

Enhancement of Dual-Rate Estimation and Its Application Demonstration

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Abstract: This paper describes a state estimation scheme for a class of linear system in which the sampling time of output measurement is multiple times of control period. Disturbance accommodation is applied to deal with model uncertainties and external disturbances. In order to maintain the correction stage between two consecutive updates of output, “pseudo measurement” is proposed. The effectiveness of the proposed estimation is verified by a case of application in vehicle state estimation using GPS.

1. INTRODUCTION

In practical control applications, there exist many systems in which the sampling time of the output measurement is equal to multiple times of control period (Fig. 1). In order to assure state feed back at every control period, multi-rate estimation methods have been widely researched for years. Multi-rate estimation can be organized into two groups as follows.

1-Prediction during inter-samples: In some previous works, inter-sample estimation is conducted by just predicting the state using plant model and input signal. For instance, instantaneous speed observer is proposed to estimate the motor speed from low precision shaft encoder (Hori, 1993). Based on the mechanical dynamics of motor, inter-sample speed is calculated. This method is improved by a dual-rate observer with observer gain matrix varying according to the number of inter-samples (Kovudhikulrunsi *et al.*, 2006). A parallel observer is proposed for multi-rate state estimation in which a low-rate observer is combined with a fast-rate observer (Thein *et al.*, 1999). Lifting method is proposed to augment all the inter-sample states into an extension state vector (Li *et al.*, 2005).

2-Correction during inter-samples: By holding the innovation and re-design the observer gain, smoother estimation is achieved (Hara *et al.*, 1999). The instantaneous speed observer is improved by predicting the inter-sample innovations for correcting the state at every control cycle (Oh *et al.*, 2004). Both methods enhance multi-rate estimation by the way that they offer pole placement for estimation error dynamics during inter-samples.

However, all the above methods do not pay enough attention to the influence of noises, model uncertainties, and external disturbances. If the plant has big model uncertainties and the external disturbances exist, impulsive behaviour and non-smoothness are possibly introduced into inter-sample prediction. Even with correction during inter-samples, considerable model uncertainty is still a serious problem.

Following the above discussion, it is essential to enhance the multi-rate estimation considering two issues: Firstly, how to

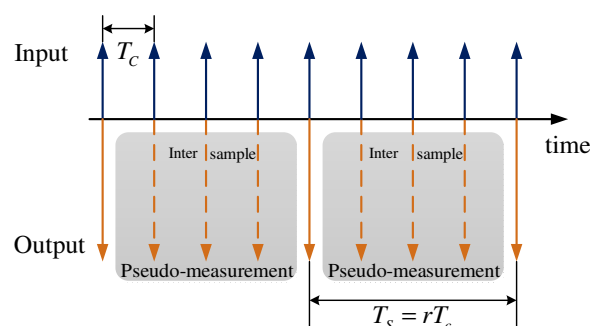


Fig. 1. Dual-rate system: signal input and signal output.

deal with model uncertainties and external disturbances? Secondly, how to provide the correction during inter-samples considering both measurement noises and process noises? Robust estimation can be a candidate for the first issue by minimizing the upper bound of estimation error covariance under model uncertainties (Xie *et al.*, 1994). A recent robust estimation is proposed such that not only robust estimation gain but also system matrices are computed (Mohamed *et al.*, 2012). Instead of using these methods which are complex for practical applications, we apply another scheme named “disturbance accommodation”. It was introduced to reject external disturbances in linear quadratic regulator (Johnson, 1974). In this paper, model uncertainties and external disturbances are lumped into extended states. In this paper, we focus on dual-rate estimation. Pseudo measurement is proposed to maintain state correction during inter-samples. We design the pseudo-measurement such that the homogeneous part of the estimation error dynamics takes the same form as that of conventional estimation with fast-rate measurement.

