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Road Slope Estimation in Bicycles without Torque Measurements ⋆

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Abstract: Electrically Power Assisted Cycles (EPACs) have been gaining increasing attention worldwide during the past few years. The delivery of a good assistance during inclines makes or destroys an EPAC. This paper addresses the low-cost estimation of road slope on bicycle without pedaling torque measurement. Two estimation algorithms are discussed. A full 6 Degrees of Freedom kinematic Extended Kalman Filter and a simpler, more cost-effective 2 DoF Kalman filter based on the longitudinal kinematic model. The proposed reduced-sensor algorithm is shown to be very accurate during straight running; however it is affected by errors during cornering. This issue is addressed by augmenting the filter with a curve correction algorithm. The curve correction algorithm, based on a time-varying low pass filter is detailed and validated on experimental data, comparing the estimated road slope against cartographic data.

Keywords: Extended Kalman Filtering, Slope estimation, Bicycle dynamics.

1. INTRODUCTION

Energy prices and environmental considerations have brought Light Electric Vehicles (LEVs) under the public interest spotlight. Strict emission regulations, congestion charges, and new paradigms for urban mobility (*e.g.* bike and car sharing) have been introduced in urban areas in order to tackle traffic issues and pollution. Electrically Power Assisted Cycles (EPACs) are an extremely interesting opportunity in this context. They are cost effective, efficient and promote a healthier life-style.

Consequently EPAC's received a considerable amount of attention in both the industrial (see for example Muetze and Tan (2007); Lataire et al. (2003) for a detailed analysis of the market) and the academic field (see Spagnol et al. (2012, 2013a,b)).

A considerable amount of EPAC models exist; they are differentiated by design, battery technology, motor packaging (in the wheel, or attached to the pedals), and different sensors layout. However, from the functional perspective, most EPAC's can be classified into two main families:

- throttle-based bicycles. They are the simplest from the control point of view. The cyclist determines the level of assistance with a throttle-like interface. This solution is cost-effective not requiring any additional sensor, but it has been criticized for its unnatural feeling.
- Torque-based bicycles. Thanks to the pedal torque measurement, a genuine torque multiplying assistance can be implemented. This simplifies the control logic and yields a very natural feeling. However torque sensors are expensive and their processing may be tricky in some conditions (see for example Spagnol et al. (2013b)).

Recently a third option has arisen. The idea is to use advanced control logic and estimation algorithms to provide a natural cycling experience without the added cost of a torque sensor. This idea (dubbed Bike+) has been firstly presented in Spagnol et al. (2012, 2013a) where, among other things, a charge maintaining strategy has been presented and analyzed.

The crucible for EPAC's feeling is the level of assistance and response they provide when going up or down an incline. Torque-based solutions do not require any ad-hoc solution; by simply amplifying the pedaling torque, they correctly and automatically adjust the level of assistance. In order to provide the same feeling, Bike+ requires an accurate slope estimation to determine the level of assistance. The estimation must be accurate (a 1 deg slope when cycling considerably increases the fatigue) and responsive.

This paper deals with the on-board road slope estimation for a bicycle not equipped with torque sensors. The slope estimation is based on a suite of acceleration and gyrometric sensors, considerably more cost-effective, reliable and requiring less maintenance than torque sensors.

The issue of road slope estimation in vehicular applications is not new. Most of the wheeled vehicles literature focus on 4-wheeled vehicles (Hsu and Chen (2010), Sebsadji et al. (2008), Schmidtbauer and Lingman (2003)), especially Heavy Duty Vehicles, in which mass can vary significantly(Vahidi et al. (2003a),Vahidi et al. (2003b),Johansson (2005)). These estimations rely on a vehicle longitudinal dynamic model (see Bae et al. (2001), Han and Rizos (1999), Sahlholm and Henrik Johansson (2010), Johansson (2005), Sahlholm et al. (2007a), Sahlholm et al. (2007b)). It is possible to prove that dynamic model-based approaches require the measurement of the input torque for the road slope to be observable.

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A similar method, requiring the measurement of pedaling torque, has been applied to bicycles by Mammar et al. (2011).

