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Attitude synchronization for multiple flexible spacecraft based on non-smooth control ⋆

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Abstract: This paper considers the problem of attitude synchronization for a group of flexible spacecraft based on distributed attitude cooperative control strategy. Based on the backstepping design, non-smooth control, and the neighbor-based design rule, a distributed attitude control law is constructed step by step. Under the proposed control law, it is shown that the attitude synchronization is achieved asymptotically and the induced vibration by flexible appendages is simultaneously suppressed.

1. INTRODUCTION

Distributed cooperative control of multi-agent systems has been attracting a lot of interest in control community recently because of its many advantages, such as greater efficiency, higher robustness, and less communication requirement Ren et al. (2007); Hong et al. (2006); Khoo et al. (2009). As an important application area of distributed control, the attitude cooperative control for spacecraft formation has also gained certain progresses.

For a group of rigid spacecraft, in Lawton et al. (2002), two kinds of distributed control strategies were designed such that the attitude synchronization is achieved under a ring communication graph. Later, this ring communication topology graph was relaxed to be a more general case in Ren et al. (2007). When the angular velocity is unmeasurable, the attitude synchronization control problem was also investigated in Lawton et al. (2002); Abdessameud et al. (2009). For the attitude cooperative tracking control problem with a single leader or multiple leaders, the distributed cooperative control laws were proposed in Dimarogonas et al. (2009); Wu et al. (2009). Recently, in order to enhance the convergence rate, precision, and robustness against disturbances, the finite-time control technique Bhat et al. (2000); Qian et al. (2005); Shen et al. (2008) has been employed to design finite-time attitude synchronization control algorithms Du et al. (2011); Meng et al. (2010).

Note that all the preceding listed literature on attitude cooperative control only concentrate on the rigid spacecraft. Nevertheless, with the development of the space science technology, the structure of spacecraft will be more complex and usually carry some flexible appendages, such as solar array, manipulator, etc. Compared with the rigid spacecraft, the control problem of flexible spacecraft becomes more complicated since not only the attitude control but also the vibration induced by the flexible appendages are required to be handled, where the coupling nonlinearities with modal variable are the main obstructions. Although for a single flexible spacecraft, many researchers have developed different nonlinear control methods, such as Gennaro (2003); Hu et al. (2010), to name just a few. However, for the attitude cooperative control for multiple flexible spacecraft, to the best our knowledge, there have been no available results.

In this paper, we focus on solving the problem of attitude synchronization for a group of flexible spacecraft. Based on the backstepping design and non-smooth control, a distritbuted attitude cooperative control law is explicitly constructed in two steps. At the first step, the angular velocity is regarded as a virtual control input and a neighbor-based distributed control law is designed, where the modal variables are first assumed to be measurable. Then to address the problem of lack of modal variables measurement, the virtual controller is redesigned together with a modal observer. At the second step, for the dynamic subsystem, a finite-time control law is designed for the control torque such that the virtual angular velocity can be tracked by the real velocity in a finite time.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Graph theory

Without loss of generality, n flexible spacecraft will be considered in this paper. Let $\Gamma = \{1, \cdots, n\}$. Each spacecraft is regarded as a node and the information exchange among n spacecraft is denoted by a directed graph $G(A) = \{V, E, A\}$. $V = \{v_i, i = 1, \cdots, n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges and $A = [a_{ij}] \in R^{n\times n}$

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is the weighted adjacency matrix of the graph $G(A)$ with non-negative adjacency elements a_{ij}. If there is an edge from node j to node i, i.e., $(v_j, v_i) \in E$, then $a_{ij} > 0$, which means there exists an available information channel from node j to node i. Moreover, we assume that $a_{ii} = 0$ for all $i \in \Gamma$. The set of neighbors of node i is denoted by $N_i = \{j : (v_j, v_i) \in E\}$. The out-degree of node v_i is defined as $\deg_{out}(v_i) = d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j\in N_i} a_{ij}$. Then the degree matrix of digraph G is $D = \text{diag}\{d_1, \cdots, d_n\}$ and the Laplacian matrix of digraph G is $L = D - A$.

A path in directed graph G from v_{i_1} to v_{i_k} is a sequence of $v_{i_1}, v_{i_2}, \cdots, v_{i_k}$ of finite nodes starting with v_{i_1} and ending with v_{i_k} such that $(v_{i_l}, v_{i_{l+1}}) \in E$ for $l = 1, 2, \cdots, k-1$. The directed graph G is strongly connected if there is a path between any two distinct vertices.

