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Robust Static Output-Feedback Control for Uncertain Linear Discrete-Time Systems via the Generalized KYP Lemma ⋆

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Abstract: This paper is concerned with robust static output-feedback (SOF) control for linear discrete-time systems subject to polytopic uncertainty and restricted frequency-domain specifications (RFDSs) via the generalized Kalman-Yakubovich-Popov (KYP) lemma. Polytopic uncertainties are assumed to enter all the system matrices, while RFDSs are motivated by the fact that practical design specifications are often described in restricted finite frequency ranges. Dilated multipliers are first introduced to relax the GKYP lemma for SOF controller synthesis and robust performance analysis. Then a two-stage approach to SOF controller synthesis is proposed: at the first stage, a robust full-information (FI) controller is designed, which is used to construct a required SOF controller at the second stage. To improve the solvability of the synthesis method, an iterative algorithm is further formulated for exploring the SOF gain. The effectiveness of the proposed design method is finally demonstrated by a numerical example.

Keywords: Restricted frequency-domain specifications (RFDSs), static output-feedback (SOF) control, polytopic uncertainty, generalized Kalman-Yakubovich-Popov (GKYP) lemma.

1. INTRODUCTION

In control theory and engineering, frequency-domain conditions are often employed for describing design specifications. However, frequency-domain specifications are not tractable for controller synthesis, and consequently traditional design methods in the classic automatic control theory mostly rely on designers' experience. In the modern and "post modern" control theories, this difficulty is overcome by converting frequency-domain specifications into other more tractable forms, for instance, Riccati equations or linear matrix inequalities (LMIs). Such an elementary and useful result for conversion is the Kalman-Yakubovich-Popov (KYP) lemma [Rantzer 1996], which bridges a frequency-domain inequality and an LMI that can be numerically treated by effective algorithms [Toh et al. 1999] and is a pervasive tool in the control theory [Karimi et al. 2008, Shi et al. 1999, Wang et al. 2009, Li et al. 2012a, Yang et al. 2014].

Note that frequency-domain design specifications in many practical applications are usually restricted to finite or semi-finite ranges, but the standard KYP lemma can treat system performances in the entire frequency domain only. To solve this issue, Iwasaki and Hara generalized the standard KYP lemma to finite frequency ranges, so that a restricted frequency-domain specification (RFDS) can be directly converted into an equivalent LMI condition [Iwasaki and Hara 2005]. Based on the generalized KYP (GKYP) lemma, fruitful results have recently been developed for controller or filter synthesis [Iwasaki and Hara 2004, 2007, Gao and Li 2011, Li and Gao 2014], as well as two-dimensional systems [Li et al. 2012b]. Especially, feedback controller design subject to general RFDSs has been considered in Iwasaki and Hara [2004, 2007], and LMI approaches have been proposed.

On one hand, it deserves pointing out that static output-feedback (SOF) control with RFDSs has not been completely solved yet. Iwasaki and Hara [2007] only investigated the full-order dynamic counterpart of output-feedback control with RFDSs, and Iwasaki and Hara [2004] only provided some explicit expressions of multiplier R therein for state-feedback (SF) control. Moreover, SOF is simpler and more economic than SF and full-order dynamic output-feedback (DOF). *On the other hand*, uncertainties inevitably exist in a practical control plant due to parameter drifting, component aging or modelling errors. Hence, it is also of applied importance for a controller to be robust enough against model uncertainties. According to the above discussion, the problem of robust SOF synthesis with RFDSs and uncertainty has not been well studied yet, which motivates the work of this paper.

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In this paper, making use of the "two-stage" idea [Peaucelle and Arzelier 2001, Mehdi et al. 2004], we will investigate the design of robust SOF controllers for uncertain linear discrete-time systems simultaneously subject to uncertainties and RFDSs. Through expanding the matrix conditions in the GKYP lemma with dilated multipliers, a new necessary and sufficient condition will be first derived for characterizing the performance of the closed-loop system with an RFDS, which is then extended to the uncertain case and further utilized for parameterizing a required robust SOF controller in the "two-stage" framework. To improve the solvability of the derived method, an iterative LMI algorithm is also proposed for exploring the SOF controller. The effectiveness of the proposed design method will be finally illustrated by a numerical example. Note that a similar "two-stage" approach has been presented in Li and Gao [2014], which, however, is concentrated on continuous-time systems and does not consider system uncertainty.

Notation: The superscripts "-1", "T", "$*$" and "$\perp$" stand for inverse, transpose, conjugate transpose and null space of a matrix, respectively. $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ are the sets of all $m \times n$ real and complex matrices, respectively. $\mathbb{H}_n$ and $\mathbb{S}_n$ stand for the sets of $n \times n$ Hermitian and symmetric matrices, respectively. The notation $P > 0$ (≥ 0) means that matrix P is positive definite (semi-definite). $\Re(\cdot)$ is the real part of a complex number. $\mathbf{I}$ denotes an identity matrix with appropriate dimension. For matrices Φ and P, $\Phi \otimes P$ is the Kronecker product. $\mathrm{diag}\{A_1, \ldots, A_n\}$ stands for the block-diagonal matrix with $A_1, \ldots, A_n$ on the diagonal. For a square matrix A, $\mathrm{sym}\{A\}$ indicates $A^* + A$. For $G \in \mathbb{C}^{n\times m}$ and $\Pi \in \mathbb{H}_{n+m}$, a function $\upsilon\colon \mathbb{C}^{n\times m} \times \mathbb{H}_{n+m} \to \mathbb{H}_m$ is defined by $\upsilon(G, \Pi) \triangleq \begin{bmatrix} G \\ \mathbf{I}_m \end{bmatrix}^* \Pi \begin{bmatrix} G \\ \mathbf{I}_m \end{bmatrix}$.

2. PROBLEM STATEMENT

Consider an uncertain linear discrete-time system $G(z, \theta)$ represented by

$$\begin{aligned} x^+(t) &= A(\theta)x(t) + B(\theta)w(t) + B_u(\theta)u(t) \\ z(t) &= C(\theta)x(t) + D(\theta)w(t) + D_u(\theta)u(t) \\ y(t) &= C_y(\theta)x(t) + D_y(\theta)w(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_\mathrm{p}}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $w(t) \in \mathbb{R}^{n_w}$ is the disturbance, $y(t) \in \mathbb{R}^{n_y}$ is the measured output, and $z(t) \in \mathbb{R}^{n_z}$ is the controlled output. $x^+(t)$ represents $x(t + 1)$. Matrices

$W(\theta) \triangleq (A(\theta), B(\theta), B_u(\theta), C(\theta), D(\theta), D_u(\theta), C_y(\theta), D_y(\theta))$

are assumed to be time-invariant and uncertain but belong to a polytopic parametric domain defined as

$$\mathbb{W} \triangleq \left\{ W(\theta) \mid W(\theta) = \sum\nolimits_{i=1}^{s} \theta_i W_i;\ \theta \in \mathbf{\Delta} \right\}$$

where

$$\begin{aligned} W_i &\triangleq (A_i, B_i, B_{u,i}, C_i, D_i, D_{u,i}, C_{y,i}, D_{y,i}) \\ \mathbf{\Delta} &\triangleq \left\{ \theta \in \mathbb{R}^r \mid \sum\nolimits_{i=1}^{r} \theta_i = 1,\ \theta_i \geq 0,\ i = 1, \ldots, r \right\}. \end{aligned}$$

Constant matrices W_i, $i = 1, \ldots, r$ denote $W(\theta)$ at the r vertices of $\mathbb{W}$, which are assumed to be known.

