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A New Adaptive Algorithm for Periodic Noise Control and its Stability Analysis

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Abstract:
The paper considers an active noise feedforward control where a noise consists of a sinusoidal signal and its harmonic components. To overcome drawbacks of filtered-x least mean square algorithm and delayed-x harmonics synthesiser algorithm, a new adaptive algorithm is proposed. Then stability property of the new algorithm is clarified by using averaging technique and it is shown that there always exists a stable equilibrium point. It is also remarkable that the new algorithm does not require any information on the second path dynamics, i.e., the intervening transfer function. Therefore, the new method could guarantee robust stability without on-line second path modelling. Finally, numerical simulation results show the effectiveness of the new algorithm.

Keywords: Active noise control, Least-squares algorithm, Nonlinear system, Stability analysis, Averaging method.

1. INTRODUCTION

There have been a lot of literatures on active noise or vibration control [Elliott and Sutton (1996), Fuller and von Flotow (1995), Kuo and Morgan (1999), George and Panda (2013)]. In this paper, a noise to be rejected is periodic, i.e., the noise consists of sinusoidal signals with a fundamental frequency and its some harmonic components. This type of noise or vibration control problem is well known to occur in many fields, e.g., cabin noise and vibration of automobiles or aircrafts, sounds in ducts of factories etc. For narrow-band noise control, as the control architectures, two main approaches are well known; *feedback* from sensors to control actuators and *feedforward* to the control actuators of a signal correlated with the disturbance. Controllers [Landau et al. (2005), Sievers and von Flotow (1992)] based on *internal model principle* belong to the former and the *least mean square* (LMS) algorithm belongs to the latter.

LMS algorithm has been still playing an important role in the field of active noise feedforward control. To deal with the case that the canceling signal cannot be directly applied to the primary signal due to an intervening transfer function, the filtered-x LMS (FxLMS) algorithm has been proposed and its characteristics have been investigated in detail [Morgan (1980), Burgess (1981), Morgan and Sanford (1992), Bodson et al. (1994)].

Concerning to FxLMS algorithm, it was pointed out that stability is not always guaranteed in the case of low SNR error input or fluctuation of secondary path. Therefore, its convergence property has still received many attentions [Vicente and Masgrau (2006), Xiao et al. (2008), Ardekani and Abdulla (2010)].

Of course, FxLMS is extended to carry on-line second path modelling, but it needs an extra noise injection to the actuator and requires a lot of computational load. Many techniques have still been proposed to improve FxLMS algorithm [Akhtar et al. (2008), Lan et al. (2002), Lin and Liao (2008), Hinamoto and Sakai (2006), Wang et al. (2006), Xiao (2011)].

Then, in order to overcome those drawbacks of FxLMS, the delayed-x harmonics synthesiser (DXHS) algorithm has been proposed as well as its extension for on-line second path modelling [Shimada et al. (1998), Shimada et al. (1999)].

However, stability consideration in DXHS algorithm is still not enough because it may become unstable due to estimation errors of the intervening transfer function.

In this paper, a new DXHS algorithm is proposed and its stability analysis is carried by using the averaging method [Sastry and Bodson (1989)]. Then it is clarified that the new method makes many equilibrium points in the space of adjustable parameters and guarantees that some of them are always stable equilibrium points. Moreover, the new method does not require any information on the intervening transfer function, and therefore the on-line second path modelling is not needed.

The paper is organized as follows: in Section 2, an active noise control system is set up. Then the conventional and the new DXHS algorithms are shown. Section 3 carries the stability analysis by characterizing all the equilibrium points, deriving an averaged system and then showing that there always exists stable equilibrium point. In Section 4, the numerical simulation results show the effectiveness of the new method and also it is shown that the new method would guarantee global stability. Section 5 is for some concluding remarks.

2. CONVENTIONAL AND NEW DXHS ALGORITHMS

First the conventional DXHS algorithm is summarized, then a new DXHS algorithm will be proposed.

2.1 The Conventional DXHS Algorithm

Figure 1 shows a block diagram of active noise control system based on DXHS algorithm where $d(t)$ is a noise signal to be suppressed, $y(t)$ is a control signal, $u(t)$ is a command signal to the actuator as the secondary source, and $e(t)$ is an error signal at control point. $G(s)$ expresses a transfer function which consists of both the actuator's dynamics and a transmission characteristic of the second path.