2.2 Flexible spacecraft attitude model

The model of flexible spacecraft attitude consists of two parts: kinematic model and dynamic model. Based on the quaternion Shuster (1993), the kinematic equation of i-th spacecraft is described by

$$\dot{q}_i = \frac{1}{2}E(q_i)\omega_i, \quad i \in \Gamma = \{1, \cdots, n\}, \tag{1}$$

where $q_i = [q_{i,0}, q_{i,1}, q_{i,2}, q_{i,3}]^T = [q_{i,0}, q_{i,v}^T]^T$ is unit quaternion, $\omega_i = [\omega_{i,1}, \omega_{i,2}, \omega_{i,3}]^T$ is the angular velocity vector, and

$$E(q_i) = \begin{pmatrix} -q_{i,v}^T \\ -s(q_{i,v}) + q_{i,0}I_3 \end{pmatrix},$$

where I_3 denotes the 3×3 identity matrix and $s(\cdot)$ denotes the skew matrix. The skew matrix is defined as

$$s(x) = \begin{bmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{bmatrix}$$

for any $x = [x_1, x_2, x_3]^T \in R^3$, which satisfies $\|s(x)\| = \|x\|$. In addition, the unit quaternion satisfies the constraint condition

$$q_{i,0}^2 + q_{i,v}^T q_{i,v} = 1. \tag{2}$$

From Gennaro (2003), the dynamic equation of i-th spacecraft is

$$\begin{aligned} J_i\dot{\omega}_i + \delta_i^T\ddot{\eta}_i &= s(\omega_i)(J_i\omega_i + \delta_i^T\dot{\eta}_i) + \tau_i, \\ \ddot{\eta}_i + C_i\dot{\eta}_i + K_i\eta_i &= -\delta_i\dot{\omega}_i \qquad\qquad i \in \Gamma, \end{aligned} \tag{3}$$

where $J_i = J_i^T$ is the positive definite inertia matrix, $\tau_i = [\tau_{i,1}, \tau_{i,2}, \tau_{i,3}]^T$ is the control torque vector, δ_i is the coupling matrix between the rigid body and the flexible attachments, η_i is the vector of the modal coordinate, $C_i = \text{diag}\{2\xi_{i,j}\omega_{i,nj}, j = 1, \cdots, N_i\}$ is the damping (diagonal) matrix, $K_i = \text{diag}\{\omega_{i,nj}, j = 1, \cdots, N_i\}$ is the stiffness matrix, N_i is the number of flexible attachment for i-th spacecraft, $\omega_{i,nj}$ is the natural frequencies and $\xi_{i,j}$ is the associated damping.

As that in Gennaro (2003), denote $\psi_i = \dot{\eta}_i + \delta_i\omega_i$ and $J_{m,i} = J_i - \delta_i^T\delta_i$. The attitude equations (1) and (3) can be rewritten as

$$\begin{aligned} \dot{q}_i &= \frac{1}{2}E(q_i)\omega_i, \quad \dot{\eta}_i = \psi_i - \delta_i\omega_i, \\ \dot{\psi}_i &= -(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i), \\ J_{m,i}\dot{\omega}_i &= s(\omega_i)(J_{m,i}\omega_i + \delta_i^T\psi_i) \\ &\quad + \delta_i^T(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) + \tau_i, i \in \Gamma. \end{aligned} \tag{4}$$

2.3 Control objective

The goal of this paper is to design a distributed attitude control law for the n flexible spacecraft such that all the attitudes can reach consensus/synchronization and the induced oscillations of the spacecraft flexible appendages are damped out.

2.4 Useful lemma

Lemma 1. (Bhat et al. (2000)): Consider system $\dot{x} = f(x), f(0) = 0, x \in R^n$, where $f(\cdot) : R^n \to R^n$ is a continuous function. Suppose there exists a continuous, positive definite function $V(x) : U \to R$ defined on an open neighborhood U of the origin such that $\dot{V}(x) + c(V(x))^\alpha \le 0$ on U for some $c > 0$ and $\alpha \in (0, 1)$. Then the origin is a finite-time stable equilibrium of system $\dot{x} = f(x)$ and the finite settling time T satisfies $T \le \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}$. If $U = R^n$ and V is radially unbounded, the origin is a globally finite-time stable equilibrium.

Lemma 2. (Xiao et al. (2009)): If a directed graph G is strongly connected, then there is a positive vector $\gamma = [\gamma_1, \cdots, \gamma_n]^T \in R^n$ (i.e. $\gamma_i > 0, i = 1, \cdots, n$) such that $\gamma^T L = 0$, where L is the corresponding Laplacian matrix L of graph G.

