

## Non-switching reaching law based discrete time quasi-sliding mode control with application to warehouse management problem

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**Abstract:** In this paper a new non-switching type reaching law for sliding mode control of discrete time systems is introduced and applied to the problem of periodic review inventory management. The considered inventory is refilled by multiple suppliers through delivery channels subject to non-negligible commodity losses. The reaching law proposed in this paper is inspired by the earlier work of Gao, Wang and Homaifa, however, in this paper the original formulation is essentially modified in order to avoid undesirable switching, and to ensure good dynamic performance with limited commodity orders. These improvements are obtained by the application of a modified definition of the quasi-sliding mode, and also by the introduction of an adjustable, state dependent sliding variable decrease rate factor. The sliding mode inventory management strategy proposed in this paper ensures full customer demand satisfaction and enables the designer to specify upper limit of the on-hand stock level.

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### 1. INTRODUCTION

Continuous time variable structure control systems with sliding modes have originally been investigated more than half a century ago in the former Soviet Union (Emelyanov, 1967; Utkin, 1977). Because of their robustness (Draženović, 1969) and computational efficiency, immediately they have become very popular among global control engineering community (DeCarlo, Žak and Mathews, 1988; Edwards and Spurgeon, 1998; Gao and Hung, 1993; Bartoszewicz and Nowacka-Leverson, 2009). Some years later discrete time sliding mode controllers have also been proposed in (Milosavljević, 1985) and (Utkin and Drakunov, 1989) and then analyzed by Furuta (1990), Bartolini, Ferrara and Utkin (1995), Gao, Wang and Homaifa (1995), Bartoszewicz (1998), Corradini and Orlando (1998), Golo and Milosavljević (2000), Yu and Chen (2003), Bandyopadhyay and Janardhanan (2006), Janardhanan and Bandyopadhyay (2006), Milosavljević et al. (2006), Janardhanan and Bandyopadhyay (2007), Pan and Furuta (2007), Janardhanan and Kariwala (2008), Yu et al. (2008), Bandyopadhyay and Fulwani (2009), Mehta and Bandyopadhyay (2009), Mehta and Bandyopadhyay (2010), Mija and Susy (2010), Kurode, Bandyopadhyay and Gandhi (2011), Corradini et al. (2012), Yu, Wang and Li (2012), and many others.

Both continuous and discrete time sliding mode controllers drive the system state (its representative point) onto a predefined hypersurface in the state space. This can either be accomplished by selecting a control law and demonstrating that this control ensures stability of the sliding motion on the surface, or by applying the reaching law approach. In the latter approach the required evolution of the sliding variable is first proposed, and then a control law which ensures that the variable changes as specified is determined. The reaching law approach was first introduced for continuous (Gao and Hung, 1993) and then extended to discrete time systems

(Gao, Wang and Homaifa (1995) (see also Bartoszewicz (1996) for further comments). Since then the reaching law approach has been used by many researchers (Golo and Milosavljević, 2000; Milosavljević et al. 2006; Mija and Susy, 2010; Kurode, Bandyopadhyay and Gandhi, 2011). Even though much work in this field has been done, the original approach proposed by Gao, Wang and Homaifa (1995) is still very popular. Therefore, in this paper we extend the results of (Gao, Wang and Homaifa, 1995) in order to obtain a non-switching discrete time sliding mode controller (Bartolini, Ferrara and Utkin, 1995; Bartoszewicz, 1998) and to ensure faster convergence of the controlled system without increasing the magnitude of the control signal. The first of the two objectives is achieved with the application of the quasi-sliding mode definition proposed by Bartoszewicz (1998), and the latter one is accomplished by the introduction of an adjustable, state dependent convergence rate factor in the proposed reaching law. In the second part of the paper, we apply the proposed reaching law to design a new periodic review inventory management strategy (Riddalls, Bennett and Tipi, 2000; Hoberg, Bradley and Thonemann, 2007; Boccadoro, Martinelli and Valigi, 2008; Karaesmen, Scheller-Wolf and Deniz, 2008; Sarimveis et al., 2008; Subramanian, 2013) for a warehouse with multiple remote suppliers and non-negligible commodity losses in delivery channels. We demonstrate favorable properties of the designed strategy which could not be achieved with the application of the original ‘constant plus proportional’ reaching law. In particular, we show that our reaching law ensures non-negative upper bounded supply orders which do not depend on the warehouse capacity, and therefore are fairly desirable in the considered system. Furthermore, we demonstrate that our reaching law based controller eliminates the risk of exceeding warehouse capacity and may ensure 100% customers’ demand satisfaction.

