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A clutch based transmission for mechanical flywheel applications ⋆

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Abstract: Continuously variable transmissions play an important role in mechanical kinetic energy recovery systems. In this work, a clutch based continuously variable transmission comprising two stages of three clutches is considered. A detailed mathematical model is derived and a control strategy for the load torque is given. The model covers losses in bearings, clutches and fixed transmissions as well as flywheel losses due to gas friction.

Keywords: continuously variable transmission; modeling; simulation; automotive control; torque control

1. INTRODUCTION

Flywheel based mechanical hybrid systems are known in literature for many years. They consist of an internal combustion engine, a flywheel storage system and a variable transmission that adapts the corresponding speed levels. The flywheel technology has been applied in motorsport for many years and is now adapted for passenger cars, see Douglas and Brockbank [2009]. Remarkable effort has been dedicated to exploit the fuel saving potential for different powertrain topologies, see e.g. Brockbank and Body [2010].

Plenty of papers exist dealing with the fuel optimal control of such systems, e.g. Pfiffner et al. [2003]. Typically, the model complexity is reduced to a minimum, in order to enable the development of energy management strategies (Serrarens et al. [1998], van Berkel et al. [2012]). Commonly, simplified clutch representations (on/off) are used. Additionally, the control of flywheel hybrid powertrains with continuously variable transmissions (CVT) is investigated extensively (Shen and Veldpaus [2004]).

Usually, metal belt and chain CVTs (Srivastava and Haque [2009]) are used for such powertrains. Alternatively, there are toroidal (Nakano et al. [2000]) and spherical (Kim et al. [2002]) CVTs as well as electric variable transmissions (Hoeijmakers and Ferreira [2006]) available.

In this work, a clutch based continuously variable transmission (C-CVT) is presented. It consists of 6 clutches as shown in Fig. 1. A first stage (clutches 1, 2, 3) is coupled with a flywheel, a second stage (clutches 4, 5, 6) is linked to a load.

The clutches are coupled via different fixed transmissions. As a consequence, 9 fixed transmission ratios are possible, if one clutch from the first stage and one from the second stage are used simultaneously. This 9 ratios split the possible range between the vehicle speed and flywheel speed into 10 segments, as shown in Fig. 2.

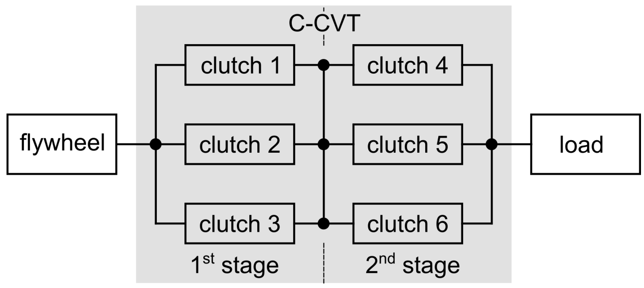

Fig. 1. Overall structure with 6 clutches

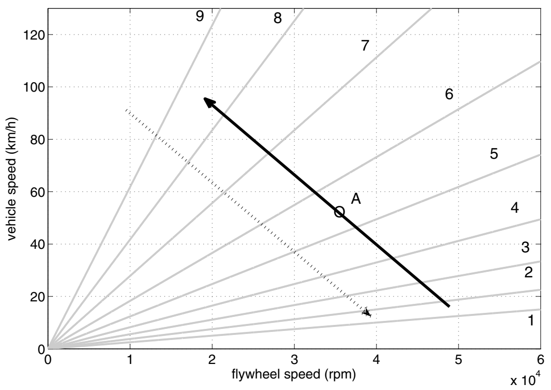

Fig. 2. Functional principle with 9 gears - unload flywheel (solid), load flywheel (dashed)

Two modes are possible during dynamic operation of the C-CVT. Energy can be transfered from the flywheel to the load (mode 'unload flywheel', solid line in Fig. 2) as well as

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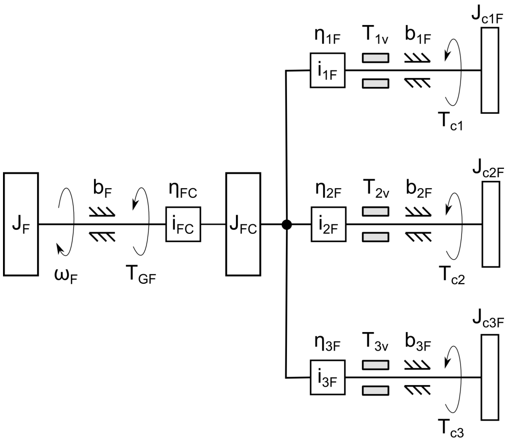