Lemma 3. (hardy et al. (1952)): For any $x \in R, y \in R, c > 0, d > 0$, $|x|^c|y|^d \le c/(c+d)|x|^{c+d} + d/(c+d)|y|^{c+d}$.

3. MAIN RESULTS

The controller design method is mainly based on the backstepping design. Specifically speaking, the design procedure is divided into two steps:
i) For the kinematic subsystem and modal dynamics

$$\begin{aligned} \dot{q}_i &= \frac{1}{2}E(q_i)\omega_i, \quad \dot{\eta}_i = \psi_i - \delta_i\omega_i, \\ \dot{\psi}_i &= -(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i), \quad i \in \Gamma, \end{aligned} \tag{5}$$

considering ω_i as the virtual input, a virtual angular velocity ω_i^* is designed such that the attitudes of kinematic subsystem achieve consensus.
ii) For the dynamic subsystem, a finite-time control law τ_i is designed such that the virtual velocity can be tracked by the real angular velocity in a finite time.

3.1 Virtual angular velocity design

In this subsection, the angular velocity ω_i is regarded as a virtual control input and is designed such that the attitude synchronization can be achieved.

Lemma 4. For the subsystem (5), if the directed graph $G(A)$ is strongly connected and the virtual angular velocity is designed as

$$\begin{aligned} \omega_i^* = &-k_1 \sum_{j\in N_i} a_{ij}\Big[(q_{i,v} - q_{j,v}) + [(\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]^T \\ &- [(\psi_j^T C_j - 2\eta_j^T K_j)\delta_j]^T\Big], \qquad i \in \Gamma, \end{aligned} \tag{6}$$

where $k_1 > 0$, then the attitude synchronization can be achieved asymptotically.

Proof. According to Lemma 2, if the directed graph $G(A)$ is strongly connected, there exists a positive column vector

$\gamma = [\gamma_1, \cdots, \gamma_n]^T \in R^n$ such that $\gamma^T L = 0$. Consider the following candidate Lyapunov function

$$V_1 = \sum_{i=1}^{n} \gamma_i W_i,\ W_i = \Big[(2 - 2q_{i,0}) + \frac{1}{2}\psi_i^T \psi_i + \eta_i^T K_i \eta_i + \frac{1}{2}(\psi_i + C_i\eta_i)^T(\psi_i + C_i\eta_i)\Big]. \quad (7)$$

Based on the definition of $E(q_i)$, the derivation of W_i along system (5) is

$$\dot{W}_i = -\eta_i^T C_i K_i \eta_i - \psi_i^T C_i \psi_i + [q_{i,v}^T + (\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]\omega_i. \quad (8)$$

Denote

$$\beta_i = q_{i,v} + [(\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]^T, \quad (9)$$

which implies that $\omega_i^* = -k_1 \sum_{j\in N_i} a_{ij}(\beta_i - \beta_j)$. By (7), and substituting this virtual control law into (8) yields

$$\begin{aligned}\dot{V}_1 =& -\sum_{i=1}^{n}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) - k_1 \sum_{i=1}^{n}\sum_{j\in N_i}\gamma_i a_{ij}(\beta_i^T\beta_i - \beta_i^T\beta_j)\\ =& -\sum_{i=1}^{n}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) - \frac{k_1}{2}\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}(\beta_i^T\beta_i - \beta_j^T\beta_j)\\ &- \frac{k_1}{2}\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}(\beta_i - \beta_j)^T(\beta_i - \beta_j). \quad (10)\end{aligned}$$

Define $\beta = [\beta_1^T\beta_1, \cdots, \beta_n^T\beta_n]^T$. By the definition of L, $\sum_{j\in N_i} a_{ij}(\beta_i^T\beta_i - \beta_j^T\beta_j) = (L\beta)_i$, where $(L\beta)_i$ denotes the i-th element of vector $L\beta$. Since $\gamma^T L = 0$, then

$$\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}(\beta_i^T\beta_i - \beta_j^T\beta_j) = \gamma^T L\beta = 0. \quad (11)$$

With this relation in mind, it follows from (10) that

$$\dot{V}_1 = -\sum_{i=1}^{n}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) - \frac{k_1}{2}\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}(\beta_i - \beta_j)^T(\beta_i - \beta_j) \le 0. \quad (12)$$