Kinematic model-based estimation better fits the needs of Bike+. They rely on the relationship between acceleration, velocity and angular rates to estimate the vehicle attitude. Their main advantage is that they are less affected by model uncertainties. Solution based on Extended Kalman Filters (see Corno and Lovera (2009)) have a long story of successes in the aerospace field going back to the Apollo program. More recently they have also been applied to powered 2-wheelers by Boniolo et al. (2010) and Boniolo and Savaresi (2010). The authors use an Extended Kalman Filter to estimate the lean and pitch angles of a motorcycle. This estimation is based on a kinematic model of sensor measurements with respect to the body reference frame of the motorcycle, using a complete (3 accelerometers and 3 gyros) inertial measurement unit. In the context of the present contribution, this solution is taken as a benchmark.

In this work a slope estimation algorithm based on just one accelerometer, one gyro and the knowledge of vehicle speed is proposed and compared against the 6 Degrees of Freedom (DOF) benchmark. The performances of both algorithms are not satisfactory when the bicycle is in a curve; a method to account for this fact is proposed.

The paper is structured as follows. Section 2 recalls some basic concepts on the complete 6 degrees of freedom attitude estimation algorithm. The architecture is presented and a simplified model employed to explain its main features. Section 3 presents the reduced sensor estimation algorithm along with the curve correction algorithm. Section 4 validates the proposed approach on data collected on urban cycling. Conclusions are then drawn in Section 5.

2. COMPLETE EXTENDED KALMAN FILTER

The following approach is described in details in Boniolo and Savaresi (2010) and in Boniolo et al. (2010) where it is applied to a motorcycle. The vehicle attitude

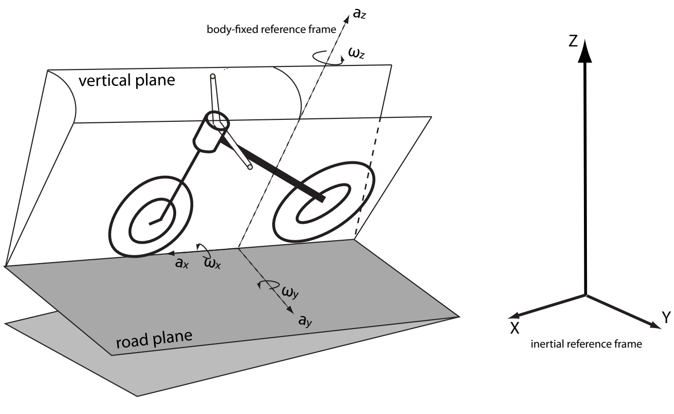

Fig. 1. Body reference frame definition and orientation of the measurement axes

is parametrized through the Roll-Pitch-Yaw angles. The rotational matrix R_{ZXY} in (1) defines the geometrical relation between the body reference frame and the inertial reference (see Figure 1) where ϕ is the roll angle, θ is the pitch angle and ψ is the yaw angle of the vehicle. c_*,s_*,t_* represent the cosine, sine or tangent of angle $*$.

$$R_{ZXY} = \begin{pmatrix} c_\theta c_\psi - s_\phi s_\theta s_\psi & c_\theta s_\psi + s_\phi s_\theta c_\psi & -c_\phi s_\theta \\ -c_\phi s_\psi & c_\phi c_\psi & s_\phi \\ s_\theta c_\psi + s_\phi c_\theta s_\psi & s_\theta s_\psi - s_\phi c_\theta c_\psi & c_\phi c_\theta \end{pmatrix}. \quad (1)$$

From R_{ZXY} the relation between the time derivatives of the Euler angles and the measured angular rates (ω_*) on the vehicle is derived as

$$\begin{pmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{pmatrix} = \begin{pmatrix} c_\theta & 0 & -s_\theta \\ t_\phi s_\theta & 1 & -t_\phi c_\theta \\ -s_\theta / c_\phi & 0 & c_\theta / c_\phi \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_x \end{pmatrix}. \quad (2)$$

A simplified formulation of the kinematic accelerations can be deduced observing that the principal terms affecting the measured accelerations in a two-wheeled vehicle are the gravitational acceleration g, the longitudinal acceleration ($\dot v_x$) and the centrifugal acceleration ($\dot\psi v_x$). The gravitational acceleration is expressed in the inertial reference frame, while the longitudinal and centrifugal acceleration can be expressed in a reference frame that is rotated by an angle ψ around the absolute Z axis with respect to the inertial frame, then:

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \begin{pmatrix} -c_\phi s_\theta g + c_\theta \dot v_x + s_\phi s_\theta \dot\psi v_x \\ s_\phi g + c_\phi \dot\psi v_x \\ c_\phi c_\theta g + s_\theta \dot v_x - s_\phi c_\theta \dot\psi v_x \end{pmatrix}. \quad (3)$$

Substituting (2) in (3), one gets a set of differential equations. These equations are arranged in a nonlinear dynamic system

$$\begin{aligned} \dot x &= f(x, u) + \eta_{x_{NL}} \\ y &= g(x, u) + \eta_{y_{NL}} \end{aligned} \quad (4)$$

defining

$$x = \begin{pmatrix} \phi \\ \theta \end{pmatrix} \quad u = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ g \\ \dot v_x \\ v_x \end{pmatrix} \quad y = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \quad (5)$$

and the zero mean process and measured noise $\eta_{x_{NL}}$ and $\eta_{y_{NL}}$.

Model (4) is the basis to develop a discrete-time Extended Kalman Filter that estimates the roll and pitch angle with respect to the inertial reference. Under the hypotheses of a rigid frame, the vehicle pitch angle is the road slope.

The derivation of the model is based on the main simplifying assumption that the accelerations are measured at the center of gravity. In most applications (and especially in bicycles) this cannot be guaranteed and, as a consequence, the estimation algorithm does not account for the following contributions:

- Translational lateral acceleration: effect of the sideslip of the vehicle
- Translational vertical acceleration: contribution of the heave dynamic of the vehicle and COG elevation
- Angular accelerations: centrifugal and tangential contribution of the accelerations due to roll rate and pitch rate
- Displacement: the difference between the position of the Center of Gravity (COG) of the vehicle and the mounting position of the accelerometers is neglected.

Both simulation and experimental tests showed excellent performance both in terms of lean and pitch tracking error with a root mean square of the estimation error around 2%.

3. REDUCED SENSOR SET KALMAN FILTER

The above estimation algorithm is considered the state-of-the-art for motorcycle applications. Its main drawback is that it requires a complete IMU. In the following a kinematic road slope estimation requiring a reduced set of sensor is presented. Fig. 2 shows the main idea of the algorithm: the acceleration measured by the accelerometer

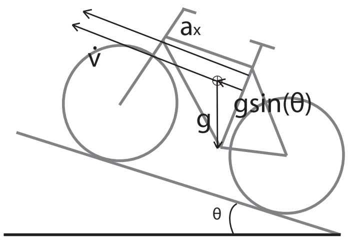

Fig. 2. Kinematic representation of a bike during slope

a_x is the sum of the longitudinal velocity rate of change $\dot{v}$ (obtained through numerical differentiation of the wheel velocity and thus measured along the inclined direction) and the gravitational contribution that depends on the road grade angle $g\sin(\theta)$. Simple trigonometry yields:

$$\sin(\theta) = \frac{a_x - \dot{v}}{g} \tag{6}$$

The simple use of the trigonometry law (presented in H.Ohnishi et al. (2000)) is heavily affected by accelerometer noise and noise generated by the differentiation of the wheel speed. An alternative solution is that of implementing a Kalman filter on the longitudinal kinematic model exemplified by Figure 2. The model is augmented with a fictitious road slope dynamics:

$$\begin{cases} \dot{v}(t) = -g\theta^*(t) + a_x(t) + \eta_{x1}(t) \\ \dot{\theta}^*(t) = \eta_{x2}(t) \\ y(t) = v(t) + +\eta_y(t) \end{cases} \tag{7}$$

where for simplicity $\theta^*(t) = \sin(\theta(t))$. This simple substitution yields a linear system with state vector, and the input and outputs:

$$x = \begin{pmatrix} v \\ \theta^* \end{pmatrix}, \quad u = a_x, \quad y = v. \tag{8}$$

The system is observable and the noises η_x and η_y are assumed to be uncorrelated with covariance matrices Q and R. Under these hypotheses, a Kalman filter can be designed to estimate the state vector x. Q and R have been tuned minimizing $J_{RMSE}(\tilde{\theta})$ - the RMSE between $\hat{\theta}$ (estimated using the reference 6 DOF EKF) and the $\tilde{\theta}$ (estimation with the longitudinal acceleration and velocity), yielding

$$Q = \begin{pmatrix} 2.406e-3 & 0 \\ 0 & 2.7e-3 \end{pmatrix} \quad R = 0.089. \tag{9}$$

Figure 3 and 4 compare the two estimates. The first figure shows the urban itinerary employed in the testing; the second shows the outputs of the two algorithms. The RMS difference between the two slope estimates is around 0.5°.