## 2. NON-SWITCHING REACHING LAW

In this section we consider a perturbed discrete-time system described by the following equation

$$\mathbf{x}[(k+1)T] = \mathbf{A}\mathbf{x}(kT) + \Delta\mathbf{A}\mathbf{x}(kT) + \mathbf{b}u(kT) + \mathbf{f}(kT) \quad (1)$$

where  $\mathbf{x}(k)$  is the state vector ( $\dim(\mathbf{x}) = n \times 1$ ),  $\mathbf{A}$  is the state matrix,  $\Delta\mathbf{A}$  is the model uncertainty matrix,  $\mathbf{b}$  is the input vector,  $u(kT)$  is a scalar input, and  $\mathbf{f}(kT)$  is a disturbance vector. We denote the demand state vector by  $\mathbf{x}_d$ , and define the closed loop system error as  $\mathbf{e}(kT) = \mathbf{x}_d - \mathbf{x}(kT)$ . Then we select the sliding variable as

$$s(kT) = \mathbf{c}^T \mathbf{e}(kT) \quad (2)$$

With this choice of variable  $s$ , equation  $s(kT) = 0$  determines the sliding hyperplane. The elements  $c_1, c_2, \dots, c_n$  of vector  $\mathbf{c}$  are selected in such a way that  $\mathbf{c}^T \mathbf{b} \neq 0$  and that the closed loop system exhibits the desired performance. This can be done in a few ways including quadratic optimization (Janardhanan and Kariwala, 2008), pole placement method (Gao, Wang and Homaifa, 1995), dead-beat design (Bartoszewicz and Żuk, 2009), etc.

In this paper the quasi-sliding mode is defined similarly as in (Bartoszewicz, 1998), i.e. it is such a motion of the system that its representative point (state) remains in a given vicinity of sliding hyperplane (2). According to this definition, the representative point (state of the system) in the quasi-sliding mode is confined to a specified layer around the hyperplane. Contrary to the definition introduced by Gao, Wang and Homaifa (1995), in our approach crossing the hyperplane is allowed but not required.

We propose the following reaching law

$$s[(k+1)T] = \{1 - q[s(kT)]\} s(kT) + \tilde{S}(kT) - \tilde{F}(kT) + F_1 + S_1 \quad (3)$$

where

$$\tilde{S}(kT) = \tilde{S}[\mathbf{x}(kT)] = \mathbf{c}^T \Delta\mathbf{A}\mathbf{x}(kT), \quad \tilde{F}(kT) = \mathbf{c}^T \mathbf{f}(kT) \quad (4)$$

represent respectively the influence of the model uncertainty on the sliding variable evolution and the effect of disturbance on this variable. Furthermore,  $S_1$  and  $F_1$  are the mean values of  $\tilde{S}$  and  $\tilde{F}$ , namely

$$S_1 = (S_U + S_L)/2, \quad F_1 = (F_U + F_L)/2 \quad (5)$$

where  $S_U, S_L$  are upper and lower bounds of  $\tilde{S}$ , and  $F_U, F_L$  are upper and lower bounds of  $\tilde{F}$ , i.e.

