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Adaptive PD Power-Level Control for Pressurized Water Reactors

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Abstract: It is well known that the pressurized water reactor (PWR) is the most widely utilized nuclear fission reactor whose safe, stable and efficient operation is meaningful to the nowadays renaissance of nuclear energy industry. Power-level regulation is a significant technique for guaranteeing both operational stability and efficiency of nuclear reactors. Since every nuclear reactor is a complex nonlinear system with high parameter uncertainties, it is necessary to develop the adaptive power-level control technique that can strengthen closed-loop stability, guarantee load-following performance and be un-sensitive to those parameter uncertainties. In this paper, an adaptive proportional-differential (PD) control is proposed for the power-level regulation of the PWRs, which is theoretically proved to be globally asymptotically stabilizable and can be tuned online with a well-designed adaptation law. Further-more, numerical simulation results show not only the feasibility of this newly-built regulator but also the relationship between the control performance and the parameter of the adaptation law.

Keywords: Adaptive Control, PD Control, Power-Level, PWR, Stability

1. INTRODUCTION

Since the pressurized water reactor (PWR) is the most widely utilized nuclear reactor, safe, stable and efficient operation of PWRs is quite important to the renaissance of fission energy industry. Power-level regulation which strengthens both the closed-loop stability and dynamic response is meaningful to provide a satisfactory operation of the PWRs. Due to the high nonlinearity of reactor dynamics especially in case of large-range power maneuver, it is quite necessary to develop nonlinear power-level control technique for guaran-teeing load-following performance. Shtessel gave a nonlinear power-level control law which is composed of a static state-feedback sliding mode controller and a sliding mode observer for space nuclear reactor TOPAZ II (Shtessel, 1998). Based on dissipation-based high gain filter (DHGF) (Dong, Feng, Huang, and Zhang, 2010), Dong gave an output feedback power-level control for the PWR (Dong, Huang and Zhang, 2011). By the use of the techniques of both the backstepping (Kokotović, 1992) and DHGF, Dong proposed a nonlinear dynamic output-feedback power-level control for the PWRs (Dong, 2011a). Through constructing the control Lyapunov functions based on the concepts of ectropy (Haddad, Chellaboina, and Nersesov, 2005) and shifted-ectroy (Dong, 2012), Dong gave a novel state-feedback PWR power-level controller which was then coupled with the DHGF to form a dynamic output feedback power-level control with the DHGF (Dong, 2013). Further, since model predictive control (MPC) technique has the ability to handle the state, input and output constraints, Eliasi et al. applied nonlinear MPC (NMPC) to power-level regulation of the PWRs (Eliasi, Menhaj, Davilu, 2011; Eliasi, Menhaj, Davilu, 2012). The above nonlinear power-level control laws can well provide the load-following performance if the dynamic models used for control design is accurate enough. However, since the system parameters of every nuclear reactor are influenced by many factors such as the power-level, fuel burnup, Xenon isotope production, control rod worth and etc., there must be modeling uncertain-ties of reactor dynamics. Therefore, it is very necessary to develop adaptive power-level control technique which can provide closed-loop stability with the existence of modeling uncertainties. Park and Cho proposed a proportional-integral (PI) control with feedback gains tuned online by an adaptive law for the nuclear power (Park, Cho, 1993). Arab-Alibeik designed an adaptive power-level control law by the use of a feed-forward neural network which is trained online (Arab-Alibeik, Setayeshi, 2005). Recently Dong (2013b) proposed a nonlinear dynamic output-feedback adaptive power-level control law which provides not only the globally asymptotic closed-loop stability but also adaption capability to system uncertainties. Though the above adaptive power level control laws may have high control performance, their complicated forms leads to the difficult implementation. Very recently, Dong (2014) gave the sufficient conditions for the PWRs to be asymptotically self-stable, and proved theoretically that a simple proportional-differential (PD) static output feedback control law could provide globally asymptotic closed-loop stability for the PWRs (Dong, 2013c). However, the feedback gains of this simple PD control are tightly related with the reactor parameters, which means that these feedback gains must be disturbed by modeling uncertainties. This is the main drawback for the practical implementation of this control.

In this paper, an adaptive PD power-level control strategy is presented. The feedback gains of this PD control are tuned on-line, and the globally asymptotic closed-loop stability is well guaranteed. Simulation results verify the feasibility of results, and show the relationship between the regulation performance and the controller parameters. This PWR power-level control law has the virtue of easy implementation and handling system uncertainties.