Fig. 3. Structure of the C-CVT model (flywheel to first clutch stage)

in the opposite direction, i.e. from load to flywheel (mode 'load flywheel', dashed line). Therefore, at least one clutch has to slip, e.g. clutch 1 is closed and clutch 5 slips.

For example starting at point A in Fig. 2, the vehicle is intended to accelerate. Energy is withdrawn from the flywheel by actuating the clutch combination that represents gear 6. This combination realizes minimal speed differences at the actuated clutches and therefore minimal clutch losses. The clutch combination has to be switched to gear 7 before line 6 is reached, in order to maintain the acceleration. Following this procedure, an acceleration from vehicle standstill up to line 9 is possible, see Fig. 2. A reversed order of clutch actuation results in a reduction of the vehicle speed to line 1 (mode 'load flywheel').

An advantage of the C-CVT is its relatively simple setup that uses well known components as e.g. gears and clutches. Additionally, the structure allows a good scalability in terms of the transmitted torque and power. The number of clutches is a compromise between the desired C-CVT efficiency and the realization respectively control effort.

In this paper, a detailed mathematical model of a C-CVT is derived in Section 2. Based on this model, a basic control strategy is developed in Section 3 and tested on different scenarios of operation. The simulation results are presented in Section 4. The main goal is to provide a detailed simulation model that can be used for the C-CVT design.

2. MATHEMATICAL MODEL

The overall structure shown in Fig. 1 is split into three parts, which can be described by means of one differential equation each. Only positive speeds are considered.

2.1 Flywheel to first clutch stage

Fig. 3 shows the first part comprising the flywheel and the corresponding side of the first clutch stage. The moments of inertia of the flywheel resp. of the transmissions between flywheel and first clutch stage are symbolized by J_F and J_{FC}. The moments of inertia of the flywheel side of the parallel clutches 1, 2 and 3 (including the clutch discs) are denoted by J_{c1F}, J_{c2F} and J_{c3F}.

Additionally, there are fixed transmissions for a first reduction with ratio i_{FC} (efficiency η_{FC}) and for all three parallel clutch branches (ratios i_{1F}, i_{2F}, i_{3F} and efficiencies η_{1F}, η_{2F}, η_{3F}).

All fixed transmissions considered in the C-CVT are modeled as follows. The ratio is defined by the quotient of input and output speed of the transmission, i.e.

$$i = \frac{\omega_1}{\omega_2} \; . \tag{1}$$

Using the input torque T_1 and output torque T_2, the input power P_1 and output power P_2 are calculated. If P_1 is greater (or equal) than P_2, the transmission is operated in positive power direction. A constant efficiency η is assumed for the positive power direction, an efficiency of

$$\bar{\eta} = \frac{1}{\eta} \tag{2}$$

for the negative power direction. Then, the resulting torques are given by

$$T_2 = T_1 \, i \, \eta \qquad \text{resp.} \qquad T_1 = \bar{\eta} \, \frac{T_2}{i} \; . \tag{3}$$

Furthermore, diverse torque losses act on the system as shown in Fig. 3. Losses T_{GF} due to gas friction within the flywheel housing are taken into account. Based on relations of the boundary layer theory of fluid mechanics, see e.g. Schlichting and Gersten [2000], T_{GF} is modeled as a function of the flywheel speed ω_F. It is approximated by a polynomial of second order.

Secondly, losses in bearings are considered. They are said to be affine in speed. For example, the losses in the flywheel bearings are modeled as

$$T_{l,F} = b_F \, \omega_F + b_{F0} \; , \tag{4}$$

where b_F and b_{F0} are constants. For the losses in clutch bearings, parameters b_{kF}, b_{kF0} and the corresponding clutch speeds ω_{kF} are used. Index $k = 1, 2, 3$ stands for the number of the clutch.

