Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Cyclist Heart Rate Control via a Continuously Varying Transmission ⋆

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Abstract: This work addresses the design of a heart rate (HR) control system for a bicycle equipped with a continuously varying transmission. The control system helps the cyclist maintain a constant physical effort throughout the trip. The bicycle longitudinal dynamics and a control-oriented model of the cyclist's HR dynamics are the basis for the development of a controller. The HR dynamics is subject to considerable uncertainties due to inter subject differences, nonlinearities and other factors that are not directly influenced by the cycling physical effort. In the control system design phase, these are addressed by designing a second order sliding mode controller. The controller is experimentally validated on several subjects comparing the HR with the without the CVT control, showing that the HR is maintained within 10 BPM from the desired one.

Keywords: Bicycle dynamics, CVT control, heart rate control, intelligent transportation system.

1. INTRODUCTION

In the past few years bicycles have regained the center stage as a means of transportation. Bicycles are extremely cost-effective, eco-friendly, healthy and in congested cities they often represent the fastest way to reach one's destination.

Traditional bike, while being very cost-effective, may not be a viable option for the elderly and for professionals commuting to their workplace because of the required physical effort. Electrically Power Assisted Bikes (EPAC) Spagnol et al. (2012, 2013), by providing electric assistance, considerably lower the physical requirements; they can be however complex to maintain and expensive. Advances in mechatronic systems are opening a third path, namely intelligent passive bikes. These kinds of bikes do not directly provide traction power, but rather modify the bicycle's response to the cyclist's input. An example of such systems is automated gear shifting, implemented through discrete gear shifting or continuously varying transmissions Giani et al. (2013).

Intelligent passive bikes have naturally less potential than EPAC's of reducing the cyclist's fatigue and physical effort, but are considerably simpler and thus may represent an interesting, more accessible, alternative to EPAC's.

This paper addresses the design, implementation and validation of a cyclist's heart rate (HR) control system through a CVT (see Figure 1). The objective of the control system is that of keeping the HR at a user-specified value. In this context, the HR is considered as a proxy of fatigue; by keeping the HR constant, one can better distribute her or his physical effort throughout the trip.

⋆ The work presented has been partially supported by Zehus s.r.l.

The paper is structured in three main parts; modeling, control and validation. In modeling, a control-oriented

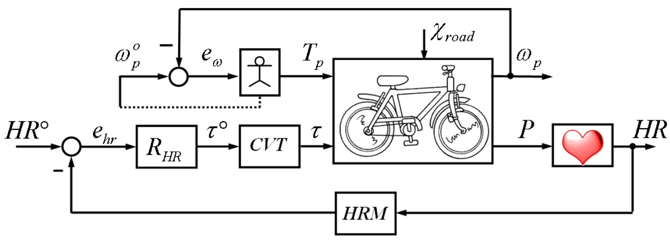

Fig. 1. Overall control system architecture.

model of the entire system is derived. The model has two main components, the model of the CVT-equipped bicycle and the model of the HR dynamics.

HR dynamics modeling is a rich field in biomechanics literature. The available models can be classified into two main categories, namely the *linearised* models and *nonlinear* modeling approaches. In the first class of models, the system dynamics is expressed as the sum of two or more terms generated by linear systems. According to Bearden and Moffatt (2001), when the exercise starts, two time constants are in general involved: the first, within 20÷40s, explains the nervous system actions, and a second, slower, dynamics is related to the changes in metabolic activity and to the lactate production. At the end of the exercise, different physiological phenomena are involved in the human body, also producing a different dynamics. One of the first and wide-accepted model is proposed by Hajek et al. (1980). Among the nonlinear models, three classes are relevant. In Cheng et al. (2007) a physiological-based second-order nonlinear state space model has been proposed to describe the HR dynamics during and after a treadmill exercise. The same model has been also investigated in Zakynthinaki and Stirling (2008); Stirling et al. (2008), demonstrating the fitting of the model output to a set of raw data for multiple constant intensity exercises for an individual at a particular level of fitness. Finally, the Hammerstein model proposed in Su et al. (2007a) seems to be an interesting compromise between the simplicity of linear models and the accuracy of nonlinear ones. In Su et al. (2007a) authors show that this modeling approach provides a reliable and effective system model and satisfactory control performances are achieved in Su et al. (2007b); Hunt and Allan (2009); Mohammad et al. (2011). The scope of the HR dynamics modeling part of this work is not that of providing a complete model of the HR dynamics, but rather have a control-oriented model to drive the control system design phase.