By LaSalle's invariance principle, and noticing that $\gamma_i > 0$, it can be concluded that $\dot{V}_1(t) \to 0$ as $t \to \infty$, which implies that $(\eta_i, \psi_i, \sum_{j\in N_i} a_{ij}(\beta_i - \beta_j)^T(\beta_i - \beta_j)) \to 0$ as $t \to \infty$. Since $a_{ij} > 0$ if $j \in N_i$, then we have for all $i \in \Gamma$, $\beta_i - \beta_j \to 0, \forall j \in N_i$, as $t \to \infty$. Since the graph $G(A)$ is strong connected, then there exists a path between any two distinct agents. As a matter of fact, $\beta_i - \beta_j \to 0$ as $t \to \infty$ for all $i, j \in \Gamma$. Based on the definition of β_i, it can be further concluded that $q_{i,v} - q_{j,v} \to 0$ as $t \to \infty$ for all $i, j \in \Gamma$. In addition, by noticing the constraint condition (2), $q_{i,v} = q_{j,v}$ implies that $q_{i,0} = q_{j,0}$ or $q_{i,0} = -q_{j,0}$. Since quaternions $(q_{i,0}, q_{i,v}^T)^T$ and $(-q_{i,0}, q_{i,v}^T)^T$ represent the same rotation in the physical space Shuster (1993), the attitude synchronization is achieved asymptotically. □

3.2 Control law design

In this section, based on the idea of backstepping design, a control law for τ_i is designed to achieve attitude consensus, which is presented in the following theorem.

Theorem 1. For the multiple flexible spacecraft systems (1) and (3), if the directed graph $G(A)$ is strongly connected and the control torque τ_i is designed as

$$\begin{aligned}\tau_i =& -s(\omega_i)(J_{m,i}\omega_i + \delta_i^T\psi_i) - \delta_i^T(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i)\\ &+ J_{m,i}\dot{\omega}_i^* - k_2\mathrm{sign}(\omega_i - \omega_i^*)\cdot|\omega_i - \omega_i^*|^\alpha, i \in \Gamma, \quad (13)\end{aligned}$$

then the attitude synchronization can be achieved asymptotically, where $k_1 > 0$, $k_2 > 0$, $0 < \alpha < 1$.

Proof. Define

$$e_i = \omega_i - \omega_i^*, i \in \Gamma,$$

as the angular velocity tracking error. It follows from (4) that

$$J_{m,i}\dot{e}_i = s(\omega_i)(J_{m,i}\omega_i + \delta_i^T\psi_i) + \delta_i^T(C_i\psi_i + K_i\eta_i - C_i\delta_i\omega_i) + \tau_i - J_{m,i}\dot{\omega}_i^*. \quad (14)$$

Substituting the control law (13) into (14) yields

$$J_{m,i}\dot{e}_i = -k_2\mathrm{sign}(e_i)|e_i|^\alpha. \quad (15)$$

Since $J_{m,i}$ is positive definite matrix, choose Lyapunov function:

$$U_i = \frac{1}{2}e_i J_{m,i} e_i \quad (16)$$

which leads to

$$\dot{U}_i = -k_2|e_i|^{1+\alpha} \le -k_2\Big(\frac{2}{\lambda_{max}(J_{m,i})}\Big)^{\frac{1+\alpha}{2}} U_i^{\frac{1+\alpha}{2}} \quad (17)$$

Hence, it follows from Lemma 1 that e_i will converge to zero in a finite time T. After the time instant T, ω_i will equivalent to ω_i^*. As a result, it follows from Lemma 4 that the attitude synchronization will be reached asymptotically after the time T.

Next, we will consider the system states at the interval $[0, T]$ and prove that the global boundedness of the system states of closed-loop system (4) with (13) at this interval.

To achieve this objective, by (7) and (16), choose Lyapunov function

$$V_2 = V_1 + \sum_{i=1}^{n}\gamma_i U_i. \quad (18)$$

According to (8) and (17), we obtain

$$\begin{aligned}\dot{V}_2 =& -\sum_{i=1}^{n}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) - \frac{k_1}{2}\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}(\beta_i - \beta_j)^T(\beta_i - \beta_j)\\ &+ \sum_{i=1}^{n}\gamma_i[q_{i,v}^T + (\psi_i^T C_i - 2\eta_i^T K_i)\delta_i](\omega_i - \omega_i^*) - k_2\sum_{i=1}^{n}\gamma_i|e_i|^{1+\alpha}\\ \le& -\sum_{i=1}^{n}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) - k_2\sum_{i=1}^{n}\gamma_i|e_i|^{1+\alpha}\\ &+ \sum_{i=1}^{n}\gamma_i[q_{i,v}^T + (\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]e_i. \quad (19)\end{aligned}$$