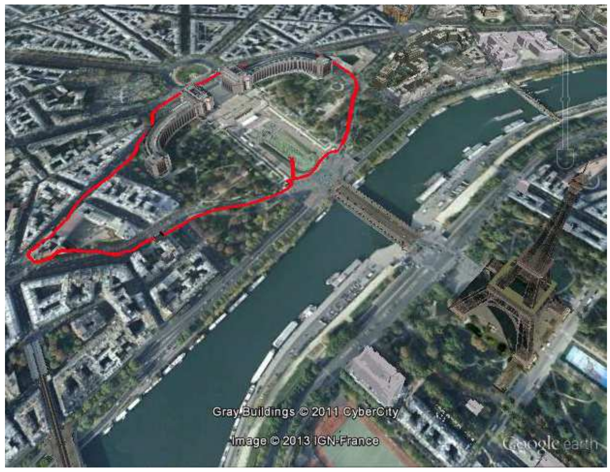

Fig. 3. Test itinerary.

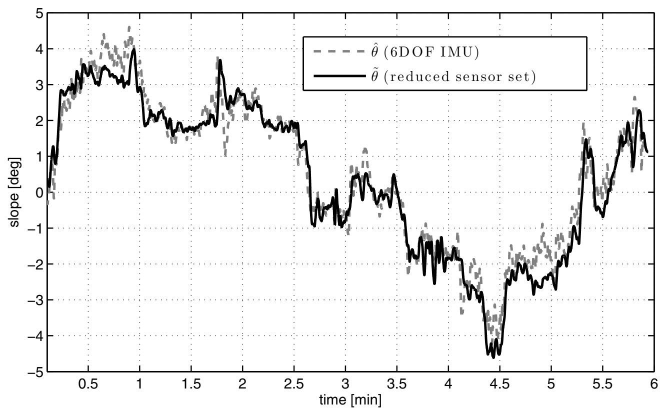

Fig. 4. Experimental comparison between $\hat{\theta}$ and $\tilde{\theta}$

The comparison shows that, in bicycles, a complete 6 DOF IMU does not yield an improvement in the estimation. Despite the overall good agreement of the two estimates, locally errors up to 1° are observed. In what follows these errors are further analyzed and addressed.

Figure 5 plots the algorithms output during a sine sweep test performed at approximately constant velocity on a flat road. Despite the road being flat, both algorithms output a maximum slope of -4°. This discrepancy is not sensitive to the filters tuning and is strongly correlated with the steering action. When the bicycle is in a curve, the discrepancies are bigger. During cornering and slope variation a number of hypotheses in both approaches break down. First of all the steering geometry of a bicycle is such that for non-null steering angle the bicycle pitches. Secondly, more specifically, It is easy to understand that the reduced sensor approach, being based on a longitudinal model, cannot be accurate when the gravitational acceleration has an out-of-plane component. As for the complete approach, its main limitation is due to the hypotheses that the rotations happen around the center of gravity where the IMU is installed. This is not exact because the roll rotation instantaneous center of rotation is along the line connecting the contacts points of the two tires. The errors introduced by these phenomena are negligible for motorcycle applications, but may be relevant to bicycles.