Furthermore, viscous oil losses are taken into account. These losses are proportional to speed differences $\Delta\omega_k$ of the plates of clutches $k = 1, 2, 3$ via constants v_{kF}. The torques

$$T_{kv} = \begin{cases} v_{kF} \, \Delta\omega_k & \text{if} \quad |\Delta\omega_k| < \Delta\omega_{k,max} \\ \bar{v}_{kF} \; \text{sign} \, \Delta\omega_k & \text{else} \end{cases} \tag{5}$$

are limited to values of $\pm\bar{v}_{kF}$, that are reached at speeds $\pm\Delta\omega_{k,max}$. Here, the speed difference, e.g. of clutch 1, is defined by

$$\Delta\omega_1 = \frac{\omega_F}{i_{FC} \, i_{1F}} - \omega_B \, i_{AB} \; . \tag{6}$$

The rotary velocity $\omega_B \, i_{AB}$ represents the speed of the right side of clutch 1. Ratio i_{AB} and speed ω_B are quantities of the interconnection between the two clutch stages, according to Fig. 4. Torque losses (5) are positive, if the flywheel drives the second clutch stage, i.e. $\Delta\omega_k > 0$. If the flywheel is driven by the second clutch stage, the coupling via the oil in the clutches acts as a propulsion for the flywheel, i.e. T_{kv} appears with the opposite sign.

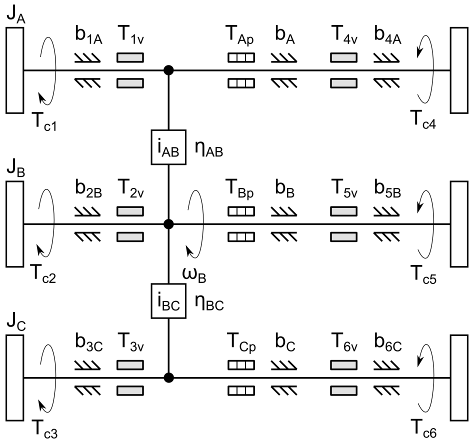

Fig. 4. Structure of the C-CVT model (first to second clutch stage)

With the combination of bearing and viscous oil losses

$$T_{l,kF} = b_{kF}\,\omega_{kF} + b_{kF0} + T_{kv}\ , \tag{7}$$

the dynamics of the first C-CVT part is governed by differential equation

$$\begin{aligned} J_{F,tot}\frac{d\omega_F}{dt} &= -T_{GF} - T_{l,F} - \\ \frac{1}{i_{FC}\,\eta_{FC}} &\left[\frac{1}{i_{1F}\,\eta_{1F}}\left(T_{c1} + T_{l,1F}\right) + \right. \\ &\left.\frac{1}{i_{2F}\,\eta_{2F}}\left(T_{c2} + T_{l,2F}\right) + \frac{1}{i_{3F}\,\eta_{3F}}\left(T_{c3} + T_{l,3F}\right)\right]\ . \end{aligned} \tag{8}$$

Here, T_{c1}, T_{c2} and T_{c3} represent the clutch torques transmitted due to friction. The total moment of inertia

$$\begin{aligned} J_{F,tot} = J_F + \frac{1}{i_{FC}^2\,\eta_{FC}} &\left(J_{FC} + \frac{1}{i_{1F}^2\,\eta_{1F}}\,J_{c1F} + \right. \\ &\left.\frac{1}{i_{2F}^2\,\eta_{2F}}\,J_{c2F} + \frac{1}{i_{3F}^2\,\eta_{3F}}\,J_{c3F}\right)\ . \end{aligned} \tag{9}$$

is dependent on the actual power direction.

2.2 First clutch stage to second clutch stage

The second part of the C-CVT is shown in Fig. 4. It consists of the interconnection between the first clutch stage (right sides of clutches 1, 2 and 3) and the second clutch stage (left sides of clutches 4, 5 and 6), where T_{ck} ($k = 1, 2, \ldots, 6$) symbolize the friction torques at the clutches.

Each parameter J_A, J_B and J_C combines the moments of inertia of two coupled clutches, e.g. clutch 1 and 4 (including the clutch baskets), see Fig. 4. A fixed transmission i_{AB} between clutches 1-4 and 2-5 (efficiency η_{AB}) and a fixed transmission i_{BC} between clutches 3-6 and 2-5 (efficiency η_{BC}) are used.