2. PROBLEM FORMULATION

2.1 Nonlinear State-Space Model for Control Design

The dynamic model for power-level control law design still adopts the point kinetics with one equivalent delayed neutron group and reactivity feedback induced by the average fuel and coolant temperatures (Schultz, 1961; Dong, 2013bc):

$$\begin{cases} \dot{n}_{\mathrm{r}} = \dfrac{\rho_{\mathrm{r}} - \beta}{\Lambda} n_{\mathrm{r}} + \dfrac{\beta}{\Lambda} c_{\mathrm{r}} + \dfrac{n_{\mathrm{r}}}{\Lambda}\left[\alpha_{\mathrm{f}}\left(T_{\mathrm{f}} - T_{\mathrm{f,m}}\right) + \alpha_{\mathrm{c}}\left(T_{\mathrm{cav}} - T_{\mathrm{cav,m}}\right)\right], \\ \dot{c}_{\mathrm{r}} = \lambda n_{\mathrm{r}} - \lambda c_{\mathrm{r}}, \\ \dot{T}_{\mathrm{f}} = -\dfrac{\Omega}{\mu_{\mathrm{f}}} T_{\mathrm{f}} + \dfrac{\Omega}{\mu_{\mathrm{f}}} T_{\mathrm{cav}} + \dfrac{P_0}{\mu_{\mathrm{f}}} n_{\mathrm{r}}, \\ \dot{T}_{\mathrm{cav}} = -\dfrac{2M+\Omega}{\mu_{\mathrm{c}}} T_{\mathrm{cav}} + \dfrac{\Omega}{\mu_{\mathrm{c}}} T_{\mathrm{f}} + \dfrac{2M}{\mu_{\mathrm{c}}} T_{\mathrm{cin}}, \\ \dot{\rho}_{\mathrm{r}} = G_{\mathrm{r}} z_{\mathrm{r}}. \end{cases} \tag{1}$$

where n_r is the relative nuclear power, β is the fraction of the delayed neutrons, c_r is the relative concentration of delayed neutron precursor, Λ is the effective prompt neutron lifetime, λ is the decay constant of delayed neutron precursor, α_f and α_c are the reactivity feedback coefficients of the fuel and coolant temperatures respectively, T_f is the average fuel temperature, T_{cav} and T_{cin} are respectively the average and the inlet coolant temperature, $T_{f,m}$ and $T_{cav,m}$ are the initial steady values of T_{cav} and T_f respectively, Ω is the heat transfer coefficient between the fuel elements and coolant, M is the coolant mass flowrate times the coolant heat capacity, P_0 is the rated reactor thermal power, μ_f is the total heat capacity of the fuel elements, μ_c is the total coolant heat capacity of the reactor core, ρ_r is the the control rod reactivity, G_r is the differential reactivity worth of the control rods, and z_r is the control rod speed signal.

Define the variations of n_r, c_r, T_f, T_{cav}, T_{cin} and ρ_r relative to their steady values, i.e. n_{r0}, c_{r0}, T_{f0}, T_{cav0}, T_{cin0} and ρ_{r0} as $\delta n_r = n_r - n_{r0}$, $\delta c_r = c_r - c_{r0}$, $\delta T_f = T_f - T_{f0}$, $\delta T_{cav} = T_{cav} - T_{cav0}$, $\delta T_{cin} = T_{cin} - T_{cin0}$, and $\delta\rho_r = \rho_r - \rho_{r0}$. Moreover, let

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \delta n_{\mathrm{r}} & \delta c_{\mathrm{r}} & \delta T_{\mathrm{f}} & \delta T_{\mathrm{cav}} \end{bmatrix}^{\mathrm{T}}, \tag{2}$$

$$\xi = \delta\rho_{\mathrm{r}}, \tag{3}$$

and

$$u = G_{\mathrm{r}} z_{\mathrm{r}}. \tag{4}$$

Here, $\boldsymbol{x}$ is called the reactor state, and both δn_r and δT_{cav} can be obtained directly by measurement. Since δT_{cin} reflects the influence of the secondary loop to primary loop, it is omitted here by assuming that the secondary loop is well operated.

Then, the nonlinear state-space model for control design can be written as

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\xi, \\ \dot{\xi} = u, \\ y = \boldsymbol{h}(\boldsymbol{x}), \end{cases} \tag{5}$$

where

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} -\dfrac{\beta}{\Lambda}(x_1 - x_2) + \dfrac{n_{\mathrm{r}}}{\Lambda}(\alpha_{\mathrm{f}} x_3 + \alpha_{\mathrm{c}} x_4) \\ \lambda(x_1 - x_2) \\ -\dfrac{\Omega}{\mu_{\mathrm{f}}}(x_3 - x_4) + \dfrac{P_0}{\mu_{\mathrm{f}}} x_1 \\ \dfrac{\Omega}{\mu_{\mathrm{c}}}(x_3 - x_4) - \dfrac{2M}{\mu_{\mathrm{c}}} x_4 \end{bmatrix}, \tag{6}$$

$$\boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} \dfrac{n_{\mathrm{r0}} + \delta n_{\mathrm{r}}}{\Lambda} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{7}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} x_1 & x_4 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \delta n_{\mathrm{r}} & \delta T_{\mathrm{cav}} \end{bmatrix}^{\mathrm{T}}. \tag{8}$$

Since δT_{cin} reflects the influence of the secondary loop to the primary loop, it is omitted in forming state-space model (5).

