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Collision-free vehicle formation control using graph Laplacian and edge-tension function

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Abstract: This paper is concerned with collision-free vehicle formation control (FC) when the communication between vehicles is model by a graph. Unlike previous FC works (dealing with either non-trivial vehicle dynamics with no consideration of collision avoidance (CA) or trivial first-order vehicle dynamics with consideration of CA), this paper discusses non-trivial (second-order) vehicle FC with consideration of CA. This collision-free vehicle FC is done by manipulating entries of the graph Laplacian and by constructing a proper edge-tension function. Theoretical and numerical evidences are provided to show that the proposed control law effectively address both CA and FC.

Keywords: Formation control (FC); Collision avoidance (CA); Weighted graph Laplacian; Edge-tension function.

1. INTRODUCTION

Recently there is a tremendous surge of interest among researchers in formation control (FC) of autonomous vehicles due to its broad applications in military and civil areas. For example, a group of autonomous vehicles can be used for air traffic control, surveillance, firefighting, exploration, cleaning up oil spills and rich spatial awareness by distributing in a suitable formation (Martin et al. [2001], Bender. [1991]). Therefore, it may often be required that multiple vehicles move along a pre-defined trajectory while maintaining a desired formation. Moving in formation has many advantages as it can reduce the system cost, increase the robustness and efficiency of the system while providing redundancy and reconfiguration ability (Serrani. [2003], Daniel et al. [2004] and Stilwell et al. [2000]). Many control approaches, for example, a range-based method (Cao et al. [2011]) and a virtual structure approach (Ren. [2003]) have been used to achieve a desired formation. Also, Dashkovskiy et al. [2008] presents a framework of ISS (input-to-state stability) and a small-gain theorem which can be used for effective vehicle FC. One noticeable work is Lafferrire et al. [2005] in which a decentralized control scheme was proposed to achieve formation. Although these existing works propose sound FC schemes for non-trivial vehicles with second-order dynamics, they do not take into account the practically important issue of collision avoidance (CA) in their control designs.

CA is an old topic and has attracted many researchers, especially in aerospace engineering. See Keviczky et al. [2008], Chao et al. [2011], Michael et al. [2011], Kang et al. [2013], Mastellone et al. [2008], Lalish et al. [2008a], Lalish et al. [2008b], Sabattini et al. [2011], Yan et al. [2011], Falconi et al. [2011], Sabattini et al. [2011], Kan et al. [2012], Ammir. [2012] for some recent works in this direction. Yet, these existing works still have various limitations that have to be removed for the present application. Some of the existing works such as Keviczky et al. [2008] use the Receding Horizon Control (RHC) scheme which often requires solving a computationally intense optimization problem and thus has the limitation for real-time applications. Besides, this RHC scheme requires complicated emergency controllers and their invariant sets to define protection zones for CA when RHC problems become infeasible. Michael et al. [2011] also involves solving an optimization problem for CA and has a similar feasibility issue. The works such as Lalish et al. [2008a], Lalish et al. [2008b] are not specifically targeted for formation flying and do require all-to-all communication to collect all vehicles' states. Mastellone et al. [2008] and Sabattini et al. [2011] present interesting research outputs closely related to the topic of present interest, but they require that a pre-defined reference state (or its accurate estimate) must be known to all vehicles to achieve the objective. In particular, Sabattini et al. [2011] is purely for ground robot applications as it requires the robots to stop and form a desired formation. Note that many aerospace applications do not allow vehicles (e.g. fixed-wing airplanes) to stop; instead, the vehicles are expected to move together in a desired formation with zero relative velocities. Other recent works such as Yan et al. [2011], Falconi et al. [2011], Kan et al. [2012] and Ammir. [2012] also discuss the topic of present interest, but all are restricted to the first-order vehicle dynamics.