Analog to Section 2.1, several kinds of losses are modeled. The losses in the bearings between baskets and housing are assumed to be affine in speeds ω_j ($j \in \{A, B, C\}$; parameter b_j, b_{j0}). All losses in bearings between baskets and plates are affine in the according speed differences $\Delta\omega_k$ ($k = 1, 2, \ldots, 6$; parameter b_{kj}, b_{kj0}). For example, b_{2B} represents the proportional factor of the losses in the bearing next to clutch 2 on shaft B, see Fig. 4.

The viscous oil losses within the clutches T_{kv} are proportional to speed differences $\Delta\omega_k$ up to a maximum torque, according to (5).

Additionally, losses caused by the pumping effect of the clutch baskets are considered. This pumping losses have to be taken into account only if minimal speeds $\omega_{j,min}$ are exceeded. They are negligible below this limit. For higher speeds, the power losses are assumed to be proportional to speeds ω_j because of a constant oil flow. As a consequence, the pumping losses are modeled as

$$T_{jp} = \begin{cases} 0 & \text{if} \quad \omega_j < \omega_{j,min} \\ \alpha_{j1} - \dfrac{\alpha_{j2}}{\omega_j} & \text{else} \end{cases}\ . \tag{10}$$

The constant parameters α_{j1} and α_{j2} ($j \in \{A, B, C\}$) are dependent on the oil flow through the clutches.

Combining all introduced losses leads to the total torque losses acting on shaft A, i.e.

$$\begin{aligned} T_{l,A} = b_A\,\omega_A + b_{A0} + b_{1A}\,\Delta\omega_1 + b_{1A0} + b_{4A}\,\Delta\omega_4 + \\ b_{4A0} + T_{Ap} - T_{1v} + T_{4v}\ . \end{aligned} \tag{11}$$

The negative sign in (11) appears because of the propulsion effect of clutch 1 for the positive power direction. The total torque losses $T_{l,B}$ and $T_{l,C}$ at shafts B and C are modeled analog to (11).

The mathematical description of the second C-CVT part is stated as differential equation

$$\begin{aligned} J_{B,tot}\frac{d\omega_B}{dt} &= T_{c2} - T_{c5} - T_{l,B} + \\ i_{AB}\,\eta_{AB}&\left(T_{c1} - T_{c4} - T_{l,A}\right) + \\ i_{BC}\,\eta_{BC}&\left(T_{c3} - T_{c6} - T_{l,C}\right) \end{aligned} \tag{12}$$

for speed ω_B, where the total moment of inertia $J_{B,tot}$ is given by

$$J_{B,tot} = J_B + i_{AB}^2\,\eta_{AB}\,J_A + i_{BC}^2\,\eta_{BC}\,J_C\ . \tag{13}$$

2.3 Second clutch stage to load

In Fig. 5, the third part of the C-CVT is depicted. All load sides of clutches 4, 5 and 6 (moments of inertias J_{c4L}, J_{c5L}, J_{c6L}) are coupled via three fixed transmissions with ratios i_{4L}, i_{5L}, i_{6L} to the final reduction ratio i_{CL} of the C-CVT. The corresponding transmission efficiencies η_{4L}, η_{5L}, η_{6L} and η_{CL} are included into the model as in Section 2.1.

Parameter J_{CL} symbolizes the moment of inertia of all parts between the second clutch stage and the load. The moment of inertia of the load

$$J_L = \frac{m\,r^2}{i_{Fd}^2} \tag{14}$$

represents an equivalent vehicle with mass m, rolling radius r and final drive ratio i_{Fd}. The speed of the equivalent vehicle v is linked to the speed ω_L of the load shaft via

$$v = \omega_L\,\frac{r}{i_{Fd}}\ . \tag{15}$$

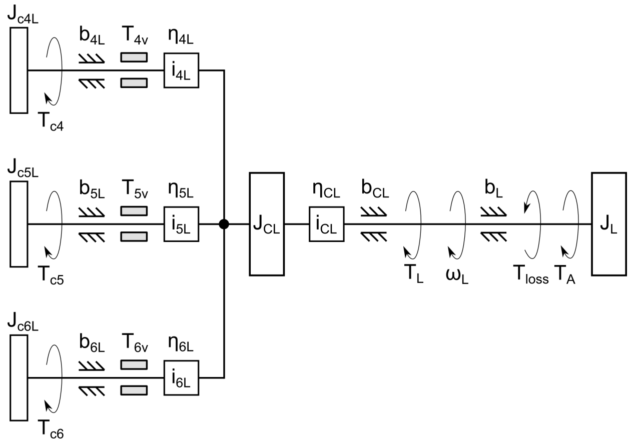

Fig. 5. Structure of the C-CVT model (second clutch stage to load)