In order to demonstrate the effectiveness of the proposed method, we will apply it to estimate the sideslip angle of vehicle for lateral stability control system. The estimation is designed using the measurement of course angle through GPS receiver. In this system, the sampling time of GPS signal is much longer than the control period of actuators including in-wheel motors and active steering motor. Using the fusion of course angle from GPS receiver and yaw rate, it

is possible to estimate vehicle sideslip angle (Anderson *et al.*, 2010). However, Anderson's works does not address the problem of model uncertainties and external disturbances. We improve this sensor fusion by introducing disturbance accommodation (Nguyen *et al.*, 2012, 2013). This paper shows that by using only course angle measurement, it is possible to obtain sideslip angle. Simulations and experiments are performed to prove that by using the "pseudo measurement" with "disturbance accommodation", the accuracy of sideslip angle estimation is effectively enhanced.

2. SYSTEM MODELING

2.1 Dual-Rate System

Consider a linear stochastic system such that the sampling time of output measurement is equal to r times of the control period (r is assumed to be a constant integer).

$$x_{k+1} = (a_k + \Delta a_k)x_k + (b_k + \Delta b_k)u_k + d_k + w_k \quad (1)$$

$$y_k = \delta_k (cx_k + v_k) \quad (2)$$

$$\delta_k = \begin{cases} 1 & \text{if } k = jr \\ 0 & \text{if } k \neq jr \end{cases} \quad (3)$$

where x_k is the state vector, u_k is the input vector, y_k is the output vector, a_k is the state matrix, b_k is the input matrix, c is the output matrix, Δa_k and Δb_k represent model uncertainties, d_k is the unknown external disturbance vector. The process noise measurement noise are assumed as Gaussian noises with zero mean, or $w_k \sim \mathcal{N}(0, Q_k)$ and $v_k \sim \mathcal{N}(0, R_k)$, respectively. They are assumed to be uncorrelated with each other. Q_k and R_k are noise covariance matrices. The size of the vectors and matrices are as follows: $x_k \in \mathbb{R}^m$, $u_k \in \mathbb{R}^p$, $y_k \in \mathbb{R}^q$, $w_k \in \mathbb{R}^m$, $v_k \in \mathbb{R}^q$, $d_k \in \mathbb{R}^m$, $\dim[a_k]=m \times m$, $\dim[b_k]=m \times p$, $\dim[c_k]=q \times m$, $\dim[Q_k]=m \times m$, $\dim[R_k]=q \times q$. The number j is the nonnegative integer. At the period $k = jr$, the measurement is updated.

2.2 Disturbance Accommodation

Define the extended state $x_{d,k}$ that represent both the external disturbances and model uncertainties:

$$x_{d,k} = \Delta a_k x_k + \Delta b_k u_k + d_k \quad (4)$$

From (1) and (4), dynamics of x_k can be rewritten as:

$$x_{k+1} = a_k x_k + b_k u_k + x_{d,k} + w_k \quad (5)$$

From (4) and (5), we have:

$$x_{d,k+1} = \Delta a_{k+1} a_k x_k + \Delta a_{k+1} x_{d,k} + (\Delta a_{k+1} b_k u_k + \Delta b_{k+1} u_{k+1} + d_{k+1}) + \Delta a_{k+1} w_k \quad (6)$$

Actually, we have no precise understanding of the dynamics of the extended state. For the sake of simplicity, we introduce the following dynamics of the extended state:

$$x_{d,k+1} = a_{d,k} x_{d,k} + u_{d,k} + w_{d,k} \quad (7)$$

where $a_{d,k}$ expresses one-step transition of the extended state, $u_{d,k}$ is the rate of the extended state, $w_{d,k}$ is assumed to be Gaussian noise with zero mean $w_{d,k} \sim \mathcal{N}(0, Q_{d,k})$. We assume that this process noise is uncorrelated with the measurement noise v_k . The size of these terms are as follows: $u_{d,k} \in \mathbb{R}^m$, $w_{d,k} \in \mathbb{R}^m$, $\dim[a_{d,k}]=m \times m$, $\dim[Q_{d,k}]=m \times m$. In practical control application, $a_{d,k}$, $u_{d,k}$, and $Q_{d,k}$ are tuned to achieve fine estimation. For instance, if we set $a_{d,k}$ as unity matrix and $u_{d,k}$ to be zero, the extend state becomes random walk process. The tuning process is based on trial-and-error. This method, although heuristic, offers more degrees of tuning to enhance estimation performance.