$$S_L \leq \tilde{S} \leq S_U, \quad F_L \leq \tilde{F} \leq F_U \quad (6)$$

Convergence rate factor  $q[s(kT)]$  in (3) is given by

$$q[s(kT)] = s_0 / [s_0 + |s(kT)|] \quad (7)$$

where  $s_0$  is a design parameter. The parameter is chosen so that  $s_0 > S_2 + F_2$ , where  $S_2$  and  $F_2$  represent the greatest possible deviation of  $\tilde{S}$  and  $\tilde{F}$  from their mean values  $S_1, F_1$

$$S_2 = (S_U - S_L)/2, \quad F_2 = (F_U - F_L)/2 \quad (8)$$

Parameter  $s_0$  allows to find a satisfactory compromise between the critical magnitude of the control signal generated in the system, and sluggish convergence to the vicinity of  $s(kT) = 0$ . The proposed reaching law has two major advantages over the one presented in (Gao, Wang and Homaifa, 1995). Firstly, it does not contain a discontinuous term, so it does not lead to chattering. Secondly, since  $q[s(kT)]$  increases with the decrease of  $s(kT)$ , our reaching law results in faster convergence and better robustness with the same bounds on the control signal magnitude.

In order to find the control signal  $u(kT)$  which ensures that the sliding variable evolution is indeed described by (3), we use (1) to rewrite (2) as follows

$$s[(k+1)T] = \mathbf{c}^T \mathbf{x}_d + \mathbf{c}^T [\mathbf{A}\mathbf{x}(kT) + \Delta\mathbf{A}\mathbf{x}(kT) + \mathbf{b}u(kT) + \mathbf{f}(kT)] \quad (9)$$

Then, comparing (3) and (9) we obtain

$$u(kT) = -(\mathbf{c}^T \mathbf{b})^{-1} \{ [1 - q(s(kT))] s(kT) + \mathbf{c}^T \mathbf{A}\mathbf{x}(kT) + F_1 + S_1 - \mathbf{c}^T \mathbf{x}_d \} \quad (10)$$

As all terms in (10) are either constants, or variables which do not depend on unknown terms  $\Delta\mathbf{A}$  or  $\mathbf{f}(kT)$ , this control signal can actually be applied in the considered system.

In the next two theorems we demonstrate, that once the representative point of system (1) has reached a band around the sliding hyperplane  $s(kT) = 0$ , it remains inside the band, and also that the proposed reaching law makes the point always move towards this band.

**Theorem 1.** If the following inequality

$$|s(kT)| \leq s_0 (S_2 + F_2) / [s_0 - (S_2 + F_2)] \quad (11)$$

is satisfied at some instant  $k = k_0$ , then it is also true for any  $k > k_0$ .

**Theorem 2.** If the absolute value of  $s(kT)$  is greater than the right hand side of (11), then  $s(kT)$  converges, at least asymptotically, to the band specified by (11).

## 3. INVENTORY SUPPLY MODEL

In this section we consider a periodic review inventory supply system with  $m$  remote commodity providers. Each provider delivers goods with its own lead time  $L_p$  through a transportation channel with non-negligible losses. The providers are required to satisfy an *a priori* unknown consumer demand. The commodity orders are determined by the controller placed at the distribution center. The signal generated by the controller is denoted by  $u$ , and it

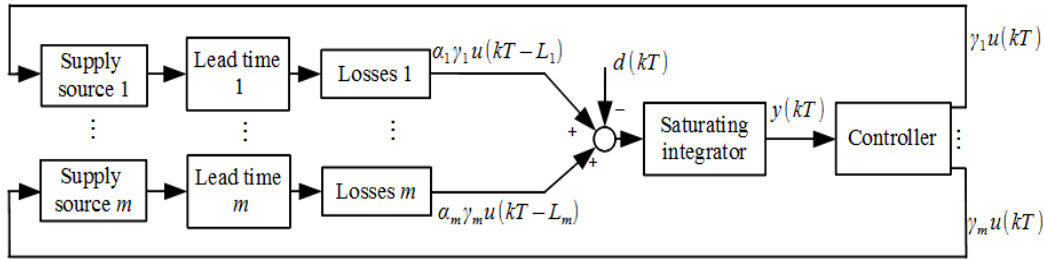


Fig. 1. Supply system.

corresponds to the total amount of goods requested from all of the suppliers. This value is divided among the suppliers so that each supplier  $p$  receives a replenishment order which is equal to  $\gamma_p$  of  $u$ , where  $0 \leq \gamma_p \leq 1$ , and  $\sum_{p=1}^m \gamma_p = 1$ . The block diagram of the considered system is depicted in Fig. 1. It is assumed, that during transport some goods are broken, so that only  $\alpha_p$  commodities from supply source  $p$  arrive at the distribution center, where  $\alpha_p \in (0, 1]$  for  $p = 1, \dots, m$ .