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In this work, the existing works are improved in the following manner. First, an *edge-tension function* is utilized for collision-free FC, thereby *avoiding solving complex optimizations* (unlike the RHC approaches). Note that the edge-tension function is originally proposed in Jiand. [2007], and successfully used in Falconi et al. [2011] for the collision-free FC of first-order systems. In this paper, this edge-tension function is further exploited for the collision-free formation *control of second-order systems*. Second, the control structure to be proposed in this paper is *similar to the state feedback control law* proposed in Lafferrire et al. [2005], and so the control law is *simple*, *easy-to-implement* and may allow vehicles to *move in a desired formation with zero relative velocities.*

The rest of the paper is organized as follows. In §2, multi-vehicle dynamics is described and the simple control law proposed in Lafferrire et al. [2005] is stated as an example of FC (while not considering CA yet). Then, the clear problem statement is given along the definition of an edge-tension function to be useful for collision-free FC. In §3, our control law is proposed and proven to guarantee stability and desired performance (collision-free flying to a desired formation). This proposed control law is initially for a complete network topology and is subsequently extended for a star (leader-following) network topology. The extended work for a star (leader-following) network topology shall be presented in a journal version of this paper, though. Numerical examples are then provided in §4 to demonstrate the developments in the preceding sections, and concluding remarks follow in §5.

2. VEHICLE DYNAMICS AND PROBLEM FORMULATION

Consider N vehicles as vertices of a graph, with an edge set determined by the relative positions between the respective vehicles. Specifically, let $\mathcal{G}$ denote the set of graphs of order N with vertex set $\mathcal{V} = \{1, 2, \ldots, N\}$ and edge set $\mathcal{E} = \{e_{ij} : i = 1, 2, \ldots, N-1, j = 2, \ldots, N; i < j\}$, and the edge weight w_{ij} assigned to each edge e_{ij} is a function of the distance l_{ij} between the two vehicles i and j. It is assumed in this paper that $w_{ij} = l_{ij}$, and $e_{ij} \in \mathcal{E}$ implies $e_{ji} \in \mathcal{E}$. The weighted graph Laplacian matrix L_w is defined as below:

$$L_w(x) = \begin{cases} \sum_{k \neq i} w_{ik} & \text{for } i = j; \\ -w_{ij} & \text{for } i \neq j. \end{cases}$$

The dynamics of each vehicle is described by the following equation:

$$\ddot{p}_i = a_{22}\dot{p}_i + u_i = a_{22}v_i + u_i, \tag{1}$$

where a constant a_{22} needs not to be zero (unlike $a_{22} = 0$ in Lafferrire et al. [2005]). Assuming that vehicle dynamics along each axis is decoupled, the dynamics of each vehicle can be written as

$$\dot{x}_i = A_{veh}x_i + B_{veh}u_i, \quad i = 1, 2, \ldots, N,$$

where the entries of $x_i = [p_i^T \ v_i^T]^T \in \mathbf{R}^{2n}$ represent n configuration variables for vehicle i and their derivatives; u_i represents the control inputs; and

$$A_{veh} = I_n \otimes \begin{bmatrix} 0 & 1 \\ 0 & a_{22} \end{bmatrix}; \quad B_{veh} = I_n \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{2}$$

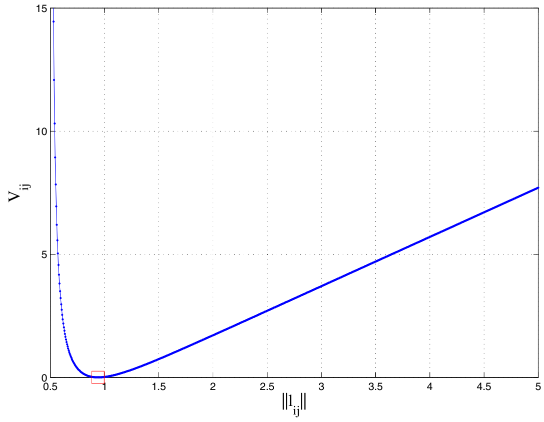

Fig. 1. Edge-tension function V_{ij} with $\delta = 0.5$, $\alpha_{ij} = 1.0$, $K_{ij} = 1.0$ and $V_{ij}^{min} = 3.2956$

Here, I_n is an identity matrix with dimension n and $\otimes$ the Kronecker product. Note that in Lafferrire et al. [2005] the following simple static feedback control law

$$u = -FL(x - h) \tag{3}$$

with constant matrices F and L, is proposed to let N second-order vehicles change their positions and velocities to achieve a desired formation described by h. Here, u and x are the vectors of u_i's and x_i's, and $L = L_\mathcal{G} \otimes I_{2n}$ ($L_\mathcal{G}$ is the standard Laplacian matrix of a connected undirected inter-vehicle communication network topology $\mathcal{G}$ - see Kim et al. [2010] for details).