To stabilize the plant $G(z, \theta)$ in (1), we are interested in finding an SOF controller:

$$u(t) = K_{\mathrm{sof}}\, y(t) \tag{2}$$

where K_{sof} is the SOF gain matrix to be determined. Substituting (2) into (1) results in the closed-loop system $G_{\mathrm{cl}}(z, \theta)$ that is given by

$$\begin{aligned} x^+(t) &= A_{\mathrm{cl}}(\theta)x_{\mathrm{c}}(t) + B_{\mathrm{cl}}(\theta)w(t) \\ z(t) &= C_{\mathrm{cl}}(\theta)x_{\mathrm{c}}(t) + D_{\mathrm{cl}}(\theta)w(t) \end{aligned} \tag{3}$$

where

$$\begin{aligned} A_{\mathrm{cl}}(\theta) &= A(\theta) + B_u(\theta)K_{\mathrm{sof}}C_y(\theta) \\ B_{\mathrm{cl}}(\theta) &= B(\theta) + B_u(\theta)K_{\mathrm{sof}}D_y(\theta) \\ C_{\mathrm{cl}}(\theta) &= C(\theta) + D_u(\theta)K_{\mathrm{sof}}C_y(\theta) \\ D_{\mathrm{cl}}(\theta) &= D(\theta) + D_u(\theta)K_{\mathrm{sof}}D_y(\theta). \end{aligned} \tag{4}$$

For any fixed $\theta \in \mathbf{\Delta}$, the transfer function of the closed-loop system is given by

$$G_{\mathrm{cl}}(z, \theta) = C_{\mathrm{cl}}(\theta) \left(z\mathbf{I} - A_{\mathrm{cl}}(\theta) \right)^{-1} B_{\mathrm{cl}}(\theta) + D_{\mathrm{cl}}(\theta).$$

Let $\Pi \in \mathbb{H}_{n_z+n_w}$ be given, and define $\mathbf{\Omega} \triangleq [-\omega_l, \omega_l]$ for low frequency (LF), $[\omega_1, \omega_2]$ for middle frequency (MF) and $[\omega_h, \pi]$ for high frequency (HF), respectively, where ω_l, ω_h, ω_1 and ω_2 are known scalars in $[-\pi, \pi]$. In the paper, the uncertain system in (1), controlled by the SOF controller in (2) to be designed, is required to satisfy the following two specifications:

i) the closed-loop system in (3) is robustly asymptotically stable for all $\theta \in \mathbf{\Delta}$, and

ii) the closed-loop system in (3) satisfies

$$\upsilon \left(G_{\mathrm{cl}}(\mathrm{e}^{\mathrm{j}\omega}, \theta), \Pi \right) < 0,\ \forall \omega \in \mathbf{\Omega},\ \theta \in \mathbf{\Delta}. \tag{5}$$

Remark 1. The RFDS in (5) is motivated by the frequency-domain description in Iwasaki and Hara [2005]. In particular, with $\Pi = \mathrm{diag}\left\{ \mathbf{I}_{n_z}, -\gamma^2 \mathbf{I}_{n_w} \right\}$ and $\mathbf{\Omega} = [-\pi, \pi]$, the specification in (5) reduces to a standard H_∞ norm specification $\left\| G_{\mathrm{cl}}(\mathrm{e}^{\mathrm{j}\omega}, \theta) \right\|_\infty < \gamma$. In this context, the considered problem includes the standard H_∞ control as a special case, which has been extensively studied (see, for instance, Grigoriadis and Skelton [1996], Shu and Lam [2009] for continuous-time systems and Agulhari et al. [2010] for discrete-time systems). Hence, the considered control problem with an RFDS is more general than the standard H_∞ control problem.

To end this section, we present the following GKYP lemma that will be used in the sequel.

Lemma 1. ([Iwasaki and Hara 2005]). Let matrices $\Pi \in \mathbb{H}_{n_z+n_w}$ and $\Phi, \Psi \in \mathbb{H}_2$, and the state-space realization of stable $G_{\mathrm{cl}}(z, \theta)$ in (3) be given. For arbitrarily fixed $\theta \in \mathbf{\Delta}$, the following statements are equivalent.

i) $\upsilon \left(G_{\mathrm{cl}}(\mathrm{e}^{\mathrm{j}\omega}, \theta), \Pi \right) < 0$ holds for all $\omega \in \mathbf{\Omega}$.

ii) There exist matrices P, $Q \in \mathbb{H}_{n_\mathrm{p}}$ such that $Q > 0$ and

$$\begin{bmatrix} A_{\mathrm{cl}} & B_{\mathrm{cl}} \\ \mathbf{I} & 0 \end{bmatrix}^{\mathrm{T}} (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} A_{\mathrm{cl}} & B_{\mathrm{cl}} \\ \mathbf{I} & 0 \end{bmatrix} + \begin{bmatrix} C_{\mathrm{cl}} & D_{\mathrm{cl}} \\ 0 & \mathbf{I} \end{bmatrix}^{\mathrm{T}} \Pi \begin{bmatrix} C_{\mathrm{cl}} & D_{\mathrm{cl}} \\ 0 & \mathbf{I} \end{bmatrix} < 0. \tag{6}$$

where $\Phi = [1\ 0;\ 0\ -1]$, and Ψ is given in the following table with $\omega_c = (\omega_1 + \omega_2)/2$ and $\omega_r = (\omega_2 - \omega_1)/2$:

$\mathbf{\Omega}$	$[-\omega_l, \omega_l]$	$[\omega_1, \omega_2]$	$[\omega_h, \pi]$
Ψ	$\begin{bmatrix} 0 & 1 \\ 1 & -2\cos\omega_l \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{\mathrm{j}\omega_c} \\ \mathrm{e}^{-\mathrm{j}\omega_c} & -2\cos\omega_r \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 2\cos\omega_h \end{bmatrix}$

3. CONSTRUCTION OF ROBUST SOF CONTROLLERS SUBJECT TO AN RFDS

In this section, a necessary and sufficient condition will be first developed for the stability and the frequency-domain performance in (5) of the closed-loop system $G_{\rm cl}(z,\theta)$ without uncertainty, which is then used to analyze robust performances subject to polytopic uncertainty, and further to construct an SOF gain $K_{\rm sof}$ that guarantees the required robust performances.