From now, the following extended system is used for estimation design.

$$X_{k+1} = A_k X_k + B_k U_k + W_k \quad (8)$$

$$Y_k = \delta_k (C X_k + V_k) \quad (9)$$

$$X_k = [x_k \quad x_{d,k}]^T, U_k = [u_k \quad u_{d,k}]^T, W_k = [w_k \quad w_{d,k}]^T \quad (10)$$

$$Y_k = y_k, V_k = v_k$$

$$A_k = \begin{bmatrix} a_k & I_{(m \times m)} \\ O_{(m \times m)} & a_{d,k} \end{bmatrix}, B_k = \begin{bmatrix} b_k & O_{(m \times m)} \\ O_{(m \times p)} & I_{(m \times m)} \end{bmatrix} \quad (11)$$

$$C = [c \quad O_{(q \times m)}]$$

The process and measurement noise covariance matrices of the extended system are expressed as:

$$Q_{e,k} = \begin{bmatrix} Q_k & O_{(m \times m)} \\ O_{(m \times m)} & Q_{d,k} \end{bmatrix}, R_{e,k} = R_k \quad (12)$$

3. PROPOSAL OF PSEUDO MEASUREMENT

3.1 Pseudo Measurement

Assume that at period k , a measurement is updated. Consequently, an innovation is calculated as:

$$\varepsilon_k = Y_k - C \hat{X}_{k|k-1} = C \tilde{X}_{k|k-1} + V_k \quad (13)$$

where $\hat{X}_{k|k-1}$ is the predicted state at period k by one step transition from period $k-1$, $\tilde{X}_{k|k-1}$ is the prediction error. An estimation gain L_k can be designed, for instance, using Kalman filter or pole placement. Then, the corrected state $\hat{X}_{k|k}$ is obtained. This trivial works is neglected in this paper. However, at period $k+i$ (for i from 1 to $r-1$), no measurement

is available. In order to maintain the correction stage, the following pseudo measurement is introduced.

$$Y_{k+i}^* = C\hat{X}_{k+i|k+i-1} + G_{k+i}\epsilon_k \quad (14)$$

As (14), pseudo measurement is computed using the predicted state at $k+i$ and storing the innovation at k . G_{k+i} is an invertible matrix to be designed. Define the pseudo measurement noise:

$$V_{k+i}^* = G_{k+i}\epsilon_k - C\tilde{X}_{k+i|k+i-1} \quad (15)$$

The pseudo measurement can be expressed as:

$$Y_{k+i}^* = CX_{k+i} + V_{k+i}^* \quad (16)$$

3.2 Dynamics of Estimation Error with Pseudo Measurement

For i from 1 to $r-1$, the estimation is conducted as follows:

Prediction:

$$\hat{X}_{k+i|k+i-1} = A_{k+i-1}\hat{X}_{k+i-1|k+i-1} + B_{k+i-1}U_{k+i-1} \quad (17)$$

$$\tilde{X}_{k+i|k+i-1} = X_{k+i} - \hat{X}_{k+i|k+i-1} = A_{k+i-1}\tilde{X}_{k+i-1|k+i-1} + W_{k+i-1} \quad (18)$$

Correction:

$$\hat{X}_{k+i|k+i} = \hat{X}_{k+i|k+i-1} + L_{k+i}(Y_{k+i}^* - C\hat{X}_{k+i|k+i-1}) \quad (19)$$

$$\tilde{X}_{k+i|k+i} = X_{k+i} - \hat{X}_{k+i|k+i} = (I_{m \times m} - L_{k+i}C)\tilde{X}_{k+i-1|k+i-1} - L_{k+i}V_{k+i}^* \quad (20)$$

where L_{k+i} is the estimation gain associated with pseudo measurement at period $k+i$. From (18) and (20), the dynamics of estimation error takes the following form:

$$\tilde{X}_{k+i|k+i-1} = \Omega_{k+i}\tilde{X}_{k+i-1|k+i-1} + \Gamma_{k+i}W_{k+i-1} - L_{k+i}V_{k+i}^* \quad (21)$$

$$\Omega_{k+i} = (I_{m \times m} - L_{k+i}C)A_{k+i-1} \quad (22)$$

$$\Gamma_{k+i} = I_{m \times m} - L_{k+i}C \quad (23)$$

3.3 Pseudo Measurement Noise

From (13) and (15), the pseudo measurement noise can be expressed as:

$$V_{k+i}^* = G_{k+i}C\tilde{X}_{k|k-1} - C\tilde{X}_{k+i|k+i-1} + V_k \quad (24)$$