As mentioned earlier, the FC law in (3) may allow collisions between vehicles in the course of flying to a desired formation. In this paper, a new control law similar to (3) is proposed to guarantee both FC and CA. To this end, some preliminary definitions are in order.

Definition 1. (CA) Let δ be a safety distance (minimum separation) between each pair of vehicles. If the distance between each pair of vehicles is greater than δ, it is said that collision is avoided. i.e. $l_{ij} = \|p_i - p_j\| > \delta \ \forall i, j \in \mathcal{V}$. Furthermore, the collision-free realization of $\mathcal{G}$ is defined as

$$\mathcal{G}_\delta = \Big\{ x \in \mathbf{R}^{nN} : l_{ij} > \delta, \forall e_{ij} \in \mathcal{E} \Big\}. \tag{4}$$

Definition 2. (Edge-tension function) For given positive constants δ, α_{ij} and K_{ij}

$$V_{ij}(l_{ij}) = \alpha_{ij} \left\{ \coth\left(\frac{l_{ij} - \delta}{K_{ij}} \right) + \frac{l_{ij}}{K_{ij}} \right\} - V_{ij}^{min}. \tag{5}$$

Here, V_{ij}^{min} is a positive constant which renders the minimum of V_{ij} zero. The edge-tension function V_{ij}, as depicted in Fig. 1 with some δ, α_{ij} and K_{ij}, is a differentiable nonnegative function of l_{ij} such that

(1) $V_{ij} \to \infty$ as $l_{ij} \to \delta$.
(2) V_{ij} attains a unique minimum at $l_{ij} = d_{ij}$, where

$$d_{ij} = \delta + \frac{K_{ij}}{2} \log(3 + 2\sqrt{2}). \tag{6}$$

It should be noted that the edge-tension function (as well as its derivative) becomes infinite as δ reaches zero, so it may look an odd choice. However, this edge-tension function shall be used in such a way that no edge length reaches δ (where $dV_{ij}/dl_{ij} = \infty$) and every edge length l_{ij}

converges to its desired one d_{ij} (where $V_{ij} = 0$). Clearly, K_{ij} in (6) can be used to *design* d_{ij} for a given δ. The total tension energy of a graph $\mathcal{G}$ is defined as

$$V = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} V_{ij} \tag{7}$$

where $\mathcal{N}_i$ denotes the set of neighbouring vehicles who can talk to vehicle i. We are now ready to present our FC laws such that each pair of vehicles i, j achieve a pre-defined desired distance d_{ij} with zero relative velocities while guaranteeing CA.

3. CONTROL LAW DESIGN

This section begins with the following theorem that allow N vehicles to converge to formation without collision.

Theorem 3. Suppose two undirected network topologies of N vehicles are given as $\mathcal{G}_i, \mathcal{G}_f$ whose realizations belong to $\mathcal{G}_\delta$, and set K_{ij} in (6) such that V_{ij} attains its minimum at $l_{ij} = d_{ij}$, where d_{ij} is the desired relative distance between agents i and j of $\mathcal{G}_f$. Then, the control law u_i $(i = 1, \cdots, N)$ for ith vehicle

$$u_i = -\frac{1}{N}\sum_{j\in\mathcal{N}_i}\{(1 + a_{22})v_{ij} + \nabla_{p_i} V_{ij}\} \tag{8}$$

drives the vehicles from the initial configuration $\mathcal{G}_i$ to the desired formation $\mathcal{G}_f$ without collision, provided that all vehicles remain connected to each other at all times. Here, $\mathcal{N}_i$ is the set of natural numbers from 1 to N except i, v_{ij} is the relative velocity between vehicles i and j, and $\nabla_{p_i} V_{ij}$ is the gradient of V_{ij} with respect to p_i.

Proof. The proof shall be presented in a journal version of this paper.