2.2 Theoretic Problem

It is clear that static output feedback control is much easier to be implemented than the dynamic output feedback controllers given by incorporating a static state-feedback controller and a state-observer. Thus, it is attractive for a static output feedback power-level controller to guarantee closed-loop stability while to be adaptive to parameter perturbations. Motivated by this need, the theoretic problem to be solved is summarized as follows.

Problem. How to design an adaptive output-feedback PD control of nonlinear system (5) so that $\boldsymbol{x}\to\boldsymbol{O}$ as $t\to\infty$?

3. ADAPTIVE PD CONTROL DESIGN

In this section, a static output feedback power-level control with an adaptive feedback gain scheduling algorithm is given. The design of this control is summarized as the following Theorem 1, which is the main result of this paper and solves the above Problem.

Theorem 1. Consider nonlinear system (5) with power-level controller taking the form as

$$u = -\left(k_{\mathrm{np}} x_1 + k_{\mathrm{nd}} \dot{x}_1 + k_{\mathrm{cp}} x_4 + k_{\mathrm{cd}} \dot{x}_4\right), \tag{9}$$

where

$$k_{\mathrm{np}} = k_{\mathrm{np0}} + q_{\mathrm{d}} x_4^2 + q_{\mathrm{p}} \left[\int_{t_0}^{t} x_4(\tau)\mathrm{d}\tau\right]^2, \tag{10}$$

$$k_{\mathrm{nd}} \geq \frac{h_{\mathrm{f}}^2 P_0 (1+\gamma)(\Omega + 2M)}{4M\Omega\theta_{\mathrm{f}}}, \tag{11}$$

$$k_{\mathrm{cp}} = k_{\mathrm{cp0}} + \hat{k}_{\mathrm{cp}}, \tag{12}$$

$$k_{\mathrm{cd}} = k_{\mathrm{cd0}} + \hat{k}_{\mathrm{cd}}, \tag{13}$$

$$\theta_{\mathrm{f}} = h_{\mathrm{f}} - \alpha_{\mathrm{f}} > 0, \tag{14}$$

k_{nd}, k_{np0}, k_{cp0}, k_{cd0}, q_d, q_p, h_f and γ are all positive constants, and $\hat{k}_{cp}$ and $\hat{k}_{cd}$ are tuned online by adaptation laws given by

$$\dot{\hat{k}}_{\mathrm{cd}} = q_{\mathrm{d}} x_1 x_4, \tag{15}$$

$$\dot{\hat{k}}_{\mathrm{cp}} = q_{\mathrm{p}} x_1 \int_{t_0}^{t} x_4(\tau)\mathrm{d}\tau. \tag{16}$$

respectively. Then reactor state $\boldsymbol{x}$ is globally asymptotically stable, i.e. $\boldsymbol{x}\rightarrow\boldsymbol{O}$ as $t\rightarrow\infty$.

Proof: Firstly, consider the adaptive control design problem of subsystem

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{g}(\boldsymbol{x})\xi_{\mathrm{r}},\\ y=\boldsymbol{h}(\boldsymbol{x}),\end{cases} \tag{17}$$

where ξ_{r} is a virtual control input, state $\boldsymbol{x}$ is defined by (2), and vector-valued functions $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$ are given by equations (6), (7) and (8) respectively.

The shifted-ectropies of reactor neutron kinetics and thermal-hydraulics can be written respectively as

$$\zeta_{\mathrm{C,N}}(x_1,x_2)=\Lambda\left[\left(1+\frac{x_1}{n_{\mathrm{r0}}}\right)-\ln\left(1+\frac{x_1}{n_{\mathrm{r0}}}\right)\right]+\frac{\beta}{\lambda}\left[\left(1+\frac{x_2}{n_{\mathrm{r0}}}\right)-\ln\left(1+\frac{x_2}{n_{\mathrm{r0}}}\right)\right], \tag{18}$$

$$\zeta_{\mathrm{C,T}}(x_3,x_4)=\frac{1}{2}\left(\mu_{\mathrm{f}}x_3^2+\mu_{\mathrm{c}}x_4^2\right), \tag{19}$$

which are apparently both positive-definite functions.