For the modeling of the losses in bearings (parameter b_L, b_{L0}, b_{CL}, b_{CL0}, b_{kL}, b_{kL0} with $k = 4, 5, 6$) and viscous oil losses T_{kv}, the same assumptions are made as in Section 2.1. As a result, these losses are given by

$$T_{l,CL} = b_L\,\omega_L + b_{L0} + b_{CL}\,\omega_L + b_{CL0} \tag{16}$$

and

$$T_{l,kL} = b_{kL}\,\omega_{kL} + b_{kL0} - T_{kv}\ . \tag{17}$$

Additional losses T_{loss} of the equivalent vehicle are considered as well, see e.g. Kiencke and Nielsen [2010]. These losses are modeled as

$$T_{loss} = \frac{r}{i_{Fd}}\left[\frac{1}{2}\,\rho\,A_f\,c_d\,v^2 + c_r\,m\,g\,\cos(\alpha) + m\,g\,\sin(\alpha)\right]\ , \tag{18}$$

with the following parameters: density of the ambient air ρ, frontal area of the vehicle A_f, aerodynamic drag coefficient c_d, rolling friction coefficient c_r, acceleration of gravity g and road angle α.

The inputs for the third part of the C-CVT are the clutch torques T_{c4}, T_{c5}, T_{c6} as well as an additional torque T_A that can be applied e.g. by means of an internal combustion engine.

Hence, the dynamics of the third part of the C-CVT can be expressed as

$$J_{L,tot}\frac{d\omega_L}{dt} = T_A - T_{loss} - T_{l,CL} + i_{CL}\,\eta_{CL}\Big[\,i_{4L}\,\eta_{4L}\,(T_{c4} - T_{l,4L}) + i_{5L}\,\eta_{5L}\,(T_{c5} - T_{l,5L}) + i_{6L}\,\eta_{6L}\,(T_{c6} - T_{l,6L})\Big]\ , \tag{19}$$

with a total moment of inertia of

$$J_{L,tot} = J_L + i_{CL}^2\,\eta_{CL}\left(J_{CL} + i_{4L}^2\,\eta_{4L}\,J_{c4L} + i_{5L}^2\,\eta_{5L}\,J_{c5L} + i_{6L}^2\,\eta_{6L}\,J_{c6L}\right)\ . \tag{20}$$

The load torque T_L at the C-CVT interface equals

$$T_L = J_L\frac{d\,\omega_L}{dt} - T_A + T_{loss} + b_L\,\omega_L\ . \tag{21}$$

2.4 Calculation of the transmitted clutch torques

A central point of the model presented in this paper is the calculation of the clutch torques T_{ck} $(k = 1, 2, \ldots, 6)$ based on the basic clutch model described in Appendix A. It covers the Coulomb friction and the stiction effect.

If only one clutch is closed (e.g. clutch 1), the speeds of the left and right side of the clutch are identical, i.e. $\omega_F \equiv \omega_B\,i_{AB}\,i_{1F}\,i_{FC}$. Consequently, the accelerations are also the same. This yields

$$\frac{1}{J_{F,tot}}\,T_1 = \frac{1}{J_{B,tot}}\,T_2\,i_{AB}\,i_{1F}\,i_{FC}\ , \tag{22}$$

where T_1 is the right side of (8), T_2 the right side of (12). An extraction of the unknown stiction torque T_{c1} results in

$$\left(J_{F,tot}\,i_{1F}\,i_{FC}\,i_{AB}^2\,\eta_{AB} + J_{B,tot}\,\frac{1}{i_{FC}\,\eta_{FC}\,i_{1F}\,\eta_{1F}}\right)T_{c1} = J_{B,tot}\,T_1|_{\{\setminus T_{c1}\}} - J_{F,tot}\,T_2|_{\{\setminus T_{c1}\}}\,i_{AB}\,i_{1F}\,i_{FC}\ . \tag{23}$$

In (23), e.g. $T_1|_{\{\setminus T_{c1}\}}$ is used as a short notation for T_1 according to (8), where all terms containing T_{c1} are removed. All other clutches may slip while clutch 1 is closed.