We assume, that each lead time  $L_p$  is a multiple of the discretization period  $T$ , i.e.  $L_p = \mu_p T$ , where  $\mu_p$  is a positive integer. The stock level at time  $kT$  is denoted by  $y(kT)$ . The maximum value of the consumer demand  $d(kT)$  is represented by  $d_{\max}$ . The amount of goods that are actually sold at time  $kT$  is denoted by  $h(kT)$ . This value cannot exceed the consumer demand, but it can be smaller than the demand if there are not enough goods in the warehouse. Therefore,  $0 \leq h(kT) \leq d(kT) \leq d_{\max}$  for any  $k \geq 0$ .

The warehouse is empty before the start of the control process, i.e.  $y(kT < 0) = 0$ , and the first replenishment order is generated at  $kT = 0$ , i.e.  $u(kT < 0) = 0$ . The on-hand stock level for  $kT > 0$  can be expressed as the difference between the amounts of incoming and outgoing goods

$$y(kT) = \sum_{p=1}^m \sum_{j=0}^{k-1} \alpha_p \gamma_p u(jT - L_p) - \sum_{j=0}^{k-1} h(jT) \quad (12)$$

In order to simplify the model, we represent all providers with equal lead times as a single supplier. The amount of goods that will arrive at the warehouse from this supplier is equal to  $a_i u$ , where  $a_i = \sum_{p: \mu_p = i} \alpha_p \gamma_p$ , for  $i = 1, \dots, n-1$  and

$n = \max(\mu_p) + 1$ . Of course, if there is no supplier with the lead time  $iT$ , then the appropriate coefficient  $a_i = 0$ . This allows us to represent the stock level as follows

$$y(kT) = \sum_{i=1}^{n-1} \sum_{j=0}^{k-1} a_i u[(j-i)T] - \sum_{j=0}^{k-1} h(jT) \quad (13)$$

We can express this relation in the standard state space form

$$\begin{aligned} \mathbf{x}[(k+1)T] &= \mathbf{A}\mathbf{x}(kT) + \mathbf{b}u(kT) + \mathbf{o}h(kT) \\ y(kT) &= \mathbf{r}^T \mathbf{x}(kT) \end{aligned} \quad (14)$$

where  $\mathbf{x}(kT) = [x_1(kT) \ x_2(kT) \ \dots \ x_n(kT)]^T$  is the state vector,  $y(kT) = x_1(kT)$  is the on-hand stock level. The state variables except for the first one are the delayed values of the control signal i.e. for  $i = 2, \dots, n$

$$x_i(kT) = u[(k-n+i-1)T]. \quad (15)$$

$\mathbf{A}$  is  $n \times n$  state matrix, and  $\mathbf{b}$ ,  $\mathbf{o}$ , and  $\mathbf{r}$  are  $n \times 1$  vectors

$$\mathbf{A} = \begin{bmatrix} 1 & a_{n-1} & a_{n-2} & \dots & a_1 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & & 0 \end{bmatrix}, \quad \begin{aligned} \mathbf{b} &= [0 \ \dots \ 0 \ 1]^T, \\ \mathbf{o} &= [-1 \ 0 \ \dots \ 0]^T, \\ \mathbf{r} &= [1 \ 0 \ \dots \ 0]^T. \end{aligned} \quad (16)$$

The desired state of the system  $\mathbf{x}_d = [y_d \ 0 \ \dots \ 0]$ , where  $y_d$  is the demand warehouse stock level.

