilde{f}}(t,x) \\ \hat{\Lambda}_w(t) \end{pmatrix}dt\right. \\ &\left.-((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) + (H - H\mathcal{C}_0)\tilde{q}_J dt\right\}, \end{aligned} \tag{32}$$

where $Q = \int q(x,z)(\cdot)dz$,

$$\mathbf{\Phi} = (H - H\mathcal{C}_0)((H - H\mathcal{C}_0)Q)^*, \tag{33}$$

and

$$\frac{\partial \tilde{\mathbf{P}}_{ff}(t)}{\partial t} = (\frac{\partial}{\partial x} - \mathcal{C}_\delta)\tilde{\mathbf{P}}_{ff}(t) + \tilde{\mathbf{P}}_{ff}(t)(\frac{\partial}{\partial x} - \mathcal{C}_\delta)^* + (1-\mathcal{C}_0)Q(1-\mathcal{C}_0)^*$$
$$-\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}\mathbf{\Phi}^{-1}$$
$$\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}^*, \tag{34}$$

$$\frac{\partial \tilde{\mathbf{P}}_{f\Lambda}(t)}{\partial t} = (\frac{\partial}{\partial x} - \mathcal{C}_\delta)\tilde{\mathbf{P}}_{f\Lambda}(t) - \tilde{\mathbf{P}}_{f\Lambda}(t)\kappa_\lambda$$
$$-\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}\mathbf{\Phi}^{-1}$$
$$\left\{\tilde{\mathbf{P}}_{\Lambda f}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^*\right\}^*. \tag{35}$$

6.1 *Original form of the Kalman filter for $f(t,x)$*

Noting that

$$\hat{\hat{f}}(t,x) = \hat{f}(t,x) - \frac{B(x)}{B^0}y_0(t),$$

we get

$$d\hat{f}(t,x) = (\frac{\partial}{\partial x} - \frac{B(x)}{B^0}H_\delta^0)\hat{f}(t,x)dt$$
$$-(1 - \frac{B(x)}{B^0}H^0)(B(x)\hat{\Lambda}_w(t)$$
$$+\tilde{q}_J(x))dt + \frac{B(x)}{B^0}dy_0(t) + K(t)d\ell(t) \tag{36}$$

where the innovation process $\ell(t)$ is given by

$$d\ell(t) = d\tilde{\mathbf{Y}}(t) - (H_\delta - H\mathcal{C}_\delta)\hat{f}(t,\cdot)dt - H(1-\mathcal{C}_0)\tilde{q}_J dt,$$

and

$$K(t) = \left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - H\mathcal{C}_\delta)^* + (1-\mathcal{C}_0)QH(1-\mathcal{C}_0)^*\right\}\mathbf{\Phi}^{-1}.$$

7. LIKELIHOOD FUNCTIONAL

Our objective now is to estimate the unknown system parameters. Our first difficulty is the covariance kernel $q(x,y)$ and seasonality function $se(t)$. If we can parameterize it with one or more parameter(s), say c, then we can estimate $q(x,y)$ from Aihara and Bagchi [2010] in advance. It is also possible to identify $se(t)$ as explained in Section2. Now we specify the randomness of the jump processes.The distribution of the jump intensity $\psi^\bullet(d\nu)$ is Gaussian with the mean $m_J^\bullet$ and the covariance $\sigma_J^\bullet$. Hence we have

$$C^\bullet(x) = \int_R \exp(e^{-\kappa x}\nu)\psi^\bullet(d\nu) - 1$$
$$= e^{e^{-\kappa x m_J^\bullet} + \frac{(\sigma_J^\bullet)^2}{2}e^{-2\kappa x}} - 1$$