From the control design point of view, no matter how well a model describes the HR response of any given subject, inter subject variability necessarily generates uncertainties. In the proposed control algorithm a Second Order Sliding Mode controller (FOSM) is employed to account for those uncertainties. This choice guarantees robustness and smoothness of control action.

The validity of the proposed control system is tested on a series of trials.

The paper is structured as follows: in Section 2 the modeling of the overall system is discussed; in Section 3 the control system is designed and detailed; the control system is finally validated in Section 4. Section 5 draws the final conclusions.

2. SYSTEM DESCRIPTION AND MODELING

The overall control system architecture, shown in Figure 1, has several components. The test vehicle considered in this work is a city bike equipped with a NuVinci® roller-based CVT controlled by a custom ECU. The ECU also measures some vehicle variables, as pedal speed and rear wheel speed and pedal torque The experimental layout also includes an Onyx® II 9560BT HR Monitor (HRM) for heart rate measurement.

The HR control algorithm runs on the smartphone, while the ECU performs the low level CVT control Giani et al. (2013). The three devices (HRM, smartphone and ECU) communicates via bluetooth. The vehicle is also equipped with a small battery pack.

From the modeling standpoint, three different sub-systems are considered: bicycle, cyclist and HR dynamics.

2.1 Bicycle

The bicycle model is composed of the transmission model and the road load model. The balance of the forces acting of the bicycle yields the road load equation

$$M_v\dot{v} = -\frac{1}{2}\rho C_x A v^2 - D_v v + \quad (1)$$

$$- M_v g\left(\sin(\mathcal{X}_{road}) + C_r\cos(\mathcal{X}_{road})\right) + F_w \quad (2)$$

where M_v is the bicycle mass and v its longitudinal velocity. The road load is the sum of several contributions:

- $(\rho C_x A v^2)/2$ represents the aerodynamic drag, which is computable as the product of the air density ρ, the reference vehicle front area A and drag coefficient C_x;
- $D_v v$ describes the viscous friction;
- $M_v g\sin(\mathcal{X}_{road})$ represents the gravitational force, where $\mathcal{X}_{road}$ is the road slope and g is the acceleration of gravity,
- $M_v g C_r\cos(\mathcal{X}_{road})$ is an approximation of the rolling resistance, depending on the normal force on the tires-road contact surface and on the friction coefficient C_r;
- F_w is the force generated by the wheel.

Note that only the longitudinal dynamics is modeled. The parameters of the model are identified from a so-called coasting down experiment (see Spagnol et al. (2012)) performed on a flat road. The results of the parameter identification procedure are shown in Figure 2.

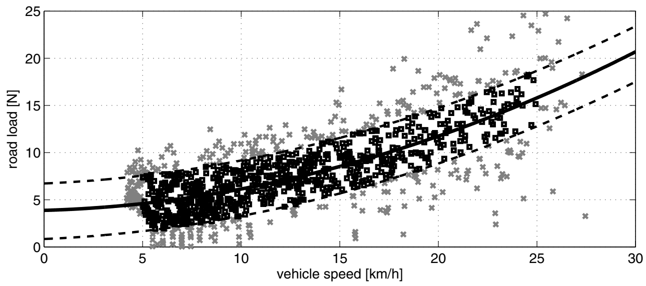

Fig. 2. Identification of the overall friction forces acting on the bicycle.

The wheel force F_w is the cyclist's pedal torque T_p as transmitted by the bicycle transmission. The transmission dynamics is given by:

$$J\dot{\omega}_p = T_p - D\omega_p - \frac{\tau r_w}{\eta}F_w \quad (3)$$

where ω_p is the pedal speed, r_w is the rear wheel rolling radius, η is the transmission efficiency (set to 0.875) and τ is the overall transmission ratio which can be controlled through the CVT. J and D are the transmission mass moment of inertia and friction factor.