#### 4. CONTROLLER DESIGN

Now we will select the elements of vector  $\mathbf{c}$ , which describe the sliding hyperplane (2), so as to obtain the dead-beat system performance. We begin, by deriving the control signal that is needed to satisfy  $s[(k+1)T] = 0$  and we substitute it into (14). In this way, we obtain the closed loop system matrix  $\mathbf{A}_c = [\mathbf{I}_n - \mathbf{b}(\mathbf{c}^T \mathbf{b})^{-1} \mathbf{c}^T] \mathbf{A}$ . This matrix has the following characteristic polynomial

$$\det(z\mathbf{I}_n - \mathbf{A}_c) = z^n + z^{n-1}(c_1 a_1 + c_{n-1} - c_n)/c_n + \dots + z(c_1 a_{n-1} - c_2)/c_n \quad (17)$$

As we have already assumed  $\mathbf{c}^T \mathbf{b} \neq 0$ . This condition and relation (16), imply that  $c_n \neq 0$ . A linear time-invariant discrete-time system is asymptotically stable if and only if all of its eigenvalues are located inside a unit circle. Moreover, in order to obtain finite time error convergence to zero the characteristic polynomial (17) must have the form  $\det(z\mathbf{I}_n - \mathbf{A}_c) = z^n$ . We find, that this can be obtained with the following choice of vector  $\mathbf{c}$

$$c_1 = 1, \quad c_i = \sum_{j=1}^{i-1} a_{n-j} \quad \text{for } i = 2, \dots, n \quad (18)$$

Now we will apply the presented reaching law to design the controller that will drive the representative point of the system to the vicinity of the sliding hyperplane  $\mathbf{c}^T \mathbf{e}(kT) = 0$ , where  $\mathbf{c}^T$  is defined by relation (18). For the considered system, the perturbations of the sliding variable caused by the model uncertainty and disturbance are

$$S_1 = S_2 = 0, \quad F_1 = -d_{\max}/2, \quad F_2 = d_{\max}/2. \quad (19)$$

Using (2) and (19) with (10) we get

$$u(kT) = -(\mathbf{c}^T \mathbf{b})^{-1} \left\{ \left[ 1 - q[s(kT)] \right] s(kT) + \mathbf{c}^T \mathbf{A} \mathbf{x}(kT) - \mathbf{c}^T \mathbf{x}_d - d_{\max}/2 \right\} \quad (20)$$

$$= (\mathbf{c}^T \mathbf{b})^{-1} \left\{ q[s(kT)] s(kT) - \mathbf{c}^T (\mathbf{A} - \mathbf{I}_n) \mathbf{x}(kT) + d_{\max}/2 \right\}$$

Moreover, we notice that by choosing  $\mathbf{c}$  according to (18) we have obtained  $\mathbf{c}^T (\mathbf{A} - \mathbf{I}_n) = [0 \dots 0]$ . Therefore, using (16) and (18) we can calculate the following control signal, which ensures the desired sliding variable evolution

$$u(k) = \left\{ q[s(kT)] s(kT) + d_{\max}/2 \right\} / \sum_{i=1}^{n-1} a_i \quad (21)$$

We will now prove essential properties of the considered system. First, we will show that control signal (21) is always non-negative and upper bounded by an *a priori* specified constant. As this signal directly corresponds to resupply orders sent to suppliers, both of these conclusions are important for the application of the proposed strategy.

**Theorem 3.** The control signal generated by the proposed controller will, for any  $k \geq 0$ , satisfy

$$0 \leq u(kT) \leq \left[ s_0 y_d / (y_d + s_0) + d_{\max}/2 \right] / \sum_{i=1}^{n-1} a_i \quad (22)$$

**Proof:** Sliding variable  $s(kT)$ , as shown in theorems 1 and 2, will originally have some initial value  $s(0)$ , and then its absolute value will decrease in each step unless (11) is satisfied. Furthermore, once (11) becomes satisfied, it remains true for the remainder of the control process. Using  $s(0) = \mathbf{c}^T \mathbf{x}_d = y_d$  with (11) and (19), we conclude, that

$$s(kT) \in \left[ -s_0 d_{\max} / (2s_0 - d_{\max}), y_d \right] \quad (23)$$

for all  $k \geq 0$ .

We now observe, that control signal (21) always increases with the increase of  $s(kT)$ . Therefore, its maximum value will be generated for the greatest possible  $s(kT)$ , and the minimum value for the smallest  $s(kT)$ . Using this observation and substituting limits of interval (23) into (21) we conclude that (22) is indeed true.