3.1 Dilated Multiplier Relaxation

For convenience, consider the closed-loop system $G_{\rm cl}(z,\theta)$ without uncertainty first, hence we omit the dependence of the uncertain matrices on θ. The LMI in (6) can be rewritten into the following form:

$$\tilde{E}^{\rm T}\tilde{\Xi}\tilde{E} < 0 \tag{7}$$

where

$$\tilde{\Xi} \triangleq \begin{bmatrix} \Phi\otimes P + \Psi\otimes Q & 0 \\ 0 & \Pi \end{bmatrix},\ \tilde{E} \triangleq \left[\begin{array}{cc|cc} A_{\rm cl}^{\rm T} & \mathbf{I} & C_{\rm cl}^{\rm T} & 0 \\ C_{\rm cl}^{\rm T} & 0 & D_{\rm cl}^{\rm T} & \mathbf{I} \end{array}\right]^{\rm T}.$$

To guarantee the stability of $G_{\rm cl}(z)$, we rewrite the Lyapunov inequality $A_{\rm cl}^{\rm T}P_{\rm s}A_{\rm cl} - P_{\rm s} < 0$ as

$$\tilde{E}_{\rm s}^{\rm T}\left(\Phi\otimes P_{\rm s}\right)\tilde{E}_{\rm s} < 0 \tag{8}$$

where Φ is the same one as the one in the GKYP lemma, $P_{\rm s} \in \mathbb{S}_{n_{\rm p}}$ is some positive definite matrix and $\tilde{E}_{\rm s} = \left[\, A_{\rm cl}^{\rm T}\ \mathbf{I}\,\right]^{\rm T}$. Introduce two matrices $K_1 \in \mathbb{R}^{n_u\times n}$ and $K_2 \in \mathbb{R}^{n_u\times n_w}$; then (7) and (8) are equivalent to the following dilated matrix inequality conditions, respectively,

$$\left[\, \tilde{E}^{\rm T}\ \Gamma^{\rm T}\,\right]\Xi\left[\, \tilde{E}^{\rm T}\ \Gamma^{\rm T}\,\right]^{\rm T} < 0 \tag{9}$$

$$\left[\, \tilde{E}_{\rm s}^{\rm T}\ \Gamma_{\rm s}^{\rm T}\,\right]\Xi_{\rm s}\left[\, \tilde{E}_{\rm s}^{\rm T}\ \Gamma_{\rm s}^{\rm T}\,\right]^{\rm T} < 0 \tag{10}$$

where

$$\Gamma \triangleq \left[\, K_{\rm sof}C_y - K_1\ \ K_{\rm sof}D_y - K_2\,\right],\ \Gamma_{\rm s} \triangleq K_{\rm sof}C_y - K_1$$
$$\Xi \triangleq {\rm diag}\{\tilde{\Xi}, 0\},\ \Xi_{\rm s} \triangleq {\rm diag}\{\Phi\otimes P_{\rm s}, 0\}.$$

In the sequel, the dilated conditions in (9) and (10) will be used to develop a controller synthesis method. First, we have the following condition for the stability and the RFDS in (5) of the closed-loop system $G_{\rm cl}(z)$ without uncertainty.

Theorem 1. Let matrices $\Pi \in \mathbb{H}_{n_z+n_w}$, $K_{\rm sof} \in \mathbb{R}^{n_u\times n_y}$ and the state-space realization of $G(z,\theta)$ in (1) be given. For arbitrarily fixed $\theta \in \mathbf{\Delta}$, the closed-loop system $G_{\rm cl}(z,\theta)$ in (3) is asymptotically stable and satisfies the RFDS in (5) if and only if there exist matrices $P = P^*$, $Q = Q^*$, $P_{\rm s} = P_{\rm s}^{\rm T}$, complex matrices F, H, M, N, and real matrices $F_{\rm s}$, K_1, K_2, R such that $P_{\rm s}, Q > 0$ and

$$\Xi + X\Sigma + \Sigma^* X^* < 0 \tag{11}$$

$$\Xi_{\rm s} + X_{\rm s}\Sigma_{\rm s} + \Sigma_{\rm s}^{\rm T}X_{\rm s}^{\rm T} < 0 \tag{12}$$

where Ξ and $\Xi_{\rm s}$ are given in (9) and (10), respectively, and

$$X \triangleq \begin{bmatrix} F & M & 0 \\ H & N & 0 \\ 0 & 0 & R \end{bmatrix},\ X_{\rm s} \triangleq \begin{bmatrix} F_{\rm s} & 0 \\ 0 & R \end{bmatrix}$$

$$\Sigma \triangleq \left[\begin{array}{cc|cc|c} -\mathbf{I} & A + B_uK_1 & 0 & B + B_uK_2 & B_u \\ \hline 0 & C + D_uK_1 & -\mathbf{I} & D + D_uK_2 & D_u \\ \hline 0 & K_{\rm sof}C_y - K_1 & 0 & K_{\rm sof}D_y - K_2 & -\mathbf{I} \end{array}\right]$$

$$\Sigma_{\rm s} \triangleq \left[\begin{array}{cc|c} -\mathbf{I} & A + B_uK_1 & B_u \\ \hline 0 & K_{\rm sof}C_y - K_1 & -\mathbf{I} \end{array}\right]. \tag{13}$$

Sketch of the Proof. (*Sufficiency*) Multiplying the condition in (11) by $\Sigma^{\perp{\rm T}}$ on the left and $\Sigma^{\perp}$ on the right, and the condition (12) by $\Sigma_{\rm s}^{\perp{\rm T}}$ on the left and $\Sigma_{\rm s}^{\perp}$ on the right, respectively, and specifically choosing $\Sigma^{\perp}$ and $\Sigma_{\rm s}^{\perp}$, respectively, as

$$\Sigma^{\perp} = \left[\begin{array}{c} \tilde{E} \\ \hline K_{\rm sof}C_y - K_1\ \ K_{\rm sof}D_y - K_2 \end{array}\right],\ \Sigma_{\rm s}^{\perp} = \left[\begin{array}{c} \tilde{E}_{\rm s} \\ K_{\rm sof}C_y - K_1 \end{array}\right],$$

one can complete the sufficiency part.