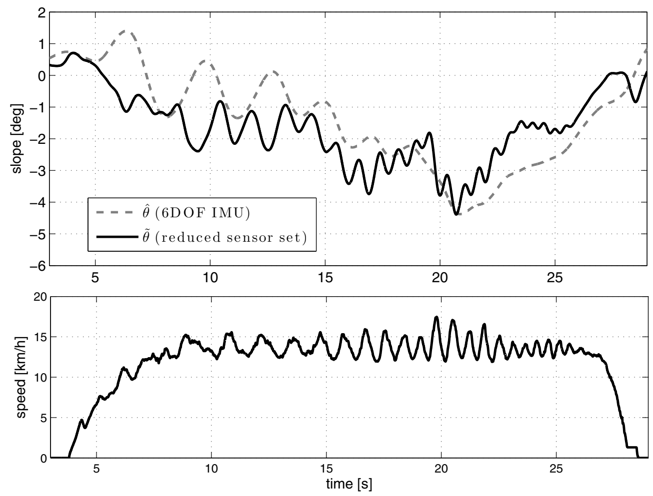

Fig. 5. Slope estimation by means of EKF and KF for a steering sweep test.

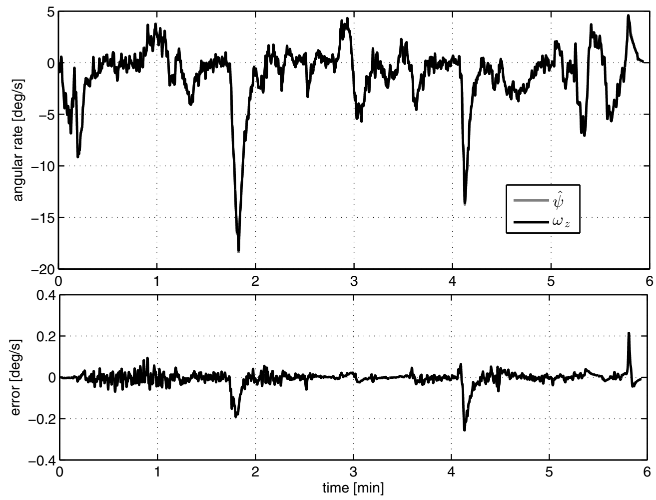

Fig. 6. Comparison between yaw rate estimation $\hat{\dot{\psi}}$ and gyro measurement ω_z.

The issue is addressed by adding a mechanism that weights the confidence of the current slope estimation as a function of the cornering state of the vehicle. This is reasonable because:

- aggressive curves, at high speed and small radius are rare events during normal cycling.
- It is not advisable to change the level of electric assistance while cornering, even if a sudden change of slope is detected. Unexpected chances in the traction force may startle the cyclist and destabilize the vehicle (see Tanelli et al. (2009); De Filippi et al. (2013) for a detailed analysis of single-track vehicle stability).

A time-varying filter implements this confidence scheduling mechanism. The filter bandwidth is modified according to the level of *aggressiveness* of the curve so that, if the bike is cornering, the estimate is updated more slowly.

The yaw rate $\dot{\psi}$, *i.e.* the angular rate around the inertial Z axis quantifies the *aggressiveness* of the curve. As the roll angles and road slope are small the yaw rare along Z is approximated by w_z. Figure 6 compares $\dot{\psi}$ and ω_x showing that when used as a scheduling index the two variables are essentially the same. The index of interest is therefore

$$\tilde{J}_{curve} = G(s)|\omega_z|. \tag{10}$$

The absolute value takes care of the fact that right and left hand turns need to be treated in the same way; and $G(s)$ is a linear low pass filter. $\tilde{J}_{curve}$ determines the cut-off frequency, f_c, of an additional low-pass filter that filters the slope estimate:

$$\begin{cases} f_c = \dfrac{f_{c_{max}}}{\alpha\left(\tilde{J}_{curve} - \bar{J}_{curve}\right)} \\ f_{c_{min}} < f_c < f_{c_{max}} \end{cases} \tag{11}$$

where $f_{c_{min}}$ and $f_{c_{max}}$ are respectively the minimum and maximum allowed cutoff frequencies [Hz]. Table 1 summarizes the tuning parameters.

Figure 7 depicts the complete reduced sensor road slope estimation scheme.

Parameter	Value
$\bar{J}_{curve}$	$11.5\frac{\circ}{s}$
$f_{c_{min}}$	0.005 Hz
$f_{c_{max}}$	1 Hz
α	15

Table 1. Parameters values for curve correction algorithm.

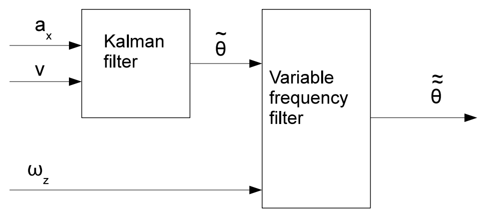

Fig. 7. Slope estimation algorithm scheme.