Combining the transmission model with the road load model, one obtains the complete longitudinal model:

$$\begin{aligned} J\dot{\omega}_p &= T_p - D\omega_p + \\ &-\frac{\tau^2 r_w^2}{\eta}\left(M_v\dot{\omega}_p + \frac{1}{2}\tau r_w \rho C_x A\omega_p^2 + D_v\omega_p\right) + \\ &-\frac{\tau r_w}{\eta}\left(M_v g\sin\left(\mathcal{X}_{road}\right) + M_v g C_r\cos\left(\mathcal{X}_{road}\right)\right). \end{aligned} \quad (4)$$

The overall model is validated comparing the simulated velocity with the measured one as a response to measured pedal torque inputs. Figure 3 shows that the fitting between the two models is satisfactory. From the longitudinal dynamics model, the current cyclist power is immediately computed multiplying the cyclist's torque and the pedaling velocity. The cyclist's power is fed into the HR model.

2.2 Cyclist

From the control point of view the bicycle has three inputs: the controllable transmission ratio that is modeled by (4),

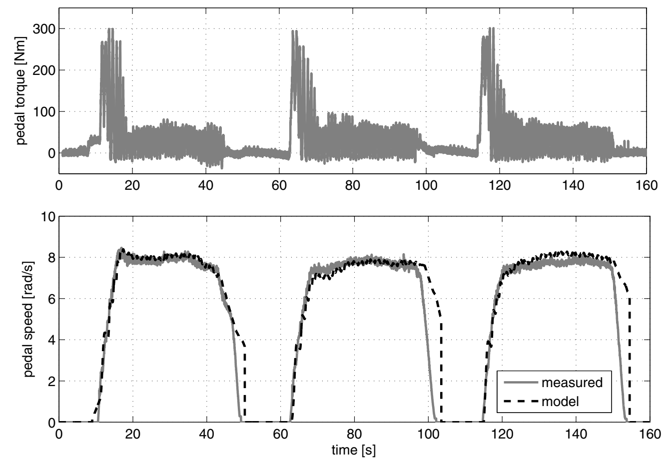

Fig. 3. Longitudinal dynamics model validation.

the cyclist torque and the road slope. The CVT ratio is the control input. The road slope is an unmeasurable disturbance; it is exogenous and not influenced by any other inputs. On the other hand, the cyclist's torque, which is also an unmeasurable disturbance (the torque sensor is only employed for analysis porpuses), is correlated with the other inputs. The cyclist responds to variation in transmission ratio and road slope. This causality direction needs to be accounted for and modeled.

The cyclist is modeled as a velocity controller. The cyclist decides the pedal torque comparing the desired velocity with the actual one. A set of experiments is used to identify the linear velocity controller model. In the experiments the rider was asked to accelerate to a constant velocity starting from standstill. The final steady state velocity is assumed to be the reference velocity. The model is identified minimizing the prediction error between the measured torque and the computed one when fed with the same velocity tracking error. The resulting velocity controller has the following structure:

$$R_{H\omega}(s) = \mu_{H\omega} \frac{(1 + sT_{zH\omega})}{(1 + sT_{pH\omega})^2} \tag{5}$$

whose parameters are $\mu_{H\omega} = 60$, $T_{zH\omega} = 0.08$, $T_{pH\omega} = 0.65$. Figure 4 plots the validation of the identified model.

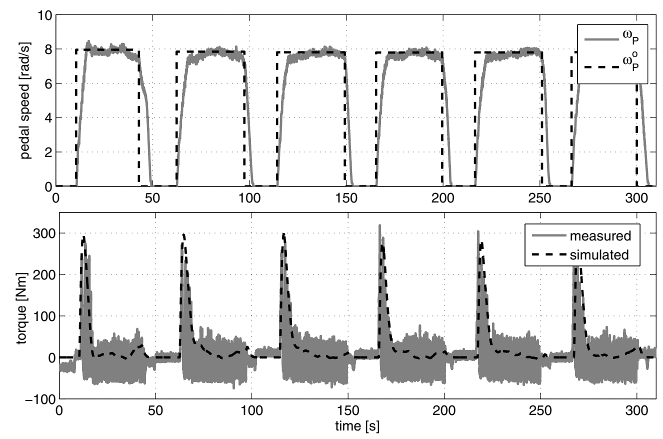

Fig. 4. Validation of the cyclist model.

Note that, the cyclist model is needed to account for the effect of a variation in the transmission gear onto the pedaling power. The HR controller does not need to have information on the current and desired velocity.