Here it is assumed that the noise $d(t)$ consists of sinusoidal signals with a fundamental frequency ω and its harmonic components where ω is known. Then $d(t)$ cannot be observed directly, only the error signal $e(t)$ is available to generate the command signal $u(t)$ adaptively.

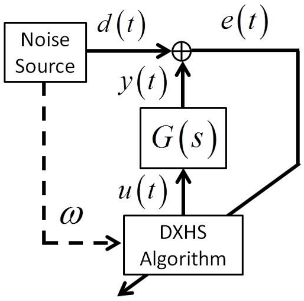

Fig. 1. Active noise control based on DXHS algorithm

Suppose $d(t)$ and $u(t)$ are given as

$$d(t) = \sum_{k=1}^{m} \alpha_k \sin(k\omega t + \delta_k) \tag{1}$$

$$u(t) = \sum_{k=1}^{m} a_k(t) \sin(k\omega t + \phi_k(t)), \tag{2}$$

where $a_k(t)$ and $\phi_k(t)$ are *adjustable parameters* in the DXHS algorithm. The control signal $y(t)$ is generated by $u(t)$ through $G(s)$. If the dynamics of $G(s)$ is much faster than the adaptive dynamics of $a_k(t)$ and $\phi_k(t)$, i.e., the adaptation speed is slow enough, then $y(t)$ and $e(t)$ can be expressed as

$$\begin{aligned} y(t) &= \sum_{k=1}^{m} g_k a_k(t) \sin(k\omega t + \phi_k(t) + \theta_k) \qquad (3)\\ e(t) &= d(t) + y(t) \\ &= \sum_{k=1}^{m} \alpha_k \sin(k\omega t + \delta_k) \\ &\quad + \sum_{k=1}^{m} g_k a_k(t) \sin(k\omega t + \phi_k(t) + \theta_k) \qquad (4)\end{aligned}$$

where g_k and θ_k are defined by

$$g_k := |G(jk\omega)|, \quad \theta_k := \angle G(jk\omega). \tag{5}$$

Therefore, in order to force $e(t)$ to be zero, $a_k(t)$ and $\phi_k(t)$ are needed to be immediately tuned as

$$a_k(t) = \alpha_k / g_k, \quad \phi_k(t) = \pi + \delta_k - \theta_k. \tag{6}$$

The adaptive rule of $a_k(t)$ and $\phi_k(t)$ in the conventional DXHS algorithm [Shimada et al.(1998 and 1999)] is expressed in continuous-time version as follows.

$$\begin{aligned} \dot{a}_k(t) &= -\gamma_{a_k} \frac{\partial e(t)^2}{\partial a_k(t)} = -2\gamma_{a_k} e(t) \frac{\partial e(t)}{\partial a_k(t)} \\ &= -2\gamma_{a_k} g_k e(t) \sin(k\omega t + \phi_k(t) + \theta_k) \qquad (7)\\ \dot{\phi}_k(t) &= -\gamma_{\phi_k} \frac{\partial e(t)^2}{\partial \phi_k(t)} = -2\gamma_{\phi_k} e(t) \frac{\partial e(t)}{\partial \phi_k(t)} \\ &= -2\gamma_{\phi_k} g_k a_k(t) e(t) \cos(k\omega t + \phi_k(t) + \theta_k) \qquad (8)\end{aligned}$$

where $k = 1, 2, \cdots, m$, the parameters γ_{a_k} and γ_{ϕ_k} are positive constants. Notice that you should use some estimated values of g_k and θ_k in (7) and (8) if their exact ones are not known.

2.2 A New DXHS Algorithm

Instead of (2), the command signal $u(t)$ in a new DXHS algorithm is proposed as

$$u(t) = \sum_{k=1}^{m} a_k(t) \sin(k\omega t + \ell_k \phi_k(t)) \tag{9}$$

where ℓ_k is any positive integer (≥ 2). Notice that this $u(t)$ could be equal to (2) if $\ell_k = 1$ for $\forall k$.