(*Necessity*) Let

$$\tilde{\Sigma} = \left[\begin{array}{cc|cc} -\mathbf{I} & A_{\rm cl} & 0 & B_{\rm cl} \\ \hline 0 & C_{\rm cl} & -\mathbf{I} & D_{\rm cl} \end{array}\right] \text{ and } \tilde{\Sigma}_{\rm s} = \left[\, -\mathbf{I}\ A_{\rm cl}\,\right].$$

Note that one can choose $\tilde{\Sigma}^{\perp} = \tilde{E}$, $\tilde{\Sigma}_{\rm s}^{\perp} = \tilde{E}_{\rm s}$, such that the inequalities in (7) and (8) can be denoted, respectively, by

$$\tilde{\Sigma}^{\perp{\rm T}}\tilde{\Xi}\tilde{\Sigma}^{\perp} < 0,\ \tilde{\Sigma}^{\perp{\rm T}}\left(\Phi\otimes P_{\rm s}\right)\tilde{\Sigma}_{\rm s}^{\perp} < 0.$$

Via Finsler's lemma [de Oliveira and Skelton 2001], it follows that

$$\exists\tilde{X},\ \tilde{\Xi} + \tilde{X}\tilde{\Sigma} + \tilde{\Sigma}^*\tilde{X}^* < 0 \tag{14}$$

$$\exists\tilde{X}_{\rm s},\ \left(\Phi\otimes P_{\rm s}\right) + \tilde{X}_{\rm s}\tilde{\Sigma}_{\rm s} + \tilde{\Sigma}_{\rm s}^{\rm T}\tilde{X}_{\rm s}^{\rm T} < 0. \tag{15}$$

For a sufficiently large scalar $\epsilon > 0$, there always hold that

$$\begin{bmatrix} \tilde{\Xi} + \tilde{X}\tilde{\Sigma} + \tilde{\Sigma}^*\tilde{X}^* & * \\ \left[\, B_u^{\rm T}\ D_u^{\rm T}\,\right]\tilde{X}^* & -2\epsilon\mathbf{I} \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \left(\Phi\otimes P_{\rm s}\right) + \tilde{X}_{\rm s}\tilde{\Sigma}_{\rm s} + \tilde{\Sigma}_{\rm s}^{\rm T}\tilde{X}_{\rm s}^{\rm T} & * \\ B_u^{\rm T}\tilde{X}_{\rm s}^{\rm T} & -2\epsilon\mathbf{I} \end{bmatrix} < 0. \tag{17}$$

By assigning matrices in Theorem 1, respectively, as

$$\left[\, F\ M;\ H\ N\,\right] = \tilde{X},\ F_{\rm s} = \tilde{X}_{\rm s},\ R = \epsilon\mathbf{I}$$
$$K_1 = K_{\rm sof}C_y,\ K_2 = K_{\rm sof}D_y \tag{18}$$

it is seen that (16) and (17) are the conditions in (11) and (12). The proof is completed. ■

Remark 2. Compared with the original conditions in the GKYP lemma and the Lyapunov inequality, Theorem 1 is more appealing for robustness analysis. Note that all the system matrices are separated from the Lyapunov matrices $(P, Q, P_{\rm s})$, which enables us to employ *parameter-dependent* Lyapunov matrices to analyze the robust performances of the closed-loop system subject to uncertainty and an RFDS [de Oliveira and Skelton 2001]. Moreover, note that the controller gain $K_{\rm sof}$ in the GKYP lemma lies in the middle of two system matrices, e.g., $B_uK_{\rm sof}C_y$ in (4), which is known to be the source causing difficulty in SOF controller synthesis, while Theorem 1 does not include such terms.

Remark 3. It is not difficult to find that matrices K_1 and K_2 can be interpreted as the gain of an FI controller $u(t) = K_1x(t) + K_2w(t)$ for the following system:

$$x^+(t) = Ax(t) + B_uu(t) + Bw(t)$$
$$z(t) = Cx(t) + D_uu(t) + Dw(t).$$

Especially, when $D_y = 0$, we can set $K_2 = 0$, and matrix K_1 is the gain of an SF controller $u(t) = K_1x(t)$. The fact is the base of the two-stage framework for SOF control.

From Theorem 1, we have the following extension to the polytopic uncertain case, which gives a necessary and sufficient condition for analyzing the robust performance of the uncertain closed-loop system.

Theorem 2. Let matrices $\Pi \in \mathbb{H}_{n_z+n_w}$, $K_{\rm sof} \in \mathbb{R}^{n_u\times n_y}$ and the state-space realization of $G(z,\theta)$ in (1) be given.

The closed-loop system $G_{\mathrm{cl}}(z,\theta)$ in (3) is robustly asymptotically stable and satisfies the RFDS in (5) for all $\theta \in \mathbf{\Delta}$, if and only if there exist matrices $P(\theta) = P^*(\theta)$, $Q(\theta) = Q^*(\theta)$, $P_{\mathrm{s}}(\theta) = P_{\mathrm{s}}^{\mathrm{T}}(\theta)$, complex matrices $F(\theta)$, $H(\theta)$, $M(\theta)$, $N(\theta)$, and real matrices $F_{\mathrm{s}}(\theta)$, $K_1(\theta)$, $K_2(\theta)$, R such that, for all $\theta \in \mathbf{\Delta}$, $P_{\mathrm{s}}(\theta)$, $Q(\theta) > 0$ and

$$\Xi(\theta) + X(\theta)\Sigma(\theta) + \Sigma^*(\theta)X^*(\theta) < 0 \quad (19)$$

$$\Xi_{\mathrm{s}}(\theta) + X_{\mathrm{s}}(\theta)\Sigma_{\mathrm{s}}(\theta) + \Sigma_{\mathrm{s}}^{\mathrm{T}}(\theta)X_{\mathrm{s}}^{\mathrm{T}}(\theta) < 0 \quad (20)$$

where $\Xi(\theta)$, $\Xi_{\mathrm{s}}(\theta)$, $X(\theta)$, $X_{\mathrm{s}}(\theta)$, $\Sigma(\theta)$ and $\Sigma_{\mathrm{s}}(\theta)$ are the parameter-dependent counterpart of Ξ, Ξ_{s}, Σ, X, $X_{\mathrm{s}}$and Σ_{s} in Theorem 1, respectively.

Proof. The proof can be completed by following lines similar to the proof of Theorem 1, but with matrices changing to be parameter-dependent. A key worth pointing out is that matrix R can be set to be parameter-independent. ■

3.2 Computation of SOF Controllers

Based on Theorem 2, the following result provides a necessary and sufficient condition for parameterizing a desired robust SOF gain K_{sof}.