2.3 Heart rate dynamics

The starting point for the HR model is the model proposed in Su et al. (2007a). A Hammerstein model is constituted by the series connection of a static non-linearity $\mathcal{N}$ and a linear dynamical system with state-space matrices A, B, C and D, as

$$\begin{cases} \dot{x}(t) = Ax(t) + B\tilde{u}(t) \\ z(t) = Cx(t) + D\tilde{u}(t) \end{cases} \tag{6}$$
$$\mathrm{HR}(t) = \mathrm{HR}_0 + \gamma z(t) \qquad \tilde{u}(t) = \mathcal{N}(u(t))$$

the model output $z(t)$ is a scaled version of the HR variations from the basal value, while $u(t)$ represents the input power. One of the advantages of using a Hammerstein model is that the static nonlinearity and linear dynamics can be decoupled (see Bai (2004)).

Two sets of experiments have been considered: firstly a series of slow varying profile of power has been required from the cyclist, assessing the steady-state relationship between pedal power and HR *i.e.*, the model nonlinearity $\mathcal{N}$. Then, some dynamical experiments have been carried out, so as to catch the main system dynamics.

For the nonlinearity identification, a 5km-long flat route has been selected: the cyclist was asked to keep a constant pedal speed, while the bicycle transmission ratio has been varied so to produce a sinusoidal power varying from 50 to 300 W. The resulting quasi-static input/output relation is shown in Figure 5 where the cyclist's HR is plotted as a function of pedaling power.

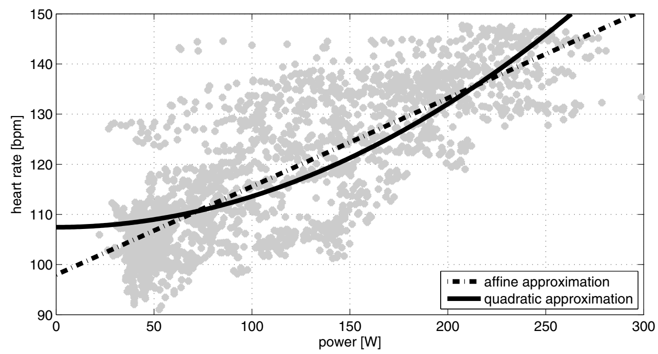

Fig. 5. Measured power/heart rate steady-state characteristic (dots), affine and quadratic approximations.

To explain the relationship between the two quantities, two different models are proposed:

$$\begin{aligned} \mathcal{N}_{lin} &: \ \hat{\mathrm{HR}} = a_1 + a_2 P \\ \mathcal{N}_{par} &: \ \hat{\mathrm{HR}} = a_3 + a_4 P^2 \end{aligned} \tag{7}$$

The first model, $\mathcal{N}_{lin}$, assumes the system to be linear, while the quadratic relation adopted in $\mathcal{N}_{par}$ follows from the hints of Mohammad et al. (2011). From Figure 5, it can be observed that both models fit the data. The resulting coefficient of determinations are $R^2_{lin} = 0.642$

and $R^2_{par} = 0.584$; the use of a more complex quadratic curve is not justified by the added accuracy. The linear approximation is thus considered in the HR model. The quadratic non-linearity would be more adequate to model the HR dynamics in case of high intense exercise, as it can ensure a more accurate fitting of the measured HR, see *e.g.*, Mohammad et al. (2011) and references therein. The linear approximation, however, provides a good accuracy for the power levels usually involved in cycling and can be sufficient for the purposes of the present work.

In view of the results obtained for the identification of the static map, the system can be assumed to be linear and its dynamics identified by means of prediction error minimization identification methods for linear systems. A series of pedal power frequency sweeps are executed. The measured HR response in one of these experiments is shown in Figure 6, together with the identified model output. From the collected data a non-parametric estimate

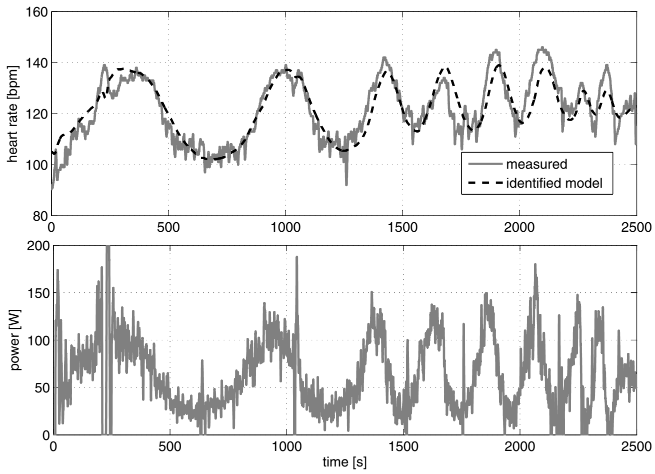

Fig. 6. Power sweep and corresponding heart rate (measured and simulated.