The error signal $e(t)$ is now given by

$$\begin{aligned} e(t) &= \sum_{k=1}^{m} \alpha_k \sin(k\omega t + \delta_k) \\ &\quad + \sum_{k=1}^{m} g_k a_k(t) \sin(k\omega t + \ell_k \phi_k(t) + \theta_k). \qquad (10)\end{aligned}$$

An adaptive rule of $a_k(t)$ and $\phi_k(t)$ in (9) is given by

$$\dot{a}_k(t) = -\mu_{a_k} e(t) \sin(k\omega t + \phi_k(t)) \tag{11}$$

$$\dot{\phi}_k(t) = -\mu_{\phi_k} e(t) \cos(k\omega t + \phi_k(t)) \tag{12}$$

where μ_{a_k}, μ_{ϕ_k} are positive constants. In oder to keep $a_k(t)$ positive, when $a_k(t)$ is tuned to be negative, $a_k(t)$ and $\phi_k(t)$ are reset as $|a_k(t)|$ and $\phi_k(t) + \pi$, respectively.

Notice that the above rules (11) and (12) do not include the dynamics of $G(s)$, i.e., g_k and θ_k, which are needed in the conventional DXHS with (7) and (8). This is an advantage of the new algorithm because g_k and θ_k are very difficult to estimate in advance and/or they are sometimes time-varying.

3. STABILITY ANALYSIS

The aim of this section is to analyze the stability of active noise control system in Fig. 1. The system is nonlinear and it is difficult to analyze the stability directly, so here the *averaging method* (Sastry and Bodson (1989)) is used. Before considering the stability analysis, let some notations be prepared.

The adjustable parameters $a_k(t), \phi_k(t)$ $(k = 1, \cdots, m)$ are collected as an adjustable parameter vector $x(t) := [x_1(t)^T, x_2(t)^T]^T \in R^{2m}$ where

$$x_1 := [a_1, a_2, \cdots, a_m]^T, \quad x_2 := [\phi_1, \phi_2, \cdots, \phi_m]^T. \quad (13)$$

Denoting $\alpha := [\alpha_1, \alpha_2, \cdots, \alpha_m]^T \in R^m$ and setting all of μ_{a_k} and μ_{ϕ_k} as $\epsilon \in (0, \epsilon_0]$ for simplicity, the dynamics of all $a_k(t), \phi_k(t)$ $(k = 1, \cdots, m)$ in (11) and (12) are expressed in a compact form by

$$\dot{x}_1(t) = -\epsilon \left\{W_{ss}(t, x_2(t))\alpha + W_{sgs}(t, x_2(t))x_1(t)\right\} \quad (14)$$

$$\dot{x}_2(t) = -\epsilon \left\{W_{cs}(t, x_2(t))\alpha + W_{cgs}(t, x_2(t))x_1(t)\right\} \quad (15)$$

where $W_{ss}(t, x_2), W_{sgs}(t, x_2), W_{cs}(t, x_2), W_{cgs}(t, x_2) \in R^{m\times m}$ and their (k, n) elements are respectively given by

$$\begin{aligned} w_{kn}^{ss} &:= \frac{1}{2}\{\cos((k-n)\omega t + \phi_k - \delta_n) \\ &\qquad - \cos((k+n)\omega t + \phi_k + \delta_n)\} \quad (16) \\ w_{kn}^{sgs} &:= \frac{g_n}{2}\{\cos((k-n)\omega t + \phi_k - \ell_n\phi_n - \theta_n) \\ &\qquad - \cos((k+n)\omega t + \phi_k + \ell_n\phi_n + \theta_n)\} \quad (17) \\ w_{kn}^{cs} &:= \frac{1}{2}\{-\sin((k-n)\omega t + \phi_k - \delta_n) \\ &\qquad + \sin((k+n)\omega t + \phi_k + \delta_n)\} \quad (18) \\ w_{kn}^{cgs} &:= \frac{g_n}{2}\{-\sin((k-n)\omega t + \phi_k - \ell_n\phi_n - \theta_n) \\ &\qquad + \sin((k+n)\omega t + \phi_k + \ell_n\phi_n + \theta_n)\}. \quad (19) \end{aligned}$$

(14) and (15) are rewritten in more compact form as

$$\dot{x}(t) = -\epsilon f(t, x(t)), \quad x(0) = x_0 \quad (20)$$

where

$$f(t, x) := \begin{bmatrix} W_{sgs}(t, x_2) \\ W_{cgs}(t, x_2) \end{bmatrix} x_1 + \begin{bmatrix} W_{ss}(t, x_2) \\ W_{cs}(t, x_2) \end{bmatrix} \alpha. \quad (21)$$