Then, based on (18) and (19), choose the control Lyapunov function of subsystem (17) as

$$V_1(\boldsymbol{x})=\zeta_{\mathrm{C,N}}(x_1,x_2)+\frac{\theta_{\mathrm{f}}}{(1+\gamma)P_0}\left[\zeta_{\mathrm{C,T}}(x_3,x_4)+\gamma\eta(x_3,x_4)\right], \tag{20}$$

where

$$\eta(x_3,x_4)=\frac{1}{2\mu_{\mathrm{f}}}\left[\mu_{\mathrm{f}}x_3+\mu_{\mathrm{c}}x_4+2M\int_{t_0}^{t}x_4(\tau)\,\mathrm{d}\tau\right]^2. \tag{21}$$

Differentiate $V_1(\boldsymbol{x})$ along the trajectory given by (17),

$$\begin{aligned}\dot{V}_1(\boldsymbol{x})=&-\frac{\beta(x_1-x_2)^2}{(n_{\mathrm{r0}}+x_1)(n_{\mathrm{r0}}+x_2)}-\frac{2M\theta_{\mathrm{f}}}{(1+\gamma)\kappa P_0}x_3^2-\\&\frac{\theta_{\mathrm{f}}\kappa}{(1+\gamma)\Omega P_0}\left(x_4-\frac{x_3}{\kappa}\right)^2+x_1\left[\xi_{\mathrm{r}}+h_{\mathrm{f}}x_3+\right.\\&\left.\left(\alpha_{\mathrm{c}}+\frac{\mu_{\mathrm{c}}}{\mu_{\mathrm{f}}}\overline{\theta}_{\mathrm{f}}\right)x_4+\frac{2M}{\mu_{\mathrm{f}}}\overline{\theta}_{\mathrm{f}}\int_{t_0}^{t}x_4(\tau)\,\mathrm{d}\tau\right],\end{aligned} \tag{22}$$

where

$$\kappa=1+\frac{2M}{\Omega}, \tag{23}$$

and

$$\overline{\theta}_{\mathrm{f}}=\frac{\gamma}{1+\gamma}\theta_{\mathrm{f}}. \tag{24}$$

Design virtual control ξ_{r} as

$$\xi_{\mathrm{r}}=-\left[k_{\mathrm{nd}}x_1+k_{\mathrm{cd}}x_4+k_{\mathrm{cp}}\int_{t_0}^{t}x_4(\tau)\,\mathrm{d}\tau\right], \tag{25}$$

where k_{nd} is positive, and k_{cp} and k_{cd} are given by (12) and (13) respectively. Substitute equations (25) to (22),

$$\begin{aligned}\dot{V}_1(\boldsymbol{x})=&-\frac{\beta(x_1-x_2)^2}{(n_{\mathrm{r0}}+x_1)(n_{\mathrm{r0}}+x_2)}-\frac{\theta_{\mathrm{f}}\Omega}{(1+\gamma)P_0}\left(x_4-\frac{x_3}{\kappa}\right)^2-\\&\frac{M\theta_{\mathrm{f}}}{(1+\gamma)\kappa P_0}\left\{x_3^2+\left[x_3-\frac{\kappa h_{\mathrm{f}}P_0(1+\gamma)}{2M\theta_{\mathrm{f}}}x_1\right]\right\}^2-\\&\tilde{k}_{\mathrm{nd}}x_1^2-\tilde{k}_{\mathrm{cd}}x_1x_4-\tilde{k}_{\mathrm{cp}}x_1\int_{t_0}^{t}x_4(\tau)\,\mathrm{d}\tau,\end{aligned} \tag{26}$$

where

$$\tilde{k}_{\mathrm{nd}}=k_{\mathrm{nd}}-\frac{h_{\mathrm{f}}^2P_0(1+\gamma)(\Omega+2M)}{4M\Omega\theta_{\mathrm{f}}}, \tag{27}$$

$$\tilde{k}_{\mathrm{cd}}=\hat{k}_{\mathrm{cd}}+k_{\mathrm{cd0}}-\left(\alpha_{\mathrm{c}}+\frac{\mu_{\mathrm{c}}}{\mu_{\mathrm{f}}}\overline{\theta}_{\mathrm{f}}\right), \tag{28}$$

$$\tilde{k}_{\mathrm{cp}}=k_{\mathrm{cp}}-\frac{2M}{\mu_{\mathrm{f}}}\overline{\theta}_{\mathrm{f}}=\hat{k}_{\mathrm{cp}}+k_{\mathrm{cp0}}-\frac{2M}{\mu_{\mathrm{f}}}\overline{\theta}_{\mathrm{f}}. \tag{29}$$