Theorem 3. Let matrix $\Pi \in \mathbb{H}_{n_z+n_w}$ and the state-space realization of $G(z,\theta)$ in (1) be given. An SOF controller in (2) exists such that the closed-loop system $G_{\mathrm{cl}}(z,\theta)$ in (3) is robustly asymptotically stable and satisfies the RFDS in (5) for all $\theta \in \mathbf{\Delta}$, if and only if there exist matrices $P(\theta) = P^*(\theta)$, $Q(\theta) = Q^*(\theta)$, $P_{\mathrm{s}}(\theta) = P_{\mathrm{s}}^{\mathrm{T}}(\theta)$, complex matrices $F(\theta)$, $H(\theta)$, $M(\theta)$, $N(\theta)$, and real matrices $F_{\mathrm{s}}(\theta)$, $K_1(\theta)$, $K_2(\theta)$, R, L such that, for all $\theta \in \mathbf{\Delta}$, $P_{\mathrm{s}}(\theta)$, $Q(\theta) > 0$ and

$$\Xi(\theta) + \Upsilon(\theta) + \Upsilon^*(\theta) < 0 \quad (21)$$

$$\Xi_{\mathrm{s}}(\theta) + \Upsilon_{\mathrm{s}}(\theta) + \Upsilon_{\mathrm{s}}^{\mathrm{T}}(\theta) < 0 \quad (22)$$

where $\Xi(\theta)$ and $\Xi_{\mathrm{s}}(\theta)$ are the parameter-dependent counterpart of Ξ and Ξ_{s} in Theorem 1, respectively, and

$$\Upsilon(\theta) \triangleq \left[\begin{array}{cc|cc|c} -F(\theta) & \Upsilon_1(\theta) & -M(\theta) & \Upsilon_4(\theta) & \Upsilon_7(\theta) \\ \hline -H(\theta) & \Upsilon_2(\theta) & -N(\theta) & \Upsilon_5(\theta) & \Upsilon_8(\theta) \\ \hline 0 & \Upsilon_3(\theta) & 0 & \Upsilon_6(\theta) & -R \end{array}\right]$$

$$\Upsilon_{\mathrm{s}}(\theta) \triangleq \left[\begin{array}{cc|c} -F_{\mathrm{s}}(\theta) & \Upsilon_9(\theta) & F_{\mathrm{s}}(\theta)B_2(\theta) \\ \hline 0 & \Upsilon_3(\theta) & -R \end{array}\right]$$

$$\Upsilon_1(\theta) \triangleq F(\theta)A(\theta) + F(\theta)B_u(\theta)K_1(\theta) + M(\theta)C(\theta) + M(\theta)D_u(\theta)K_1(\theta)$$

$$\Upsilon_2(\theta) \triangleq H(\theta)A(\theta) + H(\theta)B_u(\theta)K_1(\theta) + N(\theta)C(\theta) + N(\theta)D_u(\theta)K_1(\theta)$$

$$\Upsilon_3(\theta) \triangleq LC_y(\theta) - RK_1(\theta), \Upsilon_6(\theta) \triangleq LD_y(\theta) - RK_2(\theta)$$

$$\Upsilon_4(\theta) \triangleq F(\theta)B(\theta) + F(\theta)B_u(\theta)K_2(\theta) + M(\theta)D(\theta) + M(\theta)D_u(\theta)K_2(\theta)$$

$$\Upsilon_5(\theta) \triangleq H(\theta)B(\theta) + H(\theta)B_u(\theta)K_2(\theta) + N(\theta)D(\theta) + N(\theta)D_u(\theta)K_2(\theta)$$

$$\Upsilon_7(\theta) \triangleq F(\theta)B_u(\theta) + M(\theta)D_u(\theta)$$

$$\Upsilon_8(\theta) \triangleq H(\theta)B_u(\theta) + N(\theta)D_u(\theta)$$

$$\Upsilon_9(\theta) \triangleq F_{\mathrm{s}}(\theta)A(\theta) + F_{\mathrm{s}}(\theta)B_u(\theta)K_1(\theta)$$

Moreover, if the above conditions are satisfied, a desired SOF controller gain in (2) is given by $K_{\mathrm{sof}} = R^{-1}L$.

Sketch of the Proof. The conditions in (21) and (22) can be obtained from (19) and (20) by performing a change of variable $L = RK_{\mathrm{sof}}$. Note that $-R - R^{\mathrm{T}} < 0$ is implied by (22), thus R is invertible and accordingly, the change of variable is also invertible, and an SOF controller can be obtained by $K_{\mathrm{sof}} = R^{-1}L$. ■

The conditions in (21) and (22) are infinite dimensional due to the dependence on θ. To relax them with finite number of matrix conditions, let parameter-dependent matrices $P(\theta)$, $Q(\theta)$, $P_{\mathrm{s}}(\theta)$, $K_1(\theta)$ and $K_2(\theta)$ be matrix functions linearly with respect to θ, for instance, $P(\theta) = \sum_{i=1}^{r} \theta_i P_i$, and let matrices $F(\theta)$, $F_{\mathrm{s}}(\theta)$, $H(\theta)$, $M(\theta)$ and $N(\theta)$ be parameter-independent. This treatment results in the following corollary.

Corollary 1. Let matrix $\Pi \in \mathbb{H}_{n_z+n_w}$ and the state-space realization of $G(z,\theta)$ in (1) be given. An SOF controller in (2) exists such that the closed-loop system $G_{\mathrm{cl}}(z,\theta)$ in (3) is robustly asymptotically stable and satisfies the RFDS in (5) for all $\theta \in \mathbf{\Delta}$, if there exist matrices $P_i = P_i^*$, $Q_i = Q_i^*$, $P_{\mathrm{s},i} = P_{\mathrm{s},i}^{\mathrm{T}}$, $i = 1,\ldots,r$, complex matrices F, H, M, N, and real matrices F_{s}, $K_{1,i}$, $K_{2,i}$, $i = 1,\ldots,r$, R, L such that $P_{\mathrm{s},i}$, $Q_i > 0$, $i = 1,\ldots,r$ and

$$\Xi_i + \Xi_j + \mathrm{sym}\{\Upsilon_{i,j} + \Upsilon_{j,i}\} < 0,\ 1 \le i \le j \le r \quad (23)$$

$$\Xi_{\mathrm{s},i} + \Xi_{\mathrm{s},j} + \mathrm{sym}\{\Upsilon_{\mathrm{s},i,j} + \Upsilon_{\mathrm{s},j,i}\} < 0,\ 1 \le i \le j \le r \quad (24)$$

where Ξ_i and $\Xi_{\mathrm{s},i}$ are Ξ and Ξ_{s} in Theorem 1 with P, Q, and P_{s} replaced by P_i, Q_i and $P_{\mathrm{s},i}$, respectively, and $\Upsilon_{i,j}$ and $\Upsilon_{\mathrm{s},i,j}$ are $\Upsilon(\theta)$ and $\Upsilon_{\mathrm{s}}(\theta)$ in Theorem 3 with $F(\theta)$, $F_{\mathrm{s}}(\theta)$, $H(\theta)$, $M(\theta)$, $N(\theta)$, $K_1(\theta)$, $K_2(\theta)$, $C_y(\theta)$ and $D_y(\theta)$ replaced by F, F_{s}, H, M, N, $K_{1,i}$, $K_{2,i}$, $C_{y,i}$ and $D_{y,i}$, and with $A(\theta)$, $B(\theta)$, $B_u(\theta)$, $C(\theta)$, $D(\theta)$ and $D_u(\theta)$ replaced by A_j, B_j, $B_{u,j}$, C_j, D_j and $D_{u,j}$, respectively. Moreover, if the above conditions are satisfied, the gain of such a desired SOF controller in (2) is given by $K_{\mathrm{sof}} = R^{-1}L$.