It is easy to see that the dynamical system (20) with (21) has many equilibrium points, all of which can be given as $x_{eq} = [x_{1eq}^T, x_{2eq}^T]^T$ where

$$x_{1eq} = \left[\frac{\alpha_1}{g_1}, \frac{\alpha_2}{g_2}, \cdots, \frac{\alpha_m}{g_m}\right]^T \quad (22)$$

$$x_{2eq} = \left[\frac{(2k_1+1)\pi + \delta_1 - \theta_1}{\ell_1}, \cdots, \frac{(2k_m+1)\pi + \delta_m - \theta_m}{\ell_m}\right]^T \quad (23)$$

with $k_1, \cdots, k_m$ any integers.

Now we will discuss about stability of those equilibrium points.

The averaging method [Sastry and Bodson (1989)] relies on the fact that $f(t, x)$ has the mean value $f_{av}(x)$, i.e., there exists a continuous and strictly decreasing function $\gamma : R_+ \to R_+$ such that $\gamma(H) \to 0$ as $H \to \infty$ and

$$\left\| \frac{1}{H} \int_{t_0}^{t_0+H} f(\tau, x) - f_{av}(x) d\tau \right\| \le \gamma(H) \quad (24)$$

for all $t_0 \ge 0, H \ge 0, x \in B_\delta(x_{eq})$ where $B_\delta(x_{eq})$ is a closed ball with radius δ centered at $x_{eq} \in R^n$. And the following system is called the *averaged* system of (20).

$$\dot{x}_{av}(t) = -\epsilon f_{av}(x_{av}(t)), \quad x_{av}(0) = x_0 \quad (25)$$

Theorem 1. Associated with the original system (20) with (21), the averaged system (25) is given by

$$f_{av}(x) := \begin{bmatrix} W_{sgs}^{av}(x_2) \\ W_{cgs}^{av}(x_2) \end{bmatrix} x_1 + \begin{bmatrix} W_{ss}^{av}(x_2) \\ W_{cs}^{av}(x_2) \end{bmatrix} \alpha \quad (26)$$

where $W_{ss}^{av}(x_2), W_{sgs}^{av}(x_2), W_{cs}^{av}(x_2), W_{cgs}^{av}(x_2) \in R^{m\times m}$ are diagonal with (k, k) elements respectively given by

$$\begin{aligned} w_{kk}^{av,ss} &:= \frac{1}{2}\cos(\phi_k - \delta_k) \quad (27) \\ w_{kk}^{av,sgs} &:= \frac{g_k}{2}\cos((\ell_k - 1)\phi_k + \theta_k) \quad (28) \\ w_{kk}^{av,cs} &:= -\frac{1}{2}\sin(\phi_k - \delta_n) \quad (29) \\ w_{kk}^{av,cgs} &:= \frac{g_k}{2}\sin((\ell_k - 1)\phi_k + \theta_k). \quad (30) \end{aligned}$$

Proof. It is easy to see that

$$\begin{aligned} h(\tau, x) &:= f(\tau, x) - f_{av}(x) \\ &= \begin{bmatrix} V_{sgs}(\tau, x_2) \\ V_{cgs}(\tau, x_2) \end{bmatrix} x_1 + \begin{bmatrix} V_{ss}(\tau, x_2) \\ V_{cs}(\tau, x_2) \end{bmatrix} \alpha \end{aligned} \quad (31)$$

where $V_{ss}(\tau, x_2), V_{sgs}(\tau, x_2), V_{cs}(\tau, x_2), V_{cgs}(\tau, x_2) \in R^{m\times m}$ are respectively equal to $W_{ss}(t, x2), W_{sgs}(t, x_2), W_{cs}(t, x_2), W_{cgs}(t, x_2)$ in (16) - (19) except (k, k) elements given by

$$\begin{aligned} v_{kk}^{ss} &:= -\frac{1}{2}\cos(2k\omega\tau + \phi_k + \delta_k) \quad (32) \\ v_{kk}^{sgs} &:= -\frac{g_n}{2}\cos(2k\omega\tau + (\ell_k + 1)\phi_k + \theta_k) \quad (33) \\ v_{kk}^{cs} &:= \frac{1}{2}\sin(2k\omega\tau + \phi_k + \delta_k) \quad (34) \\ v_{kk}^{cgs} &:= \frac{g_n}{2}\sin(2k\omega\tau + (\ell_k + 1)\phi_k + \theta_k). \quad (35) \end{aligned}$$