Usually, for a given PWR, since its parameters are bounded, it is easy for us to choose a proper feedback gain k_{nd} such that inequality (11) holds, i.e. $\tilde{k}_{\mathrm{nd}}$ is a positive constant. Although the reactor parameters are bounded, there must be parameter uncertainties, which means that it is nearly impossible for us to set proper feedback gains k_{cd} and k_{cp} such that $\tilde{k}_{\mathrm{cd}}=\tilde{k}_{\mathrm{cp}}=0$. Here, the way to cope with parameter uncertainties is to see $\hat{k}_{\mathrm{cd}}$ and $\hat{k}_{\mathrm{cp}}$ as time-varying variables and design the adaptation laws. Then, choose the extended Lyapunov function as

$$V_2\left(\boldsymbol{x},\tilde{k}_{\mathrm{cp}},\tilde{k}_{\mathrm{cd}}\right)=V_1(\boldsymbol{x})+\frac{\tilde{k}_{\mathrm{cp}}^2}{2q_{\mathrm{p}}}+\frac{\tilde{k}_{\mathrm{cd}}^2}{2q_{\mathrm{d}}}. \tag{30}$$

Differentiate V_2 along the trajectory given by (17) and (25),

$$\begin{aligned}\dot{V}_2=&\dot{V}_1(\boldsymbol{x})+\frac{\tilde{k}_{\mathrm{cp}}\dot{\hat{k}}_{\mathrm{cp}}}{q_{\mathrm{p}}}+\frac{\tilde{k}_{\mathrm{cd}}\dot{\hat{k}}_{\mathrm{cd}}}{q_{\mathrm{d}}}\\=&-\frac{\beta(x_1-x_2)^2}{(n_{\mathrm{r0}}+x_1)(n_{\mathrm{r0}}+x_2)}-\frac{\theta_{\mathrm{f}}\Omega}{(1+\gamma)P_0}\left(x_4-\frac{x_3}{\kappa}\right)^2-\\&\frac{M\theta_{\mathrm{f}}}{(1+\gamma)\kappa P_0}\left\{x_3^2+\left[x_3-\frac{\kappa h_{\mathrm{f}}P_0(1+\gamma)}{2M\theta_{\mathrm{f}}}x_1\right]\right\}^2-\\&\tilde{k}_{\mathrm{nd}}x_1^2-\tilde{k}_{\mathrm{cd}}x_1x_4-\tilde{k}_{\mathrm{cp}}x_1\int_{t_0}^{t}x_4(\tau)\,\mathrm{d}\tau+\frac{\tilde{k}_{\mathrm{cp}}\dot{\hat{k}}_{\mathrm{cp}}}{q_{\mathrm{p}}}+\frac{\tilde{k}_{\mathrm{cd}}\dot{\hat{k}}_{\mathrm{cd}}}{q_{\mathrm{d}}},\end{aligned} \tag{31}$$

from which we can see that if

$$\dot{\hat{k}}_{\mathrm{cd}}=q_{\mathrm{d}}x_1x_4 \tag{32}$$

$$\dot{\hat{k}}_{\mathrm{cp}}=q_{\mathrm{p}}x_1\int_{t_0}^{t}x_4(\tau)\,\mathrm{d}\tau, \tag{33}$$

then

$$\begin{aligned}\dot{V}_2=&-\frac{\beta(x_1-x_2)^2}{(n_{\mathrm{r0}}+x_1)(n_{\mathrm{r0}}+x_2)}-\frac{\theta_{\mathrm{f}}\Omega}{(1+\gamma)P_0}\left(x_4-\frac{x_3}{\kappa}\right)^2-\\&\frac{M\theta_{\mathrm{f}}}{(1+\gamma)\kappa P_0}\left\{x_3^2+\left[x_3-\frac{\kappa h_{\mathrm{f}}P_0(1+\gamma)}{2M\theta_{\mathrm{f}}}x_1\right]\right\}^2-\tilde{k}_{\mathrm{nd}}x_1^2.\end{aligned} \tag{34}$$