Remark 4. If $K_1(\theta)$ and $K_2(\theta)$ are restricted to be parameter-independent, we have

$$\Xi(\theta) + \Upsilon(\theta) + \Upsilon^*(\theta) = \sum_{i=1}^{r} \theta_i \left(\Xi_i + \mathrm{sym}\{\Upsilon_{i,i}\}\right)$$

$$\Xi_{\mathrm{s}}(\theta) + \Upsilon_{\mathrm{s}}(\theta) + \Upsilon_{\mathrm{s}}^{\mathrm{T}}(\theta) = \sum_{i=1}^{r} \theta_i \left(\Xi_{\mathrm{s},i} + \mathrm{sym}\{\Upsilon_{\mathrm{s},i,i}\}\right).$$

Hence, the conditions in (23) and (24) reduces to

$$\Xi_i + \mathrm{sym}\{\Upsilon_{i,i}\} < 0,\ \Xi_{\mathrm{s},i} + \mathrm{sym}\{\Upsilon_{\mathrm{s},i,i}\} < 0,\ i = 1,\ldots,r.$$

Note that, when matrices $K_{1,i}$ and $K_{2,i}$ are specified *a priori*, the conditions in (23) and (24) are LMIs with respect to matrix variables P_i, Q_i, $P_{\mathrm{s},i}$, F, F_{s}, H, M, N, L and R. From Remark 3, one knows that $K_{1,i}$ and $K_{2,i}$ actually comprise a parameter-dependent robust FI controller gain $\sum_{i=1}^{r} \theta_i \left[\, K_{1,i}\ K_{2,i} \,\right]$ which guarantees that the resulting closed-loop system is robustly stable and satisfies (5) for all $\theta \in \mathbf{\Delta}$. The fact naturally leads to the following algorithm for the design of a robust SOF controller gain K_{sof} by virtue of Corollary 1, where

$$\Theta_{i,j} \triangleq \Xi_i + \Xi_j + \mathrm{sym}\{\Upsilon_{i,j} + \Upsilon_{j,i}\}$$

$$\Theta_{\mathrm{s},i,j} \triangleq \Xi_{\mathrm{s},i} + \Xi_{\mathrm{s},j} + \mathrm{sym}\{\Upsilon_{\mathrm{s},i,j} + \Upsilon_{\mathrm{s},j,i}\}$$

$$\mathbb{I} \triangleq \mathrm{diag}\left\{\mathbf{I}_{2n_{\mathrm{p}}+n_z+n_w}, \mathbf{0}_{n_u \times n_{\mathrm{pu}}}\right\}$$

$$\mathbb{I}_{\mathrm{s}} \triangleq \mathrm{diag}\left\{\mathbf{I}_{2n_{\mathrm{p}}}, \mathbf{0}_{n_u \times n_{\mathrm{pu}}}\right\}.$$

Design of Robust SOF Controller (D-RSOFC)

Stg 1 Find matrices $K_{1,i}$ and $K_{2,i}$, $i = 1,\ldots,r$ such that a robust FI controller gain $\sum_{i=1}^{r} \theta_i \left[\, K_{1,i}\ K_{2,i} \,\right]$ guarantees the robust asymptotic stability and the RFDS in (5) of $G_{\rm cl}(z,\theta)$ in (3) for all $\theta \in \mathbf{\Delta}$. Set $k=1$ and let δ be a specified tolerance.

Stg 2-1 Solve the following LMI problem to obtain $\varepsilon_1^{(k)}$:

$$\begin{aligned} &\min\ \varepsilon_1^{(k)} = \varepsilon \\ &\text{s.t.}\ \begin{cases} \Theta_{i,j} < \varepsilon \mathbb{I},\ \Theta_{{\rm s},i,j} < \varepsilon \mathbb{I}_{\rm s},\ 1 \le i \le j \le r \\ Q_i, P_{{\rm s},i} > 0,\ 1 \le i \le r \end{cases} \\ &\text{for}\ \ P_i, Q_i, P_{{\rm s},i}, F, F_{\rm s}, H, M, N, L, R \text{ and } \varepsilon \\ &\text{with } K_{1,i} = K_{1,i}^{(k)} \text{ and } K_2 = K_{2,i}^{(k)} \text{ fixed.} \end{aligned} \tag{25}$$

Denote $F^{(k)}, F_{\rm s}^{(k)}, H^{(k)}, M^{(k)}, N^{(k)}$ and $R^{(k)}$ as the obtained $F, F_{\rm s}, H, M, N$ and R, respectively. If $\varepsilon_1^{(k)} \le 0$, then $K_{\rm sof} = R^{-1}L$ is a desired SOF controller, and EXIT; otherwise, go to the next step;

Stg 2-2 Solve the following LMI problem to obtain $\varepsilon_2^{(k)}$:

$$\begin{aligned} &\min\ \varepsilon_2^{(k)} = \varepsilon \\ &\text{s.t.}\ \begin{cases} \Theta_{i,j} < \varepsilon \mathbb{I},\ \Theta_{{\rm s},i,j} < \varepsilon \mathbb{I}_{\rm s},\ 1 \le i \le j \le r \\ Q_i, P_{{\rm s},i} > 0,\ 1 \le i \le r \end{cases} \\ &\text{for}\ \ P_i, Q_i, P_{{\rm s},i}, L, K_{1,i}, K_{2,i} \text{ and } \varepsilon \\ &\text{with} \begin{cases} F = F^{(k)}, F_{\rm s} = F_{\rm s}^{(k)}, H = H^{(k)}, M = M^{(k)}, \\ N = N^{(k)} \text{ and } R = R^{(k)} \text{ fixed.} \end{cases} \end{aligned} \tag{26}$$

Denote $K_{1,i}^{(k+1)}$ and $K_{2,i}^{(k+1)}$ as the obtained $K_{1,i}$ and $K_{2,i}$, respectively. If $\varepsilon_2^{(k)} \le 0$, then $K_{\rm sof} = R^{-1}L$ is a desired SOF controller, and EXIT; otherwise, go to the next step;

Stg 2-3 If $\left|\varepsilon_1^{(k)} - \varepsilon_2^{(k)}\right| / \varepsilon_2^{(k)} < \delta$, then there may not exist a desired SOF controller for the plant $G(z,\theta)$ in (1), and EXIT; else, set $k \leftarrow k+1$, and go back to Stage 2-1.