Notice that for any $H > 0$,

$$\left|\int_{t_0}^{t_0+H} v_{kn}^{ss} d\tau\right|, \left|\int_{t_0}^{t_0+H} v_{kn}^{cs} d\tau\right|, \left|\int_{t_0}^{t_0+H} \frac{v_{kn}^{sgs}}{g_n} d\tau\right|, \left|\int_{t_0}^{t_0+H} \frac{v_{kn}^{cgs}}{g_n} d\tau\right| < c_{kn}$$

where

$$c_{kn} = \begin{cases} \dfrac{1}{2k\omega} & (\text{for } k = n) \\ \dfrac{1}{|k-n|\omega} + \dfrac{1}{(k+n)\omega} & (\text{for } k \neq n) \end{cases}.$$

Therefore, it is straightforward to see that by setting $\gamma(H) = \frac{2m(g_{max}\|x_1\| + \|\alpha\|)}{\omega}/H$ with $g_{max} := \max\{g_k; k = 1, \cdots, m\}$, (24) holds, which means that (25) with (26) is an averaged system of (20) with (21). (Q.E.D.)

Remark 1 All the points x_{eq} given in (22) and (23) are also equilibrium points of the averaged system (25) with (26). In the original system (20) with (21), the dynamics of x_{1k} and x_{2k} depend on not only x_{1k} and x_{2k} but also all x_{1n}'s and x_{2n}'s for $n = 1, 2, \cdots, k-1, k+1, \cdots, m$. But, in the averaged system (25) with (26), the dynamics of x_{av1k} and x_{av2k} are independent of all x_{av1n} and x_{av2n} for $n \neq k$. □

Now we can apply the well known theorems using the averaging techniques [Sastry and Bodson (1989)].

Theorem 2. Associated with the original system (20) with (21) and its averaged system (25) with (26) where $x(0) = x_{av}(0) \in B_\delta(x_{eq})$, there exists $\psi(\epsilon)$ such that given $H \geq 0$,

$$\|x(t) - x_{av}(t)\| \leq \psi(\epsilon) b_H \tag{36}$$

for some $b_H \geq 0$, $\epsilon_H > 0$, and for all $t \in [0, H/\epsilon]$ and $\epsilon \leq \epsilon_H$.

Proof. This theorem comes from Theorem 4.2.4 in (Sastry and Bodson (1989)) and what we need to show is that all the conditions of Theorem 4.2.4 hold.

In fact, it is easy to see that $f(t, x)$ in (21) is Lipschitz in $x \in B_\delta(x_{eq})$ as well as $f_{av}(x)$ in (26) is.

Furthermore, with respect to $h(\tau, x)$ defined in (31), it is easy to verify that

$$\left\| \frac{1}{H} \int_{t_0}^{H} h(\tau, x - x_{eq}) d\tau \right\| \leq \gamma(H) \|x - x_{eq}\|$$

$$\left\| \frac{1}{H} \int_{t_0}^{H} \frac{\partial h}{\partial x}(\tau, x - x_{eq}) d\tau \right\| \leq \gamma(H).$$

The observation above shows that all the conditions of Theorem 4.2.4 in (Sastry and Bodson (1989)) hold. (Q.E.D.)

Theorem 3. If x_{eq} given in (22) and (23) is an exponentially stable equilibrium point of the averaged system (25) with (26), then the point x_{eq} is also exponentially stable for the original system (20) with (21) under that ϵ is sufficiently small.

Proof. This theorem also comes from Theorem 4.2.5 in in (Sastry and Bodson (1989)). (Q.E.D.)

Now we will consider the stability of the equilibrium points x_{eq} given in (22) and (23). In order to do this, we will linearize the averaged system (25) with (26) in the vicinity of the equilibrium x_{eq}.