In order to expand the pseudo measurement noise, we will find the relationship between prediction error at period $k+i$ and k . From a series of predictions and corrections until the prediction at period $k+i$ and considering the expression of pseudo noise in (15), the following relationship is obtained:

$$\tilde{X}_{k+i|k+i-1} = \Psi_{k+i}\tilde{X}_{k|k-1} + \Upsilon_{k+i} - \Lambda_{k+i}V_k \quad (25)$$

$$\Psi_{k+i} = \left(\prod_{l=1}^i A_{k+i-l} \right) (I_{m \times m} - L_k C) - \sum_{l=1}^{i-1} \left[\left(\prod_{m=1}^l A_{k+i-m} \right) (L_{k+i-l} G_{k+i-l} C) \right] \quad (26)$$

$$\Upsilon_{k+i} = \sum_{l=1}^{i-1} \left[\left(\prod_{m=1}^l A_{k+i-m} \right) W_{k+i-l-1} \right] + W_{k+i-1} \quad (27)$$

$$\Lambda_{k+i} = \sum_{l=1}^i \left[\left(\prod_{m=1}^l A_{k+i-m} \right) (L_{k+i-l} G_{k+i-l}) \right] \quad (28)$$

Substitute (25) into (24), we have:

$$V_{k+i}^* = (G_{k+i}C - C\Psi_{k+i})\tilde{X}_{k|k-1} - C\Upsilon_{k+i} + (I_{m \times m} + C\Lambda_{k+i})V_k \quad (29)$$

The pseudo measurement noise at period $k+i$ is independent of prediction error at period k if the gain matrix G_{k+i} satisfies the following condition:

$$G_{k+i}C = C\Psi_{k+i} \quad (30)$$

Solving equation (30), the G_{k+i} can be calculated as:

$$G_{k+i} = C\Psi_{k+i}C^T (CC^T)^{-1} \quad (31)$$

G_{k+i} is calculated using the state matrix, output matrix, and the estimation gains till period $k+i-1$. For instance, G_{k+1} is calculated as:

$$G_{k+1} = CA_k (I_{m \times m} - L_k C)C^T (CC^T)^{-1} \quad (32)$$

If G_{k+1} is designed as (31), even during inter-samples, the dynamics of estimation error has the following form:

$$\tilde{X}_{k+i|k+i-1} = \Omega_{k+i}\tilde{X}_{k+i-1|k+i-1} + F_i(W_{k+i-1}, \dots, W_{k-1}, V_k) \quad (33)$$

The homogeneous part of the estimation error is the same as that of single-rate estimation. Moreover, F_i is a function of Gaussian noises. Therefore, the estimation error dynamics is enhanced by the proposed pseudo measurement. By calculating the pseudo measurement noise covariance using (29), L_{k+i} can be obtained using Kalman filter's way. It is also possible to use pole-placement to design L_{k+i} .

In case of without correction using pseudo-measurement, the estimation error during inter-samples has the following form:

$$\tilde{X}_{k+i|k+i}^{wo} = A_{k+i-1}\tilde{X}_{k+i-1|k+i-1}^{wo} + W_{k+i-1} \quad (34)$$

From (33), we can see that it is possible to place the pole of estimation error dynamics during inter-samples, thanks to pseudo-measurement. As can be seen from (34), it is impossible to improve the estimation convergence speed.

3.4 Other Inter-Sample Correction Methods

As mentioned in the Introduction, Hara *et al.* and Oh *et al.* also suggest the correction during inter-samples utilizing the

last available innovation. Their methods are implemented as linear observer for linear time invariant systems. Using both methods, speed of estimation performance can be improved. However, the differences to the proposal are as follows:

Method proposed by Hara *et al.*: G_{k+i} is set to be unity matrix and estimation gain is constant matrix. It is designed such that the pole of estimation error dynamics is the same as single-rate estimation using low-rate measurement.