It is interesting to note that the proposed controller (8) is similar to (3) found in Lafferrire et al. [2005]. To see this, first note that $\nabla_{p_i} V_{ij} = w_{ij}(p_i - p_j)$, where

$$w_{ij} = \alpha_{ij}\left\{-\mathrm{csch}^2\left(\frac{l_{ij} - \delta}{K_{ij}}\right) + 1\right\}\frac{1}{l_{ij}K_{ij}}.$$

The control law u_i can then be written as:

$$u_i = -\sum_{j\in\mathcal{N}_i}\left\{(p_i - p_j)\frac{w_{ij}}{N} + \frac{1 + a_{22}}{N}(v_i - v_j)\right\},$$

or in matrix form

$$u_i = -I_N \otimes [f_1,\ f_2]\left(L_w \otimes \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} + L_G \otimes \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right)x. \tag{9}$$

where $[f_1,\ f_2]$=$[1/N,\ (1+a_{22})/N]$ and x is the stack vector of positions and velocities of N vehicles. Since the motion along each axis is independent for vehicle i, (9) can be written as:

$$\begin{aligned} u_i &= -\underbrace{(I_N \otimes I_n \otimes [f_1,\ f_2])}_{F} \times \\ &\quad \underbrace{\left(L_w \otimes I_n \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} + L_{\mathcal{G}} \otimes I_n \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right)}_{L_{wg}} x \\ &= -FL_{wg}x. \end{aligned}$$

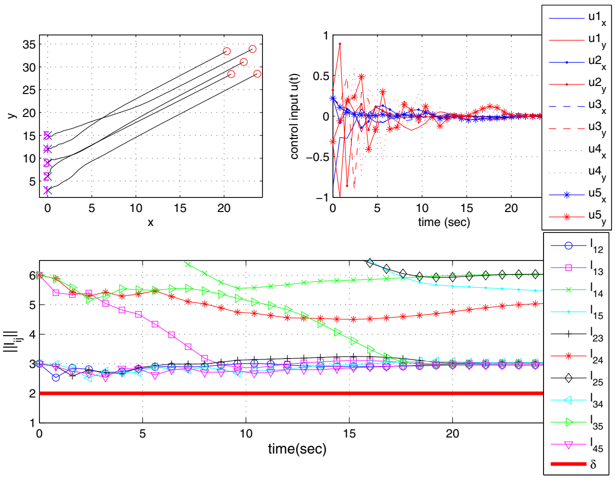

Fig. 2. Vehicles converging to a pentagon formation with (8): no collision occurs

Note that the desired formation vector h in (3) is now embedded in w_{ij} in the form of d_{ij}. The aforementioned formation control law can be extended in a way to accommodate a star (leader-following) network topology. This extension, however, shall be presented in a journal version of this paper.

4. NUMERICAL EXAMPLE

To test the proposed control (8) in a simulation environment, five vehicles were initially lined up (marked with 'x') in Fig. 2-(a)) and are required to form a pentagon formation defined by $h = [0; 1] \otimes h_p = [0; 1] \otimes [h_1; h_2; h_3; h_4; h_5]$,[1] where for some chosen d_{12} h_1=$[0; 0]$, h_2=$[d_{12}; 0]$, h_3=$[d_{12}/2; \sqrt{3}d_{12}/2]$, h_4=$[0; \sqrt{3}d_{12}]$ and h_5=$[d_{12}; \sqrt{3}d_{12}]$. Once the desired inter-vehicle distances are fixed, K_{ij} can be chosen based on (6). Also, $a_{22} = 0.1$, $\delta = 2.0$ and $\alpha_{ij} = 1.0$.

Fig. 2 shows the simulation results when (8) is used for formation control. Fig. 2-(a) shows that five vehicles start from the line formation ($\times$) and form the require pentagon formation ($\circ$) in the end. Fig. 2-(b) shows the control effort required to form the desired formation. Note that as the vehicles achieve the desired formation, the control effort becomes zero. Fig. 2-(c) shows relative distances between each pair of vehicles, along with the minimum separation line of $\delta = 2.0$. Fig. 3 shows the same formation reconfiguration scenario when (3) in Lafferrire et al. [2005] is used to achieve the desired formation. In this case, Fig. 2-(c) clearly shows that some vehicles violate the minimum separation constraint.