4. VALIDATION

Two kind of tests are performed to asses the overall road slope estimation scheme. Figure 8 plots the results of the scheme when applied to the same data of Figure 5. The figure clearly shows that when the bicycle starts the weaving trajectory the update of the road slope is slowed down and the estimation errors caused by the out-of-plane dynamics are filtered out. As soon as $\tilde{J}_{curve}$ crosses the limit $\overline{J}_{curve}$ the filter starts to limit the effect of the innovation in the estimate.

A second experiment has been performed on a hilly tract of a urban path. Figure 9 shows $\tilde{\theta}$ and $\tilde{\tilde{\theta}}$. There are two main curves in the trip. The first one leads to a sudden peak of $\tilde{\theta}$ that is removed with the curve correction algorithm. In the case of the urban trip the road slope estimate is compared with altitude data obtained from a cartographic database google inc. (2013) (with a claimed spatial resolution of of 9.5m). Cartographic data do not report slope, but rather altitude; consequently two representations of the same

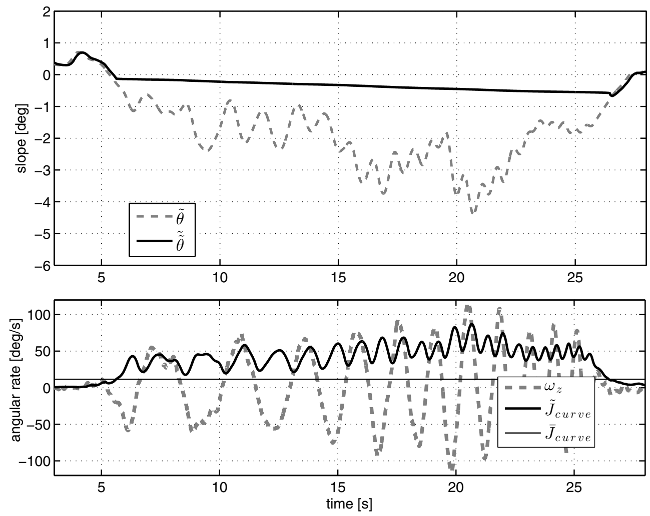

Fig. 8. Comparison between $\tilde{\theta}$ and $\tilde{\tilde{\theta}}$ during sweep test.

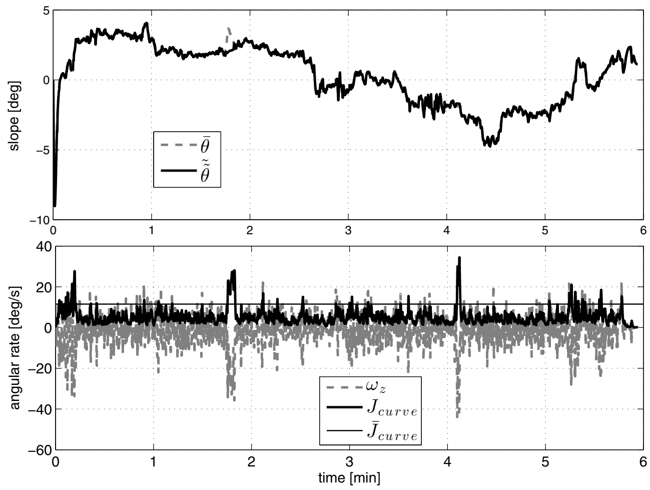

Fig. 9. Effect of the curve correction scheme on a hilly urban trip.

data are provided. In Figure 10 the cartographic altitude is compared against the integrated road slope:

$$
\begin{aligned}
h(s) &= \int_0^s \sin\left(\tilde{\tilde{\theta}}(s)\right) ds \\
s(t) &= \int_0^t v(t) dt
\end{aligned}
\tag{12}
$$

where $s(t)$, is the curvilinear abscissa describing the covered distance from the initial time to time t, $v(t)$ is the bike speed, $\tilde{\tilde{\theta}}(x)$ is the estimated slope in x, and $h(x)$ is the altitude as a function of s. In the comparison, the two initial altitudes have been set equal. This integral approach confirms the absence of any bias in the estimation. A bias would be integrated over time and cause a drift in the estimation. Table 2 reports the quantitative comparison.