of the system frequency response $G_{HR}(s)$ is obtained by windowed spectral analysis of the input/output cross-spectral densities, (see Pintelon and Schoukens (2001)). The frequency response has been fitted with a second-order parametric model expressing the main dynamics of the system,

$$G_{HR}(s) = \frac{\mu_{hr}}{(sT_{hr1}+1)(sT_{hr2}+1)} \tag{8}$$

with $T_{hr1} = 22$ and $T_{hr2} = 55$. The obtained model order is consistent with the one proposed in the literature thereby confirming the reliability of the achieved result.

As the sweep experiments last approximately 40 minutes, the fatigue effects must be considered in the model (see Hajek et al. (1980)). This contribution is effectively modeled by an integral term HR_{FF}

$$HR_{FF} = \frac{\mu_{hF}}{s} u_2 \tag{9}$$

where the integrator gain $\mu_{hF} = 0.004$ has been identified from the collected data and the binary control signal input signal u_2 simply indicates whether the cyclist's power is greater than 0. Furthermore the HR is constrained to take values within its admissible range, $HR_0 \leq HR(t) \leq HR_{\max}$, an anti-windup configuration must be considered for the integrator, so as to achieve consistent model formulation. Figure 7 shows the validation results. A free

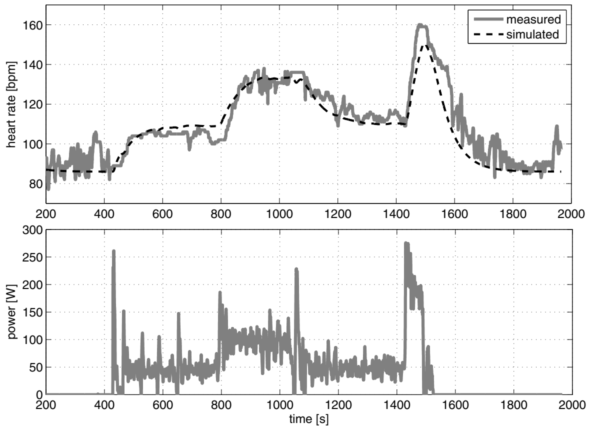

Fig. 7. Power sweep and corresponding heart rate (measured and simulated.

ride is employed. The very good fitting of the real data is apparent, yielding an overall Root Mean Square Error of 8.2%. A satisfactory results for a control-oriented model.

A complete control-oriented model system model is now available. The system is nonlinear; the most important nonlinearity is indeed the effect of the control variable τ; the system can however be linearized around an operating condition.

3. CONTROL SYSTEM DESIGN

The cyclist's HR control system should rely only on HR measurements and be robust to inter subject heart rate dynamics variability. Sliding mode control provides such features. Rather than a classical First Order Sliding Mode controller a Second Order one is implemented (see Bartolini et al. (1999); Perruquetti and Barbot (2002)). This choice is dictated by the need of avoiding discontinuities in the control variable that would be perceived negatively by the cyclist.

The error signal is chosen as the sliding variable,

$$e_{hr}(t) = HR^o(t) - HR(t) \tag{10}$$

while the control objective is to design a continuous control law τ^o capable of steering this error to zero in finite time. Linearizing the complete model, the plant can be easily rewritten in the state-space form, as

$$\begin{cases} \dot{z}(t) = Az(t) + B\tau^o(t) \\ y(t) = Cz(t) \end{cases} \tag{11}$$

where $y(t)$ is the system output, *i.e.*, the cyclist's heart rate, $z(t)$ is the system state vector and the matrices A, B and C have appropriate dimensions. Based on this representation, the first and the second derivatives of the sliding variable e_{hr} are

$$\begin{cases} \dot{e}_{hr} = \dot{y}^o + C\varrho \\ \ddot{e}_{hr} = \varphi + \gamma\dot{\tau}^o(t) \end{cases} \tag{12}$$

where $\varrho = -Az - B\tau^o$, $\varphi = \ddot{y}^o + CA\varrho$ and $\gamma = -CB$. It can be shown (see Bartolini et al. (1999); Perruquetti and Barbot (2002)) that the control law:

$$\dot{\tau}^o(t) = -\eta(t)U\text{sign}\left(e_{hr}(t) - \frac{1}{2}e_M\right)$$
$$\eta(t) = \begin{cases} \eta^* & \text{if } \left[e_{hr}(t) - \frac{e_M}{2}\right] e_M > 0 \\ 1 & \text{if } \left[e_{hr}(t) - \frac{e_M}{2}\right] e_M \leq 0 \end{cases}, \quad (13)$$

where U is a control gain, η the modulation factor and e_M a piecewise constant function taking the value of the last singular point of $e_{hr}(t)$ (*i.e.*, the most recent value e_M such that $\dot{e}_{hr}(t_M) = 0$), drives the system trajectory to the sliding manifold $e_{hr} = \dot{e}_{hr} = 0$ in finite time if

$$\eta^* \in (0,1] \cup \left(0, \frac{3\Gamma_1}{\Gamma_2}\right)$$
$$U > \max\left\{\frac{\Phi}{\eta^*\Gamma_1}, \frac{4\Phi}{3\Gamma_1 - \eta^*\Gamma_2}\right\} \quad (14)$$

where Γ_* and Φ are bounds of γ and φ such that:

$$|\varphi| \leq \Phi(z, y^o, \tau^o)$$
$$0 < \Gamma_1 \leq |\gamma| \leq \Gamma_2 \ . \quad (15)$$

In the considered applications, the nominal value of the bounds on γ in (15) are $\Gamma_1 = \Gamma_2 = CB$; which is easily bounded considering all the possible operating conditions (linearization transmission ration). The existence of $+\infty > \Phi \geq |\varphi|$ is guaranteed by the asymptotic stability of the system (11) and by the existence of reasonable bounds on $\ddot{y}^o$ and τ^o.

Note that the control law is only based on the error measurement and does not require its time derivative. Figure 8 plots the results of a HR reference step response. From figure, it is possible to conclude that the desired step

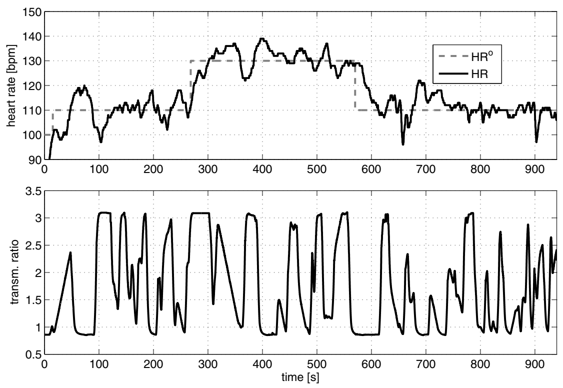

Fig. 8. Tracking performance of the SOSM controller.

is tracked with a settling time of around 30 s. The tracking is perturbed by high frequency noise that is outside the bandwidth of the controller and is part of the natural HR fluctuations that are not directly due to muscular effort.

4. VALIDATION

To further assess the effectiveness of the proposed approach, the SOSM HR controller performance has been assessed on a hilly urban test path. Two additional healthy subjects (not involved in the modeling effort) were involved in the experiments. Both subjects have ridden the same bike in two configurations:

- traditional single-gear mode. CVT set to $\tau = 1.5$;
- HR controlled mode with a constant HR set-point ($HR^o = 110$).

The urban path has been ridden twice by each cyclist in each configuration. The cyclists where not given any specific instructions on how to ride the bicycle. Figure 9 and 10 plot the results of these experiments; and Table 1 summarizes the quantitative results of the experiments.