Since system (15) is globally asymptotically stable, e_i is globally bounded, which implies that there is a positive constant $l_i < +\infty$ such that $\|e_i\| \leq l_i$. Note that the parameter matrices C_i, K_i and δ_i are bounded. Hence, With this fact and the condition (2) in mind, from Lemma 3 we have

$$\begin{aligned}
&[q_{i,v}^T + (\psi_i^T C_i - 2\eta_i^T K_i)\delta_i]e_i \\
\leq &\|q_{i,v}\| l_i + \|\psi_i^T C_i^{1/2}\| \cdot \|C_i^{1/2}\| l_i \\
&+ 2\|\eta_i^T (C_i K_i)^{1/2}\| \cdot \frac{\|K_i\delta_i\|}{\|(C_iK_i)^{1/2}\|} l_i \\
\leq &l_i + \frac{1}{2}\psi_i^T C_i \psi_i + \frac{1}{2}\|C_i\|^2 l_i^2 + \frac{1}{2}\eta_i^T C_i K_i \eta_i + \frac{\|K_i\delta_i\|^2}{2\|C_iK_i\|} l_i^2 \\
=: &\frac{1}{2}\psi_i^T C_i \psi_i + \frac{1}{2}\eta_i^T C_i K_i \eta_i + d_i, \qquad (20)
\end{aligned}$$

for a positive constant d_i. Substituting (20) into (19) yields

$$\begin{aligned}
\dot{V}_2 \leq &- \sum_{i=1}^{n} \frac{1}{2}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) \\
&- k_2 \sum_{i=1}^{n} \gamma_i |e_i|^{1+\alpha} + \sum_{i=1}^{n} \gamma_i d_i. \qquad (21)
\end{aligned}$$

Clearly, when $\sum_{i=1}^{n} \frac{1}{2}\gamma_i(\eta_i^T C_i K_i \eta_i + \psi_i^T C_i \psi_i) + k_2 \sum_{i=1}^{n} \gamma_i |e_i|^{1+\alpha} > \sum_{i=1}^{n} \gamma_i d_i$, then $\dot{V}_2 < 0$. Thus, the state (η_i, ψ_i, e_i) is globally bounded. With this fact in mind, and noticing the constraint condition on quaternion (i.e. $\|q_i\| = 1$), it can be concluded that the trajectory of system (4) with (13) is globally bounded. The proof is completed. □

Remark 1. The control law proposed in Theorem 1 is distributed, i.e. the feedback information is only based on the neighbors and itself. The advantage of distributed control law lies in its greater efficiency, higher robustness, and less communication requirement Ren et al. (2007).

Example. Consider a team with four identical flexible spacecraft described by (1) and (3). The information exchange topology among spacecraft is shown in Fig. 1. The weights of the directed edges are: $a_{13} = a_{21} = a_{34} = a_{42} = 0.5$. The model parameters are given as:

$$J_i = \begin{pmatrix} 14 & 3 & 4 \\ 3 & 18 & 0 \\ 4 & 0 & 12 \end{pmatrix}, \delta_i = \begin{pmatrix} 2.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{pmatrix},$$

$\omega_{i,41} = 1.0973, \omega_{i,42} = 1.2761, \omega_{i,43} = 1.6538, \omega_{i,44} = 2.2893, \xi_{i,1} = 0.056, \xi_{i,2} = 0.086, \xi_{i,3} = 0.08, \xi_{i,4} = 0.025, i = 1, \cdots, 4$. Let the control gains of control law

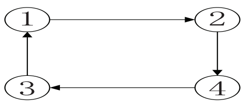

Fig. 1. The information exchange among four spacecraft.

(13) be $k_1 = 0.3, k_2 = 15$. The initial conditions are selected as:

$q_1(0) = [0.9274 - 0.1, 0.2, 0.3]^T, \omega_1(0) = [0.1, -0.35, 0.5]^T,$

$q_2(0) = [-0.9274, -0.2, -0.3, 0.1]^T, \omega_2(0) = [-0.0, 0.1, -1.0]^T$

$q_3(0) = [0.6856, 0.1, 0.6, 0.4]^T, \omega_3(0) = [-0.2, -0.3, 0.0]^T,$

$q_4(0) = [-0.8421, 0.5, 0.03, -0.2]^T, \omega_4(0) = [0.4, 0.13, 0.25]^T,$

$\eta_i(0) = [0, 0, 0, 0]^T, \psi_i(0) = \delta\omega_i(0), i = 1, 2, 3, 4.$

The control torques are limited not to exceed 25 N.m. The response curves of the closed-loop system (1)-(3) with (13) are shown in Figs. 2-4. It can be found that the attitudes of each spacecraft converge to the same attitudes, i.e. the attitude synchronization can be achieved asymptotically.