Note that the conditions in (23) and (24) may not give rise to a desired SOF controller gain $K_{\rm sof}$ for the initially given $K_{1,i}$ and $K_{2,i}$. However, from the nature of Algorithm D-RSOFC, it is easy to find that $\varepsilon_1^{(k)}$ and $\varepsilon_2^{(k)}$ satisfy $\varepsilon_1^{(k)} \ge \varepsilon_2^{(k)} \ge \varepsilon_1^{(k+1)}$ for all $k = 1, 2, \ldots$, that is, $\varepsilon_1^{(k)}$ and $\varepsilon_2^{(k)}$ both are non-increasing. Hence, it is expected to find some $\varepsilon_1^{(k)}$ or $\varepsilon_2^{(k)}$ not larger than zero, corresponding to which, the matrix computed from $K_{\rm sof} = R^{-1}L$ is a desired SOF controller gain according to Corollary 1.

3.3 Computation of Initial Robust FI Controllers

We have the following result on the existence of a parameter-dependent FI controller such that the resulting closed-loop system is robustly stable and satisfies (5) for all $\theta \in \mathbf{\Delta}$. The proof is omitted for saving space.

Theorem 4. Let matrix $\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} \in \mathbb{H}_{n_z+n_w}$ and the state-space realization of $G(z,\theta)$ in (1) be given. Suppose that the left upper $n_z \times n_z$ block Π_{11} is positive definite. An FI controller $u(t) = K_1(\theta)x(t) + K_2(\theta)w(t)$ exists such that the closed-loop system $G_{\rm cl}(z,\theta)$ in (3) with $K_{\rm sof}\,[\,C_y(\theta)\ D_y(\theta)\,]$ replaced by $[\,K_1(\theta)\ K_2(\theta)\,]$is robustly asymptotically stable and satisfies the RFDS in (5) for all $\theta \in \mathbf{\Delta}$, if there exist matrices $\bar{P}_i = \bar{P}_i^*$, $\bar{Q}_i = \bar{Q}_i^*$, $\bar{P}_{{\rm s},i} = \bar{P}_{{\rm s},i}^{\rm T}$, and real matrices $\bar{K}_{1,i}$, $K_{2,i}$, $i = 1, \ldots, r$, X such that $\bar{P}_{{\rm s},i}$, $\bar{Q}_i > 0$, $i = 1, \ldots, r$ and

$$\bar{\Xi}_i + \bar{\Xi}_j + {\rm sym}\left\{\bar{\Upsilon}_{i,j} + \bar{\Upsilon}_{j,i}\right\} < 0,\ 1 \le i \le j \le r \tag{27}$$

$$\bar{\Xi}_{{\rm s},i} + \bar{\Xi}_{{\rm s},j} + {\rm sym}\left\{\bar{\Upsilon}_{{\rm s},i,j} + \bar{\Upsilon}_{{\rm s},j,i}\right\} < 0,\ 1 \le i \le j \le r \tag{28}$$

where

$$\bar{\Xi}_i \triangleq {\rm diag}\left\{\Phi \otimes \bar{P}_i + \Psi \otimes \bar{Q}_i, -\Pi_{11}, \Pi_{22}\right\}$$

$$\bar{\Upsilon}_{i,j} \triangleq \begin{bmatrix} -a\bar{X} & aA_j\bar{X} + aB_{u,j}\bar{K}_{1,i} & 0 & aB_j + aB_{u,j}K_{2,i} \\ -b\bar{X} & bA_j\bar{X} + bB_{u,j}\bar{K}_{1,i} & 0 & bB_j + bB_{u,j}K_{2,i} \\ 0 & \Pi_{11}(C_j\bar{X} + D_{u,j}\bar{K}_{1,i}) & 0 & \Pi_{11}(D_j + D_{u,j}K_{2,i}) \\ 0 & \Pi_{12}^*(C_j\bar{X} + D_{u,j}\bar{K}_{1,i}) & 0 & \Pi_{12}^*(D_j + D_{u,j}K_{2,i}) \end{bmatrix}$$

$$\bar{\Xi}_{{\rm s},i} \triangleq \Phi \otimes \bar{P}_{{\rm s},i},\ \bar{\Upsilon}_{{\rm s},i,j} \triangleq \begin{bmatrix} -\bar{X} & A_j\bar{X} + B_{u,j}\bar{K}_{1,i} \\ 0 & 0 \end{bmatrix}$$

and scalars $a = {\rm e}^{-{\rm j}\omega_c}$ and $b = 0$ for LF and MF, and $a = 1$ and $b = -2\sin\frac{\omega_h}{2}$ for HF. If the above conditions are satisfied, the FI controller gain is given by $K_1(\theta) = \sum_{i=1}^r \theta_i \bar{K}_{1,i}\bar{X}^{-1}$ and $K_2(\theta) = \sum_{i=1}^r \theta_i K_{2,i}$.

Remark 5. Although the presented results are stated specifically for the single RFDS case, it is easy to extend them to the case of multiple RFDSs. For each RFDS, one can introduce extra conditions, respectively, similar to the one of (23) and the one in (27). Also, the proposed algorithm can be easily adapted for multiple RFDSs.

4. A NUMERICAL EXAMPLE

In this section, we provide a numerical example for illustrating the effectiveness of the proposed control method. To solve the LMI problems, the solver SDPT3 [Toh et al. 1999] and the parser YALMIP [Löfberg 2004] will be employed.

Example 1. Consider an example for system in (1) with the following parameters:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & 0.4 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix},\ B_u = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$C = [\,0.02\ 0.1\ 0.5\,],\ D = 0.1,\ D_u = 0.4$$

$$C_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ D_y = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where uncertain parameters a_1 and a_2 satisfy $0.3 \le a_1 \le 0.5$ and $-0.1 \le a_2 \le 0.1$, respectively. Hence, the example is a polytopic uncertain system with 4 vertices. Also, let $T_{zw}(z)$ and $T_{uw}(z)$ denote the transfer functions of the closed-loop system from w to z and to u, respectively. Parameter δ in Algorithm D-RSOFC is set to be $\delta = 10^{-4}$. The goal of this example is to design an SOF controller gain $K_{\rm sof}$ in (2) such that the closed-loop system is robustly stable and satisfies

$$\begin{aligned} \Re\left(T_{zw}({\rm e}^{{\rm j}\omega})\right) > 0,\ \forall \omega \in [-2, 2] \\ \left|T_{uw}({\rm e}^{{\rm j}\omega})\right| < 0.35,\ \forall \omega \in [0, \pi]. \end{aligned} \tag{29}$$

That is, it is expected to passify $T_{zw}(z)$ over the finite frequency range $[-2, 2]$ rad/s, and meanwhile restrict the magnitude of $T_{uw}(z)$ to be less than 0.35.