An efficient inventory management strategy should ensure that all incoming shipments can be accommodated in the warehouse. In the next theorem we will derive the upper bound of the on-hand stock. This means, that if warehouse capacity equal to this bound is secured, then the risk of hiring costly emergency storage will be eliminated.

**Theorem 4.** If the proposed control strategy is applied, then for every  $k \geq 0$ , the on-hand stock level will satisfy the following inequality

$$y(kT) \leq y_d + s_0 d_{\max} / (2s_0 - d_{\max}) \quad (24)$$

**Proof:** From (23) we get

$$s(kT) \geq -s_0 d_{\max} / (2s_0 - d_{\max}) \quad (25)$$

for any  $k \geq 0$ . Using (2) and (15) we may rewrite (25) as

$$y(kT) \leq y_d + \frac{s_0 d_{\max}}{2s_0 - d_{\max}} - \sum_{i=2}^n c_i u[(k-n+i-1)T] \quad (26)$$

As the control signal is always non-negative we conclude that (26) implies (24).

In order to maximize profit, it is important to eliminate lost sales opportunities. In other words, if possible the consumer demand should be fully satisfied. In the next theorem we derive the minimum value of the demand stock level that ensures that after some initial time the warehouse will never be empty. As we can observe from (13) this is equivalent to full customers' demand satisfaction.

**Theorem 5.** If the demand queue length satisfies

$$y_d > d_{\max} \sum_{i=1}^{n-1} i a_i / \sum_{i=1}^{n-1} a_i + s_0 d_{\max} / (2s_0 - d_{\max}) \quad (27)$$

then  $y(kT) > 0$  for any  $k \geq k_0 + n - 1$ , where  $k_0$  is the first time instant when (11) is satisfied.

**Proof:** Using (11), for any  $k \geq k_0$ , we can obtain

$$y(kT) \geq y_d - \sum_{i=2}^n c_i u[(k-n+i-1)T] - \frac{s_0 d_{\max}}{2s_0 - d_{\max}} \quad (28)$$

Furthermore, substituting (11) into (21) we obtain,

$$u(kT) \leq d_{\max} / \sum_{i=1}^{n-1} a_i \quad (29)$$

which is true for any  $k \geq k_0$ . Combining (28) and (29) we get

$$y(kT) \geq y_d - d_{\max} \sum_{i=1}^{n-1} i a_i / \sum_{i=1}^{n-1} a_i - \frac{s_0 d_{\max}}{2s_0 - d_{\max}} \quad (30)$$

for any  $k \geq k_0 + n - 1$ . Therefore, if (27) holds, then the right hand side of the above inequality is always strictly positive.

## 5. SIMULATION RESULTS

In order to verify the properties of the proposed control law computer simulations of an inventory replenishment system with four suppliers are performed. The review period is selected as one day. The greatest lead time considered in the simulation is nine days, therefore  $\max(\mu_p) = 9$  and  $n = 10$ . The suppliers' parameters are shown in Table 1. The corresponding parameters  $a_i$  are  $a_4 = 0.1$ ,  $a_6 = 0.2$ ,  $a_7 = 0.15$ ,  $a_9 = 0.5$  and the remaining  $a_i$  are equal to zero. The maximum consumer demand  $d_{\max} = 30$  items and the actual one is shown in Fig. 2. The design parameter  $s_0$  is selected as 35 items in order to obtain a satisfactory compromise between fast convergence rate and reasonable values of the control signal. The minimum demand stock level that ensures full consumer demand satisfaction, according to Theorem 5, is equal to 252 items and for the simulation a slightly larger value was used, namely  $y_d = 270$  items. The control signal is depicted in Fig. 3. As we can observe, it is always non-negative and as predicted by Theorem 3, it never exceeds 48.4 items. The warehouse stock level is shown in Fig. 4. It does not exceed the value of 296.25 items predicted by Theorem 5, and after the initial period it is always greater

than zero. This means, that the risk of hiring (quite often very expensive) emergency storage is eliminated, and the consumer demand is fully satisfied. The sliding variable is shown in Fig. 5. It can be seen from the figure that after converging to the region  $|s(kT)| \leq 26.25$  the variable remains inside it for the rest of the control process.