4. CONCLUSION

This paper have discussed the attitude synchronization problem for a group of flexible spacecraft. By using the backstepping control and graph theory, a distributed attitude cooperative control law is proposed. Rigorous proof has shown that the attitude synchronization can be achieved asymptotically and the vibrations are damped out at the same time. Further work will be focused on the study of the robustness problem with model uncertainties.

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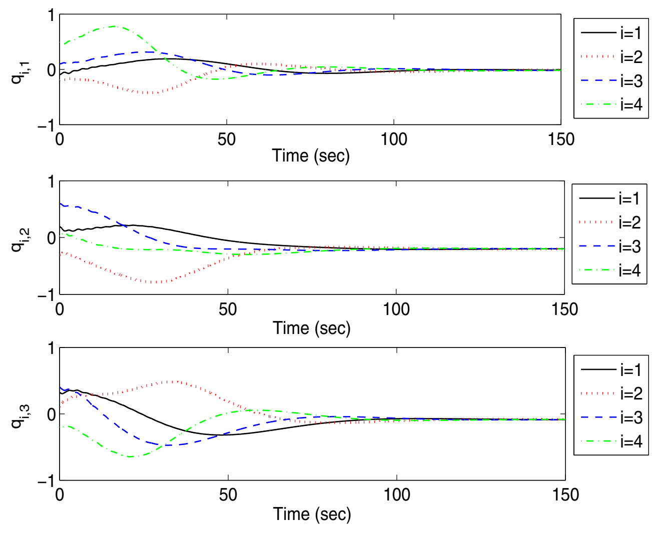

Fig. 2. Attitudes of all spacecraft.

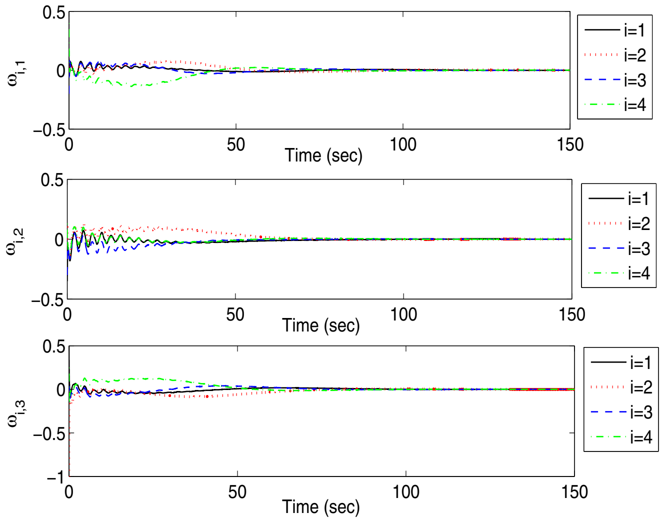

Fig. 3. Angular velocities of all spacecraft.

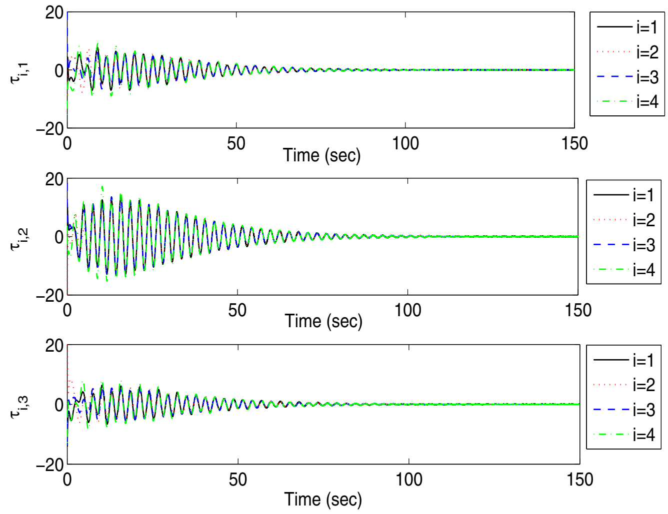

Fig. 4. Control torques of all spacecraft.

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