Recall Remark 1, the averaged system (25) with (26) can be regarded as a set of independent subsystems, kth subsystem of which is

$$\dot{a}_k(t) = -\frac{\epsilon g_k}{2} a_k(t) \cos((\ell_k - 1)\phi_k(t) + \theta_k) - \frac{\epsilon \alpha_k}{2} \cos(\phi_k(t) - \delta_k) \tag{37}$$

$$\dot{\phi}_k(t) = -\frac{\epsilon g_k}{2} a_k(t) \sin((\ell_k - 1)\phi_k(t) + \theta_k) + \frac{\epsilon \alpha_k}{2} \sin(\phi_k(t) - \delta_k). \tag{38}$$

Suppose that $(a_{eq,k}, \phi_{eq,k})$ is an equilibrium point and let $a_k(t)$ and $\phi_k(t)$ be described by $a_k(t) = a_{eq,k} + \Delta a_k(t)$ and $\phi_k(t) = \phi_{eq,k} + \Delta\phi_k(t)$. Then the kth averaged subsystem (37) and (38) can be linearized in the vicinity of $(\Delta a_k, \Delta\phi_k) = (0, 0)$ as

$$\begin{bmatrix} \Delta\dot{a}_k(t) \\ \Delta\dot{\phi}_k(t) \end{bmatrix} = A_k \begin{bmatrix} \Delta a_k(t) \\ \Delta\phi_k(t) \end{bmatrix} \tag{39}$$

where

$$A_k = \frac{\epsilon}{2} \begin{bmatrix} g_k \cos(\phi_{eq,k} - \delta_k) & \alpha_k \ell_k \sin(\phi_{eq,k} - \delta_k) \\ -g_k \sin(\phi_{eq,k} - \delta_k) & \alpha_k \ell_k \cos(\phi_{eq,k} - \delta_k) \end{bmatrix} \tag{40}$$

Then the following theorem on stability of the equilibrium points x_{eq} is obtained.

Theorem 4. Consider the original system (20) with (21) and its equilibrium points x_{eq} in (22) and (23). The equilibrium point x_{eq} is asymptotically stable if for $k = 1, 2, \cdots, m$

$$\cos(\phi_{eq,k} - \delta_k) < 0. \tag{41}$$

Proof. It is easy to see that the linearized model (39) is asymptotically stable, i.e., A_k in (40) is stable, if and only if the condition (41) holds. Therefore, the equilibrium point x_{eq} is asymptotically stable in the averaged system (25) with (26) as well as in the original system (20) with (21) because of Theorem 3. (Q.E.D.)

Figure 2 shows distribution of the equilibrium points in the a_k-ϕ_k plane in the case of $\ell_k = 5$ and $\delta_k = 0$. Note that there exists ℓ_k equilibrium points in the ϕ_k's interval $(-\pi, \pi]$ on the line of $a_k = \alpha_k/g_k$. In the case of $\delta_k = 0$, the equilibrium point in the region of either $(\frac{\pi}{2}, \pi]$ or $(-\pi, -\frac{\pi}{2})$ is stable, the other points are unstable; red circles are stable and red crosses unstable. Therefore, when ℓ_k is even, the number of stable equilibrium points is always $\ell_k/2$. On the other hand, when ℓ_k is odd, it is either $(\ell_k - 1)/2$ or $(\ell_k + 1)/2$ (See that Fig. 2 shows the case of $(\ell_k - 1)/2$).

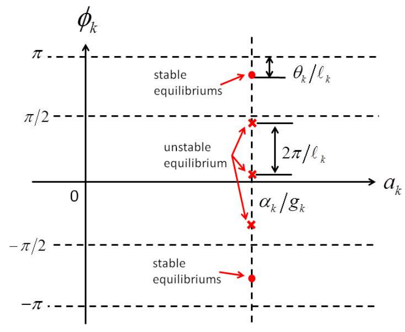

Fig. 2. Distribution of equilibrium points ($\ell_k = 5, \delta_k = 0$)

Remark 2 If $\ell_k = 1$ for $k = 1, 2, \cdots, m$, which corresponds to the conventional DXHS algorithm, the system (20) with (21) has a unique equilibrium point in the range $(-\pi, \pi]$ of ϕ_k and it is stable if and only if $-\pi/2 < \theta_k < \pi/2$ in the case of $\delta_k = 0$. But if $\ell_k \geq 2$, there always exists at least one stable equilibrium point. This is important advantage of the proposed method, compared with the conventional algorithm. □

Recall that in the adaptive rule of $a_k(t), \phi_k(t)$ in (11) and (12), to keep $a_k(t)$ positive, when $a_k(t)$ is tuned to be negative, $a_k(t)$ and $\phi_k(t)$ are reset as $|a_k(t)|$ and $\phi_k(t)+\pi$, respectively. If this process would cause $a_k(t)$ to remain on the line of $a_k = 0$, the proposed algorithm could not guarantee the global stability. In this sense, it is important to see whether $a_k(t)$ remains on the line of $a_k = 0$ or not.