Hence the parameters we need to estimate are $\kappa, \sigma_r, \lambda^+, \lambda^-$, $m_J^P, m_J^M, \sigma_J^P, \sigma_J^M$, $\sigma, \kappa_\lambda, \bar{\Lambda}, \sigma_\Lambda$ and $\bar{r}$. The standard approach is to use the method of maximum likelihood, for which we need to calculate the likelihood functional from the observation data $\{\tilde{\mathbf{Y}}(t); 0 \le t \le t_f\}$, where t_f denotes a final time. The likelihood functional for our problem is

$$L(t_f, Y) = \int_0^{t_f} \mathbf{\Phi}^{-1}(s)(\mathcal{H}\begin{bmatrix}\hat{\hat{f}}(s,x)\\ \hat{\Lambda}_w(s)\end{bmatrix} + \hat{G})^* d\tilde{\mathbf{Y}}(s)$$
$$-\frac{1}{2}\int_0^{t_f} ||\mathbf{\Phi}^{-1/2}(s)(\mathcal{H}\begin{bmatrix}\hat{\hat{f}}(s,x)\\ \hat{\Lambda}_w(s)\end{bmatrix} + \hat{G})||^2 ds, \tag{37}$$

where $\hat{\hat{f}}(s,x)$ and $\hat{\Lambda}_w(s)$ are the "best" estimates of the states $\hat{f}(s,x)$ and $\Lambda_w(s)$ given by the observation data $\sigma\{\tilde{\mathbf{Y}}(\tau); 0 \le \tau \le s\}$, $\mathcal{H} = [H_\delta - H\mathcal{C}_0, 0]$, and $\hat{G} = ((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) + (H - H\mathcal{C}_0)\tilde{q}_J$. The MLE of the unknown parameter $\theta = [\kappa \;\; \sigma_r \;\; \lambda^+ \;\; \lambda^- \;\; m_J^P \;\; m_J^M \;\; \sigma_J^P \;\; \sigma \;\; \kappa_\lambda \;\; \bar{\Lambda} \;\; \sigma_\Lambda \;\; \bar{r}]$ is then given by

$$\hat{\theta} = \text{argmax } L(t_f, Y). \tag{38}$$

8. SIMULATION STUDIES

8.1 *Filtering*

First we check the feasibility of our developed filtering algorithm. The system parameters are set as in Table-2.

Table 2. Systems parameters

κ	$\bar{r}$	σ_r	σ	m_J^P	λ^+,	σ_J^P
5.00	3.00	0.80	0.001	0.50	8.00	0.20

m_J^M	λ^-	σ_J^M	κ_λ	$\bar{\Lambda}$	σ_Λ
-0.50	8.00	0.20	4.00	0.30	3.00

Setting the seasonality function is set as shown in Table -1, we simulate (13) by using the finite difference method with $dx = 0.01, dt = 0.005$. For details, see Imreizeeq [2011].

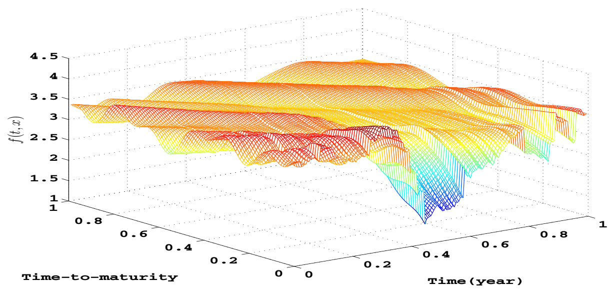

Fig. 3. $f(t,x)$- process

We also generate the observation data $\boldsymbol{Y}(t) = [10y_i(t)]_i$ for $i = 1, 2, \cdots, 7$ with $\tau_1 = 0, \tau_2 = 0.1, \cdots, \tau_7 = 0.6$ shown in Fig.4.

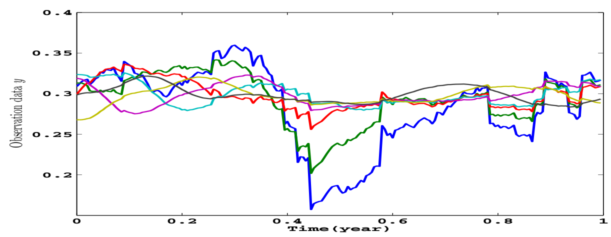

Fig. 4. Observed data $\boldsymbol{Y}$

The filtering for the stochastic market price of risk is demonstrated in Fig. 5.

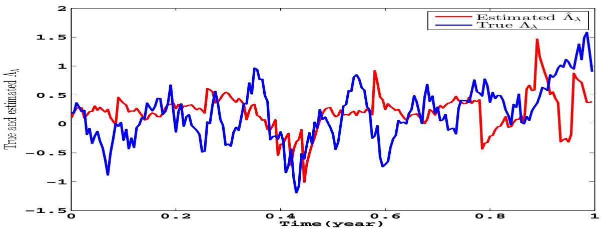

Fig. 5. True and estimated $\Lambda_w(t)$

The estimated $\hat{f}(t,x)$ is also shonw in Fig.6.