In the second scenario, vehicles are required to form a circular formation from an initial horizontal configuration, where vector h_p is given by θ=$2\pi/N$ and h_{i+1}=$[d_{12}\cos(i\theta);\ d_{12}\sin(i\theta)]$ $(i = 0, \cdots, N - 1)$. Fig. 4 shows that vehicles achieve the circular formation while maintaining the distances above the pre-defined safety distance of $\delta = 1.0$.

[1] $[a; b]$ denotes a column vector of a and b; this is a Matlab-like notation.

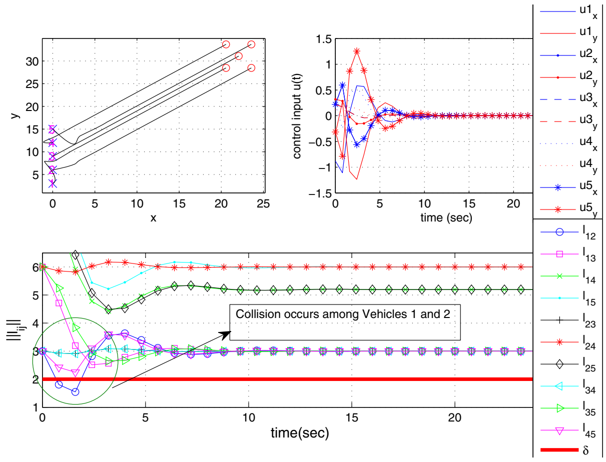

Fig. 3. Vehicles converging to a pentagon formation with (3) in Lafferrire et al. [2005]: collision occurs

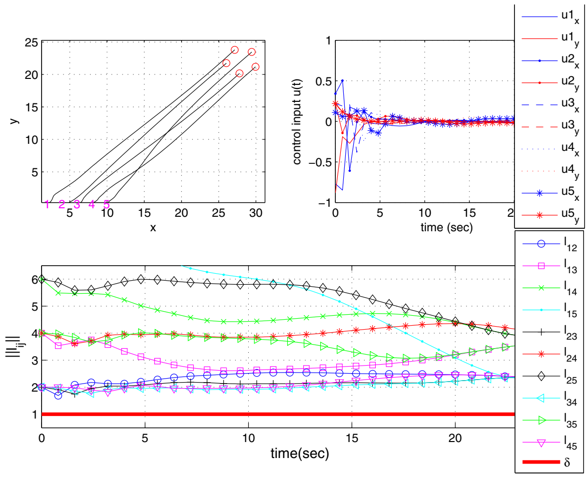

Fig. 4. Vehicles converging to a circular formation with (8): no collision occurs

5. CONCLUSION

In this paper, collision-free formation control for second-order vehicles was considered when the inter-vehicle network topology is complete. Unlike typical collision-free FC approaches, the proposed control approach utilizes graph Laplacian and an edge-tension function to synthesize a simple and easy-to-implement feedback control scheme for achieving both desired formation and collision avoidance; this approach does not involve solving complex optimizations problems. The present work considers complete and star network topologies only. The future work will be focused on control design for more general topologies with accounting for practical issues such as time-delay and link failures in the network.

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REFERENCES

S. K. M. Martin, P. Klupar, and J. Winter. TechSat 21 and Revolutionizing Space Missions Using Microsatellites. *In Proc. of AIAA Conference*, Logan, UT, Aug. 2001.

J. G. Bender. An overview of systems studies of automated highway systems. *IEEE Transactions on Vehicular Technology*, Vol. 40, No. 1,pp. 82– 99,1991.

A. Serrani. Robust coordinated control of satellite formations subject to gravity perturbations. *In Proc. of the American ControlConference*, Vol. 1, pp. 302–307, June 2003.

Daniel P. Scharf, F. Y. Hadaegh, and S. R. Ploen. A survey of space formation flying guidance and control (part 2). *In Proc. of the American Control Conference*, Boston, Massachusetts, June 2004.

D. J. Stilwell and B. E. Bishop. Platoons of underwater vehicles. *IEEE Control Systems Magzine*, Vol. 20, pp.45-52, 2000.

M. Cao, C. Yu and B. D. O. Anderson. Formation control using range-only measurements. *Automatica*, Vol. 47, pp.776–781, 2011.