For the case of two closed clutches (one from the first and one from the second stage), two speed identities have to be fulfilled, e.g. $\omega_F \equiv \omega_B\,i_{AB}\,i_{1F}\,i_{FC}$ and $\omega_B \equiv \omega_L\,i_{5L}\,i_{CL}$, if clutch 1 and 5 are closed. The corresponding stiction torques T_{c1} and T_{c5} are determined via

$$\left(J_{F,tot}\,i_{1F}\,i_{FC}\,i_{AB}^2\,\eta_{AB} + J_{B,tot}\,\frac{1}{i_{FC}\,\eta_{FC}\,i_{1F}\,\eta_{1F}}\right)T_{c1} - J_{F,tot}\,i_{1F}\,i_{FC}\,i_{AB}\,T_{c5} = J_{B,tot}\,T_1|_{\{\setminus T_{c1}\}} - J_{F,tot}\,T_2|_{\{\setminus T_{c1}\wedge T_{c5}\}}\,i_{AB}\,i_{1F}\,i_{FC} \tag{24}$$

and

$$-J_{L,tot}\,i_{AB}\,\eta_{AB}\,T_{c1} + \left(J_{B,tot}\,i_{5L}^2\,\eta_{5L}\,i_{CL}^2\,\eta_{CL} + J_{L,tot}\right)T_{c5} = J_{L,tot}\,T_2|_{\{\setminus T_{c1}\wedge T_{c5}\}} - J_{B,tot}\,T_3|_{\{\setminus T_{c5}\}}\,i_{5L}\,i_{CL}\ . \tag{25}$$

With the right side T_3 of differential equation (19), both stiction torques can be determined.

The stated procedure for the calculation of the stiction torques is realized for all admissible combinations of clutches 1 to 6. A clutch logic checks, which clutches are opened and closed depending on the current clutch actuations, torques and speeds.

3. CONTROL STRATEGY

For the operation of the C-CVT, a strategy for the switching between different clutch combinations is necessary.

In Fig. 6, a procedure for a switching between clutch combination 1-6 and 4-1 is shown exemplarily. Before starting the switching, clutches 1 and 6 are activated (see top plot in Fig. 6) and clutch 1 is closed (middle plot), clutch 6 is slipping. The switching is triggered if

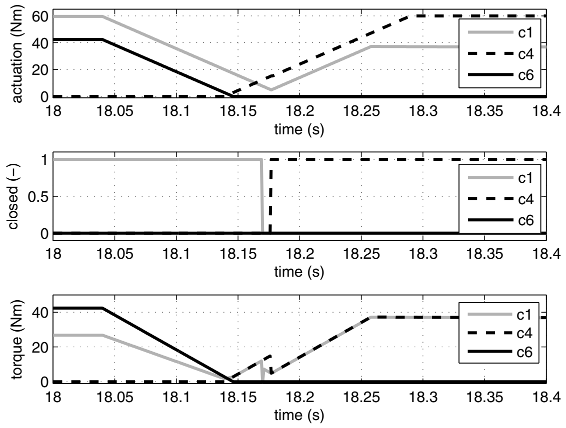

Fig. 6. Example for a switching procedure: clutch actuation (top), clutch closed signals (middle), actual clutch torques (bottom)

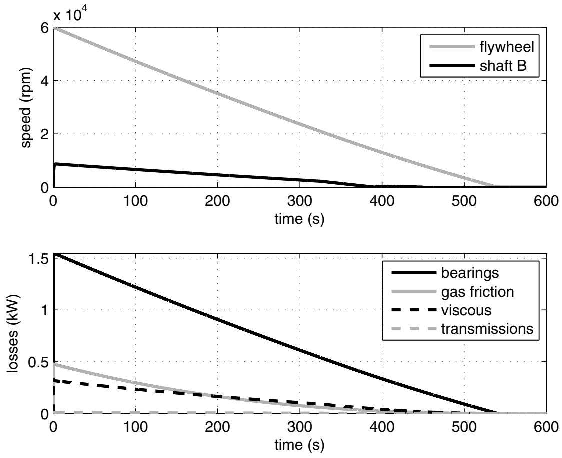

Fig. 7. Free run: flywheel speed ω_F and scaled clutch speed $10 \cdot \omega_B$ (top), losses in the C-CVT and flywheel (bottom)