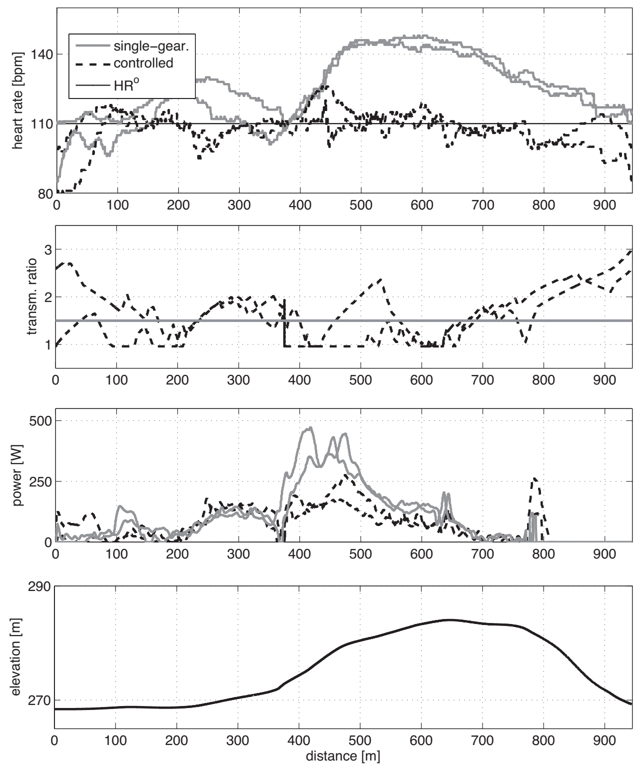

Fig. 9. HR controller validation on a urban path. Subject 1.

From figures, some considerations are due:

- In the single-gear case, the HR is strongly correlated with the road slope (see the bottom plot in Figure): when cycling at constant speed, slope is the main factor determining power variations.
- The power profiles are similar until the uphill section. In this section the controller effectively limits the cyclist's power.
- The performance for subject 1 and 2 are similar, showing that the proposed control is robust.
- The controller successfully keeps the HR close to the desired set-point HR^o with small fluctuations.
- The average of the HR values obtained when the cyclists ride the traditional bicycle configuration are quite different from one rider to the other. This is due to the riders physical characteristics and their fitness level, that make their hearts to behave in different ways.
- Standard deviations of the HRs in this case are sensibly higher than what observed with the previous bicycle configuration: this is again related to the open-loop behaviour of the riders' hearts, which are excited by the slope variations and the related power adjustments.

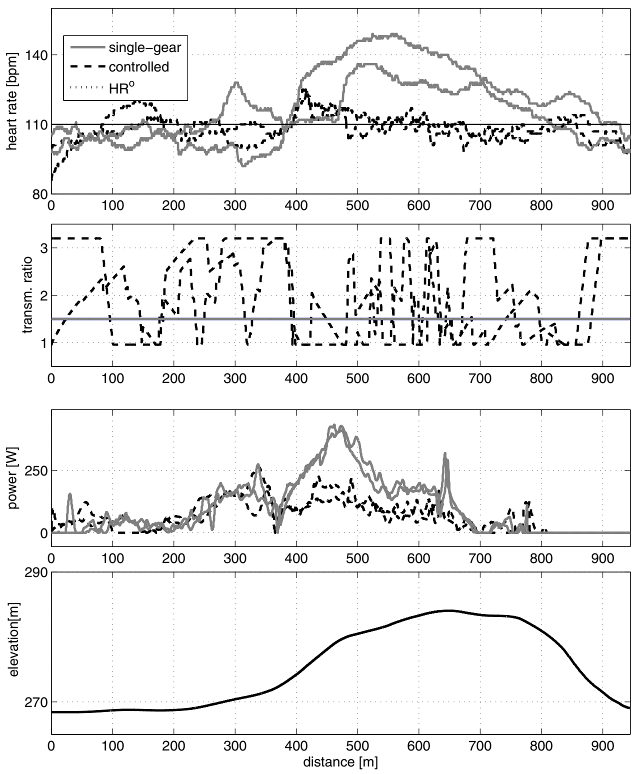

Fig. 10. HR controller validation on a urban path. Subject 2.

- Unsurprisingly, the reduced HR variability and average values come at a cost of a lower average speed for the trip.

	Subject 1		Subject 2	
	single-gear	CVT	single-gear	CVT
HR (bpm) (std.dev.)	124 (17.3)	111 (8.6)	117 (18)	109 (7)
mean velocity [km/h]	14.3	11.2	15.2	10. 6

Table 1. Quantitative analysis of the validation results.

5. CONCLUSIONS

In this paper a Second Order Sliding Mode HR control for cycling application has been proposed. The controller, by adjusting the bicycle transmission ratio via a CVT, is capable to modulating the cyclist's effort so to track a desired HR target. The design of the control system is guided by a thorough analysis of the dynamics to be controlled; which is made possible by a control oriented model of the longitudinal dynamics and HR dynamics. Both the model and the control system are validated using human subjects.

In this work, a constant HR reference has been used. One could imagine different approaches where the HR reference is scheduled according to other possible physiological criteria, for example, for training.

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