**Table 1. Parameters of the supply sources**

$p$	1	2	3	4
$L_p$	4	6	7	9
$\gamma_p$	0.1	0.21	0.16	0.53
$\alpha_p$	1	0.95	0.94	0.94

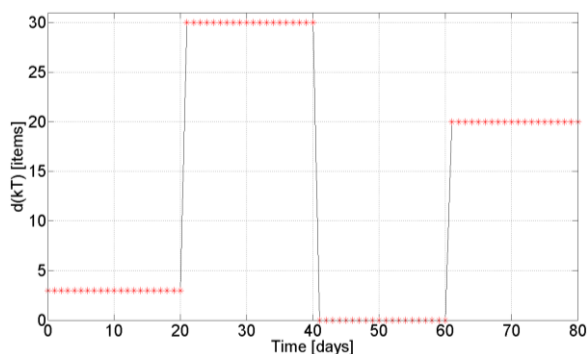


Fig. 2. Consumers' demand

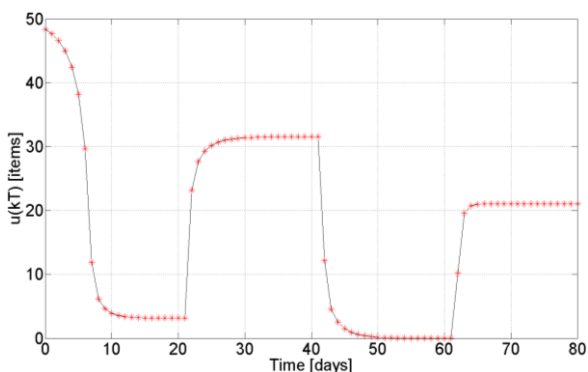


Fig. 3. Control signal.

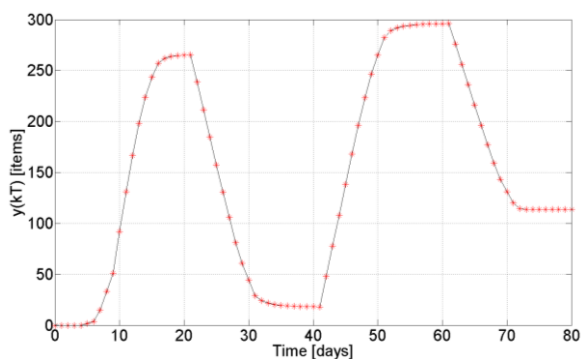


Fig. 4. Stock level.

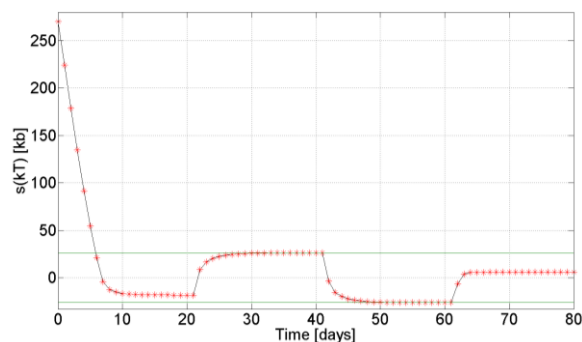


Fig. 5. Sliding variable.

## 6. CONCLUSIONS

In this paper we have presented a novel reaching law for discrete time sliding mode control systems. Contrary to previous works, it enforces a state dependent sliding variable decrease rate factor, and it does not require crossing the sliding hyperplane in each step during the quasi-sliding motion. These modifications improve system robustness, ensure bounded control signal, and eliminate chattering. In the second part of the paper, the proposed reaching law has been applied to control a periodic review inventory system. Important properties of the obtained controller – i.e. full satisfaction of the consumers' demand, predictable upper bounded order volumes, and a priori known maximum stock level – have been proved analytically and verified in computer simulations. As the reaching law proposed in this paper does not cause chattering and offers fast convergence with limited magnitude of the control signal, it may also be a feasible option for many other engineering and non-engineering applications.

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