The integral approach focuses on low frequency accuracy. The high frequency accuracy is evaluated by Figure 11; which plots the comparison between the istantaneous estimated road slope and the differentiated altitude from the cartographic data. The altitude differentiation introduces

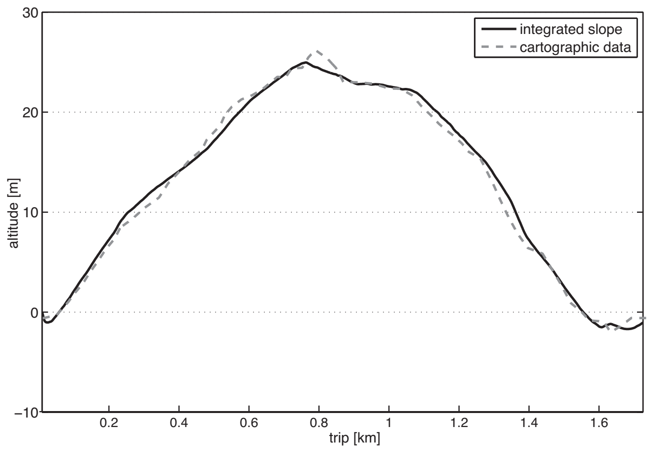

Fig. 10. Comparison between cartographic altitudes and integrated altitude as a function of the curvilinear abscissa.

Index	Value
$J_{L_2^2}(h(x))$	1%
$J_{RMSE}(h(x))$	0.94 m

Table 2. Integral performance indexes $J_{L_2^2}(h(x))$ and $J_{RMSE}(h(x))$

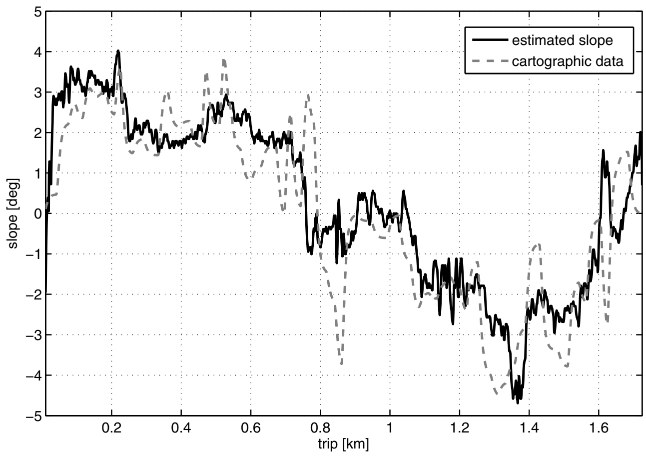

Fig. 11. Comparison between slope calculated form GoogleMaps altitude data and $\tilde{\tilde{\theta}}$. The general behavior is quite similar, but there are huge punctual difference due to lack of precision and spatial discretization and interpolation of GoogleMaps altitude data

high frequency noise, which makes the quantitative comparison of the estimate difficult; nevertheless note that the road slope is rather well described by the algorithm.

5. CONCLUSIONS

In this paper the issue of road slope estimation for bicycles without torque measurement has been addressed. It has been shown that, given the impossibility of relying on a dynamic model, a kinematic approach is the preferred approach. Two estimation algorithms have been discussed. A full 6DoF Extended Kalman Filter approach and a simpler

2 DoF approach based on only the longitudinal kinematic model. It has been shown that the performance of the two approaches is similar, thus opening the possibility of a more cost-effective solution.

The features of bicycle dynamics lead to error in both methods during cornering; a curve correction algorithm has been thus presented. Using an additional gyro, it is possible to detect when the bicycle is cornering and adjust the slope estimation accordingly.

The reduced sensor set approach has been finally tested comparing it against the full sensor suite approach and cartographic data.

REFERENCES

Bae, H.S., Ryu, J., and Gerdes, J.C. (2001). Road grade and vehicle parameter estimation for longitudinal control using GPS. In *IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC*, 166–171.

Boniolo, I. and Savaresi, S. (2010). *Estimate of the Lean Angle of Motorcycles: Design and Analysis of Systems for Measuring and Estimating the Attitude Parameters of Motorcycles*. VDM Publishing.