Method proposed by Oh *et al.*: The gain matrix G_{k+i} is designed as:

$$G_{k+i} = CA[(I_{m \times m} - LC)A]^{i-1} E_1 \quad (35)$$

However, estimation error dynamics is not the same as that of estimation with fast-rate measurement. Considering the time invariant model, the homogeneous part of estimation error dynamics is expressed as:

$$\tilde{X}_{k+i|k+i} = \left\{ A^i [(I_{m \times m} - LC)A] E_2 + [(I_{m \times m} - LC)A]^{i+1} E_1 \right\} \tilde{X}_{k-1|k-1} \quad (36)$$

where E_1 and E_2 are gain matrices to be designed.

Proposed method: The dynamics part of estimation error is the same as estimation with fast-rate measurement.

4. APPLICATION:

VEHICLE SIDELSLIP ANGLE ESTIMATION

4.1 Motivation

Sideslip angle of vehicle is the angle between the velocity vector and its longitudinal component. Sideslip angle must be controlled to prevent the vehicle accidents which may happen in critical driving situation, such as vehicle cornering into slippery road at high speed (Nam *et al.*, 2012). In fact, current vehicles are not equipped with an ability of measuring sideslip angle directly. Corrsys-Datron provides the noncontact optical sensor for calculating sideslip angle based on longitudinal and lateral velocity measurement. Because of its high cost, this noncontact optical sensor cannot be a practical solution.

Sideslip angle can be estimated using linear observer and the measurement of yaw rate and lateral acceleration (Aoki *et al.*, 2004). In fact, the variation of road condition introduces uncertainties into estimation model, especially in cornering stiffness. Moreover, gyroscope and accelerometer are interfered by strong noise and bias. Therefore, this method is not robust enough for vehicle control system. Since the last decade, attitude information from on-board GPS receiver can be used for sideslip angle estimation. For instance, a Kalman filter is designed to combine GPS receiver and gyro sensor for estimating sideslip angle (Anderson *et al.*, 2010). In comparison with gyroscope and accelerometer, a RTK-GPS

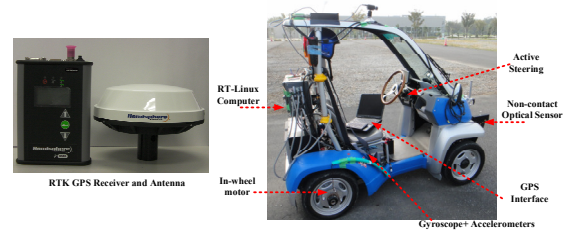


Fig. 2. Experimental electric vehicle and RTK-GPS.

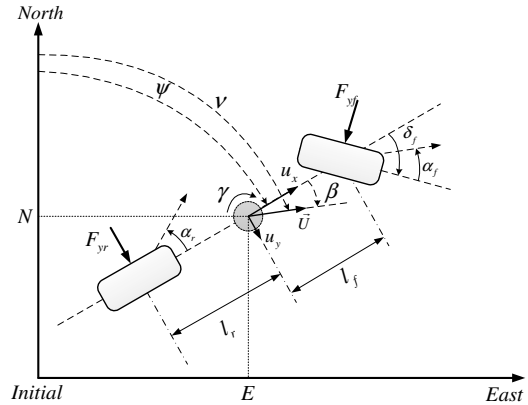


Fig. 3. Planar bicycle model with front steering.

receiver can provide accurate attitude measurement with no bias and less noise in long term.

Thanks to the application of motor-based-actuators as in-wheel motor and electric power steering, vehicle stability system can achieve the control period of one millisecond. To satisfy the controller, it is required to estimate sideslip angle every one millisecond. However, the sampling time of course angle from GPS receiver is much longer. Therefore, between two consecutive updates of GPS signal, the performance of GPS-based-estimator is degraded due model uncertainties and disturbances.

In this paper, we aim to two contributions: First, we show that sideslip angle can be estimated using only GPS receiver. Secondly, we apply the proposed method to enhance the sideslip angle estimation. Experimental vehicle and GPS receiver are shown in Fig. 2. This is an electric vehicle with two rear in-wheel motors and active front steering system as actuators. The RTK-GPS receiver is produced by Hemisphere.