a certain speed difference at clutch 6 is reached. Then, the actuation torques are reduced, where the rate is limited due to the used actuators. If the actuation torque of the closed clutch equals a predefined threshold, e.g. $20\,Nm$ in Fig. 6, clutch 4 is actuated. The actual torques at the clutches depend on the current speed and torque levels in the system, see Fig. 6 (bottom). After closing clutch 4, clutch 1 has to be controlled in order to achieve a desired load torque T_L.

Please note that there are several strategies for the switching between two clutches of the same stage to reduce the torque drop. For the switching between two clutches of different stages, e.g. from clutches 1 and 4 to clutches 3 and 5, the torque gap cannot be avoided.

The torque controller is based on a simplified version of model (8,12,19). Here, the assumption of vanishing losses yields

$$J_1 \frac{d\omega_F}{dt} = -\frac{1}{i_{FC}\, i_{1F}}\, T_{c1} - \frac{1}{i_{FC}\, i_{2F}}\, T_{c2} - \frac{1}{i_{FC}\, i_{3F}}\, T_{c3} \quad (26)$$

$$J_2 \frac{d\omega_B}{dt} = i_{AB}\, T_{c1} + T_{c2} + i_{BC}\, T_{c3} - i_{AB}\, T_{c4} + T_{c5} + i_{BC}\, T_{c6} \quad (27)$$

$$J_3 \frac{d\omega_L}{dt} = i_{CL}\, i_{4L}\, T_{c4} + i_{CL}\, i_{5L}\, T_{c5} + i_{CL}\, i_{6L}\, T_{c6} \quad (28)$$

$$T_L = J_L \frac{d\,\omega_L}{dt}\ , \quad (29)$$

where J_1, J_2 and J_3 represent $J_{F,tot}$, $J_{B,tot}$ and $J_{L,tot}$ under the assumption that all efficiencies are 1. The simplified model (26)-(29) can also be used for a rough estimation of the load torque.

A feedforward is derived by rearranging (26)-(29). For example, clutch 5 is closed, i.e. $\omega_B = \omega_L\, i_{5L}\, i_{CL}$, and clutch 1 is slipping, i.e.

$$T_{c1} = k_{c1}\ \mathrm{sign}\left(\frac{\omega_F}{i_{FC}\, i_{1F}} - \omega_B\, i_{AB}\right)\ . \quad (30)$$

Clutch torque T_{c1} is proportional to the actuation factor k_{c1}, which is assumed to be proportional to the pressure of the actuation piston. Then, the load torque is given by

$$T_L = \frac{J_L}{J_3}\, i_{CL}\, i_{5L} \left(-J_2\, i_{CL}\, i_{5L} \frac{d\,\omega_L}{dt} + i_{AB}\, T_{c1}\right)\ . \quad (31)$$

With a desired load torque $T_{L,des}$, the feedforward can be expressed as

$$T_{c1} = \frac{1}{i_{AB}} \left(J_2\, i_{CL}\, i_{5L} \frac{d\,\omega_L}{dt} + \frac{J_3}{J_L\, i_{CL}\, i_{5L}}\, T_{L,des}\right)\ . \quad (32)$$

The same procedure is used to determine a feedforward for all remaining combinations of slipping and closed clutches.

Additionally to the feedforward, a standard PI-controller is applied to compensate the effect of unmodeled dynamics, disturbances and parameter variations. The controller is tuned manually in simulation.

4. SIMULATION

In Fig. 7, the results of a simulation without any clutch actuation are shown. The flywheel (initial speed $\omega_F = 60.000\,rpm$) accelerates the parts between first and second clutch stage due to the viscous oil losses in clutches 1, 2 and 3. Furthermore, the power losses in bearings, flywheel housing and clutches are shown. The losses in fixed transmissions are negligible because of the low torques transmitted. The power transmitted to the load via viscous losses is not sufficient to accelerate the load, i.e. the load speed remains zero for the simulation.