Note that the dynamical equations of $a_k(t), \phi_k(t)$ in (25) with (26) on the line of $a_k = 0$ satisfy

$$\dot{a}_k(t) = -\frac{\epsilon\alpha_k}{2}\cos(\phi_k(t)-\delta_k),\ \dot{\phi}_k(t) = \frac{\epsilon\alpha_k}{2}\sin(\phi_k(t)-\delta_k).$$

Figure 3 shows the behavior on the line of $a_k = 0$ in the case of $\delta_k = 0$, where red arrows denote vector field and blue dotted arrows show the resetting of $a_k(t)$ and $\phi_k(t)$. It is easy to see that $a_k(t)$ gets closer to the line of $a_k = 0$ only when $\phi_k(t) \in (-\frac{\pi}{2}, \frac{\pi}{2})$. After $a_k(t)$ reaches to $a_k = 0$, $a_k(t)$ is reset to $a_k = 0$ with either $\phi_k(t) \in (\frac{\pi}{2}, \pi]$ or $\phi_k(t) \in [-\pi, -\frac{\pi}{2})$, and then $a_k(t)$ goes away from the line of $a_k = 0$. Therefore it concludes that the resetting process never force $a_k(t)$ to remain on the line of $a_k = 0$.

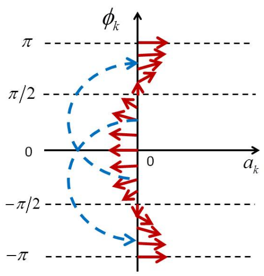

Fig. 3. Behavior on the line of $a_k = 0$ in the case of $\delta_k = 0$

4. NUMERICAL SIMULATIONS

4.1 The averaged system and its equilibriums

Figure 4 (a)-(d) show behaviors of the averaged system in the a_k-ϕ_k plane; (a) and (b) for $\ell_k = 1$ with $\theta_k = -45$ [deg] and -135 [deg] respectively, and (c) and (d) for $\ell_k = 5$ with $\theta_k = -45$ [deg] and -135 [deg] respectively. For simplicity, the parameters are set as $\alpha_k = g_k = 1$, $\delta_k = 0, \epsilon = 10$. The red circles in the figures denote stable equilibrium points and the red crosses are unstable ones.

Figure 5 shows time-histories $a_k(t), \phi_k(t)$ of the averaged system with $\ell_k = 5$ with four different initial values $(a_k(0), \phi_k(0))$, i.e., (1) $(0.1, 130)$, (2) $(0.5, 50)$, (3) $(0.5, -10)$, and (4) $(0.7, -90)$. The dotted lines denote stable equilibrium values. The jump phenomenon in $\phi_k(t)$ can be seen in the case of (3) at an instant when $a_k(t)$ reaches to 0.

From the observation above, it seems to suggest that the proposed algorithm provides not only local stability but also global stability.

4.2 The original system and its averaged system

Here we will compare the behaviors of the original system with its averaged system, where the noise signal's parameters are set as $\omega = 2\pi \times 40$[rad/sec], $m = 2, \alpha_1 = 1, \alpha_2 =$