Moreover, from equations (28) and (29), it is clear that

$$\dot{\tilde{k}}_{\mathrm{cd}} = \dot{\hat{k}}_{\mathrm{cd}} = \dot{k}_{\mathrm{cd}} , \quad (35)$$

$$\dot{\tilde{k}}_{\mathrm{cp}} = \dot{\hat{k}}_{\mathrm{cp}} = \dot{k}_{\mathrm{cp}} , \quad (36)$$

which directly leads to adaptation laws (15) and (16). Actually, equations (25), (15) and (16) forms the adaptive PI control law for subsystem (17). Finally, we give the adaptive stabilizer for entire system (5). Here, we define

$$e_{\xi} = \xi - \xi_{\mathrm{r}} , \quad (37)$$

and choose the control Lyapunov function of system (5) as

$$V_3\left(\boldsymbol{x}, \tilde{k}_{\mathrm{cp}}, \tilde{k}_{\mathrm{cd}}, e_{\xi}\right) = V_2\left(\boldsymbol{x}, \tilde{k}_{\mathrm{cp}}, \tilde{k}_{\mathrm{cd}}\right) + \frac{e_{\xi}^2}{2k_{\mathrm{n,p0}}} . \quad (38)$$

Differentiate V_3 along the trajectory given by (5),

$$\dot{V}_3 = -\frac{\beta\left(x_1 - x_2\right)^2}{\left(n_{\mathrm{r0}} + x_1\right)\left(n_{\mathrm{r0}} + x_2\right)} - \frac{\theta_{\mathrm{f}} \Omega}{\left(1+\gamma\right) P_0}\left(x_4 - \frac{x_3}{\kappa}\right)^2 - \frac{M\theta_{\mathrm{f}}}{\left(1+\gamma\right)\kappa P_0}\left\{x_3^2 + \left[x_3 - \frac{\kappa h_{\mathrm{f}} P_0\left(1+\gamma\right)}{2M\theta_{\mathrm{f}}} x_1\right]\right\}^2 - \tilde{k}_{\mathrm{nd}} x_1^2 + x_1 e_{\xi} + k_{\mathrm{np0}}^{-1} e_{\xi}\left(u - \dot{\xi}_{\mathrm{r}}\right). \quad (39)$$

Design control input u as

$$u = \dot{\xi}_{\mathrm{r}} - k_{\mathrm{np0}} x_1 , \quad (40)$$

where ξ_{r} is given by (25). Substitute (40) and (25) to (39),

$$\dot{V}_2\left(\boldsymbol{x}, e_{\xi}\right) \le -\frac{\beta\left(x_1 - x_2\right)^2}{\left(n_{\mathrm{r0}} + x_1\right)\left(n_{\mathrm{r0}} + x_2\right)} - \frac{\theta_{\mathrm{f}} \Omega}{\left(1+\gamma\right) P_0}\left(x_4 - \frac{x_3}{\kappa}\right)^2 - \frac{M\theta_{\mathrm{f}}}{\left(1+\gamma\right)\kappa P_0} x_3^2 - \tilde{k}_{\mathrm{nd}} x_1^2 , \quad (41)$$

from which the reactor state is globally asymptotically stable.

Finally, substitute (25) to (40), and based on equations (35) and (36), we have

$$u = -\frac{\mathrm{d}}{\mathrm{d}t}\left[k_{\mathrm{nd}} x_1 + k_{\mathrm{cd}} x_4 + k_{\mathrm{cp}} \int_{t_0}^{t} x_4\left(\tau\right) \mathrm{d}\tau\right] - k_{\mathrm{np0}} x_1$$

$$= -\left[k_{\mathrm{np0}} + q_{\mathrm{d}} x_4^2 + q_{\mathrm{p}}\left(\int_{t_0}^{t} x_4\left(\tau\right) \mathrm{d}\tau\right)^2\right] x_1 - k_{\mathrm{nd}} \dot{x}_1 - k_{\mathrm{cp}} x_4 - k_{\mathrm{cd}} \dot{x}_4 , \quad (42)$$

which is just PD power-level control (9). This completes the proof of this theorem.

4. SIMULATION RESULTS WITH DISCUSSIONS

To verify the feasibility and performance of the adaptive PD control law given by equations (9), (10), (15) and (16), it is applied to the power-level regulation of a new type of PWR, i.e. the nuclear heating reactor (NHR). The NHR is designed by Institute of Nuclear and New Energy Technology (INET) of Tsinghua University and has some advanced features such as the integral arrangement, self-pressurizing, entire-range natural circulation, hydraulic driving control rods and passive residual heat removing. Moreover, the NHR can be applied to not only power production but also some other areas such as district heating and seawater desalination (Wang, et al., 1992). The power-level of the NHR should tightly follow the load given by the grid in the case of electricity production, by the environment temperature in the case of district heating and by fresh-water demand in the case of sea-water desalination.

4.1 Description of the Numerical Simulation

The simulation model of the NHR is composed of the point kinetics model with six delayed neutron groups and the lumped dynamic models corresponding to reactor thermal hydraulics, primary heat exchanger, U-tube steam generator (UTSG), feedwater pump of the UTSG and some pipe or volume cells (Dong, Huang, Feng, and Zhang, 2009). Here, the UTSG water-level control adopts that one presented in (Dong, Huang, and Feng, 2009). The main parameters of the NHR in the middle stage of the fuel cycle at the full power-level are shown in Table 1.