As a second example, a vehicle acceleration maneuver is plotted in Fig. 8. The desired load torque $T_L = 100\,Nm$ can be provided by the C-CVT except for the short switching periods. Because the presented model is designated for design studies, the achieved performance of the control loop is sufficiently good.

Fig. 9 illustrates the acceleration maneuver with respect to flywheel and vehicle speed. Four switchings are used to decelerated the flywheel resp. to accelerate the vehicle to speed $v = 36\,km/h$ (x in Fig. 9). In contrast to the previous

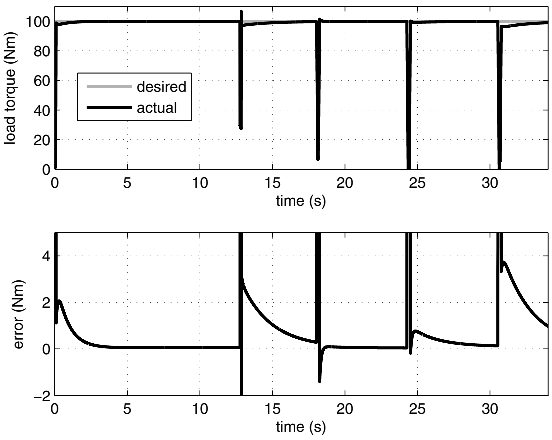

Fig. 8. Accelerate vehicle: desired and actual load torque T_L)

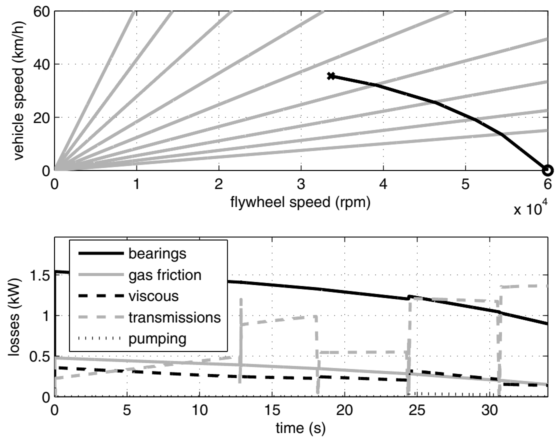

Fig. 9. Accelerate vehicle: flywheel speed ω_F versus vehicle speed v (top), losses in the C-CVT and flywheel (bottom)

simulation, the power loss in the fixed transmissions has to be taken into account as shown in Fig. 9.

It is planned to design and manufacture a prototype C-CVT to investigate the behavior of the controlled flywheel system on a test rig.

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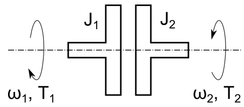

Fig. A.1. Scheme of the basic clutch model

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Appendix A. BASIC CLUTCH MODEL

Fig. A.1 shows the scheme of a simple clutch model. Here, J_1 denotes the moment of inertia of the first clutch side (e.g. including the disks) and J_2 symbolizes the moment of inertia of the second clutch side (e.g. including the basket). The principle of angular momentum yields differential equations for the input speed ω_1 and output speed ω_2

$$J_1 \frac{d\omega_1}{dt} = T_1 - T_C \quad \text{and} \quad J_2 \frac{d\omega_2}{dt} = T_C - T_2 \ , \qquad \text{(A.1)}$$

where the input torque T_1 and output torque T_2 are assumed to be known. Both equations are coupled via torque T_C that is transmitted due to friction. For a slipping clutch, i.e. ω_1 differs ω_2, friction torque T_C is modeled by $T_C = k \ \mathrm{sign}(\omega_1 - \omega_2)$, where parameter k depends on the hydraulic clutch actuation. If ω_1 equals ω_2, i.e. the case of closed clutch, both speeds and the corresponding accelerations have to be identical. This yields

$$T_C = \frac{T_1 J_2 - T_2 J_1}{J_1 + J_2} \ . \qquad \text{(A.2)}$$

The clutch remains closed, if torque T_C is smaller (or equal) than the maximal transmittable torque, i.e. $|T_C| \leq k \ k_{st}$. The stiction factor k_{St} is assumed to be constant. If the maximal torque is exceeded, the clutch opens and the transmitted torque equals

$$T_C = k \ \mathrm{sign}\left(\frac{T_1 J_2 - T_2 J_1}{J_1 + J_2}\right) \ . \qquad \text{(A.3)}$$