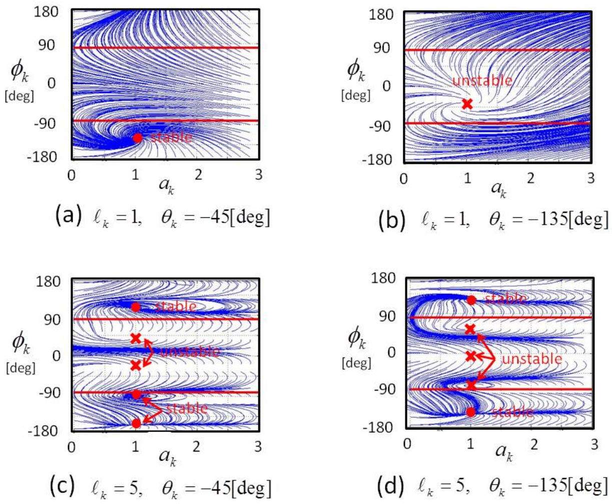

Fig. 4. Averaged system's behaviors and its equilibrium points

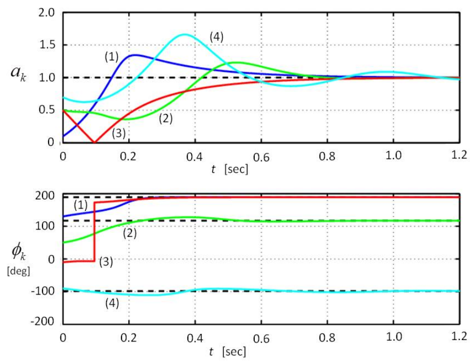

Fig. 5. Averaged system's time-histories ($\ell = 5$)

$0.5, \delta_1 = 0$, and $\delta_2 = -\pi/4$. The parameters with respect to $G(s)$ are set as $g_1 = 1, g_2 = 0.8, \theta_1 = -\pi/3, \theta_2 = -2\pi/3$. Note that the parameters on the adaptive algorithm are $\ell_1 = \ell_2 = 5$ and $\epsilon = 10$.

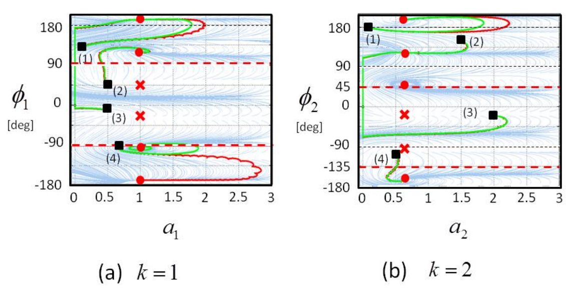

Fig. 6. Behaviors of Original(red curves) and Averaged(green curves) systems ($\ell_1 = \ell_2 = 5$)

Figure 6 shows four behaviors of the original system (red curves) and its averaged system (green curves) in the a_1-ϕ_1 plane in Fig. (a) as well as in the a_2-ϕ_2 plane in Fig. (b), where each behavior's initial point $x_0 = (a_1(0), a_2(0), \phi_1(0), \phi_2(0))^T$ is set as follows and denoted by black square in the figures: (1) $x_0 =$

$(0.1, 0.1, 130, 180)^T$, (2) $x_0 = (0.5, 1.5, 50, 150)^T$, (3) $x_0 = (0.5, 2.0, -10, -20)^T$ and (4) $x_0 = (0.7, 0.5, -90, -110)^T$. Notice that red circles and red crosses denote stable and unstable equilibrium points, which are given by (22) and (23), i.e., $x_{1eq} = (\alpha_1/g_1, \alpha_2/g_2)^T = (1, 5/8)^T$ and $x_{2eq} = \left(\frac{(2k_1+1)+1/3}{5}\pi,\ \frac{(2k_2+1)+5/12}{5}\pi\right)^T$. Table 1 shows all equilibrium points of x_{2eq} in the range of $(-\pi, \pi]$ and whether they are stable or unstable.

From the case (1) in Fig. 6, we can see that the averaged system's behavior (green curve) is almost equal to the original system's behavior (red curve) except the middle and they converge to the same stable equilibrium point. In the cases (2) and (3), the averaged system's behaviors look same as the original system's ones, that is why red curves cannot be seen there. Note that the average system sometimes converges to an equilibrium point different from the original system (see the a_1-ϕ_1 behavior of the case (4) in Fig. 6).

Table 1. Equilibrium points x_{2eq}

	stable	unstable	stable
ϕ_{1eq} [deg]	(192) 120	48, -24	-96, -168
ϕ_{2eq} [deg]	(195) 123 51	-21, -93	-165

5. CONCLUSION

The paper proposed the new DXHS algorithm, by which the active noise feedforward control is always stable without any information on the second path dynamics, i.e., the intervening transfer function. And also by using the averaging method, this stability mechanism was investigated under the assumption of slow adaptation.

The future researches are to investigate theoretically whether the proposed algorithm has a property of global stability or not, and to verify how well the proposed algorithm works in practical situation (in discrete-time version).

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