W. Ren and R. W. Beard. A decentralized scheme for spacecraft formation flying via the virtual structure approach. *In Proc. of American Control conference*, Denver, Colorado, June 2003.

S.N. Dashkovskiy, B.S. Rüffer, F.R. Wirth. "Stability of autonomous vehicle formations using an ISS small-gain theorem for networks. *Proc. Appl. Math. Mech.*, Vol. 8, No. 1, pp. 10911–10912, 2008.

G. Lafferrire, A. Williams, J. Caughman and J. J. P. Veerman. Decentralized control of vehicle formations. *Systems & Control Letters*, Vol. 54, pp. 899–910, 2005.

T. Keviczky, F. Borrelli, K. Fregene, D. Godbole, and G. J. Balas. Decentralized Receding Horizon Control and Coordination of Autonomous Vehicle Formations. *IEEE Transactions on Control Systems Technology*, Vol. 16, No. 1, JANUARY 2008.

Z. Chao, L. Ming, Z. Shaolei and Z. Wenguang. Collision-free UAV Formation Flight Control based on Nonlinear MPC. *In Proc. International Conference on Electronics, Communications and Control*, pp. 1951–1956, 2011.

N. Michael and V. Kumar. Control of Ensembles of Aerial Robots. *Proceedings of the IEEE*, Vol. 99, No. 9, pp. 1587–1602, 2011.

S. Kang, H. Choi and Y. Kim. Formation Flight and Collision Avoidance for Multiple UAVs using Concept of Elastic Weighting Factor. *Intrnational Journal of Aeronautical and Space Sciencies*, Vol. 14, No. 1, pp. 75–84, 2013.

S. Mastellone, D. M. Stipanovic, C. R. Graunke, K. A. Intlekofer and M. W. Spong. Formation Control and Collision Avoidance for Multi-agent Non-holonomic Systems:Theory and Experiments. *The International Journal of Robotics Research*, Vol. 107, No. 27, 2008.

E. Lalish and K. A. Morgansen. Decentralized Reactive Collision Avoidance for Multivehicle Systems. *Proceedings of the 47th IEEE Conference on Decision and Control*, Mexico, Dec. 09-11, 2008.

E. Lalish, K. A. Morgansen and T. Tsukamaki. Decentralized Reactive Collision Avoidance for Multiple Unicycle-Type Vehicles. *American Control Conference*, USA, JUN. 11-13, 2008.

L. Sabattini, C. Secchi, and C. Fantuzzi. Arbitrarily shaped formations of mobile robots: artificial potential fields and coordinate transformation. *Autonomous Robot*, Vol. 30,No. 4, pp. 385–397,May 2011.

J. Yan, X. Guan, X. Luo and F. Tan. Formation and obstacle avoidance control for multiagent systems. *Journal of Control Theory Application*, No. 2, Vol. 9, pp. 141–147, 2011.

R. Falconi, L. Sabattini, C. Secchi, C. Fantuzzi and C. Melchiorri. A Graph-Based Collision-Free Distributed Formation Control Strategy. *In Proc. 18th IFAC World Congress*, pp. 6011–6016, 2011.

Z. Kan, A. P. Dani, J. M. Shea, and W. E. Dixon. Network Connectivity Preserving Formation Stabilization and Obstacle Avoidance via a Decentralized Controller. *IEEE Transactions on Automatic Control*, Vol. 57, No. 7, 2012.

A. Ajorlou and A. G. Aghdam. A Bounded Connectivity Preserving Aggregation Strategy with Collision Avoidance Property for Single-Integrator Agents. *IEEE Conference on Decision and Control*,Hawaii, USA, December 10–13, 2012.

C. Secchi and C. Fantuzzi. Formation control over delayed communication networks. *In Proc.of the IEEE Conference on Robotics and Automation*,pp.563–568, 2008.

M. Jiand M. Egerstedt. Distributed Coordination Control of Multiagent Systems While Preserving Connectedness. *IEEE Transactions on Robotics*, Vol. 23, No. 4, August 2007.

Y. Kim. Bisection algorithm of increasing algebraic connectivity by adding an edge. *IEEE Transactions on Automatic Control*, Vol. 55, No. 1, pp.170–174, 2010.