Table 1. NHR Parameters at the Middle of the Fuel Cycle in100% Power-Level

Symbol	Quantity	Symbol	Quantity
β	0.0069	α_{f}	-3.85e-5 (1/℃)
Λ	1.0367e-4 (s)	α_{c}	-2.3e-4 (1/℃)
λ	0.08 (1/s)	G_{r}	0.0005
μ_{f}	588.544 (kWs/℃)	M	304.89 (kW/℃)
μ_{c}	25151 (kWs/℃)	Ω	125.68 (kW/℃)

4.2 Simulation Results

In this simulation, set k_{nd}=k_{np0}=1.0, k_{cd0}=k_{cp0}=0.1, q_{d}=0.01, h_{f}=0.0001, and γ=1.0. The maximal control rod speed is set to be 1cm/s. The following two case studies are done to show the control performance:

Case A (Load Reject): The load steps down from 100% to 20% immediately with different q_{p}.

Case B (Large Load Lift): The load increases from 20% to 100% in 60s linearly with different q_{p}.

(1) Load Rejection

This verification represents a hard operation for the NHR. In this study, the load immediately steps down from 100% to 20%.The responses of the relative nuclear power, average fuel temperature, outlet coolant temperature of the reactor core T_{cout} and designed control rod speed with different q_{p} are all given in Fig. 1.

(2) Large Load Lift

This case also represents a stressed operation for the NHR. In this case, load signal changes linearly from 20% to 100% in 60 seconds. The responses of the concerned process variables with different q_{p} are illustrated in Fig. 2.

4.3 Discussions

In the case of load rejection, step-down of the load signal causes the rapid increase of δn_{r} which results in immediate generation of a negative control rod speed signal, which causes insertion of then control rods and decreases of both the nuclear power and fuel temperatures. The closed-loop system enters into a steady state if the reactivity given by the control rods cancels that given by the fuel and coolant

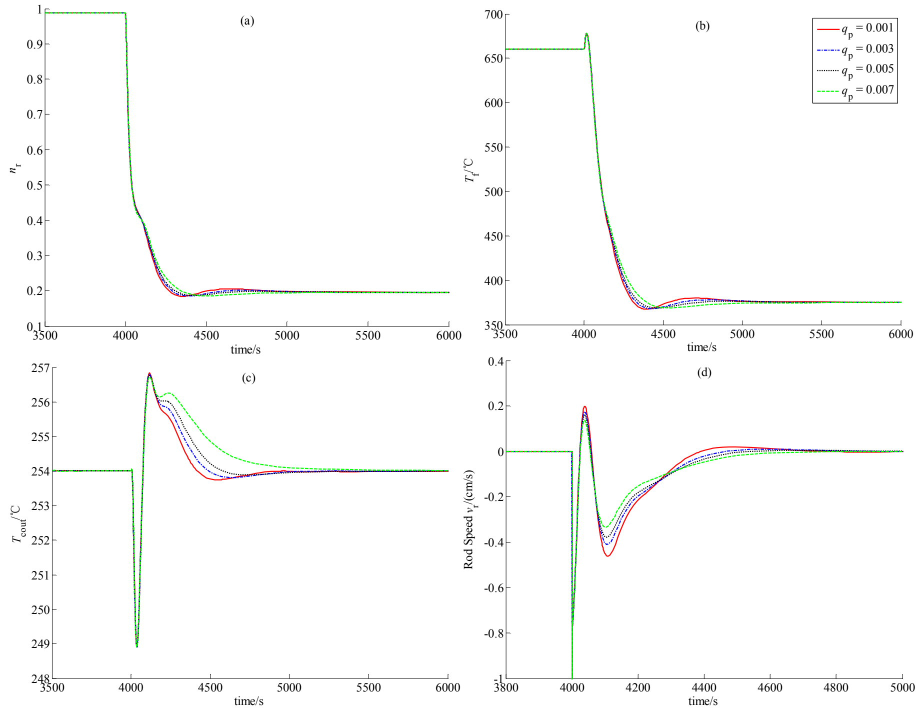

Fig. 1. Simulation results in case A: (a) relative nuclear power, (b) average fuel temperature, (c) outlet coolant temperature, (d) designed control rod speed.

temperature feedback effects. From Fig. 2, the adaptive PD power-level controller in this paper can guarantee satisfactory load-following performance. Also from Fig. 2, q_p is larger, the transition time of outlet coolant temperature T_{cout} is larger. Actually, based on (30), it is clear that a larger q_p induces smaller influence of $\tilde{k}_{cp}$ to V_2, which further induces a larger estimation error of gain k_{cp} and deteriorates dynamic response of T_{cout}. Due to reactivity feedback of the coolant temperature, there are variations in dynamic responses of relative nuclear power n_r and average fuel temperature T_f if q_p is different.