By Theorem 4, the obtained robust SF controller is given by

$$K_{\rm sf} = [\,-0.1027\ -0.0935\ -0.5713\,]. \tag{30}$$

Then by invoking Algorithm D-RSOFC with the SF controller in (30) as the initial value, a required SOF controller is produced, which is given by

$$K_{\rm sof} = [\,0.0804\ -0.4522\,]. \tag{31}$$

Connecting the SOF controller in (31) to the open-loop system, we depict the frequency responses, $\Re\left(T_{zw}({\rm e}^{{\rm j}\omega})\right)$

and $|T_{uw}(e^{j\omega})|$, of the closed-loop transfer functions at the vertices in Figure 1, which clearly shows the effectiveness of the designed SOF controller in (31).

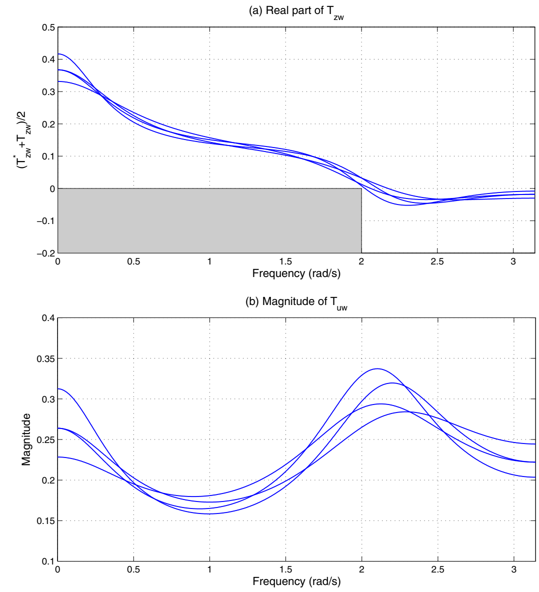

Fig. 1. Real part of $T_{zw}(z)$ and magnitude of $T_{uw}(z)$ of the closed-loop system at the vertices in Example 1

5. CONCLUSION

In the paper, we have studied the problem of robust SOF control with RFDSs for polytopic uncertain linear discrete-time systems, and a two-stage method has been developed for computing a desired SOF controller. To this end, through introducing dilated multipliers, a new necessary and sufficient condition has been derived for characterizing the robust performance of the closed-loop system with RFDSs, where the product terms between Lyapunov matrices and system and controller matrices are eliminated. With the proposed two-stage method, an initial robust FI controller needs to be designed at the first stage, and then a robust SOF one is constructed at the second stage. To improve the solvability of the method, an iterative algorithm has been formulated for exploring the SOF controller. Finally, a numerical example have clearly shown the effectiveness of the proposed design method.

REFERENCES

Agulhari, C.M., Oliveira, R.C.L.F., and Peres, P.L.D. (2010). Robust H_∞ static output-feedback design for time-invariant discrete-time polytopic systems from parameter-dependent state-feedback gains. In *Proc. of 2010 American Control Conf.*, 4677–4682. Marriott Waterfront, Baltimore, MD, USA.

de Oliveira, M.C. and Skelton, R.E. (2001). Stability tests for constrained linear systems. In *Perspectives in Robust Control*, volume 268 of *Lecture Notes in Control and Information Sciences*, 241–257. Springer, Berlin.

Gao, H. and Li, X. (2011). H_∞ filtering for discrete-time state-delayed systems with finite frequency specifications. *IEEE Trans. Autom. Control*, 56(12), 2935–2941.

Grigoriadis, K.M. and Skelton, R.E. (1996). Low-order control design for LMI problems using alternating projection methods. *Automatica*, 32(8), 1117–1125.

Iwasaki, T. and Hara, S. (2004). Robust control synthesis with general frequency domain specifications: Static gain feedback case. In *Proc. 2004 American Control Conf.*, 4613–1618. Boston, Massachusetts.

Iwasaki, T. and Hara, S. (2005). Generalized KYP lemma: Unified frequency domain inequalities with design applications. *IEEE Trans. Autom. Control*, 50(1), 41–59.

Iwasaki, T. and Hara, S. (2007). Feedback control synthesis of multiple frequency domain specifications via generalized KYP lemma. *Int. J. Robust Nonlin. Control*, 17, 415–434.

Karimi, H.R., Zapateiro, M., and Luo, N. (2008). Robust mixed H_2/H_∞ delayed state feedback control of uncertain neutral systems with time-varying delays. *Asian J. Control*, 10(5), 569–580.

Li, H., Liu, H., Hilton, C., and Hand, S. (2012a). Non-fragile H_∞ control for half-vehicle active suspension systems with actuator uncertainties. *J. Vibration Control*, 19(4), 560–575.

Li, X., Gao, H., and Wang, C. (2012b). Generalized Kalman-Yakubovich-Popov Lemma for 2-D FM LSS model. *IEEE Trans. Autom. Control*, 57(12), 3090–3103.

Li, X. and Gao, H. (2014). A heuristic approach to static output-feedback controller synthesis with restricted frequency-domain specifications. *IEEE Trans. Autom. Control*, 59(4), 1008–1014.

Löfberg, J. (2004). YALMIP: a toolbox for modeling and optimization in MATLAB. In *Proceedings of the 2004 IEEE International Symposium on Computer Aided Control System Design*, 284–289. Taipei, Taiwan.

Mehdi, D., Boukas, E.K., and Bachelier, O. (2004). Static output feedback design for uncertain linear discrete time systems. *IMA J. Math. Control Inform.*, 21(1), 1–13.

Peaucelle, D. and Arzelier, D. (2001). An efficient numerical solution for H_2 static output feedback synthesis. In *Proceeding of the 2001 European Control Conference*, 3800–3805. Porto, Portugal.

Rantzer, A. (1996). On the Kalman-Yakubovich-Popov lemma. *Syst. Control Lett.*, 28, 7–10.

Shi, P., Boukas, E.L., and Agarwal, R.K. (1999). Control of Markovian jump discrete-time systems with norm bounded uncertainty and unknown delay. *IEEE Trans. Autom. Control*, 44(11), 2139–2144.

Shu, Z. and Lam, J. (2009). An augmented system approach to static output-feedback stabilization with H_∞ performance for continuous-time plants. *Int. J. Robust Nonlin. Control*, 19, 768–785.

Toh, K.C., Todd, M.J., and Tütüncü, R.H. (1999). SDPT3 - a Matlab software package for semidefinite programming. *Optimization Methods and Software*, 11(1), 545–581.

Wang, Z., Ho, D.W.C., Liu, Y., and Liu, X. (2009). Robust H_∞ control for a class of nonlinear discrete time-delay stochastic systems with missing measurements. *Automatica*, 45, 684–691.

Yang, R., Liu, G.P., Shi, P., Thomas, C., and Basin, M.V. (2014). Predictive output feedback control for networked control systems. *IEEE Trans. Ind. Electron.*, 61(1), 512–520.