4.2 Modeling

Planar bicycle model of vehicle is shown in Fig. 3. Course angle of a vehicle is angle between its moving direction and geodetic North. Course angle ν can also be defined as the summary of yaw angle ψ and sideslip angle β . The following continuous time model is established.

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_f \\ N_z \end{bmatrix} \quad (37)$$

$$v = \begin{bmatrix} 1 & 0 & 1 \\ \beta \\ \gamma \\ \psi \end{bmatrix} \quad (38)$$

where:

$$\begin{aligned} a_{11} &= \frac{-2(C_f + C_r)}{mv_x}, & a_{12} &= -1 - \frac{2(C_f l_f - C_r l_r)}{mv_x^2} \\ a_{21} &= \frac{-2(C_f l_f - C_r l_r)}{I_z}, & a_{22} &= \frac{-2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \\ b_{11} &= \frac{2C_f}{mv_x}, & b_{21} &= \frac{2C_f l_f}{I_z}, & b_{22} &= \frac{1}{I_z} \end{aligned} \quad (39)$$

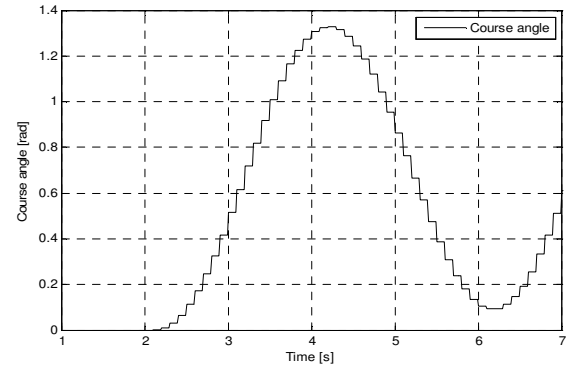
C_f and C_r are the front and rear tire cornering stiffness; m is the mass of vehicle; I_z is the yaw moment of inertia; l_f and l_r are the distances from center of gravity to front and rear wheel axis; v_x is the longitudinal velocity; the control inputs include front steering angle δ_f and yaw moment N_z generated by the torque difference between left and right in-wheel motors.

4.3 Estimation Design

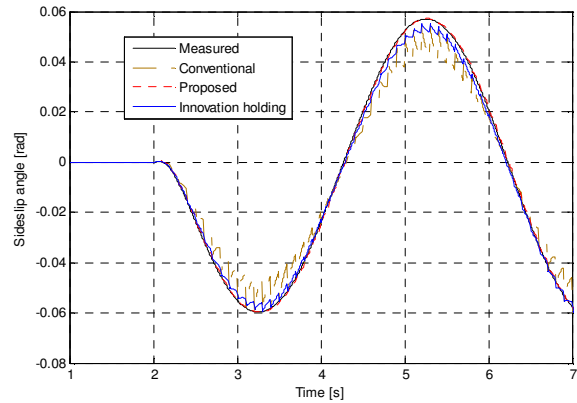
From the continuous time model (37)~(39), a discrete time stochastic model is established. The state matrix and input matrix can be updated using the longitudinal velocity v_x . The course angle from GPS receiver is updated at 10 Hz, in other words, its sampling time is 100 milliseconds. In order to evaluate the proposed estimation, the cornering stiffness of the estimation model is selected such that it is different to the real value: $C_{f_model} = 1.3C_{f_real}$, $C_{r_model} = 1.3C_{r_real}$. This means that uncertainties are introduced into estimation model. The model is extended by disturbance accommodation in which the extended state is assumed to be random walk process. Notice that the extended output matrix satisfies that CC^T is invertible. Therefore, it is possible to designed G_{k+i} such that the pseudo noise is independent of prediction error. Besides the proposed method, other two methods are also designed and evaluated: 1) *Conventional* method: The sideslip angle is only predicted during inter-samples. 2) *Innovation holding* method: As proposed by Hara *et al.*, the innovation of course angle is memorized for correcting the sideslip angle during inter-samples.