Furthermore, the load increase leads δn_r to be negative and decreasing which drives the power-level control strategy to compensate for this error signal by generating a proper control rod speed. The closed-loop system finally enters into a steady state if the reactivity given by the control rods cancels that induced by temperature feedback effect. The generation of the control rod speed signal is driven by the variations of both the relative nuclear power and the average coolant temperature obtained from measurement. From Fig. 3, it is easy for us to see that the transition period of the dynamic response of T_{cout} is shorter if scalar q_p is smaller. The reason is the same with that given in the case of load rejection. Also due to the reactivity feedback of coolant temperature, shorter transition period of T_{cout} results in the shorter transition periods of both relative nuclear power n_r and average fuel temperature T_f.

Based on the above analysis and discussion, we can see that the adaptive PD power-level controller given by (9), (15) and (16) provides globally asymptotic closed-loop stability of the reactor state-variables and the satisfactory load following performance without the accurate knowledge about the system parameters. Further, from the proof of Theorem 1, the control design is given by physically-based method, which induces the simple form of this newly-built controller. Both the simplicity and adaptation ability are very attractive in practical engineering.

5. CONCLUSIONS

Both the fast increase in the electricity consumption and the severe pollution problem caused by burning fossil fuels leads to the renaissance of nuclear energy. Due to the widely usage of the PWR, its safe, stable and efficient operation is meaningful to the development of nuclear energy. Power-level control, which strengthens both the stability and the dynamic response of the closed-loop system, is important to give a high operation performance. Since every PWR is a complex nonlinear system with high parameter uncertainties, it is attractive to develop the adaptive power-level control technique. In this paper, an adaptive PD power-level control is proposed. It has been proved theoretically that this newly-built adaptive control guarantees globally asymptotic closed-loop stability without sensitivity to those reactor parameters. Numerical simulation results not only verify the correctness of the theoretic results but also illustrate the relationship between control performance and the parameters of the adaptation law. The expression of this control strategy is very simple, which means that it can be easily implemented as a simple program running on digital control system platforms.

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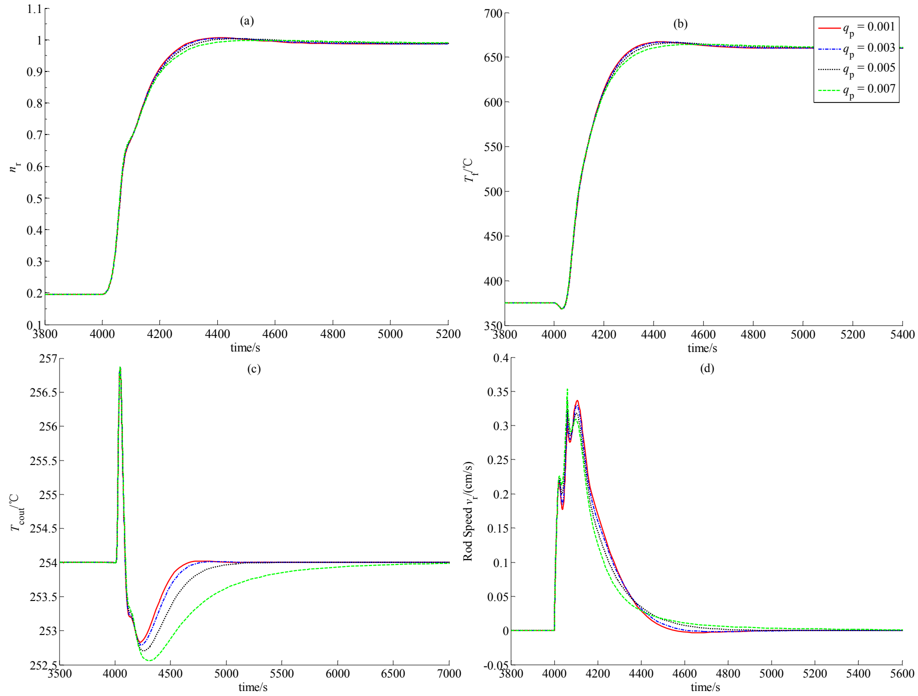

Fig. 2. Simulation results in case B: (a) relative nuclear power, (b) average fuel temperature, (c) outlet coolant temperature, (d) designed control rod speed.

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