Boniolo, I., Corbetta, S., and Savaresi, S. (2010). Attitude Estimation of a Motorcycle in a Kalman Filtering Framework. In *Advances in Automotive Control*, 779–784.

Corno, M. and Lovera, M. (2009). Spacecraft attitude dynamics and control in the presence of large magnetic residuals. *Control Engineering Practice*, 17(4), 456–468.

De Filippi, P., Tanelli, M., Corno, M., Savaresi, S.M., and Santucci, M.D. (2013). Electronic stability control for powered two-wheelers. *IEEE Transactions on Control Systems Technology*, PP(99), 1–1.

google inc. (2013). Google earth.

Han, S. and Rizos, C. (1999). Road slope information from GPS-derived trajectory data. *Journal of surveying engineering*, 125(2), 59–68.

H.Ohnishi, J.Ishii, M.Kayano, and H.Katayama (2000). A study on road slope estimation for automatic transmission control. *JSAE Review*.

Hsu, L.Y. and Chen, T.L. (2010). Estimating road angles with the knowledge of the vehicle yaw angle. *Journal of dynamic systems, measurement, and control*, 132(3).

Johansson, K. (2005). Road slope estimation with standard truck sensors. *KTH, Sweden*.

Lataire, P., Timmermans, J., Maggetto, G., Van den Bossche, P., et al. (2003). Electrically Assisted Cycling around the World. In *20 th International Electric VehicleSymposium (EVS 20)*.

Mammar, S., Oufroukh, N., and Glaser, S. (2011). Reducing rider effort for electric bicycles by environment disturbance compensation. In *Control Applications (CCA), 2011 IEEE International Conference on*, 550–555.

Muetze, A. and Tan, Y. (2007). Electric bicycles - A performance evaluation. *Industry Applications Magazine, IEEE*, 13(4), 12 –21. doi:10.1109/MIA.2007.4283505.

Sahlholm, P. and Henrik Johansson, K. (2010). Road grade estimation for look-ahead vehicle control using multiple measurement runs. *Control Engineering Practice*, 18(11), 1328–1341.

Sahlholm, P., Jansson, H., and Johansson, K.H. (2007a). Road grade estimation results using sensor and data fusion. In *ITSW*.

Sahlholm, P., Jansson, H., Kozica, E., and Johansson, K.H. (2007b). A sensor and data fusion algorithm for road grade estimation. In *ITSW*.

Schmidtbauer, B. and Lingman, P. (2003). Road slope and vehicle mass estimation using Kalman filtering. In *Proceedings of the 17th Iavsd Symposium. Lyngby*, volume 37, 12–23. Taylor & Francis.

Sebsadji, Y., Glaser, S., Mammar, S., and Dakhlallah, J. (2008). Road slope and vehicle dynamics estimation. In *American Control Conference, 2008*, 4603–4608. IEEE.

Spagnol, P., Alli, G., Spelta, C., Lisanti, P., Todeschini, F., Savaresi, S., and Morelli, A. (2012). A full hybrid electric bike: How to increase human efficiency. In *American Control Conference (ACC), 2012*, 2761–2766.

Spagnol, P., Corno, M., Mura, R., and Savaresi, S.M. (2013a). Self-sustaining strategy for a hybrid electric bike. In *American Control Conference (ACC), 2013*, 3479–3484.

Spagnol, P., Corno, M., and Savaresi, S.M. (2013b). Pedaling torque reconstruction for half pedaling sensor. In *Proceedings of the European Control Conference (ECC), 2013*, 275–280.

Tanelli, M., Corno, M., Boniolo, I., and Savaresi, S.M. (2009). Active braking control of two-wheeled vehicles on curves. *International Journal of Vehicle Autonomous Systems*, 7(3), 243–269.

Vahidi, A., Druzhinina, M., Stefanopoulou, A., and Peng, H. (2003a). Simultaneous mass and time-varying grade estimation for heavy-duty vehicles. In *Proceedings of the American Control Conference*, 4951–4956.

Vahidi, A., Stefanopoulou, A., and Peng, H. (2003b). Experiments for online estimation of heavy vehicles mass and time-varying road grade. *Proceedings IMECE DSCD*.