4.4 Simulation and Experimental Results

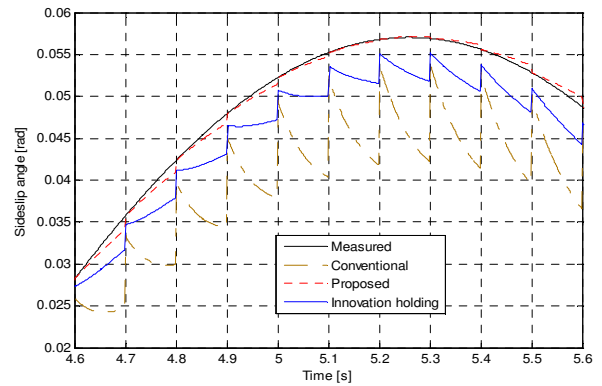
Simulation results of a slalom test are illustrated in Fig. 4. In the simulation, not only model uncertainties but also lateral wind force is generated as external disturbances. The conventional method shows impulsive behaviour such that estimation error increases considerably during inter-samples. In contrast, by maintaining the corrections, the innovation holding method can provide smoother estimation results. The best estimation is offered by the proposed method, thanks to both “disturbance accommodation” and “pseudo-measurement”.



(a) Course angle



(b) Sideslip angle



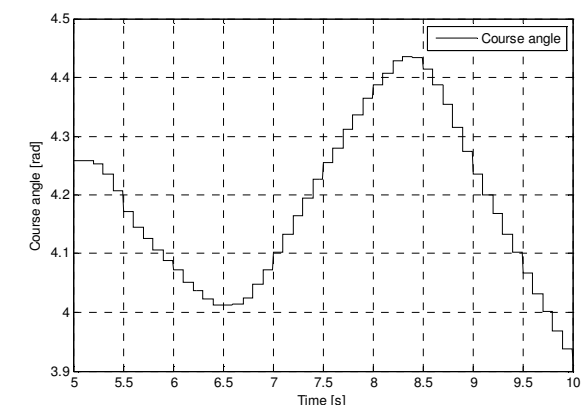
(c) Inter-sample performance

Fig. 4. Simulation results.

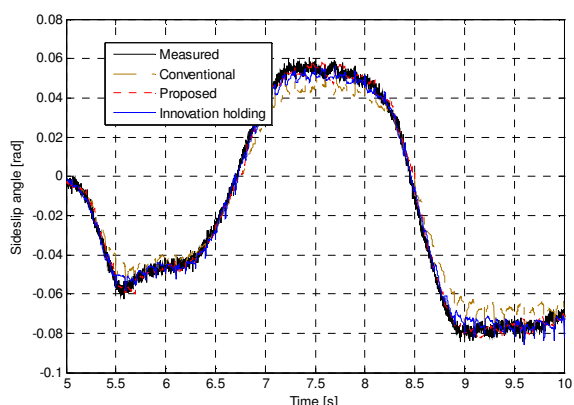
Fig. 5 demonstrates the experimental results of slalom test. As the simulation, the conventional method provides the poorest estimated sideslip angle while the most accurate estimation comes from the proposed method.

6. CONCLUSIONS

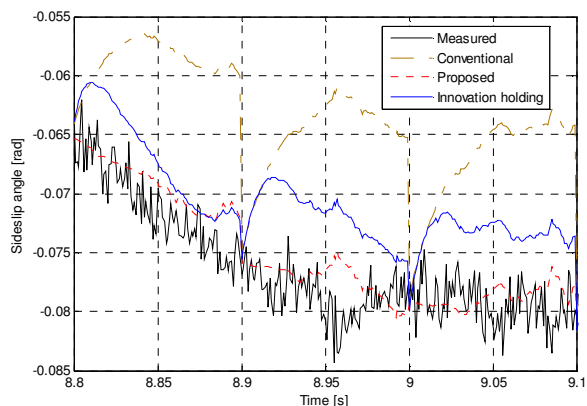
From the view point of theory, this paper shows that it is possible to enhance the performance of estimation with low rate measurement by simultaneously extending the model using disturbance accommodation and maintaining the correction at every control period using pseudo-measurement. From the view point of application, this paper suggests that sideslip angle can be robustly obtained using only GPS receiver. Simulations and experiments are conducted to



(a) Course angle



(b) Sideslip angle



(c) Inter-sample performance

Fig. 5. Experimental results.

verify the proposed estimation scheme. In future works, we will develop this scheme to the system with multi sources of measurement considering time delay.

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