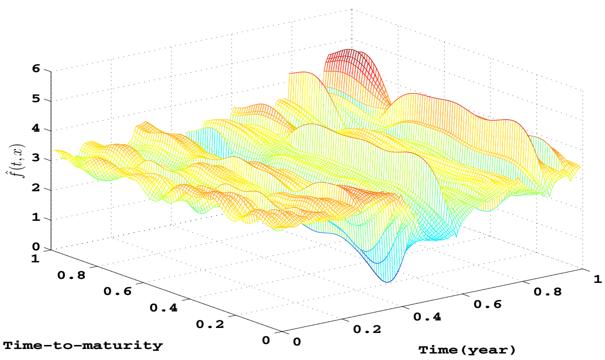

Fig. 6. Estimated $\hat{f}(t,x)$

At each $x = 0, 0.1$ points, we show the true and estimated f from Fig.7,and 8.

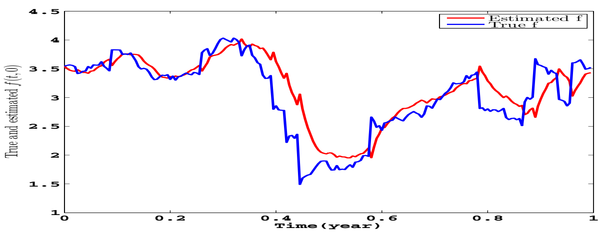

Fig. 7. True $f(t,0)$ and estimated $\hat{f}(t,0)$

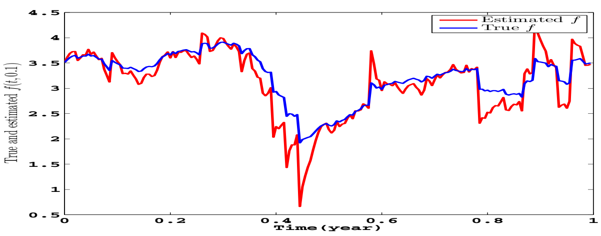

Fig. 8. True $f(t,0.1)$ and estimated $\hat{f}(t,0.1)$

8.2 MLE

By using the Generic Algorithm toolbox in MATLAB, the MLE estimates for unknown parameters listed in Table 2 are performed. The results are shown in Table-3 and Fig. 9.

Table 3. MLE of systems parameters

$\hat{\kappa}$	$\hat{\bar{r}}$	$\hat{\sigma}_r$	$\hat{\sigma}$	$\hat{m}_J^P$	$\hat{\lambda}^+$	$\hat{\sigma}_J^P$
1.5000	1.8339	0.9971	0.0008	0.7338	3.4000	0.4126

$\hat{m}_J^M$	$\hat{\lambda}^-$	$\hat{\sigma}_J^M$	$\hat{\kappa}_\lambda$	$\hat{\bar{\Lambda}}$	$\hat{\sigma}_\lambda$
-0.9722	3.5784	0.6000	8.7175	0.1638	0.9064

The MLE results have still some estimated errors in only one year data. To support consistency we need for more long range historical data.

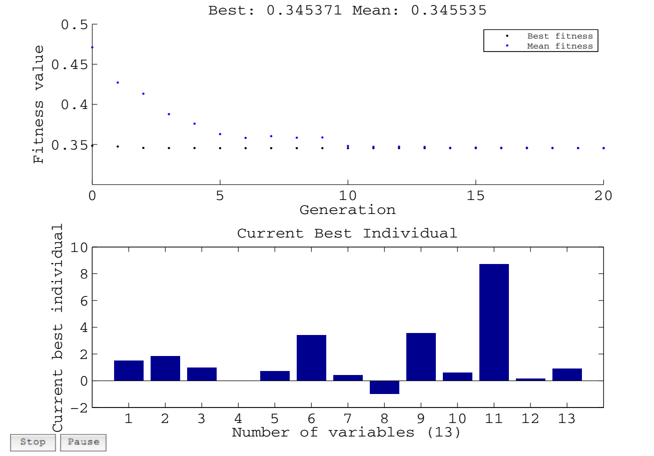

Fig. 9. GA output using MATLAB GA toolbox

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